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Effects on growth of environmental policy in a small open economy

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Abstract

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This paper examines the effect of environmental policy on economic growth in a small open economy in a neoclassical framework with pollution as an input. We show that environmental policy imposes a drag on long run growth in both the open and closed economy cases. The effect of environmental policy on growth is stronger in the open economy case relative to the closed economy model if the country has strong aversion to pollution and thus serves as a net exporter of capital in the international capital market. On the other hand, if the agents in the economy have low aversion to pollution and thus import capital, the effect of environmental care on growth is stronger in the closed economy relative to the open economy. Thus, from our set-up, environmental policy is harmful to growth but environmental sustainability need not be incompatible with continued economic growth.

JEL Classification O40, O41, Q56

Key words: Economic growth, Pollution tax, Capital-output ratio, Open economy, Capital flight

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1. Introduction

The pollution haven hypothesis (PHH) is one of the most contentious and hotly debated predictions in all of international environmental economics. The central prediction of the PHH is that liberalized trade in goods will lead to the relocation of pollution intensive production from high income and stringent environmental regulation countries to low income and lax environmental regulation countries.

The existing literature on the pollution haven hypothesis can be divided into two: theoretical and empirical studies on inter-country trade flows (Copeland and Taylor, 1994, 2003; Levinson and Taylor, 2008) and studies (mainly empirical) on plant and industrial location (foreign direct investment) decisions (Kalamova and Johnstone, 2011; Javorcik and Wei, 2004; List and Co, 2000). In the studies that focus on regulatory stringency and trade flows, the conclusions seem to back the existence of pollution haven effect. For instance Levinson and Taylor (2008), Copeland and Taylor (2004), Brunnermeier and Levinson (2004) and Ederington and Minier (2003) all found that environmental policy has a significant impact on trade flows that is consistent with the pollution haven hypothesis, after using slightly different methodologies.

In this paper, we will theoretically analyze how environmental preferences influence the decision to invest abroad and at home, respectively. Thus, more relevant to the present paper is a strand within the PHH literature that focuses on the role of capital mobility, in the form of foreign direct investment. Millimet and List (2004) find that the impacts of environmental policy on

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industry location depend crucially on heterogeneity of location-specific attributes. List and Co (2000) suggest that stringer environmental regulation does influence negatively the location decisions of inward FDI in the US. Keller and Levinson (2002), however, find a less robust evidence of the pollution haven effect at the industry level. Xing and Kolstad (2002) find that US outbound flows move significantly to host countries with more lax environmental regulations in the heavily polluting industries; this result is not valid for less polluting industries. The industry-level evidence shows that environmental regulation can influence negatively the location decision of a specific industry, while having no effect on another polluting industry (e.g. Keller and Levinson (2007), Henderson and Millimet (2007), Waldkirch and Gopinath (2008)).

There are only a few papers using FDI data to study pollution havens at the global level. Javorcik and Wei (2004) study the determinants of actual and planned investment by 534 major multinational firms in Central and Eastern Europe and in the former Soviet Union. They find no robust support for the pollution-haven hypothesis. The theoretical model of Eskeland and Harrison (2003) shows that, depending on possible complementarities between capital and pollution abatement, environmental regulation can lead to an increase or a decline in investment in both the host (developing) country and the originating (developed) country. In their empirical analysis they find some evidence that foreign investors are concentrated in sectors with high levels of air pollution in Mexico, Venezuela, Morocco and Ivory Coast, although the evidence is weak. In their recent study, Kalamova and Johnstone (2011) established two major results regarding the effect of stringent environmental policy and FDI flows. First, they show that a relatively lax policy in the host country has a positive (although small) effect on incoming FDI flows in both developed and developing countries. However, this effect tends to exhibit an inverse U-shape, and thus reverses below a certain level of environmental stringency in the sample of non-OECD host countries. Thus, once the environmental regime of a host country becomes too lax, this country loses its attractiveness as an FDI location.

To back the empirical literature linking environmental regulation to FDI flows, we undertake a cursory examination of the pattern of FDI flows in 2010 against some measure of environmental regulatory standards for a cross section of 163 countries.

Figure 1. Ratio of Outward FDI Flows to GDP versus EPI Score

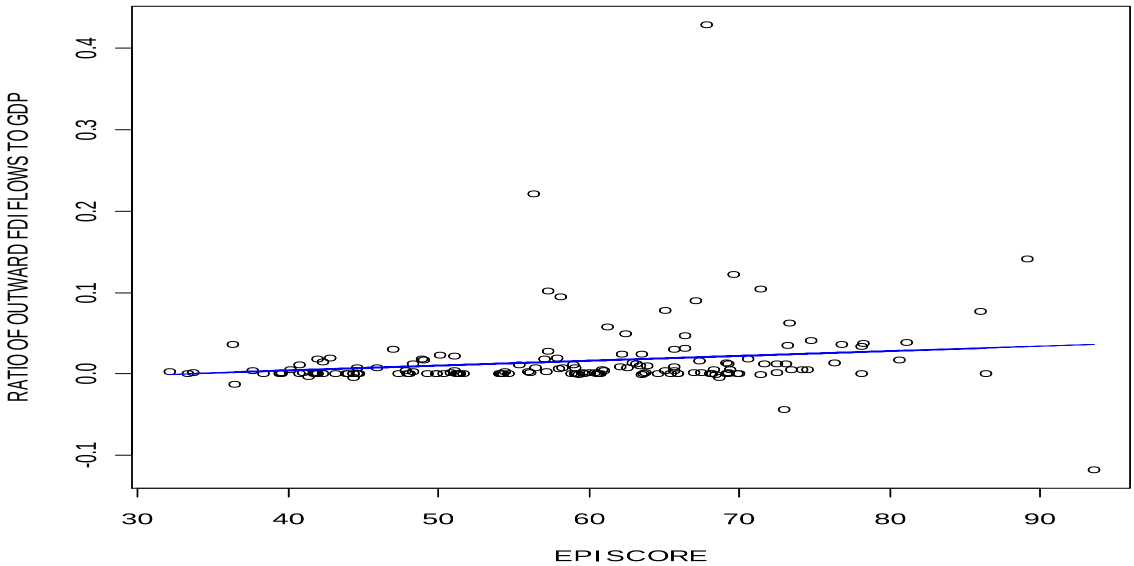
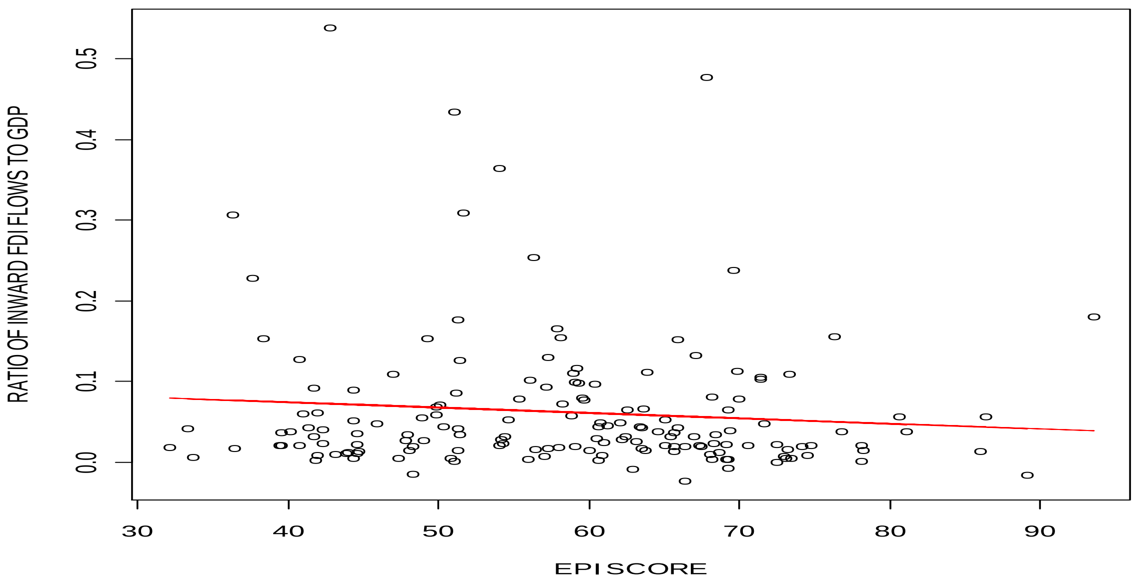


Figure 2. Ratio of Inward FDI Flows to GDP versus EPI Score



Our main measure of environmental policy stringency is the environmental performance index (EPI)[1] score. Our preliminary examinations of the data reveal the relationship presented in Figures 1 and 2 in which we plot the ratio of outward and inward FDI flows to GDP respectively against the EPI score. The revelation is that the EPI score is inversely related to inward FDI flows expressed as a ratio to GDP while the relationship is positive for outward flows also expressed as a ratio to GDP. The observed relationship in the data have the implication that inward (outward) FDI flows are higher in countries with lower (higher) EPI score and thus lends support to the pollution haven hypothesis. We do not push this far as this is only a pair of bivariate plots and the observed relationship may change as we allow for more controls, but together with the reviewed literature above it gives a reason enough for a theoretical investigation of the relation between preferred environmental policy and international capital flows.

This paper complements the existing theoretical literature on the PHH phenomenon. In particular, we extend the basic static multi-sector trade models in the previous literature (e.g. Copeland and Taylor 1994; 2003) into a dynamic model. To do this in a simplified framework, we use a one-sector aggregated growth model with pollution, similar to Brock (1977)[2], which allows for international capital flows. In the previous literature, a higher demand for environmental quality forces (some of) the dirty production sector abroad. In this paper, a high demand for environmental quality directs savings away from polluting domestic production investments to investments on the "clean" (from the small country's perspective) international capital markets. The model contains only one physical kind of good, but foreigners can buy domestic output and domestic residents can buy foreign output. The function

[1] The 2010 Environmental Performance Index (EPI) ranks 163 countries on 25 performance indicators tracked across ten policy categories covering both environmental public health and ecosystem vitality. These indicators provide a gauge at a national government scale of how close countries are to establish governmental policy goals. The EPI's proximity-to-target methodology facilitates cross-country comparisons as well as analysis of how the global community is doing collectively on each particular policy issue. Source: Yale Center for Environmental Law & Policy (http://epi.yale.edu/Countries)

[2] See also Keeler, Spence and Zeckhauser, 1971; Forster, 1973; Gruver, 1976; Brock, 1977; Becker, 1982; Musu, 1989; Tahvonen and Kuuluvainen, 1993; van der Ploeg and Withagen, 1991; Selden and Song, 1995, for similar studies. However, none of these studies allows for international capital flows.

of international (and inter-temporal) trade in our model is to allow domestic production to diverge from domestic expenditure on consumption and investment. Thus we consider the intertemporal aspects of international trade but neglect the implications for patterns of inter-sectoral specialization and comparative advantage in production. The latter case is well addressed in the previous literature (see for instance Copeland and Taylor, 2003; Levinson and Taylor, 2008).

The objective of this paper is to examine the effect of optimal environmental pollution on economic growth in a small open economy in which pollution serves as an input. The idea of modelling pollution as serving as an input in the production function is not novel; see for instance Brock (1977) and Becker (1980). Copeland and Taylor (1994; 2003) suggest a formal motivation for this approach. The novelty of this paper is its extension of the basic model to a small open economy in which capital can flow across national borders and the complementary explanation of the pollution haven hypothesis that it offers. The central question that this paper answers is: what determines whether a country imports capital (and thereby hosts considerable volumes of polluting production) or exports capital (leaving considerable part of polluting production to the rest of the world)? In addition to this, we examine how the preference for a clean environment (environmental policy) influences national income and its growth rate.

To ensure that the economy's intertemporal budget constraint binds, we assume that the parameters are such that both the capital stock and aggregate consumption grow at rates lower than the world interest rate. This will ensure that there is no Ponzi game with respect to foreign debt. In order for our hypothetical economy to stay "small" as time runs, we assume that the growth rates of consumption and the capital stock are lower than the growth rate of the rest of the world. This assumption requires that our hypothetical economy has high preference for environmental quality relative to the rest of the world.

Our analysis reveals the following conclusions. Within our framework we show that it is not changes in comparative advantage due to differences in environmental policy per se that generates the pollution havens, but the fact that the environmental quality is a normal good. Environmental care imposes a drag on long run economic growth, by increasing the capital-output ratio and lowering the returns to capital. Furthermore, our analysis shows that the drag that the demand for environmental quality imposes on growth is larger in the open economy if and only if consumption grows faster than the capital stock. In the reverse case where the capital stock grows faster than consumption, the drag is smaller in the open economy case relative to the closed economy.

Our analysis of the open economy case also reveals that whilst the growth rates of consumption, output, the capital stock and pollution are constant, there are three possible qualitative scenarios with respect to the debt depending on the relation between the growth rates of consumption and the capital stock. First, we considered a benchmark scenario in which consumption and capital grow at a common rate. Under this scenario, we show that the debt and capital stock grow at a common rate and hence the debt-capital ratio is constant at all points in time. The implication of this is that the history of the debt and the capital stock puts a constraint on the initial consumption level. We show that a country which has had a history of generous lending will benefit from it twofold. It can afford to choose a high consumption path and also have a considerable share of its income from the international capital market, which does not cause any negative pollution effect on this country. In the second case where consumption grows faster than the capital stock, we show that the small open economy eventually exports capital (negative debt) to the rest of the world by accumulating assets abroad over time. This process is faster if the elasticity of marginal disutility of pollution is high relative to the rest of the world so that it has higher drag on (production) growth. As a final scenario, we considered the opposite scenario where the capital stock grows faster than consumption. In this case we show that the economy imports capital due to the low aversion to pollution, and thus accumulate debt. To summarize, we show that a high demand for environmental quality in our small open economy is found to induce capital flight to countries with lower demand for environmental quality. This result offers an alternative restatement of the pollution haven hypothesis.

The remainder of this paper is organised as follows: Section 2 sets up the basic open economy growth model in which pollution augments the primary inputs (man-made capital and labour) in the production process. The model solution and its implication for long run growth are discussed in Section 3. Section 4 modifies our basic model of Section 2 and Section 3 to a closed economy model with optimal policy and also compares the results in the open economy with the closed economy outcome. Section 5 concludes the paper. All the analyses in the paper are based on command optimal solution. The decentralized solution is presented in the appendix and the equivalence between command optimum and the decentralized solution is established therein.

2. The Basic Model

Even though we will provide the solution to a central-planner problem, the model is here presented in a typical decentralized economy style. The purpose is that it will be useful for the brief description of the decentralized solution in the appendix. Besides, the decentralized expressions are easily transformed to the "planner expressions", while it is more difficult to go the other way around.

Our setup begins with an open economy neoclassical-type model with environmental concern. The role of government is limited to taxing polluting firms, and redistributing the proceeds to households as a way of internalizing (and compensating for) externalities generated through production. We thus abstract from government purchases; only households and firms interact at the market place. The behaviour of firms and households in this model economy are described below.

2.1 Firms

The firms produce goods with effective labour (LT), capital (K) and pollution (Z), according to a Cobb-Douglas production function, which exhibits constant returns to scale in its three arguments. The production function takes the form of equation (1).[3]

$$Y(t)\left[K(t), Z(t), L(t)T(t)\right] = K(t)^{\alpha} Z(t)^{\beta} (L(t)T(t))^{1-\alpha-\beta} . \tag{1}$$

Here *Y(t)* is the flow of output at time *t*, and all other variables are as defined already. The index of technological progress, *T*, is assumed to grow exogenously at the rate x. Raw labour, *L*, grows at the exogenous rate, *n*.

Firms are atomistic and pay each input its marginal contribution to output. The problem facing a representative firm then is to maximize profits given by equation (2). We set up the maximization problem in terms of aggregate variables, which, given the representative firm, is without any loss of generality. Output price has been normalized at unity and the representative firm faces both competitive factor markets and product market. The profit function is:

$$\pi = K^{\alpha} Z^{\beta} (LT)^{1-\alpha-\beta} - RK - wL - \tau Z , \tag{2}$$

where R is the rental price of capital services, w is the real wage and τ is the tax per unit of pollution.

Profit maximization requires that the conditions given by equations (3)-(5) are fulfilled.

$$\alpha \frac{Y(t)}{K(t)} = R(t) \tag{3}$$

$$\beta \frac{Y(t)}{Z(t)} = \tau(t) \Leftrightarrow Z(t) = \beta \frac{Y(t)}{\tau(t)} \tag{4}$$

$$(1-\alpha-\beta) \frac{Y(t)}{L(t)} = w(t) \tag{5}$$

[3] The idea behind this formulation is that "techniques of production are less costly in terms of capital inputs if more pollution is allowed". To give the rational for augmenting the aggregate production function as an input in a more formal way, suppose that gross output is produce with capital and effective labour and takes the following general form $\tilde{Y}=F(K,LT)$. Suppose further that pollution is proportional to gross output according to the relation $Z=f(\Omega)\tilde{Y}$.A constant fraction of the gross output Ω is used as input for abatement. This leaves a net out Put of $Y=(1-\Omega)\tilde{Y}$ which is available for consumption, investment and export. Assume that $f(\Omega)=(1-\Omega)^{1/\alpha}$, where $0<\alpha<1$. This implies that $Z=(1-\Omega)^{1/\alpha}F(K,LT) \Leftrightarrow 1-\Omega=Z^{\alpha}[F(K,LT)]^{-\alpha}$. This and the expression for net output gives $Y=Z^{\alpha}[F(K,LT)]^{1-\alpha}$, which is constant returns in *Z, K* and *L* and can conveniently be written in the form of (1) as above without loss of generality. For a motivation of having *Z* as an input, see Chapter two of Copeland and Taylor (2003).

Equation (3) is the derivative of the profit function with respect to capital. This condition states that the firm's optimal choice of capital equates the value of the marginal product of capital to the rental rate of capital. Equation (4) is the optimality condition with respect to pollution: the firm sets the value of the marginal product of pollution equal to the tax rate. Since the marginal product of pollution is decreasing in the levels of emissions, a low tax rate will induce high pollution. Note that the pollution tax payment, $\tau Z / Y = \beta$, is a constant share of the produced value, irrespective of the tax rate, τ. The optimality condition with respect to labour input is given in (5): the firm sets the value of the marginal product of labour equal to the wage rate.

We now follow some implications that will occur frequently in this paper. First, defining the capital-output ratio, $v \equiv K / Y$ and using $r = R - \delta$, where r is the market interest rate[4], equation (3) can be re written as

$$r = \frac{\alpha}{v} - \delta \text{ or } \frac{1}{v} = \frac{r+\delta}{\alpha} \tag{6}$$

Note that the capital-output ratio is an inverse measure of the average productivity of capital. From equation (6), it is clear that the net rate of return on capital is inversely related to the capital-output ratio. This is as expected: an increase in the net rate of return to capital means high cost of capital and hence lowers demand for capital since capital exhibits diminishing marginal returns.

From equation (1) we derive another implication that will be used frequently below, namely, the proportional growth rate of aggregate output is

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + \beta \frac{\dot{Z}}{Z} + (1-\alpha-\beta)(n+x)\,. \tag{7}$$

Furthermore by the optimality condition in equation (4), the growth rate of pollution satisfies the expression in (8).

[4] Here, *r(t)* is the net rate of return; *R(t)* is the gross return on capital and δ is the rate of depreciation of capital. The reason why this relationship holds among these variables is that in the absence of uncertainty, capital and loans are perfect substitutes as stores of value and, as a result, they must deliver the same return in equilibrium (Barro and Sala-i-Martin, 2004).

$$\frac{\dot{Z}}{Z}=\frac{\dot{Y}}{Y}-\frac{\dot{\tau}}{\tau} \tag{8}$$

According to equation (8), the growth rate of pollution is the difference between the growth rates of two variables: aggregate output and the environmental tax rate. Pollution is constant if the growth rates of these two variables are equal. However, if output grows faster than the environmental tax rate, pollution will increase over time: because the higher Y raises the marginal product of pollution. On the contrary, an ambitious environmental policy that allows the tax rate to grow faster than production will make pollution decline over time.

Another useful implication from the firm side is obtained by substituting equation (8) into (7). The growth rate of aggregate output is then rewritten as equation (9).

$$\frac{\dot{Y}}{Y}=\frac{1}{1-\beta}\left(\alpha\frac{\dot{K}}{K}-\beta\frac{\dot{\tau}}{\tau}+(1-\alpha-\beta)(n+x)\right) \tag{9}$$

One can quickly see the effect of the growth in the environmental tax on economic growth from equation (9). The growth rate of the tax term enters the expression for the growth rate of aggregate production negatively. This suggests that the environmental tax exerts a drag on long run growth. We take up the detailed analysis of this in Sections 3 and 4 of this paper.

A final useful implication from the firm side is obtained by taking the derivative of the log of equation (5). This gives:

$$\frac{\dot{w}}{w}=\frac{\dot{y}}{y}=\frac{\dot{Y}}{Y}-n \tag{10}$$

According to equation (10), the wage and per capita income grow at the same rate, which is equal to the growth rate of GDP less the population growth rate. Equations (6), (8), (9) and (10) together with the optimality conditions (3)-(5) will play key role in the derivations in the rest of the paper and we shall frequently make references back to them.

2.2 Households

We begin by considering an infinite-horizon economy and suppose that the economy admits a normative representative household with the instantaneous utility function

$$u(C/L, Z) = \frac{(C/L)^{1-\theta} - 1}{1-\theta} - \phi \frac{Z^{1+\eta}}{1+\eta}, \qquad \theta \geq 0,\ \eta \geq 0, \phi > 0. \tag{11}$$

The utility function is additively separable in the level of consumption per person *C/L* and the level of pollution. The parameter ϕ is the weight of the disutility of pollution, θ is the elasticity of marginal utility of consumption[5] and η is the elasticity of marginal disutility of pollution. The instantaneous utility function is twice continuously differentiable in both of its arguments and strictly concave in *C/L* and strictly convex in *Z*.

Households are atomistic and take prices in both the output and factor markets as given. Each individual owns one unit of labour which he supplies inelastically at any wage rate. The household receives a lump-sum transfer $\varphi(t)$ from the government as a compensation for the deterioration in environmental quality, due to activities of firms in period *t*. There are negative externalities in the form of environmental pollution but the single household cannot influence that. Households use income that they do not consume to accumulate more assets according to the following dynamic equation

$$\dot{A}(t) = r(t)A(t) + w(t)L(t) + \varphi(t) - C(t). \tag{12}$$

Here $\dot{A}(t) = dA(t)/dt$ is the change in households' asset at time *t*, *A(t)* is the total households assets at time *t*, *C(t)* is the aggregate household consumption at time *t*. All other variables are as defined already. To develop equation (12), we first assume that the government balances the budget in each period so that the budget constraint is given by

$$\tau Z = \varphi, \tag{13}$$

where τZ is the total revenue from environmental taxation.

We now take the economy's international affairs into account, by introducing, *D* for the international debt. The international debt corresponds to foreign

[5] In this paper we assume $\theta \geq 1$ as these values of θ has empirical support (see: Hall, 1988; Hahm, 1998; Gali, 2008; Jones, 2009).

claims on the domestic economy. However, if households assets holdings exceeds the capital stock (a negative *D*), domestic residents have claims on the rest of the world and hence accumulate assets abroad. This means that the assets of this open economy are *A=K-D,* and thus $\dot{A} = \dot{K} - \dot{D}$. Equation (12) can therefore be rewritten as equation (14).

$$\dot{K} - \dot{D} = rK + wL + \tau Z - rD - C \tag{14}$$

Using now $R = r + \delta$, the optimality conditions in (3)-(5) and the government's budget constrain in (13), equation (14) simplifies to

$$\dot{K} + \delta K = Y + \dot{D} - rD - C\,. \tag{15}$$

Equation (15) is the constraint on aggregate resources in the open economy. The aggregate resource constraint in (15) can be disaggregated into two sub constraints: debt and capital accumulation constraints. The change in the foreign debt at any point in time is the difference between domestic absorption and domestic production (GDP). That is;

$$\dot{D} = C + I + rD - Y\,. \tag{16}$$

The debt grows if domestic absorption is higher than production in that period. The first constraint in (19) is the equation of motion for foreign debt. According to this constraint, the change in foreign debt at any point in time is the sum of consumption, investment and interest payments on debt less production in that period (change in the current account deficit). The world interest rate, *r* is taken as given by our small open economy.

The constraint on capital accumulation satisfies:

$$\dot{K} = I - \delta K\,. \tag{17}$$

According to equation (17), gross investment covers replacement investment and capital expansion. The capital stock grows over time if gross investment more than offset gross depreciation.

A brief refresher in national income accounting identities and definitions may be useful at this point. The gross domestic product (GDP) is the total money value of all final goods and services produced in an economy over a period of time, usually one year. Mathematically, *GDP= C+I+NX*, where *NX* is net exports (exports-imports). GNP is the total money value of all final goods and services produced with inputs owned by the citizens of a given economy, irrespective of the physical location of the inputs. We obtain the GNP by adding the current account balance (negative of the change in the foreign debt over time) to the GDP. In our case, the mathematical expression for GNP is; *GNP=GDP-rD.* We show later how the international capital market can be used to compensate for a decline in GDP due to environmental policy for a country that is a net lender in the international capital market.

3. The social planner's problem

In the growth literature, it has been established that the command optimal allocation and the decentralized equilibrium allocation are equivalent, if externalities are properly internalized. By assuming an optimal policy that fully internalized the pollution externality right away, we exploit this equivalence here and thus will only solve the command optimal allocations in both the open and closed versions of the model (See appendix *A* and *B* for the solution to the decentralized allocation).

3.1 The problem and first-order conditions

The problem facing a benevolent social planner is to maximize the discounted sum of utility of the representative household over all periods. Thus, the social planner maximizes

$$\int_0^\infty e^{-(\rho-n)t}\left(\frac{(C/L)^{1-\theta}-1}{1-\theta}-\phi\frac{Z^{1+\eta}}{1+\eta}\right)dt\,, \tag{18}$$

subject to $D(0)=D_0$, $K(0)=K_0$ and;

$$\dot{D}=C+I+rD-Y \text{ and } \dot{K}=I-\delta K \tag{19}$$

Here; ρ is the subjective rate of time preference.

The current value Hamiltonian for the command optimal allocation problem is:

$$H(C/L,Z,K,D,\mu,q)=\frac{(C/L)^{1-\theta}}{1-\theta}-\phi\frac{Z^{1+\eta}}{1+\eta}+\mu(C+I+rD-Y)+q(I-\delta K) \tag{20}$$

The shadow prices (costate variables) on the equation of motion of the foreign debt and the capital accumulation equation in (19) are μ and q respectively. The shadow price of debt accumulation is expected to be negative as adding more debt reduces future utility. The planner's choice of consumption, investment and pollution that maximize the discounted sum of utility for the representative household satisfies the following optimality conditions.

$$\frac{\partial H}{\partial C} = 0 \Leftrightarrow C^{-\theta} = -\mu L^{1-\theta} \tag{21}$$

$$\frac{\partial H}{\partial I} = 0 \Leftrightarrow -\mu = q \tag{22}$$

$$\frac{\partial H}{\partial Z} = 0 \Leftrightarrow \phi Z^{\eta} = -\mu\beta \frac{Y}{Z} \tag{23}$$

$$\frac{\dot{\mu}}{\mu} = (\rho - n - r) \tag{24}$$

$$\frac{\dot{q}}{q} = \rho + \delta - n - \alpha / v \tag{25}$$

$$\lim_{t\to\infty} \mu(t)D(t)e^{-(\rho-n)t} = 0 \tag{26}$$

$$\lim_{t\to\infty} q(t)K(t)e^{-(\rho-n)t} = 0 \tag{27}$$

Equations (21)-(23) are the optimality conditions with respect to consumption, investment and pollution. The rates of change of the shadow prices satisfy equations (24) and (25). It is not hard to see that the shadow prices grow at the same rate. This is necessary for condition (22) to hold in all periods: the absolute values of the shadow prices are equal in all periods only if they have an equal growth rate. By (22), not only are the growth rates of the co-state variables equal, but also their initial (absolute) values; hence $-\mu(0) = q(0)$. Equations (26) and (27) are the transversality conditions for the debt and capital stock respectively.

By (22) we have $\dot{q}/q = \dot{\mu}/\mu$. This, (24) and (25) imply the following steady state value for the capital-output ratio in the open economy model.

$$\frac{1}{v^*} = \frac{r+\delta}{\alpha} \tag{28}$$

Recall that by the small open economy assumption, r is constant. Hence, the capital-output ratio goes to its steady state value instantly, due to the rapid capital flows that equate the international and domestic interest rates. The steady state capital-output ratio decreases in the exogenous world interest rate and the rate of depreciation, but increases in the elasticity of output with respect to the capital stock. These results are quite plausible. A high world interest rate encourages investments abroad just as a high rate of depreciation will. This holds capital accumulation back and therefore the capital-output ratio falls. On the other hand, higher elasticity of output with respect to capital makes firms demand more capital for a given level of output.

To develop some implications of the above conditions, we start by taking logs of equation (21), differentiating it with respect to time, and using the equation of motion for the costate variable in (24) and the steady state expression for the capital-output ratio in equation (28). We then obtain the consumption Euler's equation.

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r-\rho)+n. \tag{29}$$

Remarkably, the consumption growth rate is constant from time zero on, because the world interest rate is constant. The growth rate of aggregate consumption is positive if and only if $r+\theta n>\rho$. Per capita consumption grows if and only if $r>\rho$, as usual. We will assume this latter inequality holds from here onwards. As in the standard neoclassical model, the consumption growth rate increases with the world interest rate and population growth rate but decreases with the subjective rate of time preference and the elasticity of marginal utility of consumption.

Another useful implication of the necessary conditions concerns pollution. The time derivative of (23) combined with (24) yields the optimal growth rate of pollution as

$$\frac{\dot{Z}}{Z} = \frac{1}{1+\eta}\left(\rho-n-r+\frac{\dot{Y}}{Y}\right). \tag{30}$$

This expression says that the socially optimal growth rate of pollution increases with the growth rate of the shadow cost of the debt and the growth rate of aggregate production but decreases in the elasticity of marginal disutility of pollution. This is quite intuitive: as the growth rate of the shadow cost of the debt increases, the planner increases domestic production to reduce the imports of consumables. A high growth rate of production will require more of every input including pollution. On the other hand, a higher elasticity of marginal disutility of pollution raises the social cost of pollution and thus causes a reduction in the socially optimal emissions growth rate.

Turning to the transversality conditions, we first integrate equations (24) and (25) to obtain the following time paths for the shadow cost of the debt and the shadow price of capital respectively:

$$\mu(t) = \mu(0)e^{(\rho-n-r)t} \qquad \text{and} \qquad q(t) = q(0)e^{(\rho-n-r)t} \,. \tag{31}$$

Combining equation (31) with the transversality conditions in (26) and (27), we obtain the following conditions that hold along an optimal path.

$$\lim_{t\to\infty} \mu(0)e^{-rt}D(t) = 0 \qquad \text{and} \qquad \lim_{t\to\infty} q(0)e^{-rt}K(t) = 0 \tag{32}$$

The expressions in (32) imply the following asymptotic conditions on the growth rates of the debt and the capital stock: $\dot{D}/D < r$ and $\dot{K}/K < r$. That is the socially optimal steady state growth rates of the debt and the capital stock should be lower than the world interest rate asymptotically. This implies that they do not grow in present value terms, asymptotically.

3.2 Solving the model

We now derive the socially optimal long-run growth rates of aggregate domestic production, the capital stock, pollution, consumption and the debt. We find that the growth rates of *K*, *Y*, *C* and *Z* are constant from time zero and remain so. For *D* and $\chi = C/K$, however, growth may be non-constant (if for instance income and consumption grow at different rates), but we must examine whether such paths are consistent with the transversality conditions and the small country assumption. We present the cases of constant growth rates in Section 3.2.1 while the differential equations for the consumption-capital ratio ($\chi = C/K$) and the debt (*D*) are presented in Section 3.2.2.

3.2.1 The growth rates of the capital stock and pollution

We begin by combining equation (30) with the expression for the growth rate of aggregate production in (7). This and the steady state condition that

$\dot{Y}/Y = \dot{K}/K$, combined with equation (24), give the following expression for the steady state growth rate of the capital stock and domestic production.

$$g_Y = g_K = \frac{\dot{K}}{K} = n + x - \beta \frac{\eta(n+x) + (n + r - \rho)}{(1-\alpha)(1+\eta) - \beta} \tag{33}$$

Before we analyse the effect of environmental care on economic growth, it is important to impose a condition on the interest rate using the transversality condition. Equation (33) and the transversality condition impose this restriction on the world interest rate:

$$r > n + x - \beta \frac{\left[(n+x)(1+\eta) + n + \rho\right]}{(1-\alpha)(1+\eta)}.$$

Substitution of (33) into (30) and some algebraic simplifications yield the following expression for the socially optimal steady state rate of change of emissions in the open economy model.

$$\frac{\dot{Z}}{Z} = -\frac{(1-\alpha)(r-\rho) + \beta(n+x) - (1-\alpha)x}{(1+\eta)(1-\alpha) - \beta} \tag{34}$$

Note that pollution is a direct policy variable here, since we are analyzing the central-planner problem and therefore do not include any taxes. Hence equation (34) defines the steady state optimal policy rule. As can be seen from (33) and (34), the growth rates of *Y*, *K* and *Z* are all independent of the elasticity of the marginal utility of consumption, θ.

The sign of the growth rate of pollution over time depends on the sign of the numerator term since the denominator is known to be positive. The sign of the numerator term in (34) is ambiguous. If $(1-\alpha)(r-\rho) + \beta(n+x) > (1-\alpha)x$, then the overall term in the numerator is unambiguously negative and the socially optimal growth rate of pollution will decrease over time. On the other hand, if $(1-\alpha)(r-\rho) + \beta(n+x) < (1-\alpha)x$, then sign of the numerator is positive and the socially optimal growth rate of pollution increases over time. Thus the steady state growth rate of emissions in the open economy hinges on the following condition;

$$(1-\alpha)(r-\rho) + \beta(n+x) > (<)(1-\alpha)x \Rightarrow \dot{Z}/Z < (>)0.$$

The steady state growth rate of pollution is a decreasing function of the world interest rate, and the rate of population growth. A higher elasticity of marginal disutility of pollution lowers the absolute rate of change *Z*, irrespective of whether it is positive or negative. However, the growth rate of pollution increases in the subjective rate of time preference and the rate of technological progress. The higher the elasticity of output with respects to effective labour, the stronger the latter effect.

We now turn to the examination of the effect of environmental care on economic growth. The numerator in the final term in (33) is unambiguously negative since $r > \rho$. By the constant returns to scale assumption, denominator in the final term in (33) can be shown to be positive. This makes the final term in (33) negative and thus makes the growth rate obtained here lower than what the standard model predicts (which would be $n + x$). Hence environmental protection imposes a drag on steady state growth rates. The magnitude of this negative effect depends among other things on the elasticity of output with respect to pollution and the elasticity of marginal disutility of pollution.

The effect of the elasticity of marginal disutility of pollution on the common growth rate of the capital stock and output is ambiguous. To show this we compute the derivative of the equation (33) with respect to η. This gives the following results.

$$\frac{\partial g_Y}{\partial \eta} = \frac{(1-\alpha)(r-\rho) + \beta(n+x) - (1-\alpha)x}{\left((1-\alpha)(1+\eta) - \beta\right)^2} \tag{35}$$

The sign of the above derivative depends on the sign of the numerator term as the denominator is known to be positive. The first two terms in the numerator is positive whilst the final term is negative. However, the sign of the overall term in the numerator is ambiguous. However a negative sign of the derivative in (35) would not be totally unreasonable, because the higher elasticity of marginal disutility of pollution will at least lead to a lower level of pollution according to (23). The elasticity of output with respect to pollution has much role to play here. If this elasticity is high, then the numerator will more likely be positive. Thus with higher elasticity of output with respect to pollution, the first two terms in the numerator could dominate the final term. Thus, it is possible for a higher η to increase the growth rate of the economy. A higher value of η will increase (lower) the growth rate of the economy if and only if

$$(1-\alpha)x < (>)(1-\alpha)(r-\rho)+\beta(n+x).$$

Thus, the effect on growth of the elasticity of marginal disutility of pollution is positive if and only if the growth rate of pollution is negative, an indication of a high demand for environmental quality (see equation (34)).

3.2.2 Dynamics of the time path of consumption and the debt

We turn now to the steady state socially optimal growth rate of the consumption-capital ratio. By the definition of the consumption-capital ratio, $\chi = C/K$, its growth rate is $\dot{\chi}/\chi = \dot{C}/C - \dot{K}/K$.

This, (30) and (33) gives the following steady state growth rate of the consumption-capital ratio

$$g_\chi \equiv \frac{\dot{\chi}}{\chi} = \frac{1}{\theta}(r-\rho) - x + \beta\frac{\eta(n+x)+(n+r-\rho)}{(1-\alpha)(1+\eta)-\beta}. \tag{36}$$

Since the right-hand side is constant, the expression in (36) can be integrated to:

$$\chi(t) = \chi(0)e^{g_\chi t}. \tag{37}$$

Clearly, g_χ can be positive, negative or zero, depending on the values of the parameters. The growth rate of the consumption-capital ratio is positive (negative) if consumption growth rate is greater (less than) than the growth rate of the capital stock. If the growth rates of consumption and capital are equal, then the growth rate of the consumption-capital ratio is zero. Any of these cases is possible in an open economy. We discuss this in detail in Section 3.3 under two scenarios and their implications for the asset position of the economy in the international capital markets.

For this we also need the time path of debt. Starting with the derivation of the dynamic equation of the debt-capital ratio we first divide through equation (15) by K to obtain

$$\frac{\dot{K}}{K} - \frac{\dot{D}}{K} = \frac{Y}{K} - \frac{C}{K} - r\frac{D}{K} - \delta. \tag{38}$$

Now define $d \equiv D/K$, which implies that

$\dot{d}/d = \dot{D}/D - \dot{K}/K \Leftrightarrow \dot{D}/K = \dot{d} + d(\dot{K}/K)$. This and (38) combines to give the following expression.

$$\dot{d} = g_K - (r + (1-\alpha)\delta)/\alpha + \chi(0)e^{g_\chi t} - \left(g_K - r\right)d \tag{39}$$

The solution to this differential equation is

$$d = B - \frac{\chi(0)}{r - g_K - g_\chi} e^{g_\chi t} + \Gamma e^{-(g_K - r)t}, \tag{40}$$

where

$$\Gamma = d(0) - B + \frac{\chi(0)}{r - g_K - g_\chi}, \text{ and } B \equiv \left[\frac{(1-\alpha)(r+\delta)}{\alpha} + r - g_K\right]\frac{1}{r - g_K}.$$

The expression for Γ may be positive or negative, while B is positive because of the transversality condition.

To describe the development of the debt in absolute terms, we recall that $d = D/K$, where $K = K(0)e^{g_K t}$. Thus (40) can be rewritten as

$$D(t) = K(0)Be^{g_K t} - \frac{K(0)\chi(0)}{r - g_K - g_\chi} e^{g_C t} + \Gamma K(0)e^{rt}. \tag{41}$$

Since the growth rate of the final term is *r*, the transversality condition cannot be satisfied unless $\chi(0)$ is chosen by the social planner such that $\Gamma = 0$. This gives the following expressions for $\chi(0)$ and $C(0)$ respectively;

$\chi(0) = \left(r - g_K - g_\chi\right)\left(B - d(0)\right)$ and $C(0) = \chi(0)K(0) = \left(r - g_K - g_\chi\right)\left(B - d(0)\right)K(0)$,

The final solution thus is

$$D(t) = K(0)Be^{g_K t} - \left(B - d(0)\right)K(0)e^{g_C t}. \tag{42}$$

It is assumed that $\left(B - d(0)\right) > 0$; otherwise the economy is so heavily indebted from the start that it has to choose negative consumption. This is counterfactual.

3.3 Two scenarios

The dynamics of the open economy model is now given by the four equations, (29), (33), (34) and (42). The first three give constant growth rates for *C, K, Y* and *Z*. There are however two possible, and qualitative different, scenarios with respect to *D*, depending on the relation between g_C and g_K. One possibility is of course that *K, C* and *D* all grow at the same rate, but it is quite unlikely that the parameters are related in such a way. We will therefore also examine another possibility where consumption and capital grow at different rates.

In relation to this, there are two important conditions that must be fulfilled: the TVC and that the economy stays 'small' as time runs. To ensure this we impose the following assumption.

Assumption 1

i. $g_C < r$ and $g_K < r$ *so that the long run value of* $\dot{D}/D$ *is always lower than the international interest rate.*

ii. For some $\varepsilon \geq 0$*, the growth rate of the rest of the world is* $g_W = n + x - \varepsilon$*; moreover,* $g_C < g_W$ *and* $g_K < g_W$

iii. Our representative economy has a strong aversion to pollution. Also ρ *and* θ *are sufficiently large so that the drag on growth of environmental policy is large relative to the rest of the world.*

3.3.1 The benchmark case: $g_K = g_C$

We now assume that the parameters in equation (36) are such that $g_K = g_Y = g_C$ implying that $g_\chi = 0$ at all points in time. That is[6]

$$[1-\alpha-(1+\eta)^{-1}(1-\theta)\beta](r-\rho) = \theta[(1-\alpha-\beta)x-\beta n] \tag{43}$$

The bracket on the left hand side is clearly positive. It is reasonable to assume that $(1-\alpha-\beta)x-\beta n > 0$, because only then can we have $r-\rho > 0$ which is necessary for growth in per capita consumption.

Some central expressions now take specific values. Solving for $r-\rho$ in (43) and substituting this into (29) we obtain the growth rate of consumption as:

$$g_C = \frac{(1-\alpha-\beta)(x+n)-(1+\eta)^{-1}(1-\theta)\beta n}{[1-\alpha-(1+\eta)^{-1}(1-\theta)\beta]}, \tag{44}$$

[6] One way to get this, is to assume that $r = \rho$ and $n = x = 0$, which is what Blanchard and Fischer (1989) do. There are however other parameter constellations that will make $g_\chi = 0$ in (37), and we can allow for these possibilities as well.

which of course also is the growth rate of K and Y. The growth rate is clearly positive. Next substituting for $r-\rho$ in equation (34) and simplifying the resulting expression, we have:

$$\frac{\dot{Z}}{Z}=\frac{(1-\theta)[1-\alpha-(1+\eta)^{-1}\beta][(1-\alpha-\beta)x-\beta n]}{[(1+\eta)(1-\alpha)-\beta][1-\alpha-(1+\eta)^{-1}(1-\theta)\beta]} \tag{45}$$

Because $g_K=g_C$, (42) boils down to

$$D(t)=D(0)e^{g_K t}$$

This implies that $d=d(0)$ at all points in time. This further pin down the initial consumption to

$$C(0)=K(0)\left[\frac{(1-\alpha)(r+\delta)}{\alpha}\right]+\left(K(0)-D(0)\right)(r-g_C).$$

In this case it is very obvious how the history of debt and capital puts constraint on the initial consumption level. There is no room for deciding how large a debt-capital ratio to have; it is given by history. An economy which has had a history of generous lending ($d(0)<0$) will benefit from it twofold. First, it can afford to choose a high consumption path. Second, it will have a considerable share of its income from the international capital market, which does not cause any negative pollution effects on this country. Note however, that this share of income is not a result of optimal current choice, but it is given by history.

The value of the elasticity of marginal utility of consumption plays an important role in the above discussions. To understand the importance of this parameter, we set $\theta=1$ as a starting point. Then, Z is constant and the growth rate in (44) collapses to

$$g_C=\frac{(1-\alpha-\beta)(x+n)}{1-\alpha}<x+n.$$

When Z is constant, the remaining production factors in the production function, K, and L experience diminishing returns to scale. This creates drag on growth[7].

[7] If $\beta=0$, then $g_C=x+n$, which is the growth rate in the standard model without environmental protection. Thus if pollution is useless (has zero partial elasticity) then environmental policy does not create drag on growth.

In order to examine the influence of the preference parameters here, we examine the derivatives of (44) and (45), given that equation (43) continues to hold. We begin these investigations by first considering the effect of changes in θ on the growth rate of consumption (equation 44) and the growth rate of pollution (equation 45). Taking the partial derivatives of (44) and (45) with respect to θ and some amount of algebraic simplifications, we obtain their respective derivative expressions as (a) and (b);

$$\frac{\partial g_C}{\partial \theta} = \frac{\beta(1+\eta)^{-1}[\beta n - (1-\alpha-\beta)x]}{[1-\alpha-(1+\eta)^{-1}(1-\theta)\beta]^2} \tag{a}$$

$$\frac{\partial(\dot{Z}/Z)}{\partial \theta} = \frac{(1-\alpha-(1+\eta)^{-1}\beta)(\beta(1+\eta)^{-1}\theta-\alpha)[\beta n-(1-\alpha-\beta)x]}{[1-\alpha-(1+\eta)^{-1}(1-\theta)\beta]\Lambda}, \tag{b}$$

where $\Lambda = [(1+\eta)(1-\alpha)-\beta][1-\alpha-(1+\eta)^{-1}(1-\theta)\beta]$.The numerator in the above derivative expression in (a) is negative since $(1-\alpha-\beta)x > \beta n$ by assumption while the denominator is guaranteed to be positive. It is not hard to see that the denominator of the derivative expression (b) above is positive since the term in bracket has the same sign as Λ. By our assumption that per capita consumption grows, the numerator in (b) may very well be negative. Hence, the sign on the derivative expressions in (a) and (b) are both negative. Thus, an increase in the elasticity of marginal utility of consumption makes the growth rates of both consumption and pollution lower. That is, pollution falls more rapidly and consumption grows slower as θ becomes larger. The reason here is not farfetched: an increase in θ reduces the growth rate of consumption and hence output growth, which necessarily requires reduction in the growth rate of inputs used in production if the production process is to remain technically efficient.

To see the impact of the elasticity of marginal disutility of pollution on the growth rate of consumption and pollution respectively, we take the partial derivative of equations (44) and (45) with respect to η. After some amount of algebra, we obtain the respective derivative expressions in (c) and (d).

$$\frac{\partial g_C}{\partial \eta} = \frac{\beta(1+\eta)^{-2}(1-\theta)[\beta n-(1-\alpha-\beta)x]}{[1-\alpha-(1+\eta)^{-1}(1-\theta)\beta]^2} \tag{c}$$

$$\frac{\partial(\dot{Z}/Z)}{\partial \eta} = \frac{(1-\theta)(1+\eta)^{-2}\beta[\beta+\beta n-(1-\alpha-\beta)x]-(1-\alpha)^2}{\Lambda^2} \tag{d}$$

As before, the denominators of both equations (c) and (d) are known to be positive. Clearly, the numerator on the derivative expression in (c) is nonnegative while the sign on the numerator in (d) is not straight forward to see. Further investigations however show that the numerator is non-negative. Thus, both derivative expressions are positively signed. This implies that a higher value of η makes the growth rates of both consumption and pollution higher, other things equal. This means that agents require a greater increase in consumption to compensate them from the disutility of pollution. The fact that a higher η increases pollution growth rate is hard to discern since a stronger aversion to pollution is expected to make the pollution growth lower. However, looking at (23), if the initial pollution level is extremely low, for a given marginal utility, this makes pollution lower to some extent.

3.3.2 Deviations from the benchmark case

We now consider two possible deviations from the benchmark case discussed above. Specifically, we consider cases in which the growth rates of consumption and the capital stock differ (i.e. $g_K \neq g_C$), starting with the case where consumption grows faster than the capital stock.

Consumption grows faster than capital ($g_K < g_C$)

Consider now a growth path where the growth rate of consumption is higher than the growth rate of the capital stock ($g_K < g_C$). Since the exponential expression of the final term in (42) then grows faster, *D* will sooner or later turn negative and decline. In this case, history does not matter much: a country that has had debt in the past can repay all and invest abroad over time. This process will be faster if for instance the elasticity of marginal disutility of pollution (η) is large (cf. (34)) which implies a strong aversion to pollution.

In the first case where consumption grows faster than the capital stock, the economy increasingly makes use of the international capital market to generate income, and eventually the share of capital in assets is negligible. However, this does not need to make the economy a large player in the international capital market, which would violate the small open economy assumption and make the exogenous-interest-rate assumption irrelevant. To see this, suppose the rest of the world does not control emissions to the same extent as "our" country. Then a situation with $g_K < g_C < n + x - \varepsilon$ is possible, which would make $\dot{D}/D$ lower than the international growth rate. This country's claim on the rest of the world would not grow compared to the rest of the world, and thereby it remains a small player in the international capital market.

It is interesting to investigate now, the behaviour of the optimal pollution growth path under this scenario. With consumption growing faster than the capital stock ($g_K < g_C$), the growth rate of the consumption-capital ratio must be positive (i.e. $g_\chi > 0$). By (37) a sufficient condition for a positive growth rate of the consumption-capital ratio is

$$r > \rho + \frac{(1-\alpha-\beta)x - \beta n}{\left[(1-\alpha)+(\theta-1)(1+\eta)^{-1}\beta\right]\theta^{-1}}, \quad \theta \geq 1.$$

The denominator term in the above condition is positive. The sign of the numerator in the second term on the right hand side of the inequality sign is ambiguous. Thus, if consumption grows faster, than the capital stock, the socially optimal growth rate of emissions will decrease over time if $(1-\alpha-\beta)x < \beta n$, so that the above condition will always be met.

Consumption grows slower than Capital ($g_K > g_C$)

We turn now to the opposite deviation from the benchmark case where consumption grows slower than capital. This is highly possible in a country with low aversion to pollution. Then by (42), D will be growing in the long run, but not too fast to violate the TVC or the smallness criterion. Due to the low aversions to pollution, the country accepts to 'produce for others' ($\dot{D} > 0$)

and live with the extra pollution that this implies. The increase in the domestic capital stock makes the domestic income large but a greater share of it is used for servicing debt.

In this case, the small economy becomes a net importer of capital and thus accumulates debt over time. The economy will then act as net exporter of goods, but its pollution as well as debt grows over time. However, given that the transversality condition is satisfied, our small open economy does not become overly indebted as time runs (which would mean that it could no more borrow from the international capital markets). Note also that the implication for the consumption-capital ratio along this growth path is that is growth rate will be negative. What does this imply for the optimal growth path of pollution? By (37), a negative growth rate of the consumption-capital ratio requires that

$$r < \rho + \frac{(1-\alpha-\beta)x - \beta n}{\left[(1-\alpha) + (\theta-1)(1+\eta)^{-1}\beta\right]\theta^{-1}}.$$

This will be consistent with our earlier conditions on the interest rate if $\beta n < (1-\alpha-\beta)x$, since $r > \rho$. What is the implied direction of optimal pollution path under this scenario? Whether pollution grows (declines) over time depends on whether the wedge between $(1-\alpha-\beta)x$ and βn is lager (less) than $(1-\alpha)(r-\rho)$. Sufficient for this condition to hold is that $(1-\alpha)(1+\eta) \le \beta$ which is never satisfied under constant returns to scale. While we cannot tell the direction of the growth rate of emissions, the possibility of positive growth cannot be ruled out. The reason is that high rate of capital accumulation will lead to increase in the scale of domestic production which may lead to a greater use of pollution (because the marginal product of this factor is pulled upward).

These findings reecho the environmental Kuznets curve (EKC) hypothesis and the pollution haven hypothesis. At the initial levels of development countries depend much on domestic production to finance its consumption and investments. Over time however, it can afford to invest in the international capital markets and thus cut down on the accumulation of domestic capital and hence production. This is possible if and only if consumption grows faster than the capital stock, though. Other things equal, this implies an inverted-U time path for emissions.

4. Command optimum in the closed economy model

We now turn to the closed economy case. The analysis in this section, in principle can be found elsewhere in the earlier literature. However, since we do not know any paper that has exactly the same formulation as the present one, the analysis in this section is necessary for comparison between the open and the closed cases. In the closed economy, $A = K$ and $\dot{D} = D = 0$. Thus, in the closed economy, the debt accumulation constraint disappears from the planner's optimization problem, hence we have $\mu = 0$. By recalling that in the closed economy without government purchases, $I = Y - C$, the optimality conditions in (21)-(27) are modified to the following set of conditions.

$$\frac{\partial H}{\partial C} = 0 \Leftrightarrow C^{-\theta} = qL^{1-\theta} \tag{46}$$

$$\frac{\partial H}{\partial Z} = 0 \Leftrightarrow \phi Z^{\eta} = \beta q \frac{Y}{Z} \tag{47}$$

$$\frac{\dot{q}}{q} = \left(\rho + \delta - n - \alpha / v\right) \tag{48}$$

$$\lim_{t\to\infty} \left[q(t)K(t)\right] e^{-(\rho-n)t} = 0 \tag{49}$$

In the closed economy, the capital-output ratio v, is not a constant, and hence the growth rate of the shadow price of capital will also vary over time. This follows from the fact that in the closed economy the interest rate is not constant.

As in the case of the open economy, to develop some implications of the above conditions, we start by taking logs of both sides of equation (46) and differentiating with respect to time. Using the condition in equation (48), we obtain the consumption Euler's equation for the planner's optimization problem as equation (50).

$$-\theta \frac{\dot{C}}{C} = \frac{\dot{q}}{q} + (1-\theta)n \qquad \Rightarrow \frac{\dot{C}}{C} = \frac{1}{\theta}\left[\alpha(1/v) + \theta n - (\rho + \delta)\right] \tag{50}$$

Similar to the results obtained in the open economy case, the growth rate of aggregate consumption depends on the parameters in the utility and the production functions.

Yet another useful implication concerns pollution. Taking logs of the optimality condition with respect to pollution in (47) and differentiating the results with respect to time, we obtain the social optimal pollution control rule as

$$\frac{\dot{Z}}{Z} = \frac{1}{1+\eta}\left(\frac{\dot{q}}{q} + \frac{\dot{Y}}{Y}\right) \tag{51}$$

The above expression says that emissions grow in constant proportion with the sum of the growth rates of the shadow price of capital and real GDP. An increase in the shadow price of capital makes the planner substitute capital for pollution whilst high GDP growth rate requires using more productive factors including emissions.

4.1. Dynamic systems

In this section we develop a two dimensional dynamic system in consumption-capital ratio and capital-output ratio space. To begin with, we substitute $I = Y - C$ into (17) and divide the results by K to obtain the growth rate of the capital stock as

$$\frac{\dot{K}}{K} = \frac{1}{v} - \chi - \delta . \tag{52}$$

Combining equations (50) and (52) and some algebraic simplifications give the growth rate of the consumption-capital ratio as:

$$\frac{\dot{\chi}}{\chi} = \chi - \left(\frac{\theta - \alpha}{\theta}\right)\frac{1}{v} + \frac{1}{\theta}\left[\theta n + (\theta - 1)\delta - \rho\right] \tag{53}$$

To close the system, we derive the equation of motion for the capital-output ratio for the command optimal allocation as in equation (54). To derive (54) we combine equations (9), (48) and (52) together with equation (6). After a fair amount of algebra, we obtain the differential equation in (54).

$$\dot{v} = (1-\alpha) - \frac{(1-\alpha)(1+\eta)-\beta}{1+\eta-\beta}\chi v$$
$$-\frac{1+\eta}{1+\eta-\beta}\left((1-\alpha)\delta + \left(\beta/(1+\eta)\right)(\rho - n) + (1-\alpha-\beta)(n+x)\right)v \tag{54}$$

Equations (53) and (54) constitute our dynamic system. In the appendix C to this paper, a condition that ensures that the dynamic system above is saddle path stable is derived. Here, we establish the existence of unique steady state by studying the iso-clines of the dynamic systems along which $\dot{\chi} = \dot{v} = 0$. We do this by analyzing the derivatives of (53) and (54) along the $\dot{\chi} = 0$ and $\dot{v} = 0$ loci in a $\chi - v$ plane. When $\dot{\chi} = 0$, the differential equation in (53) simplifies to $\chi = \frac{\theta-\alpha}{\theta}\frac{1}{v} + \frac{1}{\theta}\left[\rho + (1-\theta)\delta - \theta n\right]$. The first and second derivative with respect to v are $-\left((\theta-\alpha)/\theta\right)/v^2$ and $2\left((\theta-\alpha)/\theta\right)/v^3$ respectively. Since the elasticity of marginal utility of consumption is higher than the elasticity of output with respect to capital, the first derivative is negative and the second, positive. Hence the $\dot{\chi} = 0$ iso-cline is monotonically decreasing, but at an increasing rate in v.

Turning to the differential equation in (54), $\dot{v} = 0$ implies that

$$\chi = \frac{(1-\alpha)(1+\eta-\beta)}{(1-\alpha)(1+\eta)-\beta}\frac{1}{v} - \frac{1+\eta}{(1-\alpha)(1+\eta)-\beta}\left((1-\alpha)\delta + \left(\frac{\beta}{1+\eta}\right)(\rho-n) + (1-\alpha-\beta)(n+x)\right)$$

. This then means that the $\dot{v} = 0$ locus is monotonically decreasing in v. The absolute slope of the $\dot{v} = 0$ iso-cline is greater than the slope of the $\dot{\chi} = 0$ iso-cline if $(1-\alpha)(1+\eta) + (\theta-1)\beta > 0$, which is always satisfied.

By setting $\dot{\chi} = \dot{v} = 0$ in equations (53) and (54) and solving the resulting static two-equation systems we obtain the steady state values for the capital-output ratio and the consumption-capital ratio respectively as in equations (55) and (56) respectively.

$$v^* = \frac{\alpha\left(\beta(\theta-1) + (1-\alpha)(1+\eta)\right)}{\left(\left[(\theta-1)\beta + (1-\alpha)(1+\eta)\right](\rho+\delta) + \theta(1+\eta)(1-\alpha-\beta)x - \beta(1+\eta)\theta n\right)} \tag{55}$$

Note that from equation (55), the planner's steady state choice of capital-output ratio decreases in the elasticity of marginal disutility of pollution and hence rate of growth of pollution. This makes sense if we assume that capital intensive modes of production are dirty. The strength of the pollution growth effect on capital-output ratio depends on three parameters in our model, the elasticities of marginal utility of consumption, output with respect to capital and output with respect to pollution. High values of elasticity of marginal utility and elasticity of output with respect to pollution amplify the negative effect of pollution growth on the capital-output ratio.

$$\chi^* = \frac{(1-\alpha)\delta+\rho}{\alpha} + \frac{\left[(\theta-\alpha)(1+\eta)(1-\alpha-\beta)\right]x - \left[\beta\left(\theta(1+\eta)+\alpha(\theta-1)\right)+\alpha(1+\eta)(1-\alpha-\beta)\right]n}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}. \quad (56)$$

As seen from equation (56), the consumption-capital ratio under the command allocation increases in the subjective rate of time preference and the rate of depreciation. However, the effect of the elasticity of marginal disutility of pollution on the consumption-capital ratio is ambiguous. As before, the strength of this negative effect of pollution growth on consumption-capital ratio depends on the size of elasticities of marginal utility of consumption, output with respect to capital, labour and pollution.

Now that the steady state equilibrium of the dynamic system given by equations (53) and (54), has been identified, it is important to examine if the solution is stable when subjected to a slight perturbations to the steady state: does the system diverge from the equilibrium or return to it when perturbed?

We focus on the question of whether the steady state is *saddle point stable* and thus sensitive to slight perturbations of the capital-output ratio or the consumption-capital ratio or not. We do this by examining the eigenvalues or characteristic roots of the Jacobian of the dynamic system given in (50) and (51). Since our focus is on saddle point stability, we require the determinant of the Jacobian to be negative. That is $|J|<0$. The condition that ensures this is

$$g_Y = g_C = g_K > \frac{\alpha[(\theta-1)^{-1}-\theta]+(\theta-1)}{\alpha}(\theta-1)\delta - \frac{\alpha\theta-(\theta-1)}{\alpha}(\rho-\theta n)$$

(See appendix C for detailed mathematical derivations on local stability analysis). The first term on the right hand side of the above inequality expression may very well be positive; the second term cannot be signed *a priori*. However, stability requires that the growth rate of the economy should be higher than a certain threshold which is expected to be positive.

4.2 Steady state growth rates

We have already mentioned that pollution is a choice variable in the planner's maximization problem. The interesting question then is how does the planner's choice of pollution path affects long run growth? By equation (9) and the steady state condition that $\dot{Y}/Y = \dot{K}/K$, we obtain the growth rate of aggregate output as

$$\frac{\dot{Y}}{Y} = \frac{1}{1-\alpha}\left((1-\alpha-\beta)(n+x)+\beta\frac{\dot{Z}}{Z}\right) \tag{57}$$

From the expression above, a policy that aims to decrease the level of pollution over time ($\dot{Z}/Z < 0$), will have negative effect on long run growth. Ambitious environmental care can however allow for growth in our setting provided that Z does not decline too fast.

At the steady state aggregate production, consumption and the capital stock all grow at the same constant rate of $\dot{Y}/Y = \dot{C}/C = \dot{K}/K = 1/v^* - \chi^* - \delta$. Substituting the steady state values for v^* and χ^* into the above expression and simplifying, we obtain the following solution for the socially optimal growth rate of aggregate variables at the steady state.

$$g_C = g_Y = g_K = n + x - \beta\frac{(\theta+\eta)x+(1+\eta)n}{\beta(\theta-1)+(1-\alpha)(1+\eta)} \tag{58}$$

Now the similarity between the growth rate in the closed and open economy cases are obvious; in both cases, the growth rate is lower than in the standard model without pollution. An important difference however is that the growth rate is influence by the size of the elasticity of marginal utility of consumption. As was found in the open economy case, the elasticity of output with respect to pollution and the elasticity of marginal disutility of pollution play significant role on the size of the drag that environmental policy imposes on growth.

It is now interesting to investigate whether the drag imposed on growth by environmental care is larger or smaller in the open economy relative to the closed economy case. It can be shown that the size of the drag depends on the relationship between the growth rates of consumption and the capital stock in the open economy case. If consumption grows faster than the capital stock, the drag is always larger in the open economy case. Comparing (58) with (33), we show that the drag is larger in the open economy case if;

$$r > \rho + \frac{(1-\alpha-\beta)x - \beta n}{\theta^{-1}\left[\beta(\theta-1)(1+\eta)^{-1} + 1 - \alpha\right]}.$$

This is the same condition that we derived in the open economy case where consumption is assumed to be growing faster than the capital stock. Thus the possibility of satisfying part of the domestic consumption through imports amplifies the negative effect of demand for environmental quality on growth. The opposite holds when the capital stock grows faster than consumption. This becomes obvious when one reverses the inequality in the previous condition.

We finally turn to the steady state optimal pollution growth in the closed economy. We obtain this by substituting equations (57) and (58) into (51). This with some algebra yields the results in (56).

$$\frac{\dot{Z}}{Z} = (\theta-1)\frac{\beta n - (1-\alpha-\beta)x}{\beta(\theta-1) + (1-\alpha)(1+\eta)} \tag{59}$$

Recalling that $\theta \geq 1$, the growth rate of pollution is negative if and only if $\beta n < (1-\alpha-\beta)x$. This is in sharp contrast to the condition obtained in the open economy case. This becomes obvious if one compares equation (59) with (35). Recall that in the open economy, pollution growth at the steady state is negative if and only if

$$(1-\alpha)(r-\rho) > (1-\alpha-\beta)x - \beta n.$$

This inequality is somehow reversed in the closed economy case where negative growth rate of pollution requires that $(1-\alpha-\beta)x - \beta n > 0$. Thus given the elasticity of output with respect to effective labour and pollution, the direction of the growth rate of optimal emissions is driven by the relative strength of the rates of population growth and technological progress. Note however, that in the open economy the wedge between the world interest rate and the rate of time preference play a role in this. In the closed economy, the growth rate in pollution increases in the rate of population growth but decreases in the rate of technological progress. Remember that this is slightly the opposite of the open economy case. The reason why pollution increases with the rate of population growth is not hard to discern: more labour makes the marginal product of pollution higher and therefore reduce the optimal reduction in pollution. Also higher population implies higher consumption which can only be satisfied with increased production in a closed economy which may require more pollution for a given technology. Similarly, technological progress allows the economy to have faster improvements in instantaneous utility: it can afford to increase the utility of consumption and decrease the disutility of pollution at higher rates. In other words, technological progress improves the trade-off between consumption and pollution in favour of consumption.

5. Conclusion

The paper examined the effect on growth of the demand for environmental quality in a small open economy. Our analysis reveals the following conclusions. Environmental care imposes a drag on long run economic growth, by increasing the capital-output ratio and lowering the returns to capital. This result becomes less severe when the economy is closed and trade in goods and assets across national borders are not allowed.

Our analysis of the open economy case also reveals that whilst the growth rates of consumption, output, the capital stock and pollution are constant, there are three possible qualitative scenarios with respect to the debt. First, we considered a benchmark scenario in which consumption and capital grow at a common rate. Under this scenario, we show that the debt and capital stock grow at a common rate and hence the debt-capital ratio is constant at all points in time. The implication of this is that the history of the debt and the capital stock puts a constraint on the initial consumption level. We show that a country which has had a history of generous lending will benefit from it twofold. It can afford to choose a high consumption path and also have a considerable share of its income from the international capital market, which does not cause any negative pollution effect on this country. In the second case where consumption grows faster than the capital stock, we show that the small open economy eventually exports capital (negative debt) to the rest of the world by accumulating assets abroad over time. This process is faster if the elasticity of marginal disutility of pollution is high relative to the rest of the world so that it has higher drag on (production) growth. As a final scenario, we considered the opposite scenario where the capital stock grows faster than consumption. In this case we show that the economy imports capital due to the low aversion to pollution, and thus accumulate debt. A high demand for environmental quality is found to induce capital flight from high income countries to poor countries with lower demand for environmental quality. This result offers an alternative restatement of the pollution haven hypothesis. Furthermore, our analysis shows that the drag that the demand for environmental quality imposes on growth is larger in the open economy if and only if consumption grows faster than the capital stock. In the reverse case where the capital stock grows faster than consumption, the drag is smaller in the open economy case relative to the closed economy.

Appendix A: Decentralized solution

Then the objective of the household is to maximize

$$u=\int_0^\infty u(C/L,Z)e^{-(\rho-n)t}dt=\int_0^\infty\left[\frac{(C/L)^{1-\theta}-1}{1-\theta}-\phi\frac{Z^{1+\eta}}{1+\eta}\right]e^{-(\rho-n)t}dt \qquad \text{(A1)}$$

with respect to C, subject to

$$\dot{A}=wL+rA+\varphi-C, \qquad \text{(A2)}$$

The current value Hamiltonian for the above dynamic optimization problem is:

$$H(C/L,Z,K,\lambda)=\frac{(C/L)^{1-\theta}}{1-\theta}-\phi\frac{Z^{1+\eta}}{1+\eta}+\lambda\left[wL+rA+\varphi-C\right] \qquad \text{(A3)}$$

Note that the household cannot influence Z. It is therefore not a choice variable here. The associated optimality conditions for the representative household are:

$$\frac{\partial H}{\partial C}=0\Leftrightarrow C^{-\theta}L^{\theta-1}-\lambda=0\Leftrightarrow C^{-\theta}=\lambda L^{1-\theta} \qquad \text{(A4)}$$

$$\frac{\partial H}{\partial A}=-\left[\dot{\lambda}-(\rho-n)\lambda\right]\Leftrightarrow r\lambda=\dot{\lambda}-(\rho-n)\lambda\Rightarrow\dot{\lambda}=(\rho+\delta-n-\alpha/v)\lambda \qquad \text{(A5)}$$

$$\lim_{t\to\infty}\left[\lambda(t)A(t)\right]e^{-(\rho-n)t}=0 \qquad \text{(A6)}$$

Note the (6) has been used in equation (A5). It can conveniently be rewritten as

$$\frac{\dot{\lambda}}{\lambda}=\rho+\delta-n-\alpha/v \qquad \text{(A7)}$$

Taking logs of equation (A4), differentiating both sides with respect to time and using condition (A7), we obtain the familiar consumption Euler equation.

$$\frac{\dot{C}}{C}=\frac{1}{\theta}\left[(\alpha/v)+\theta n-(\rho+\delta)\right] \qquad \text{(A8)}$$

Now we define the consumption as a ratio to the capital stock as $\chi=C/K$. It follows that:

$$\frac{\dot{\chi}}{\chi}=\frac{\dot{C}}{C}-\frac{\dot{K}}{K} \qquad \text{(A9)}$$

From equation (62), $\dot{K}/K=(1/v)-\chi-\delta$. Using this and (A8) in (A9):

$$\frac{\dot{\chi}}{\chi}=\chi+\left(\frac{\alpha-\theta}{\theta}\right)\frac{1}{v}+\frac{1}{\theta}\left[\theta n-(\rho+(1-\theta)\delta)\right] \qquad \text{(A10)}$$

This is the first of two differential equations in χ and v, describing the dynamics of the model.

Derivation of the equation of motion for the COR in a decentralized Ramsey Closed economy model

$$\frac{\dot{v}}{v}=\frac{\dot{K}}{K}-\frac{\dot{Y}}{Y} \tag{A11}$$

According to equation (7)

$$\frac{\dot{Y}}{Y}=\alpha\frac{\dot{K}}{K}+\beta\frac{\dot{Z}}{Z}+(1-\alpha-\beta)(n+x) \tag{A12}$$

Equation (8) is repeated here as equation (A13)

$$\frac{\dot{Z}}{Z}=\frac{\dot{Y}}{Y}-\frac{\dot{\tau}}{\tau} \tag{A13}$$

Substituting equation (A13) into (A12) we obtain equation (A14)

$$\frac{\dot{Y}}{Y}=\alpha\frac{\dot{K}}{K}+\beta\frac{\dot{Y}}{Y}+(1-\alpha-\beta)(n+x)-\beta\frac{\dot{\tau}}{\tau} \tag{A14}$$

$$(1-\beta)\frac{\dot{Y}}{Y}=\alpha\frac{\dot{K}}{K}+(1-\alpha-\beta)(n+x)-\beta\frac{\dot{\tau}}{\tau}$$

Collecting terms in (A14) and simplifying gives the expression in (A15)

$$\frac{\dot{Y}}{Y}=\frac{\alpha}{1-\beta}\frac{\dot{K}}{K}+\frac{(1-\alpha-\beta)}{1-\beta}(n+x)-\frac{\beta}{1-\beta}\frac{\dot{\tau}}{\tau} \tag{A15}$$

Substituting equations (17) and (A15) into (A11), we obtain the equation of motion for the capital-output ratio in a decentralized Ramsey economy as in (A16) below.

$$\frac{\dot{v}}{v}=\frac{1}{v}-\chi-\delta-\left(\frac{\alpha}{1-\beta}\frac{\dot{K}}{K}+\frac{(1-\alpha-\beta)}{1-\beta}(n+x)-\frac{\beta}{1-\beta}\frac{\dot{\tau}}{\tau}\right)$$

$$\frac{\dot{v}}{v}=\frac{1}{v}-\chi-\delta-\frac{\alpha}{1-\beta}\left(\frac{1}{v}-\chi-\delta\right)-\frac{(1-\alpha-\beta)}{1-\beta}(n+x)+\frac{\beta}{1-\beta}\frac{\dot{\tau}}{\tau}$$

$$\frac{\dot{v}}{v}=\frac{1-\alpha-\beta}{1-\beta}\frac{1}{v}-\frac{1-\alpha-\beta}{1-\beta}(\chi+\delta)-\frac{1-\alpha-\beta}{1-\beta}(n+x)+\frac{\beta}{1-\beta}\frac{\dot{\tau}}{\tau}$$

$$\frac{\dot{v}}{v}=\frac{1-\alpha-\beta}{1-\beta}\frac{1}{v}-\frac{1-\alpha-\beta}{1-\beta}\left(n+x+\delta+\chi\right)+\frac{\beta}{1-\beta}\frac{\dot{\tau}}{\tau}$$

$$\dot{v}=\frac{1-\alpha-\beta}{1-\beta}-\frac{1}{1-\beta}\left((1-\alpha-\beta)\left(n+x+\delta+\chi\right)-\beta\frac{\dot{\tau}}{\tau}\right)v \tag{A16}$$

Appendix B: Computation of the steady state values of χ and v in the decentralized Ramsey economy

At the steady state;

$$\dot{v} = \dot{\chi} = 0. \tag{B1}$$

Invoking the steady state condition on equation (A10) and (A16), we obtain equations (B2) and (B3).

$$\dot{\chi} = 0 \Rightarrow \chi = \frac{\theta - \alpha}{\theta}\frac{1}{v} + \frac{1}{\theta}\left[\rho + (1-\theta)\delta - \theta n\right] \tag{B2}$$

$$\dot{v} = 0 \Rightarrow \chi = \frac{1}{v} + \xi\frac{\dot{\tau}}{\tau} - \left(n + x + \delta\right), \qquad \xi = \frac{\beta}{1-\alpha-\beta} \tag{B3}$$

Combine equations (B2) and (B3) to eliminate χ. We do this by equating (B2) and (B3).

$$\frac{1}{v} + \xi\frac{\dot{\tau}}{\tau} - \left(n + x + \delta\right) = \frac{\theta - \alpha}{\theta}\frac{1}{v} + \frac{1}{\theta}\left[\rho + (1-\theta)\delta - \theta n\right] \tag{B4}$$

$$\frac{1}{v}\left[1 - \frac{\theta - \alpha}{\theta}\right] = \left(n + x + \delta\right) + \frac{1}{\theta}\left[\rho + (1-\theta)\delta - \theta n\right] - \xi\frac{\dot{\tau}}{\tau}$$

$$\frac{\alpha}{\theta}\frac{1}{v} = \left(n + x + \delta\right) + \frac{1}{\theta}\left[\rho + (1-\theta)\delta - \theta n\right] - \xi\frac{\dot{\tau}}{\tau}$$

$$\frac{1}{v} = \frac{\theta}{\alpha}\left[\left(n + x + \delta\right) + \frac{1}{\theta}\left[\rho + (1-\theta)\delta - \theta n\right] - \xi\frac{\dot{\tau}}{\tau}\right] \tag{B5}$$

Equation (B5) implies;

$$v^* = \frac{1}{\left(\frac{\theta}{\alpha}\right)\left[\left(n + x + \delta\right) + \frac{1}{\theta}\left[\rho + (1-\theta)\delta - \theta n\right] - \xi\frac{\dot{\tau}}{\tau}\right]} \quad . \tag{B6}$$

Substituting (B5) into (B3) gives the steady state value of the consumption to capital ratio as

$$\chi^* = \frac{\theta - \alpha}{\alpha}\left(\left(n + x + \delta\right) - \xi\frac{\dot{\tau}}{\tau}\right) + \frac{1}{\alpha}\left[\rho + (1-\theta)\delta - \theta n\right] \tag{B7}$$

Appendix C: Local stability analysis

Consider the following first order non-linear homogeneous dynamic system.

$$\dot{\chi} = f(\chi, v) \tag{C1}$$

$$\dot{v} = g(\chi, v) \tag{C2}$$

Making Taylor approximation around the steady state and indicating steady state values with a star, we have the following:

$$\dot{\chi} = f_\chi(\chi^*, v^*)(\chi - \chi^*) + f_v(\chi^*, v^*)(v - v^*) \tag{C3}$$

$$\dot{v} = g_\chi(\chi^*, v^*)(\chi - \chi^*) + g_v(\chi^*, v^*)(v - v^*) \tag{C4}$$

These expressions can be rewritten as:

$$\dot{\chi} - f_\chi(\chi^*, v^*)\chi - f_v(\chi^*, v^*)v = -\left(f_\chi(\chi^*, v^*)\chi^* + f_v(\chi^*, v^*)v^* \right) \tag{C3'}$$

$$\dot{v} - g_\chi(\chi^*, v^*)\chi - g_v(\chi^*, v^*)v = -\left(g_\chi(\chi^*, v^*)\chi^* + g_v(\chi^*, v^*)v^* \right) \tag{C-4'}$$

Note that the right hand sides of the last two equations are constants. Dropping these constants, we have the following reduced dynamic system in matrix form.

$$\begin{pmatrix} \dot{\chi} \\ \dot{v} \end{pmatrix} - \underbrace{\begin{pmatrix} f_\chi(\chi^*, v^*) & f_v(\chi^*, v^*) \\ g_\chi(\chi^*, v^*) & g_v(\chi^*, v^*) \end{pmatrix}}_{J} \begin{pmatrix} \chi \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{C5}$$

$$J = \begin{pmatrix} f_\chi(\chi^*, v^*) & f_v(\chi^*, v^*) \\ g_\chi(\chi^*, v^*) & g_v(\chi^*, v^*) \end{pmatrix} = \begin{pmatrix} f_\chi & f_v \\ g_\chi & g_v \end{pmatrix} \tag{C6}$$

$$|J - rI| = 0$$

$$|J - rI| = \begin{pmatrix} f_\chi - r & f_v \\ g_\chi & g_v - r \end{pmatrix} \tag{C7}$$

$$|J - rI| = 0$$

$$\Rightarrow r^2 - \underbrace{(f_\chi + f_v)}_{trJ} r + \underbrace{f_\chi g_v - g_\chi f_v}_{\det J} = 0$$

$$r^2 - (trJ)r + \det J = 0$$

$$r_i = \frac{trJ}{2} \pm \frac{\sqrt{(trJ)^2 - 4\det J}}{2}, \quad i = 1, 2 \tag{C8}$$

In our case we have the following first order nonlinear dynamic systems of differential equations.

$$\dot{\chi}+\kappa\chi-\chi^2-\frac{(\alpha-\theta)}{\theta}\frac{\chi}{v}=0 \tag{C9}$$

$$\dot{v}+\frac{1-\alpha-\beta}{1-\beta}\chi v+\frac{1}{1-\beta}\left[(1-\alpha-\beta)(n+x+\delta)-\beta\frac{\dot{\tau}}{\tau}\right]v=\frac{1-\alpha-\beta}{1-\beta} \tag{C10}$$

Rewrite the last two expressions in terms of χ and v as;

$$\dot{\chi}=f(\chi,v)=\chi^2-\kappa\chi+\frac{(\alpha-\theta)}{\theta}\frac{\chi}{v} \tag{C9'}$$

$$\dot{v}=g(\chi,v)=\frac{1-\alpha-\beta}{1-\beta}-\frac{1-\alpha-\beta}{1-\beta}\chi v-\frac{1}{1-\beta}\left[(1-\alpha-\beta)(n+x+\delta)-\beta\frac{\dot{\tau}}{\tau}\right]v \tag{C10'}$$

where $\kappa\equiv\left[\rho+(1-\theta)\delta-\theta n\right]\theta^{-1}$.

Before computing the derivatives, it is useful to obtain the 'steady state' conditions which follow from putting $\dot{\chi}=\dot{v}=0$:

$$0=\chi^2-\kappa\chi+\frac{(\alpha-\theta)}{\theta}\frac{\chi}{v} \tag{C11}$$

$$0=\frac{1-\alpha-\beta}{1-\beta}-\frac{1-\alpha-\beta}{1-\beta}\chi v-\frac{1}{1-\beta}\left[(1-\alpha-\beta)(n+x+\delta)-\beta\frac{\dot{\tau}}{\tau}\right]v \tag{C12}$$

Equation (C11) implies;

$$\chi^2=\kappa\chi-\frac{(\alpha-\theta)}{\theta}\frac{\chi}{v}$$

$$\chi^*=\kappa+\frac{\theta-\alpha}{\theta}\frac{1}{v} \tag{C13}$$

Equation (C12) implies;

$$\chi v=1-\frac{1}{1-\alpha-\beta}\left[(1-\alpha-\beta)(n+x+\delta)-\beta\frac{\dot{\tau}}{\tau}\right]v$$

$$\chi=\frac{1}{v}+\frac{\beta}{1-\alpha-\beta}\frac{\dot{\tau}}{\tau}-(n+x+\delta) \tag{C14}$$

The partial derivatives of the above system in (C9') and (C10') are:

$$f_\chi(\chi^*,v^*)=2\chi^*-\frac{\theta-\alpha}{\theta}\frac{1}{v^*}-\kappa$$

Using $\chi^* = \kappa + \frac{\theta - \alpha}{\theta}\frac{1}{v}$

$$f_\chi(\chi^*, v^*) = 2\chi^* \underbrace{-\frac{\theta-\alpha}{\theta}\frac{1}{v^*}}_{\kappa-\chi} - \kappa$$

$$\chi^* = \kappa + \frac{\theta-\alpha}{\theta}\frac{1}{v} \Leftrightarrow -\frac{\theta-\alpha}{\theta}\frac{1}{v} = \kappa - \chi^*$$

$$f_\chi(\chi^*, v^*) = \chi^* > 0$$

$$f_v(\chi^*, v^*) = -\frac{(\alpha-\theta)}{\theta}\frac{\chi^*}{(v^*)^2} = \frac{\theta-\alpha}{\theta}\frac{\chi^*}{(v^*)^2}$$

$$f_v(\chi^*, v^*) = \frac{\chi^*(\chi^* - \kappa)}{v^*}$$

$$g_\chi(\chi^*, v^*) = -\frac{1-\alpha-\beta}{1-\beta}v^*$$

$$g_v(\chi^*, v^*) = -\frac{1}{1-\beta}\left[(1-\alpha-\beta)(n+x+\delta+\chi^*) - \beta\frac{\dot{\tau}}{\tau}\right]$$

Using $\chi = \frac{1}{v} + \frac{\beta}{1-\alpha-\beta}\frac{\dot{\tau}}{\tau} - (n+x+\delta)$, this derivative simplifies to

$$g_v(\chi^*, v^*) = -\frac{1-\alpha-\beta}{1-\beta}\frac{1}{v^*} < 0$$

The Jacobian is given as:

$$J = \begin{pmatrix} \chi^* & \frac{\chi^*(\chi^*-\kappa)}{v^*} \\ -\frac{1-\alpha-\beta}{1-\beta}v^* & -\frac{1-\alpha-\beta}{1-\beta}\frac{1}{v^*} \end{pmatrix} \tag{C15}$$

Computing the trace of the Jacobian (*trJ*)

$$trJ = \chi^* - \frac{1-\alpha-\beta}{1-\beta}\frac{1}{v^*}$$

To determine the sign of this, it is instructive to substitute the steady state values for the CCR and COR (See appendix *B1*)

$$\chi^* = \frac{\theta-\alpha}{\alpha}\left((n+x+\delta) - \xi\frac{\dot{\tau}}{\tau}\right) + \frac{\theta}{\alpha}\left[\rho + (1-\theta)\delta - \theta n\right]$$

$$\frac{1}{v^*}=\left(\frac{\theta}{\alpha}\right)\left[\left(n+x+\delta\right)+\frac{1}{\theta}\left[\rho+(1-\theta)\delta-\theta n\right]-\xi\frac{\dot{\tau}}{\tau}\right]$$

$$trJ=\frac{\theta-\alpha}{\alpha}\left(\left(n+x+\delta\right)-\xi\frac{\dot{\tau}}{\tau}\right)+\frac{\theta}{\alpha}\left[\rho+(1-\theta)\delta-\theta n\right]$$
$$-\left(\frac{1-\alpha-\beta}{1-\beta}\right)\left(\frac{\theta}{\alpha}\right)\left[\left(n+x+\delta\right)+\frac{1}{\theta}\left[\rho+(1-\theta)\delta-\theta n\right]-\xi\frac{\dot{\tau}}{\tau}\right]$$

$$trJ=\frac{\theta-\alpha}{\alpha}\left(\left(n+x+\delta\right)-\xi\frac{\dot{\tau}}{\tau}\right)-\left(\frac{1-\alpha-\beta}{1-\beta}\right)\left(\frac{\theta}{\alpha}\right)\left[\left(n+x+\delta\right)-\xi\frac{\dot{\tau}}{\tau}\right]$$
$$+\frac{\theta}{\alpha}\left[\rho+(1-\theta)\delta-\theta n\right]-\left(\frac{1-\alpha-\beta}{1-\beta}\right)\left(\frac{\theta}{\alpha}\right)\frac{1}{\theta}\left[\rho+(1-\theta)\delta-\theta n\right]$$

$$trJ=\left[\frac{\theta-\alpha}{\alpha}-\frac{1-\alpha-\beta}{1-\beta}\frac{\theta}{\alpha}\right]\left(\left(n+x+\delta\right)-\xi\frac{\dot{\tau}}{\tau}\right)+\left[\frac{\theta}{\alpha}-\frac{1-\alpha-\beta}{1-\beta}\frac{\theta}{\alpha}\frac{1}{\theta}\right]\left[\rho+(1-\theta)\delta-\theta n\right]$$

$$trJ=\frac{1}{1-\beta}\left(\theta-1+\beta\right)\left(\left(n+x+\delta\right)-\xi\frac{\dot{\tau}}{\tau}\right)+\frac{1}{\alpha(1-\beta)}\left((\theta-1)(1-\beta)+\alpha\right)\left[\rho+(1-\theta)\delta-\theta n\right]$$

Given that the final expressions in each term are positive, $\theta \geq 1$ is sufficient condition for positive trace of the Jacobian in (C15).

Determinant of the Jacobian

$$|J|=-\frac{1-\alpha-\beta}{1-\beta}\frac{\chi^*}{v^*}+\frac{1-\alpha-\beta}{1-\beta}\chi^*(\chi^*-\kappa)$$

$$|J|=\frac{1-\alpha-\beta}{1-\beta}\left(\frac{\chi^*(\chi^*-\kappa)}{1}-\frac{\chi^*}{v^*}\right)$$

$$|J|=\frac{1-\alpha-\beta}{1-\beta}\left(\frac{\chi^*(\chi^*-\kappa)}{1}-\frac{\chi^*}{v^*}\right)$$

$$|J|=\frac{1-\alpha-\beta}{1-\beta}\left(\frac{\chi^* v^*(\chi^*-\kappa)-\chi^*}{v^*}\right)$$

$$|J|=\frac{1-\alpha-\beta}{1-\beta}\chi^*\left(\frac{v^*(\chi^*-\kappa)-1}{v^*}\right)$$

There is an interesting relationship among the characteristic roots, the trace of the Jacobian and the determinant of Jacobian. We explore that relationship here. It holds that $r_1+r_2=trJ$ and $r_1 r_2=|J|$. For saddle point stability, the determinant must be negative which means that one of the two roots is negative and the other positive.

Condition for saddle point stability

We have saddle point path if the determinant of the Jacobian is negative, implying that the two roots of the characteristic equation of the Jacobian have reverse signs.

$$|J| < 0$$

$$|J| = \frac{1-\alpha-\beta}{1-\beta}\chi^*\left(\frac{v^*(\chi^*-\kappa)-1}{v^*}\right)$$

$$|J| < 0 \Leftrightarrow \frac{1-\alpha-\beta}{1-\beta}\chi^*\left(\frac{v^*(\chi^*-\kappa)-1}{v^*}\right) < 0$$

$$v^*(\chi^*-\kappa)-1 < 0$$

$$v^*(\chi^*-\kappa) < 1$$

$$\chi^*-\kappa < \frac{1}{v^*}$$

$$\frac{\theta-\alpha}{\alpha}\left((n+x+\delta)-\xi\frac{\dot{\tau}}{\tau}\right)+\frac{\theta}{\alpha}\left[\rho+(1-\theta)\delta-\theta n\right]-\frac{1}{\theta}\left[\rho+(1-\theta)\delta-\theta n\right]$$
$$<\left(\frac{\theta}{\alpha}\right)\left[(n+x+\delta)+\frac{1}{\theta}\left[\rho+(1-\theta)\delta-\theta n\right]-\xi\frac{\dot{\tau}}{\tau}\right]$$

$$\frac{\theta-\alpha}{\alpha}\left((n+x+\delta)-\xi\frac{\dot{\tau}}{\tau}\right)+\left(\frac{\theta}{\alpha}-\frac{1}{\theta}\right)\left(\rho+(1-\theta)\delta-\theta n\right)$$
$$<\frac{\theta}{\alpha}\left((n+x+\delta)-\xi\frac{\dot{\tau}}{\tau}\right)+\frac{1}{\alpha}\left(\rho+(1-\theta)\delta-\theta n\right)$$

$$\left(\frac{\theta-\alpha}{\alpha}-\frac{\theta}{\alpha}\right)\left((n+x+\delta)-\xi\frac{\dot{\tau}}{\tau}\right)<\left(\frac{1}{\alpha}+\frac{1}{\theta}-\frac{\theta}{\alpha}\right)\left(\rho+(1-\theta)\delta-\theta n\right)$$

$$-\left((n+x+\delta)-\xi\frac{\dot{\tau}}{\tau}\right)<\left(\frac{\theta(1-\theta)+\alpha}{\alpha\theta}\right)\left(\rho+(1-\theta)\delta-\theta n\right)$$

$$\xi\frac{\dot{\tau}}{\tau}-(n+x+\delta)<\left(\frac{(1-\theta)+\alpha\theta}{\alpha}\right)\left(\rho+(1-\theta)\delta-\theta n\right)$$

$$\xi\frac{\dot{\tau}}{\tau}<(n+x+\delta)+\left(\frac{(1-\theta)+\alpha\theta}{\alpha}\right)\left(\rho+(1-\theta)\delta-\theta n\right)$$

Using the optimal tax rule (see appendix *D1* below), we can restate the stability condition in terms of the planner's solution as follows

$$n + x - \beta \frac{(\theta+\eta)x + (1+\eta)n}{\left(\beta(\theta-1) + (1-\alpha)(1+\eta)\right)} > \frac{\alpha[(\theta-1)^{-1} - \theta] + (\theta-1)}{\alpha}(\theta-1)\delta - \frac{\alpha\theta - (\theta-1)}{\alpha}(\rho - \theta n)$$

Note that the left hand side of the above expression is the common growth rate that we obtained in (58). Hence the stability condition for the differential equation system in (53) and (54) is stated here as

$$g_Y = g_C = g_K > \frac{\alpha[(\theta-1)^{-1} - \theta] + (\theta-1)}{\alpha}(\theta-1)\delta - \frac{\alpha\theta - (\theta-1)}{\alpha}(\rho - \theta n)$$

Appendix D: Optimal tax rule

From the optimality condition in (4), we obtain the following relationship among the growth rates of pollution tax, aggregate production and aggregate emissions as follows.

$$\frac{\dot{\tau}}{\tau}=\frac{\dot{Y}}{Y}-\frac{\dot{Z}}{Z}$$

Imposing the planners steady state growth rate of aggregate production and pollution into the above condition for the growth rate of the pollution tax, we derived the socially optimal growth rate of the tax as follows.

$$\frac{\dot{Y}}{Y}=\frac{(1+\eta)(1-\alpha-\beta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x+\frac{\left(\beta(\theta-1)+(1-\alpha-\beta)(1+\eta)\right)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{Z}}{Z}=-\left(\frac{(\theta-1)(1-\alpha-\beta)x-\beta(\theta-1)n}{\beta(\theta-1)+(1-\alpha)(1+\eta)}\right)$$

$$\frac{\dot{\tau}}{\tau}=\frac{(1+\eta)(1-\alpha-\beta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x+\frac{\left(\beta(\theta-1)+(1-\alpha-\beta)(1+\eta)\right)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$
$$+\left(\frac{(\theta-1)(1-\alpha-\beta)x-\beta(\theta-1)n}{\beta(\theta-1)+(1-\alpha)(1+\eta)}\right)$$

$$\frac{\dot{\tau}}{\tau}=\left(\frac{(1+\eta)(1-\alpha-\beta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}+\frac{(\theta-1)(1-\alpha-\beta)}{\beta(\theta-1)+(1-\alpha)(1+\eta)}\right)x$$
$$+\left(\frac{\left(\beta(\theta-1)+(1-\alpha-\beta)(1+\eta)\right)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}-\frac{\beta(\theta-1)}{\beta(\theta-1)+(1-\alpha)(1+\eta)}\right)n$$

$$\frac{\dot{\tau}}{\tau}=\frac{(\theta+\eta)(1-\alpha-\beta)x+(1-\alpha-\beta)(1+\eta)n}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}$$

Appendix D2: Steady state values for v and χ under optimal policy

$$v^{*}=\frac{1}{\left(\frac{\theta}{\alpha}\right)\left[\left(n+x+\delta\right)+\frac{1}{\theta}\left[\rho+(1-\theta)\delta-\theta n\right]-\xi\frac{\dot{\tau}}{\tau}\right]}$$

$$\frac{1}{v^{*}}=\frac{\theta}{\alpha}(n+x+\delta)+\frac{1}{\alpha}(\rho-(\theta-1)\delta-\theta n)-\frac{\theta}{\alpha}\frac{\beta}{(1-\alpha-\beta)}\frac{\dot{\tau}}{\tau}$$

$$\frac{\dot{\tau}}{\tau}=\frac{(\theta+\eta)(1-\alpha-\beta)x+(1-\alpha-\beta)(1+\eta)n}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}$$

$$\frac{\dot{\tau}}{\tau}=\frac{(\theta+\eta)(1-\alpha-\beta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x+\frac{(1-\alpha-\beta)(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\xi\frac{\dot{\tau}}{\tau}=\frac{\beta}{(1-\alpha-\beta)}\frac{\dot{\tau}}{\tau}=\frac{\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x+\frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{1}{v^*}=\frac{\theta}{\alpha}n+\frac{\theta}{\alpha}x+\frac{\theta}{\alpha}\delta+\frac{1}{\alpha}\rho-\frac{(\theta-1)}{\alpha}\delta-\frac{\theta}{\alpha}n$$
$$-\frac{\theta}{\alpha}\frac{\beta}{(1-\alpha-\beta)}\frac{(\theta+\eta)(1-\alpha-\beta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x-\frac{\theta}{\alpha}\frac{\beta}{(1-\alpha-\beta)}\frac{(1-\alpha-\beta)(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{1}{v^*}=\frac{\delta+\rho}{\alpha}+\left(\frac{\theta}{\alpha}-\frac{\beta\theta(\theta+\eta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}\right)x+\frac{\beta\theta(1+\eta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{1}{v^*}=\frac{\delta+\rho}{\alpha}+\frac{\theta(1+\eta)(1-\alpha-\beta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x+\frac{\beta\theta(1+\eta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{1}{v^*}=\frac{\delta+\rho}{\alpha}+\frac{\theta(1+\eta)(1-\alpha-\beta)x+\beta\theta(1+\eta)n}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}$$

$$v^*=\frac{\alpha}{\delta+\rho}+\frac{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}{\theta(1+\eta)(1-\alpha-\beta)x+\beta\theta(1+\eta)n}$$

$$\chi^*=\frac{\theta-\alpha}{\theta}\frac{1}{v^*}+\frac{1}{\theta}\left(\rho+(1-\theta)\delta-\theta n\right)$$

$$\chi^*=\frac{1}{\theta}\rho-\frac{\theta-1}{\theta}\delta-n+\frac{\theta-\alpha}{\theta}\left(\frac{\delta+\rho}{\alpha}\right)$$
$$+\frac{\theta-\alpha}{\theta}\frac{\theta(1+\eta)(1-\alpha-\beta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x+\frac{\theta-\alpha}{\theta}\frac{\beta\theta(1+\eta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\chi^*=\left(\frac{1}{\theta}+\frac{\theta-\alpha}{\alpha\theta}\right)\rho+\left(\frac{\theta-\alpha}{\alpha\theta}-\frac{\theta-1}{\theta}\right)\delta+\frac{(\theta-\alpha)(1+\eta)(1-\alpha-\beta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x$$
$$+\left(\frac{(\theta-\alpha)\beta(1+\eta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}-1\right)n$$

$$\chi^*=\frac{(1-\alpha)\delta+\rho}{\alpha}$$
$$+\frac{(\theta-\alpha)(1+\eta)(1-\alpha-\beta)}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x-\frac{\left[\beta\left(\theta(1+\eta)+\alpha(\theta-1)\right)+\alpha(1+\eta)(1-\alpha-\beta)\right]}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\chi^* = \frac{(1-\alpha)\delta+\rho}{\alpha}$$
$$+\frac{(\theta-\alpha)(1+\eta)(1-\alpha-\beta)x-\left[\beta\left(\theta(1+\eta)+\alpha(\theta-1)\right)+\alpha(1+\eta)(1-\alpha-\beta)\right]n}{\alpha\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}$$

Appendix D3: Steady state growth rates of aggregate and per capita variables in decentralized Ramsey economy

From equation (58) aggregate output grows at the rate;

$$\frac{\dot{Y}}{Y} = n + x - \xi\frac{\dot{\tau}}{\tau}$$

The optimal tax rule satisfies,

$$\xi\frac{\dot{\tau}}{\tau} = \frac{\beta}{(1-\alpha-\beta)}\frac{\dot{\tau}}{\tau} = \frac{\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x + \frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

Substituting this into the expression for the growth rate of aggregate production, we obtain the steady state growth rate as follows:

$$\frac{\dot{Y}}{Y} = n + x - \frac{\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x - \frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{Y}}{Y} = \left(1-\frac{\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}\right)x + \left(1-\frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}\right)n$$

$$\frac{\dot{Y}}{Y} = \frac{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)-\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x + \frac{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)-\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{Y}}{Y} = \frac{(1-\alpha-\beta)(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x + \frac{\left(\beta(\theta-1)+(1-\alpha-\beta)(1+\eta)\right)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{Y}}{Y} = \frac{(1-\alpha-\beta)(1+\eta)x+\left(\beta(\theta-1)+(1-\alpha-\beta)(1+\eta)\right)n}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}$$

Note that we have the same results as the one obtained under command allocation.

According to equation (59) in the main text, per capita variables grow at the rate of

$$\frac{\dot{y}}{y} = x - \xi\frac{\dot{\tau}}{\tau}$$

in a decentralized equilibrium. This implies that;

$$\frac{\dot{y}}{y} = x - \frac{\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x - \frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{y}}{y} = \left(1 - \frac{\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}\right)x - \frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{y}}{y} = \frac{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)-\beta(\theta+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x - \frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{y}}{y} = \frac{(1-\alpha-\beta)(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}x - \frac{\beta(1+\eta)}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}n$$

$$\frac{\dot{y}}{y} = \frac{(1-\alpha-\beta)(1+\eta)x-\beta(1+\eta)n}{\left(\beta(\theta-1)+(1-\alpha)(1+\eta)\right)}$$

$$\frac{\dot{y}}{y} = \frac{(1-\alpha-\beta)x-\beta n}{\beta(\theta-1)(1+\eta)^{-1}+(1-\alpha)}$$

Again, we have the same expression for the growth rate for per capita variables as in the command optimal allocation. Thus, we can conclude that with the Pigouvian tax optimally set, the command allocation and decentralized allocation are equivalent.

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