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Application of mean-Gini stochastic efficiency analysis

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This article adds to the information base concerning the applicability of mean-Gini stochastic efficiency analysis in agriculture. The mean-Gini efficient set of decisions is characterized rigorously in terms of its corresponding absolute risk aversion. In an empirical analysis, the mean-Gini efficient set of decisions is derived for four studies from the literature and compared to the second degree stochastic dominance efficient set. An alternative quantitative measure of risk aversion is used to gain insight in a visceral sense to the risk preferences associated with mean-Gini efficient decisions.

A number of stochastic efficiency criteria consistent with the expected utility hypothesis have been developed and used empirically to compare decisions in agriculture which involve uncertain outcomes (Bar-Shira 1992). Perhaps the best known among available comparison methods of this type, the stochastic dominance approach to stochastic efficiency is generally regarded as the least restrictive from an analyst's perspective, since it requires only very general assumptions about decision-maker preferences (Hadar and Russell 1969; Anderson, Dillon and Hardaker 1977; Drynan 1986). A notable shortcoming of stochastic dominance is the propensity for inconclusive results; that is, the likelihood that more than one alternative remains in the efficient set of alternatives following application of stochastic dominance rules. At the other end of the spectrum of comparison methods, evaluation and comparison of certainty equivalents can identify a unique, efficient decision from among a set of possible choices which involve uncertain outcomes. However, a comparison based on certainty equivalents is more restrictive than stochastic dominance in requiring use of a specific utility function in order to compare decision alternatives (for example, Yassour, Zilberman and Rausser 1981). 'Intermediate' to the stochastic dominance and certainty equivalence approaches

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is use of stochastic dominance with respect to a function (Meyer 1977).[1] Application of this approach can identify a range for a quantitative risk aversion measure, absolute risk aversion, for which a decision is efficient. The stochastic dominance, stochastic dominance with respect to a function, and certainty equivalence approaches have been used extensively in agricultural decision analysis (for example, Klemme 1985; Raskin and Cochran 1986; Feinerman, Shani and Bresler 1989; Feinerman, Choi and Johnson 1990).

An alternative to the stochastic dominance, stochastic dominance with respect to a function, and certainty equivalence approaches to stochastic efficiency, which is based on the Gini's mean difference associated with the distributions of uncertain outcomes, has been developed by Yitzhaki (1982). Mean-Gini stochastic efficiency analysis may have some advantages over other approaches. First, the set of mean-Gini efficient decisions may be a proper subset of the second degree stochastic dominance efficient set. Hence, more conclusive findings may be permitted by this approach. Second, the mean-Gini approach does not require use of a specific utility function and, therefore, does not involve the degree of restrictiveness associated with the certainty equivalence approach. Finally, decisions identified by application of the mean-Gini approach are not dependent on outcome units as are the risk efficiency ranges identified by use of stochastic dominance with respect to a function (Raskin and Cochran 1986). Despite these potential advantages, relatively little use has been made of mean-Gini stochastic efficiency concepts in agricultural decision analysis.

There is at least one important limitation of the mean-Gini approach which has undoubtedly hindered its application in agriculture. There seems to be relatively little information available on the risk preferences omitted (if any) from the second degree stochastic dominance efficient set by use of mean-Gini analysis. In this regard, Buccola and Subaei (1984) have suggested that the mean-Gini approach may be expected to identify rational decisions for decision-makers whose risk preferences are confined to an interval on the lower end of the spectrum of risk aversion. In an empirical illustration involving co-operative pooling rules, they found that the mean-Gini efficient set was efficient for an interval of a quantitative risk aversion measure, absolute risk aversion, ranging from 0.0 to 0.0015 with dollar per acre net returns and a given purchasing power of the dollar. Bailey and Boisvert (1989) compared groundnut genotype yields

[1]Certainty equivalence approach as used here refers to comparison of the certainty equivalents associated with different decisions and should not be confused with the 'CE theory' developed by Handa (1977).

according to several stochastic efficiency criteria including the mean-Gini criterion. As a part of their analysis, they considered the greater discriminatory power of the mean-Gini approach relative to stochastic dominance in terms of degree of risk aversion. While their utilization of the mean-Gini (and extended mean-Gini) method did not provide an exact range for risk aversion, the empirical results were consistent with the Buccola and Subaei (1984) finding. It should be noted, however, that many of the cumulative yield distributions they compared did not satisfy an important condition associated with application of the mean-Gini approach; that is, the condition that the cumulative distributions cross at most once.

While it is known that the mean-Gini efficient set is a subset of the second degree stochastic dominance efficient set in the case where the cumulative distributions of the outcomes cross at most once, there seems to be little practical information other than the Buccola and Subaei (1984) and Bailey and Boisvert (1989) studies to shed light on the preferences of the risk-averse decision-makers who are essentially ignored (if any) by the mean-Gini approach. This lack of information has left practitioners with considerable uncertainty regarding the implications and appropriateness of mean-Gini stochastic efficiency analysis for use in practical settings.

The purpose of this article is to add to the information base concerning the applicability of mean-Gini stochastic efficiency analysis in agriculture by first, providing a rigorous characterization of the absolute risk aversion interval associated with the mean-Gini efficient set and, second, providing empirical analysis complementary to previous studies but which includes use of an alternative quantitative measure of risk aversion in a more visceral approach to evaluation. The first section briefly describes the mean-Gini approach to analysing decisions which lead to uncertain outcomes along with the specific conditions for implementing mean-Gini stochastic efficiency analysis. The second section provides a rigorous link between the mean-Gini efficient set and a quantitative risk aversion measure, absolute risk aversion. Following this, the third section examines the risk preferences associated with the stochastically efficient decisions derived from mean-Gini analysis and compares these with the decisions forthcoming from stochastic dominance analysis. An empirical comparison is facilitated by use of an alternative quantitative risk aversion measure involving four studies from the literature. Concluding remarks are given in the final section.

1. Mean-Gini stochastic efficiency

This section briefly reviews the concepts underlying mean-Gini stochastic efficiency analysis and utilizes this opportunity to clarify the nature and potential significance of the Yitzhaki (1982) findings for applied analysis of stochastic efficiency. Notation is defined now for use in this and later sections. Let F_i and F_j be the cumulative distribution functions associated with the uncertain outcomes of two distinct decisions. Denote the probability density function corresponding to F_i by f_i, the expected value of the outcomes corresponding to F_i by M_i, the standard deviation of the outcomes by S_i, and one-half the Gini's mean difference of the outcomes $((1/2) \int \int |x - y| f_i(x)f_i(y)\ dx\ dy)$ by G_i.[2] The cumulative distribution function F_i is said to dominate F_j according to second degree stochastic dominance if and only if $\int_{-\infty}^{x} [F_j(t) - F_i(t)]\ dt \geqslant 0$ for all x with the strict inequality holding at least once. With this definition and notation, the following proposition (Yitzhaki 1982) reveals the principal result underlying mean-Gini stochastic efficiency analysis.

Proposition 1: Let $\{F_i(x)\}$ be a set of distribution functions which cross at most once, $i = 1, 2, ..., I$, such that $F_i(a) = 0$, $F_i(b) = 1$, and $0 \leqslant F_i(x) \leqslant 1$ for $x \in [a,b]$. Denote the set of distribution functions for which

$$\begin{aligned}&(1) \quad M_i \geqslant M_j \text{ and}\\ &(2) \quad M_i - G_j \geqslant M_j - G_j\end{aligned}$$

for some i with at least one strict inequality by $\{F_j(x)\}$. Then the complement, $\{F_j(x)\}^c$, is a subset of the second degree stochastic dominance efficient set.

Proof: Follows from Proposition 2 (Yitzhaki 1982) and discussion in a subsequent section of Yitzhaki (1982).

The proposition reveals the implications of applying the following procedure when comparing decisions with uncertain outcomes. Given a set of cumulative distribution functions, $\{F_i(x)\}$, corresponding to different decisions, evaluate the mean, M_i, and one-half the Gini's mean difference, G_i, for each distribution in the set. Conduct pair-wise comparisons of all the distributions, F_i and F_j, eliminating all distributions, F_j, which satisfy (1) $M_i \geqslant M_j$ and (2) $M_i - G_i \geqslant M_j - G_j$ for some i and with at least one strict inequality. According to the proposition, the set of distributions not eliminated by this procedure is in the second degree stochastic dominance efficient set but may not constitute a proper subset of the set.

It is important to note that some of the decisions with uncertain outcomes eliminated by the above procedure may be in the second degree

[2]Other equivalent formulations of Gini's mean difference include $\int F_i(x)\ [1 - F_i(x)]dx$ (Yitzhaki 1982) and 2 Cov $[x, F_i(x)]$ (Lerman and Yitzhaki 1985).

stochastic dominance efficient set. While application of mean-Gini stochastic efficiency analysis offers the potential for a smaller efficient set of decisions than use of stochastic dominance rules, an important question remains concerning the decisions discarded by the mean-Gini approach. Buccola and Subaei (1984) have suggested that the mean-Gini efficient set may be expected to correspond to an interval of risk aversion on the lower end of the spectrum of risk preference. Their empirical results were consistent with this suggestion. The next section rigorously considers this issue.

2. Mean-Gini efficient decisions and absolute risk aversion

Existence of a univariate, twice differentiable, von Neumann-Morgenstern utility function, $U(W)$, with $U'(W) > 0$, $U''(W) < 0$, and wealth, W, expressed in monetary units is assumed. The coefficient of absolute risk aversion is $r(W) = -U''(W)/U'(W)$ (Pratt 1964; Menezes and Hanson 1970). This section provides a rigorous correspondence between mean-Gini efficient decisions and absolute risk aversion. Two propositions reveal the main results. Proposition 2 establishes that mean-Gini efficient decisions are the optimal decisions for a left-most constant absolute risk aversion (CARA) interval, $[0,K]$, and that optimal decisions for CARA decision-makers outside this interval are discarded by the mean-Gini approach. Proposition 3 shows that the optimal decisions for risk-averse decision-makers with absolute risk aversion confined to the CARA interval of Proposition 2 ($0 \leqslant r(W) \leqslant K$) are in the mean-Gini efficient set of decisions.

Proposition 2: Let $\{F_i(x)\}$ be a set of distribution functions which cross at most once, $i = 1, 2, \ldots, I$, such that $F_i(a) = 0$, $F_i(b) = 1$, and $0 \leqslant F_i(x) \leqslant 1$ for $x \in [a,b]$. If the set of distribution functions in $\{F_i(x)\}$ corresponding to mean-Gini efficient decisions is a proper subset of the second degree stochastic dominance efficient set, then the mean-Gini efficient set corresponds to rational decisions for expected utility maximizers with constant absolute risk aversion in the interval $[0,K]$.

Proof: Without loss of generality, assume that the second degree stochastic dominance efficient set includes all of the distribution functions $\{F_i(x)\}$, $i = 1, 2, \ldots, I$. By assumption, the mean-Gini efficient set is a proper subset of the second degree stochastic dominance efficient set. Hence, $M_i - M_j \geqslant \max(0, G_i - G_j)$ for at least one i, j pair; $i \neq j$. This inequality, in conjunction with an expression for M_i, $M_i = {}_a\int^b [1 - F_i(x)]dx$, implies ${}_a\int^b [F_j(x) - F_i(x)]dx \geqslant 0$. Since, by assumption, $F_i(x)$ and $F_j(x)$ are in the second degree stochastic dominance efficient set, the latter inequality implies that $F_i(x)$ and $F_j(x)$ cross at a point, say c, such that $F_j(x) \leqslant F_i(x)$ for $x \leqslant c$ and $F_j(x) \geqslant F_i(x)$ for $x \geqslant c$, $x \in [a,b]$. Hence $\int_a^c [F_j(x) - F_i(x)]dx < 0$ and ${}_c\int^b [F_j(x) - F_i(x)]dx > 0$. Note that $F_i(x)$ is preferred to $F_j(x)$ by expected utility maximizers with von Neumann-Morgenstern utility such that ${}_a\int^b U(x)f_i(x)dx > {}_a\int^b U(x)f_j(x)dx$. Integrating by parts, the latter condition becomes ${}_a\int^b [F_j(x) - F_i(x)]U'(x)dx > 0$.[3] Now assuming constant absolute risk aversion, the sign of

$$v_{ij}(r) = {}_a\int^c [F_j(x) - F_i(x)] \exp(-rx)dx + {}_c\int^b [F_j(x) - F_i(x)] \exp(-rx)dx$$

as a function of the CARA coefficient, r, indicates the preferred decision. First, note from the previous derivations that $v_{ij}(0) \geqslant 0$. Second, note that for $x \in [a,c]$ and $y \in (c,b]$, $\exp(-rx) > \exp(-ry)$; hence, marginal untility falls more slowly as a function of the CARA coefficient, r, for $x \in [a,c]$ than for $x \in (c,b]$. Consequently, large values of r can make the first term in $v_{ij}(r)$ larger in absolute value than the second term, ensuring that $v_{ij}(r) < 0$ as r grows large.

Third, note that

$$v_{ij}'(r) = {}_a\int^c [F_j(x) - F_i(x)](-x) \exp(-rx)\, dx + {}_c\int^b [F_j(x) - F_i(x)] (-x) \exp(-rx)\, dx < 0$$

whenever $v_{ij}(r) \geqslant 0$. To see this, observe that $v_{ij}(r) \geqslant 0$ implies that $v_{ij}'(r) < 0$ since $x < y$ for $x \in [a,c]$ and $y \in (c,b]$. Hence, $v_{ij}(r) = 0$ has a single root, say $K_{ij.}$ Finally, suppose that one or more distributions are eliminated from the second degree stochastic dominance efficient set through application of the mean-Gini criteria. Moreover, suppose that some or all of these distributions are eliminated by one or more of the mean-Gini efficient distributions. Then it is apparent that the mean-Gini efficient set is characterized by the intersection of the CARA efficiency intervals $[0, K_{ij}]$ identified by the procedure above. Hence, the mean-Gini efficient set corresponds to the CARA interval [0, K] where $K = \min\{K_{ij}\}$.

Proposition 3: Let $\{F_i(x)\}$ be a set of distribution functions which cross at most once, $i = 1, 2, \ldots, I$, such that $F_i(a) = 0$, $F_i(b) = 1$, and $0 \leqslant F_i(x) \leqslant 1$ for $x \in [a,b]$. Rational decisions for expected utility maximizers with absolute risk aversion in the interval [0, K] are elements of the mean-Gini efficient set.

Proof: Suppose that both $F_i(x)$ and $F_j(x)$ are in the second degree stochastic dominance efficient set and that $F_j(x)$ is not in the mean-Gini efficient set. Note that $U'(x) = \exp(-{}_a\int^x r(W)dW)$. Let $v_{ij} = {}_a\int^c [F_j(x) - F_i(x)]\exp(-{}_a\int^x r(W)dW))dx + {}_c\int^b [F_j(x) - F_i(x)]\exp(-{}_a\int^x r(W)dW)dx$. The result is proved if it can be shown that $r(W) \leqslant K$ implies that $v_{ij} \geqslant 0$

[3]A linear transformation of U(W) may be required to ensure that $U(a) = 0$ and $U'(a) = 1$.

where K is the CARA coefficient identified in Proposition 2. First note that $r(W) \leqslant K$ implies that $\exp(-{}_a\int^x r(W)dW)) \geqslant \exp\,(-{}_a\int^x KdW))$ for all $x \in [a,b]$. Hence, the integrands in both the first (non-positive) and second (non-negative) terms of v_{ij} increase relative to the case where $r(W) = K$; that is, the case with two corresponding terms

$${}_a\int^c[F_j(x) - F_i(x)]\exp(-{}_a\int^x KdW))dx + {}_c\int^b[F_j(x) - F_i(x)]\exp\,(-{}_a\int^x KdW)dx$$

which is known to be non-negative by Proposition 2. Second, observe that if $\exp(-{}_a\int^x r(W))dW))$ is larger relative to $\exp(-{}_a\int^x(KdW))$ for $x \in [c,b]$ than for $x \in [a,c]$, then this will guarantee that $v_{ij} \geqslant 0$. To see that this is indeed the case, consider the ratio

$$\exp(-{}_a\int^x r(W)dW))/\exp(-{}_a\int^x KdW)) = \exp(-{}_a\int^x(K - r(W))dW).$$

Note that $\exp\,(-{}_a\int^x(K - r(W))dW)) \leqslant \exp(-{}_a\int^y(K - r(W)dW))$ for all $x \in [a,c]$ and $y \in [c,b]$ implying that $v_{ij} \geqslant 0$. Hence, for $r(W) \in [0,K]$, $F_i(x)$ is preferred to $F_j(x)$. Since i and j were selected arbitrarily from the second degree stochastic dominance and mean-Gini efficient sets, the result follows.

With respect to Proposition 3, it is noteworthy that the proposition ensures that CARA efficiency intervals for a set of distribution functions which satisfy conditions required for mean-Gini stochastic efficiency analysis coincide with efficiency intervals developed by application of stochastic dominance with respect to a function (Meyer 1977). This is the case for the CARA efficiency intervals for the first three empirical studies reported in the next section.

3. CARA, risk premium, and mean-Gini efficient decisions

As reviewed earlier, when the cumulative distribution functions associated with alternative decisions cross at most once, application of mean-Gini stochastic efficiency analysis identifies an efficient set of decisions which is a subset of the second degree stochastic dominance efficient set. Hence, as Yitzhaki (1982) proves, the use of the mean-Gini criteria potentially reduces the second degree stochastic dominance efficient set by discarding some decisions that may be optimal for some expected utility-maximizing decision-makers. The mean-Gini approach might be regarded as an improvement over the second degree stochastic dominance approach; however, in practice, this would seem to depend on the expected utility-maximizing decision-makers' preferences which are discarded.

The specific problem statement pursued in this section is as follows: Does application of mean-Gini stochastic efficiency analysis typically discard risk preferences in a manner that seems in some sense reasonable *vis à vis* real world decision-makers? If mean-Gini stochastic efficiency analysis is to become a popular tool for agricultural decision analysis, more information is needed to shed light on this question. This section reports some findings related to this issue based on an empirical comparison using four studies from the literature. Specifically, the constant absolute risk aversion interval which corresponds to the mean-Gini efficient set is derived for choice among different yielding rice varieties in the Philippines (Roumasset 1976; Yassour, Zilberman and Rausser 1981), different pest management strategies in the southern United States (Liapis and Moffitt 1983), different United States Department of Agriculture commodity programme features (Kramer and Pope 1981), and different tillage practices in corn and soybean production in the midwestern region of the United States (Klemme 1985). The least upper bound of the CARA efficiency interval is then used to evaluate the risk premium as a percentage of gamble size in order to gain some additional insight in a visceral sense to the risk preferences included in the mean-Gini efficient set (Babcock, Choi and Feinerman 1993). The analysis of rice varieties and pest management strategies is based on alternative continuous net returns densities, while discrete densities of net returns are used for the analysis of alternative commodity programme features. Analysis of discrete densities of net returns to corn and soybean tillage practices is used to examine the mean-Gini approach when the conditions underlying the proposition shown earlier are not satisfied.[4]

The stochastic efficiency of conventional and high-yielding Philippine rice varieties was analysed by Roumasset (1976) using second degree stochastic dominance and analysed subsequently by Yassour, Zilberman and Rausser (1981) using a certainty equivalence approach. A total of 4 different rice varieties were analysed with the conventional variety denoted by T_1 and the three high-yielding varieties denoted by T_2, T_3, and T_4. Basic data reported by Roumasset (1976) include an average price of 16 per unit of yield and average yield per hectare (32, 70, 80, 90), total factor cost per hectare (106, 350, 410, 490), and standard deviation of yield per hectare (5, 25, 30, 35) for the 4 rice varieties T_1, T_2, T_3, and T_4, respectively.

Alternative distributional specifications for yield are possible and facilitate stochastic efficiency analysis of the different rice varieties. Both Roumasset (1976) and Yassour, Zilberman and Rausser (1981) specify the normal density for each rice variety yield given by

$$f_i(y) = (2\pi\sigma_i^2)^{-1/2} \exp(-(1/(2\sigma_i^2))(y - \mu_i)^2)$$

[4]Computations reported in this section were performed using *Mathematica* and custom Basic and FORTRAN routines. Details are available from the authors on request.

where μ_i and σ_i are the mean and standard deviation of yield, y, for each rice variety, T_i. The gamma density has also been suggested as an appropriate density for crop yield (Day 1965). This density is given by

$$f_i(y) = (\lambda_i^{\alpha_i}/\Gamma(\alpha_i))\, y^{\alpha_i - 1} \exp(-\lambda_i y)$$

where α_i and λ_i are parameters. This density was also used to analyse the different rice varieties by Yassour, Zilberman and Rausser (1981).

Method of moments estimates of the parameters in the normal and gamma densities are obtained by equating sample and population moments. Hence, method of moments estimates are found as μ_i = sample mean yield, σ_i = sample standard deviation of yield, $\alpha_i = (\mu_i/\sigma_i)^2$, and $\lambda_i = \mu_i/(\sigma_i)^2$.

The mean (M_i) and standard deviation (S_i) of net returns for each rice variety follow from the basic price, yield, and cost data and are shown in table 1. These net returns statistics may be used in conjunction with the

Table 1 Net returns statistics and stochastic efficiency analysis of Philippine rice varieties

Philippine rice variety	T_1	T_2	T_3	T_4
Mean (M_i)	406	770	870	950
Standard deviation (S_i)	80	400	480	560
Normal Yield Density				
One-half Gini's mean difference (G_i^N)	45.13	225.67	270.81	315.95
$M_i - G_i^N$	360.87	544.33	599.19	634.05
Second degree stochastic dominance efficient set (√)	√	√	√	√
Mean-Gini efficient set (√)	–	–	–	√
Constant absolute risk aversion efficiency interval	(.004,∞)	(.003, .004)	(.001, .003)	(0, .001)
Risk premium as a percentage of gamble size [a]	–	112.00	84.00	28.00
Gamma Yield Density				
One-half Gini's mean difference (G_i^Γ)	44.99	222.11	266.09	310.04
$M_i - G_i^\Gamma$	361.01	547.89	603.91	639.96
Second degree stochastic dominance efficient set (√)	√	√	√	√
Mean-Gini efficient set (√)	–	–	–	√
Constant absolute risk aversion efficiency interval	(.008,∞)	(.006, .008)	(.003, .006)	(0, .003)
Risk premium as a percentage of gamble size [a]	–	108.25	92.66	59.25

Notes: [a]Percentage evaluated at the maximum of the corresponding constant absolute risk aversion efficiency interval with gamble size reflected by the standard deviation of net returns.
Source: Roumasset (1976).

alternative distributional specifications for yield to facilitate stochastic efficiency analysis of the rice varieties. For example, application of second degree stochastic dominance criteria to the four varieties reveals that all four are in the second degree stochastic dominance efficient set under both normal and gamma yield densities. Hence, stochastic dominance analysis is inconclusive as an analytical tool in this case as indicated in table 1.

Application of mean-Gini stochastic efficiency analysis to the 4 varieties may be accomplished using the procedure described earlier. One-half Gini's mean difference for each variety for normal and gamma yield densities is shown in table 1 (Dorfman 1979; Kendall and Stuart 1963). Mean net returns minus one-half Gini's mean difference for each rice variety and specification of yield density is also shown in the table.

Pairwise comparison of the different rice varieties according to the mean-Gini criteria is accomplished by comparing the mean net return and the mean less one-half Gini's mean difference as shown in the columns of the table. Results indicate that the mean-Gini efficient set includes only the single rice variety denoted by T_4 for both normal and gamma yield densities. Note that this result occurs since

$$(1) \quad M_4 \geqslant M_j;\ j = 1, 2, 3$$

and

$$(2) \quad M_4 - G_4^N \geqslant M_j - G_j^N;\ M_4 - G_4^\Gamma \geqslant M_j - G_j^\Gamma;\ j = 1, 2, 3$$

with the strict inequality holding in all cases.

Comparison of certainty equivalents is an alternative approach to stochastic efficiency that can provide a complete ranking of net returns for the four rice varieties. The ranking in terms of certainty equivalents can also be used to identify the constant absolute risk aversion interval for which each rice variety is efficient. For example, the constant absolute risk aversion interval associated with the mean-Gini efficient variety (T_4) may be found by evaluating certainty equivalents employing the utility function, $U(\Pi) = -\exp(-r\Pi)$, where Π denotes net returns and r is the coefficient of constant absolute risk aversion. The coefficient of constant absolute risk aversion depends on the unit associated with the decision outcomes but not on an initial wealth level.

Table 1 summarizes the findings regarding the CARA efficiency interval of the four rice varieties for both normal and gamma yield densities. The highest yielding variety, T_4, is seen to be optimal for r in the range 0.0 to 0.001 in the case of normally distributed rice yield and optimal for r in the range 0.0 to 0.003 in the case of gamma distributed rice yield. Note that this interval is the constant absolute risk aversion analogue to the interval found by Buccola and Subaei (1984) in their study of co-operative pooling rules and the interval which may be gleaned approximately from comparison of the mean-Gini and certainty equivalence findings for groundnut genotypes reported by Bailey and Boisvert (1989). However, in view of Proposition 3 shown in the previous section, the CARA interval reported here also coincides with the absolute risk aversion interval of these previous studies (outcome units notwithstanding). From the table, the CARA efficiency interval associated with the mean-Gini efficient variety falls within the range of defensible risk aversion coefficients suggested by Babcock, Choi and Feinerman (1993) for both normally and gamma distributed yields.

The risk premium as a percentage of gamble size reported in table 1 is the ratio of the risk premium at the maximum of the CARA interval to the standard deviation of the mean-Gini efficient variety. Note that this ratio is increasing in the risk aversion coefficient. The value of this ratio (59 per cent) suggests that the mean-Gini efficient variety may represent a decision corresponding to reasonable risk preferences when yield is distributed according to the gamma density. For example, Babcock, Choi and Feinerman (1993) suggest that a ratio of 68 per cent corresponds to risk aversion which would be quite high for commercial farmers. The interpretation of the ratio of the risk premium to gamble size (28 per cent) with respect to degree of risk aversion is more questionable in the case of normally distributed yield. While many reasonable risk preferences are undoubtedly included in the corresponding risk aversion interval, the exclusion of reasonable risk preferences cannot be ruled out in this case.[5]

The stochastic efficiency of different pest management strategies for cotton pests in the southern United States was analysed by Liapis and Moffitt (1983) using a certainty equivalence approach. Four different strategies were analysed including a biological control strategy referred to as Trichogramma Releases in table 2 and a co-operative, area-wide strategy referred to as Community Management in table 2. Two other strategies which avoid use of pest controls were also analysed as indicated in the table. Basic data used by Liapis and Moffitt (1983) include an average price of $0.63 per unit of yield and average yield per acre (665, 604, 625, 516), pest control cost per acre (137, 29, 15, 14), and standard deviation of yield per acre (105, 205, 251, 192) for the four strategies Trichogramma Releases, Community Management, Untreated Community Management, and Untreated Outside Community, respectively (table 2).

Similar distributional and computational procedures to those indicated

[5]None of the studies considered here involve two-state gambles. The computation and use of the risk premium as a percentage of gamble size parallel that of Babcock, Choi and Feinerman (1993) in their investigation of selected studies.

Table 2 Net returns statistics and stochastic efficiency analysis of cotton pest management strategies

Cotton Pest Management Strategy	Trichogramma Releases	Community Management	Untreated Community Management	Untreated Outside Community
Mean (M_i)	278.94	348.31	376.42	309.27
Standard Deviation (S_i)	65.85	128.63	157.08	120.13
Normal yield density				
One-half Gini's mean difference (G_i^N)	46.56	90.96	111.06	84.94
$M_i - G_i^N$	232.38	257.35	265.36	224.33
Second degree stochastic dominance efficient set (√)	√	√	√	√
Mean-Gini efficient set (√)	–	–	√	–
Constant absolute risk aversion efficiency interval	$(.01, \infty)$	(.006, .01)	(0, .006)	–
Risk premium as a percentage of gamble size [a]	–	78.33	47.14	–
Gamma Yield Density				
One-half Gini's mean difference (G_i^r)	37.18	71.53	86.85	66.61
M_i - G_i	241.76	276.78	289.57	242.66
Second degree stochastic dominance efficient set (√)	√	√	√	√
Mean-Gini efficient set (√)	–	–	√	–
Constant absolute risk aversion efficiency interval	(.02, .3)	(.01, .02)	(0, .01)	$(.3, \infty)$
Risk premium as a percentage of gamble size [a]	209.72	87.96	55.97	–

Note: [a]Percentage evaluated at the maximum of the constant absolute risk aversion efficiency interval with gamble size reflected by the standard deviation of net returns.
Source: Liapis and Moffitt (1983).

for the case of Philippine rice varieties are used in a stochastic efficiency analysis of the four pest management strategies. Application of second degree stochastic dominance criteria reveals that all four strategies are in the second degree stochastic dominance efficient set under both normal and gamma yield densities. Hence, stochastic dominance analysis is again inconclusive as an analytical tool in this case as indicated in table 2.

Application of mean-Gini stochastic efficiency analysis to the four strategies is accomplished using the procedure described earlier. One-half Gini's mean difference for each strategy for normal and gamma yield densities is shown in table 2. Mean net returns minus one-half Gini's mean difference for each strategy and specification of yield density is also shown in the table. Pairwise comparison of the different strategies according to the mean-Gini criteria is accomplished by comparing the mean net return and the mean less one-half Gini's mean difference as shown in the columns of the table. Results indicate that the mean-Gini efficient set includes only the single strategy Untreated Community Management for both normal and gamma yield densities as indicated in table 2.

Table 2 summarizes the findings regarding the CARA efficiency interval of the four pest management strategies for both normal and gamma yield densities. The mean-Gini efficient strategy was found to be efficient for a constant absolute risk aversion interval which many might regard as reasonable for both normal and gamma distributed yields. The constant absolute risk aversion interval shown in table 2 for the mean-Gini efficient strategy is (0, .006) in the case of normally distributed yield and (0, .01) in the case of gamma distributed yield. Hence, the CARA efficiency interval associated with the mean-Gini efficient variety again falls within the range of defensible risk aversion coefficients suggested by Babcock, Choi and Feinerman (1993). The probable reasonableness of the degree of risk aversion implied by the CARA efficiency intervals is suggested by the magnitude of the risk premium as a percentage of gamble size associated with these intervals also reported in table 2. The latter are 47 per cent for normally distributed yield and 56 per cent for gamma distributed yield. It seems likely that many decision-makers would fall within the degree of risk aversion implied in this case.

Table 3 shows summary statistics and the results of a stochastic efficiency analysis of net returns reported by Kramer and Pope (1981) for alternative United States Department of Agriculture commodity programmes. In this case, the discrete densities of net returns are reported in detail by Kramer and Pope (1981) along with a description of the programme alternatives (table 3). Examination of the probability densities of net returns verifies that the conditions required for application of the proposition presented earlier are present for the six commodity programme alternatives considered. As indicated in table 3, stochastic efficiency analysis shows that the second degree stochastic dominance and mean-Gini efficient sets coincide and lead to a single efficient alternative (Target Prices Raised by 10 per cent). Hence, the mean-Gini efficient set is an improper subset of the second degree stochastic dominance efficient set and corresponds to an identical constant absolute risk aversion efficiency interval. Application of the mean-Gini criteria generates an efficient decision which certainly includes reasonable risk preferences in this case.

Table 4 shows summary statistics and stochastic efficiency for net returns reported by Klemme (1985) in an analysis of different tillage practices in corn and soybean production in the midwestern region of the United States. The tillage practices compared include Conventional Tillage, Chisel Plow Tillage, Till-Plant Tillage, and No-Till as indicated in table 4. In this

Table 3 Net returns statistics and stochastic efficiency analysis of commodity programme alternatives

Commodity Program Alternative	Non-participation	1979 Program	Set-asides Cut by ½	Target Prices Raised by 10%	Loan Rates Raised by 10%	Allocation Factor Raised by 10%
Mean (M_i)	51 699	50 846	53 455	54 905	50 023	51 279
Standard deviation (S_i)	26 019	20 808	21 596	18 926	21 464	20 486
One-half Gini's mean difference (G_i)	14 772	11 807	12 241	10 743	12 182	11 629
$M_i - G_i$	36 927	39 039	41 214	44 162	37 841	39 650
Second degree stochastic dominance efficient set ($\surd$)	–	–	–	$\surd$	–	–
Mean-Gini efficient set ($\surd$)	–	–	–	$\surd$	–	–
Constant absolute risk aversion interval for alternative efficiency	–	–	–	$(0,\infty)$	–	–
Risk premium as a percentage of gamble size	–	–	–	–	–	–

Source: Kramer and Pope (1981).

case, the conditions required for application of the proposition shown earlier are not satisfied by the probability distributions of net returns. The latter are reported in detail along with a description of the alternative tillage practices by Klemme (1985).

In the case of corn tillage practices, application of second degree stochastic dominance criteria reveals that two strategies, Conventional Tillage and Till-Plant Tillage, are in the second degree stochastic dominance efficient set. Though two of the four practices are eliminated, stochastic dominance analysis is again inconclusive as an analytical tool in this case as indicated in table 4.

Application of mean-Gini stochastic efficiency analysis to the four corn tillage practices is accomplished using the procedure described earlier.

Table 4 Net returns statistics and stochastic efficiency analysis of alternative tillage practices in corn and soybeans

Tillage Practice	Conventional Tillage	Chisel Plough Tillage	Till-Plant Tillage	No-Till
Corn				
Mean (M_i)	231.79	229.58	232.21	219.22
Standard deviation (S_i)	60.60	61.12	58.25	54.07
One-half Gini's mean difference (G_i)	32.58	33.80	30.88	29.58
$M_i - G_i$	199.21	195.78	201.33	189.64
Second degree stochastic dominance efficient set ($\surd$)	$\surd$	–	$\surd$	–
Mean-Gini efficient set ($\surd$)	–	–	$\surd$	–
Constant absolute risk aversion efficiency interval	$(.58, \infty)$	–	$(0, .58)$	–
Risk premium as a percentage of gamble size[a]	–	–	98.95	–
Soybeans				
Mean (M_i)	234.54	232.19	222.51	220.53
Standard deviation (S_i)	43.33	29.61	39.82	43.83
One-half Gini's mean difference (G_i)	23.12	16.72	21.15	24.08
$M_i - G_i$	211.42	215.47	201.36	196.45
Second degree stochastic dominance efficient set ($\surd$)	$\surd$	–	$\surd$	–
Mean-Gini Efficient Set ($\surd$)	$\surd$	–	$\surd$	–
Constant absolute risk aversion efficiency interval	$(0, .04)$	–	$(.04, \infty)$	–
Risk premium as a percentage of gamble size [a]	96.75	–	–	–

Note: [a]Percentage evaluated at the maximum of the constant absolute risk aversion efficiency interval with gamble size reflected by the standard deviation of net returns.
Source: Klemme (1985).

Relevant net returns statistics are shown in table 4. Results indicate that the mean-Gini efficient set includes only the single practice referred to as Till-Plant Tillage in table 4.

Though the conditions which guarantee applicability of the mean-Gini criteria are not present, the risk aversion interval corresponding to the mean-Gini efficient set may again be regarded by many as an appealing one. In the case of corn, for which the mean-Gini efficient set is a proper subset of the second degree stochastic dominance efficient set, the constant absolute risk aversion efficiency interval is (0, .58) for the mean-Gini efficient tillage practice (table 4). The CARA efficiency interval associated with the mean-Gini efficient practice includes the range of defensible risk aversion coefficients suggested by Babcock, Choi and Feinerman (1993). The probable reasonableness of the degree of risk aversion implied by the CARA efficiency interval is suggested by the magnitude of the risk premium as a percentage of gamble size associated with the interval (99 per cent) also reported in table 4. It seems likely that many decision-makers would fall within the degree of risk aversion implied in this case.

In the case of soybean tillage practices, stochastic efficiency analysis shows that the second degree stochastic dominance and mean-Gini efficient sets coincide and contain two efficient tillage practices (Conventional Tillage and Till-Plant Tillage). The mean-Gini efficient set is an improper subset of the second degree stochastic dominance efficient set and corresponds to an identical constant absolute risk aversion efficiency interval. Application of the mean-Gini criteria generates an efficient decision which certainly includes reasonable risk preferences in this case.

4. Concluding remarks

When comparing decisions with uncertain outcomes, agricultural decision analysts may choose from among a number of stochastic efficiency criteria to assist them in their task. However, the criteria employed should be chosen with care if the analysts aspire to focus their recommendations and/or predictions on decisions with real-world relevance. Mean-Gini stochastic efficiency analysis may be of particular interest to analysts in this respect. For example, mean-Gini stochastic efficiency analysis may be preferred to second degree stochastic dominance analysis to the extent that a result of the latter's generality is a failure to provide a focus on decisions with practical significance. In such cases, the insurance against 'Type I' error afforded by the stochastic dominance approach works against the analyst who is concerned primarily with identifying decisions which are important to most, though perhaps not all, decision-makers. Mean-Gini stochastic efficiency analysis may also be preferred to stochastic efficiency criteria such as those based on comparison of certainty equivalents and stochastic dominance with respect to a function. The latter criteria are known to give results which depend on a particular utility function and/or are sensitive to the outcome units of a particular decision problem. Consequently, stochastic efficiency findings must be interpreted through the results of perhaps unrelated risk preference analyses, through problem-specific elicitation or other assessments of risk preference, or through perhaps arbitrary bound setting. In contrast, mean-Gini stochastic efficiency analysis is a self-contained approach which is independent of outcome units. Such an approach would seem to be preferable to many analysts provided, of course, that the imperfect results achieved turn out to be satisfactory for practical purposes. This study has attempted to shed light on this issue through both theoretical analysis and empirical example.

Theoretical findings presented earlier relate mean-Gini efficient decisions to an interval of constant absolute risk aversion bounded below by risk neutrality. This finding rigorously establishes the relationship suggested by at least one previous study. In addition, a relationship between stochastic dominance with respect to a function and the certainty equivalence approach with constant absolute risk aversion was derived; in particular, coincidence of the corresponding efficiency intervals for decisions with uncertain outcomes which satisfy conditions needed for application of mean-Gini stochastic efficiency analysis was demonstrated.

Application of mean-Gini stochastic efficiency analysis to four studies from the literature provided what many analysts might regard as attractive results relative to the second degree stochastic dominance and certainty equivalence approaches. Mean-Gini stochastic efficiency analysis identified efficient decisions which many decision-makers might consider reasonable. The approach discounted the optimal decisions of those decision-makers which many might regard as exhibiting unreasonable levels of risk aversion in their behaviour with only one potential, though obviously worrisome, exception. These results are essentially consistent with the conclusions drawn by Buccola and Subaei (1984) and Bailey and Boisvert (1989) and, when viewed with their studies, provide additional evidence of the power and practicality of the mean-Gini approach as an applicable tool for stochastic efficiency analysis.

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