



The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

Papers downloaded from AgEcon Search may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

Does ignoring multideestination trips in the travel cost method cause a systematic bias?^{*}

Timo Kuosmanen, Eleonora Nillesen and
Justus Wesseler[†]

The present paper demonstrates that treating multideestination trips (MDT) as single-destination trips does not involve any systematic upward or downward bias in consumer surplus (CS) estimates because the direct negative effect of a price increase (treating MDT as a single-destination trip) is offset by a shift in the estimated demand curve. Still, ignoring MDT can greatly underestimate or overestimate the CS. In addition, we demonstrate that there is a sound theoretical basis for using preference information for allocating travel costs between different sites included in the MDT package. A novel extreme value approach is proposed, which does not require any overly restrictive assumptions about consumer preferences. This approach is applied to the zonal travel cost model of the Bellenden Ker National Park, Australia. Parametric and non-parametric estimation techniques are used for calculating CS estimates, and the effects of different MDT treatments and estimation methods are compared.

1. Introduction

The problem of how to handle multideestination trips (MDT) is as old as the travel cost method (TCM) itself (e.g., Ward and Beal 2000). A standard assumption of the TCM is that observed visits are single-destination trips (SDT). A problem occurs when a significant proportion of individuals visit other destinations on the way to the recreation site, near the site, or on the way back home.

The most common approaches to dealing with the MDT in TCM can be classified in three categories:

* We are grateful to three anonymous reviewers for helpful comments. The first author acknowledges financial support from the Emil Aaltonen Foundation (Nonparametric Methods in Economics of Production, Natural Resources, and the Environment (NOMEPRE) project).

† Timo Kuosmanen, Eleonora Nillesen and Justus Wesseler, Environmental Economics and Natural Resources Group, Department of Social Sciences, Wageningen University, Wageningen, The Netherlands.

1. Ignoring MDT, by either excluding MDT visitors from the sample or by treating MDT visitors as if they were single-destination visitors.
2. Correcting for MDT bias by using a proportion of the total cost attributable to the evaluated site as a proxy for the price of the trip.
3. Modelling MDT and SDT as different commodities.

Although ignoring MDT is likely to result in a biased estimate of the demand function and, hence, affect the estimate of consumer surplus (CS), the first approach remains a standard practice in applied research. Of course, this is typically the simplest approach, and the MDT bias is likely to be minimal if there are few MDT visitors. Even if there are many MDT visitors, some authors argue that it is better to simply ignore the possible MDT bias because any attempt to correct for that bias would be more or less arbitrary (e.g., Beal 1995).

The second approach of transforming MDT to be comparable with SDT is also often used in empirical research. The main challenge of this approach is to attribute the correct proportion of the total travel costs to the site under evaluation. Several approaches have been suggested. One is to use a quantifiable variable, such as 'nights spent' at the different sites, as a proxy for relative importance (Knapman and Stanley 1991; Stoeckl 1993). Another is to try to use visitors' preferences to allocate the cost (Hanley and Ruffel 1992). Bennett (1995) notes that, although the second approach is much more subjective, it does recognise the possibility that the importance of visits may not be simply a function of time allocation. However, evidence from experimental studies clearly illustrates the difficulty of expressing preferences in measurable quantities (e.g., Hajkowicz *et al.* 2000).

The third approach has been suggested by Hotelling (see Ward and Beal 2000, pp. 217–218) and has been rigorously developed by Mendelsohn *et al.* (1992). This approach includes all alternative sites, and combinations thereof, in the estimation of the demand function, to take into account any substitution possibilities. As Mendelsohn *et al.* demonstrate, this approach is applicable in practice if a few main substitutes (or complements) for the evaluated site are visited by most MDT visitors. However, the number of demand equations rises exponentially as the number of MDT sites increases. Estimating demand functions for each MDT package requires a sufficiently large number of observations on each package tour. Ideally, the approach would require data on those individuals who visited alternative sites, but not the site under evaluation. These extensive data requirements explain why the theoretically appealing third approach is not widely used in practice.

Focusing on approaches (1) and (2) above, the objective of the present paper is to shed further light on how methods for dealing with MDT can

influence the CS estimates obtained by the TCM.¹ We resolve the question of whether treating MDT observations in the same way as SDT observations will systematically overestimate (Haspel and Johnson 1982) or underestimate (Loomis *et al.* 2000) the CS. To this end, three distinct routes will be pursued. First, we develop some analytical insights into this issue by decomposing the MDT effect into two measurable, partly offsetting components: the direct effect of a price change, and the indirect effect of a shift in the empirical demand function. Second, we demonstrate that using ordinal rankings of alternative MDT sites as a basis for extracting cardinal cost-shares required by the TCM has a strong theoretical foundation in duality theory. To convert ordinal preference rankings to meaningful cardinal cost shares, the extreme value approach initially proposed by Kmietowicz and Pearman (1981) is adapted to the present TCM context. Third, we apply the extreme value approach and present some empirical evidence of the influence of the MDT on the TCM CS estimates. Using data from the recent TCM study for the Bellenden Ker National Park in Australia reported by Nillesen *et al.* (2003), we estimate the theoretical minimum and maximum bounds for the TCM, both with and without an MDT-correction, using a parametric weighted-ordinary-least-squares and a non-parametric trapezoid-rule approach for estimating the demand functions. These MDT-corrected minimum and maximum bounds allow us to analyse the effect of ignoring MDT, without imposing strong assumptions about consumer preferences.

The rest of the paper unfolds as follows: Section 2 analytically decomposes the effect of MDT on CS, and concludes that considering MDT will not necessarily increase the CS. This is followed by an introduction of the modified extreme value approach for handling MDT in Section 3. Section 4 applies this approach to a case study from Australia, where approximately half of the visitors of a National Park visited the park as part of an MDT. Section 5 closes with a discussion of our results and draws conclusions for the TCM.

2. Consumer surplus

Individuals are assumed to have a well-behaved utility function, $U = U(y)$, where vector $y = (y_1, \dots, y_n)$ summarises consumed quantities of n commodities marketed. Let the recreational activity of interest be commodity i . Note that prices of $n - 1$ substitute/complement goods, including alternative recreational activities, influence demand for commodity i . In our framework,

¹ It is known that approach (3) yields higher consumer surplus (CS) estimates than excluding MDT visitors from the sample under approach (1), as Mendelsohn *et al.* (1992) have demonstrated. Therefore, we focus on approaches (1) and (2).

MDT are modelled as commodity baskets that include a positive quantity of two or more goods. Prices of commodities included in the MDT package need not be additive and, hence, MDT may introduce nonlinearities and path-dependencies in the budget line, but these do not affect the utility function.

Consumer surplus is a measure of consumer welfare: it is the sum of money consumers were willing to pay for a particular product over what they actually had to pay. Formally, let $x_i : \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+$ denote the Marshallian demand for commodity i as a function of price vector, $p : \mathbb{R}_+^n$, and income, I . Let \bar{p}_i denote the prevailing price for commodity i . The consumer surplus (CS) of commodity i can be written as:

$$CS(i) = \int_{\bar{p}_i}^{\infty} x_i(p_1, \dots, p_i, \dots, p_n, I) dp_i. \quad (1)$$

In other words, CS is the area under the Marshallian demand curve above the current price level.²

The purpose of the TCM approach is to estimate the demand function, x_i , for the recreational site. The purpose of the present paper is to investigate how dealing with MDT influences the estimated CS. The essential problem is to establish a link between the consumption decision and the price of the commodity perceived by the individual at the moment when the consumption decision is made. Clearly, for rational and informed SDT visitors, the perceived price is the total net travel cost plus the possible entrance fees. For MDT visitors, however, the price of visiting one of the sites in the MDT package can be extremely difficult to determine because the total cost of the MDT package often includes sunk costs and joint, non-separable costs related to the whole package (e.g., insurance costs). Nevertheless, the consumer must have some subjective perception of the price of visiting the site, which may be anything between zero and the total cost of the MDT, in order to make a rational and informed decision. In theory, if these subjective prices were known for all individuals visiting the evaluated site, the standard TCM using subjective prices rather than total costs would yield consistent estimates of CS. Of course, estimating these subjective prices empirically involves many practical complications, which will be addressed in more detail in the next section.

Suppose for the moment that the true subjective prices are known for certain and, hence, the standard TCM provides a consistent estimate for CS.

² For convenience, the definition above described CS in terms of Marshallian (market) demand curves rather than Hicksian (compensated) demand curves. Marshallian curves are easier to estimate empirically, and they can reasonably approximate the Hicksian demand curves for small price changes and for goods with few substitutes and compliments.

This ‘ideal’ situation forms a good starting point for assessing the common approach of ignoring MDT, either by: (i) excluding MDT visitors from the sample; or (ii) treating MDT visitors as if they were single-destination visitors.

The first case does not necessarily involve any systematic error, provided that data availability does not introduce problems. Omitting the multideestination visitors may have an undesirable side-effect of decreased sample size, but that is a statistical matter which might be taken into account in the design of the study. A more serious difficulty is that the profiles of the single purpose respondents and omitted multideestination visitors might differ. For example, the SDT visitors tend to live closer to the nature reserve than the MDT visitors. In such circumstances, the omission of long-distance MDT travellers might leave some important influences of demographic variables undetected because of little variation in the sample. This can also influence the shape of the estimated demand curve and, hence, the estimated total CS. The existing empirical evidence (e.g., Loomis *et al.* 2000) unanimously suggests that the omission of MDT visitors from the data set leads to an underestimation of the total CS, which can amount to 50 per cent or higher.

Together, the effects of decreased sample size and the respondent profile may become an issue in zonal models, where it may be difficult to find enough SDT visitors from distant zones. Therefore, the omission of multideestination visitors seems a more viable strategy in individual traveller models where plenty of data are available. As the zonal approach is more frequently applied, we next focus on the second approach, which treats MDT visitors as if they were SDT visitors.

In this case, the treatment of MDT influences the CS in two mutually offsetting ways. We call them the direct effect and the indirect effect. When the total travel cost is used instead of the effective (correct) cost share, the price of the commodity increases. This has the direct effect of decreasing the CS: Taking the subdifferential of the CS, we see:

$$\frac{\partial CS(i)}{\partial \tilde{p}_i} = -x_i(p_1, \dots, \tilde{p}_i, \dots, p_n, I) \leq 0 \quad \forall p_1, \dots, p_n, I. \quad (2)$$

Consequently, if total travel costs are used without correcting for the MDT, the prevailing MDT price, \tilde{p}_i^M , will increase to \tilde{p}_i^O ($\tilde{p}_i^O > \tilde{p}_i^M$) and, hence, the CS will be smaller. In other words, the direct effect is always non-positive. We can quantify this direct effect as

$$DE = \int_{\tilde{p}_i^M}^{\tilde{p}_i^O} x_i(p_1, \dots, p_n, I) dp_i. \quad (3)$$

However, the previous effect does not take into account the indirect effect of the MDT to the estimated demand function, x_i . Let \hat{x}_i^M and \hat{x}_i^O denote the demand functions estimated from data adjusted for MDT and omitting adjustment, respectively. Since the MDT adjustment always results as a lower price than omission of MDT (i.e., $\bar{p}_i^O > \bar{p}_i^M$), any sensible estimation technique will yield coefficients with the property that:

$$\hat{x}_i^M(p_1, \dots, p_n, I) \leq \hat{x}_i^O(p_1, \dots, p_n, I) \quad \forall p_1, \dots, p_n, I. \quad (4)$$

That is, for any given prices and income, the estimated demand will be higher if the total cost of travel is assigned to the nature reserve, compared to the case where only the effective fraction of the costs is used. This is because we have the same demand observations in the data, but the price observations are higher in the former case. Typically, we would expect the slope of the demand curve to be flatter when the travel costs are adjusted for MDT, because a higher proportion of high-cost, long-distance trips are MDT.

The explicit analytical representation of the latter effect depends, among other things, on the estimation technique to be used, the specified function form (if applicable), and the specified error distribution. Given the empirical demand functions \hat{x}_i^M and \hat{x}_i^O , we can quantify the indirect effect as:

$$IE = \int_{\bar{p}_i^{TC}}^{\infty} (\hat{x}_i^O(p_1, \dots, p_n, I) - \hat{x}_i^M(p_1, \dots, p_n, I)) dp_i. \quad (5)$$

Figure 1 illustrates these effects graphically. The direct effect (*DE*) of accounting for multideestination trips is a result of the price increase from p^M to p^O , given the original demand curve, \hat{x}_i^M . The indirect effect (*IE*) is the shift of the demand curve from \hat{x}_i^M to \hat{x}_i^O , given the new price, p^O . Whether accounting for MDT makes a difference, and by how much, depends on the relative difference of the areas *DE* and *IE* in figure 1. This is solely an empirical question, which probably depends on the proportion of MDT in the sample. We address this issue from an empirical perspective in Section 4.

3. Weighting multideestination trips using ordinal preference rankings

The previous section revealed that ignoring MDT may or may not result in biased estimates of the CS. The main challenge in estimating the possible MDT bias is to separate the subjective price of the evaluated site from the total cost of the MDT package. This section proposes a new approach that combines observed behaviour in the form of travel costs with stated

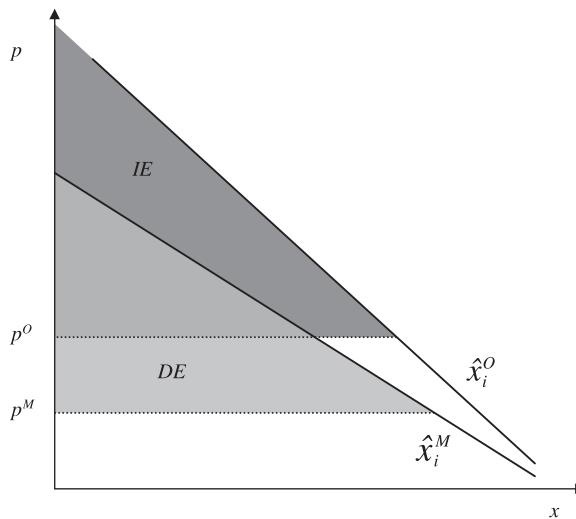


Figure 1 Illustration of the direct and the indirect effects.

preference information. Instead of trying to derive a sharp point estimate for the subjective price, we resort to an interval estimate and infer both the minimum and the maximum bound for the subjective price.

Since the consumption decision is conditional upon the individual's subjective perception of the price, it is natural to resort to survey methods. However, in contrast to most stated-preference approaches, such as contingent valuation, our approach does not require respondents to state anything other than purely ordinal information about the sites visited. That is, the respondents are asked to rank the multiple destinations that they have visited in ascending order according to their satisfaction with the park. According to Hajkowicz *et al.* (2000), who evaluated five weighting methods applied to natural resource management based on ease of use, and how much they helped clarify the problem, ordinal ranking is preferred to fixed point scoring, rating, geographical weighting and paired comparisons. Their results showed that decision-makers felt most uncomfortable when applying fixed point scoring where a fixed number of points is distributed among the criteria, as is occasionally used within TCM (e.g., Willis and Garrod 1991; Hanley and Ruffell 1992).

Given ordinal preference information on multiple destinations, TCM necessitates a conversion of ordinal preference rankings into cardinal cost-shares. First, the fundamental duality relationship between utility and expenditure functions implies that the ordinal preference rankings have a one-on-one correspondence with ordinal expenditure rankings. Recall the standard first-order optimality condition of the consumer's utility maximisation problem:

$$\frac{U'_j(y)}{U'_k(y)} = \frac{p_j}{p_k}. \quad (6)$$

That is, the utility maximising consumer keeps consuming commodities j and k until the marginal utilities of the commodities conform with the price ratio. Therefore, if the consumer's choice is optimal in the sense of the standard neoclassical microeconomic theory, as we assume, then an ordinal preference ranking implies a unique ordinal ranking in terms of expenditures (i.e., travel costs).³

The second challenge is to recover the cardinal cost shares underlying the ordinal cost rankings. To be on the safe side, we resort to the extreme value approach of Kmietowicz and Pearman (1981).⁴ We simply make calculations for two different scenarios: one using the minimum cost shares for all respondents to derive the lower-bound estimates; another involving the maximum cost shares to derive the upper-bound estimate. If the lower bound does not considerably differ from the upper bound, we can be assured that the estimated CS is robust with respect to the treatment of the MDT. Even if there is considerable deviation, we can use the lower and/or the upper bound for making safe and sound policy inference.

Assume that the respondent has visited n destinations, and can rank the destinations in non-increasing ordering according to their importance; that is, from the most important to the least important. Let the unknown travel-cost shares of each destination be denoted by vector, $\gamma = (\gamma_1 \dots \gamma_n) \in \mathfrak{R}_+^n$, satisfying the following properties: (1) $\gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_n$; and (2) $\sum_{i=1}^n \gamma_i = 1$.

Let the ranking position of the destination we are interested in be $j \in \mathfrak{R}_+$: $1 \leq j \leq n$, and the cost share of this destination γ_j . We would like to know the value of γ_j to calculate the effective travel cost to the destination for this particular visitor, but given the ordinal information only, we cannot infer the exact value. However, we can derive the minimum and the maximum value of γ_j , such that all cost shares satisfy conditions (1) and (2). It is straightforward to show (see Kmietowicz and Pearman 1981) that:

$$\min_{\gamma} \left\{ \gamma_j \mid \gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_n, \sum_{i=1}^n \gamma_i = 1 \right\} = \begin{cases} \frac{1}{n} & \text{for } j = 1 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

³ A possible practical problem is that the ordinal ranking of destinations after the trip might be based on the trip satisfaction, while the actual pre-trip consumption decisions that we are primarily interested in may have been based on a different ordinal ranking perceived prior to trip. This problem can be dealt with by a careful design of the survey.

⁴ Alternative techniques are available for this conversion. For example, Nillesen *et al.* (2003) apply the mean-expected value approach (Rietveld 1989) for that purpose.

and

$$\max_{\gamma} \left\{ \gamma_j \mid \gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_n, \sum_{i=1}^n \gamma_i = 1 \right\} = \frac{1}{j}. \quad (8)$$

These minimum and maximum values characterise the feasible range of the unknown cost share, γ_j . For example, the cost share of a site that ranks first among five sites would be somewhere between 20 and 100 per cent, while the cost share of a site that was ranked third would be somewhere between zero and 33 per cent.

From a methodological perspective, it is interesting to note that the extreme value approach is well in line with the traditional Hotelling approach that simply excludes multideestination visitors from the sample. Observe that the minimum weight equals zero, except for the topmost ranking destination. Therefore, if our nature reserve does not typically rank as the primary destination, then the traditional approach comes very close (or even coincides) with our lower-bound estimate. In this sense, the safe-play extreme value approach is built in to the traditional travel cost method. Still, our more systematic approach that uses additional ordinal information can improve even the lower-bound estimate by assigning a strictly positive weight whenever the respondent ranks our destination j as his/her primary destination.

4. Empirical application

4.1 Data

The extreme value approach is next applied to the zonal TCM model of the Bellenden Ker National Park in Australia reported by Nillesen *et al.* (2003). The national park is part of the Wet Tropics of Queensland World Heritage Area (WTWHA). The WTWHA extends along the upper north-east coast of Australia, measuring approximately 894 000 ha, and is highly important for the conservation of Australia's biological diversity and the local tourism economy (Driml 1996). The Bellenden Ker National Park includes, amongst several short tracks, two overnight walking tracks: the Goldfield track and Mount Bartle Frere track. The park is situated approximately 60 km from Cairns, which is the nearest city. The park is an area of mostly undeveloped tropical rainforest.

Our data set is based on the camping permit records of the Queensland Parks and Wildlife Service (QPWS) for those visitors that stay overnight. Australian visitors who camped in the park during the period 1995–2001 inclusive were chosen as the relevant population, for whom 1135 permits

were issued between 1995 and 2001. Permit holders were grouped according to their postal address into 18 mutually exclusive, geographically meaningful zones. For each zone, visitation rates were calculated as the ratio of permit holders to the population of the zone.

To obtain more detailed information about travel costs, modes of transportation, MDT combinations, and socioeconomic background of the visitors, a postal survey was conducted among the permit holders. A total of 482 questionnaires were posted in the first week of October 2001, of which 96 were returned unopened as a result of an invalid address. A total of 142 responses were received, representing a high response rate of 36.8 per cent for this kind of mail-survey (Brouwer and Slangen 1998). Therefore, we expect the profiles of the respondents to be well representative of the entire population.

Respondents were asked to state their modes of transportation from their home to the starting point of the walking track. The zonal average travel costs were estimated based on these responses using the route planner of the Royal Automobile Club of Queensland (see Nillesen *et al.* 2003 for further details). Following Bojö (1985), the opportunity cost of time spent for travel and on-site was assumed to be negligible because the majority of respondents reported experiencing positive utility from travel.

In the case of single destination visitors, total travel costs were used throughout the study. The proportion of MDT respondents was as high as 48 per cent in this application, which implies the treatment of MDT visitors is likely to have an impact on the CS estimate. For those who had combined trips, a proportion of the total travel costs attributable to Bellenden Ker National Park was also calculated using the extreme value approach discussed in the previous section. This leaves us with three travel cost estimates: (i) an MDT-corrected minimum bound estimate ($p_j^{M\min}$); (ii) an MDT-corrected maximum bound estimate ($p_j^{M\max}$); and (iii) an MDT-omitted (total cost) estimate (p_j^0).

Apart from the travel cost variable, socioeconomic variables were considered since demand for visits to a national park could be influenced by these variables. These variables included indicators for age, education and income (see Nillesen *et al.* 2003 for details). Prices of other recreational activities were considered, but were excluded from the model in the preliminary correlation analysis. The reasons were either minimal variation across zones or high correlation with the price of the present site. For most common recreational alternatives, such as sports, movies and gardening, prices are almost identical across zones and, hence, cannot explain variations in visitation rates. Prices can also be expected to be at the same level for more rare activities involving a high entrance fee (e.g., rock concerts) or a long travel distance (overseas destinations), as the differences in regional travel costs are a

negligible proportion of the total expenditure of the visit. However, the geographical location of the Bellenden Ker National Park along the coast of north-east Australia means that almost all multideestination visitors approach the park from the south-west. The area includes many other spectacular destinations, such as the Great Barrier Reef, which were frequently visited by long-distance travellers. However, the travel costs to these neighbouring destinations are virtually identical to the travel costs to the present site, and, hence, provide little additional explanatory power. To avoid multicollinearity, travel costs to sites in the same area were excluded.

4.2 Methods

Given the data set, the first step of the empirical analysis was to estimate the demand function. The traditional estimation technique is ordinary least squares (OLS). Unfortunately, economic theory does not provide many guidelines for the *ex ante* specification of the functional form for the recreational demand of, for example, a national park; besides the fact that the demand curve should be downward sloping since recreational trips are clearly a normal good. Therefore, we did not restrict to a single ideal model, but experimented with multiple approaches. We applied the OLS technique to various different functional forms, but also used a non-parametric trapezoid-rule estimation approach (Cooper 2000), which does not impose any functional form at all.

The advantages of the non-parametric technique are its theoretical consistency with the demand theory, and avoidance of strong functional form assumptions, which makes it very robust to specification errors. On the downside, non-parametric estimators require a large sample size, and they are generally sensitive to sampling errors and data perturbations. We achieved a reasonable sample size of 142 observations by ignoring the zonal structure imposed in Nillesen *et al.* (2003). Moreover, we investigated the exposure to sampling errors by bootstrapping. Nevertheless, the problem of possible data perturbations still remains. We find the demand quantities highly reliable, but for some visitors, the actual travel costs can differ from the travel costs Nillesen *et al.* (2003) had estimated for each zone. The OLS error term may be better able to accommodate possible data errors. The main problem with OLS is the sensitivity of the CS estimates to the (ad hoc) specification of the functional form.

In conclusion, we have no preference for one estimation approach over another, so we report both the OLS and non-parametric estimates. This also serves to validate our results: in principle, if two very different approaches yield similar results, we can be more confident that the estimates are not completely unreasonable.

4.3 Regression analysis

The general form of the regression model is best summarised by the relation F :

$$VI_j = F(p_j^r, A_j, E_j, I_j) \quad (9)$$

where:

- VI_j = visitation rate for the years 1995–2001 per zone j
- = number of visitors from zone j (VIS_j)/the population of zone j (POP_j)),
- p_j^k = travel cost per zone j in orientation r : $r \in M_{\min}, M_{\max}, O$,
- A_j = average age per zone j ,
- E_j = average education level per zone j , and
- I_j = average household income adjusted for real income changes per zone j .

Six different functional forms for F were tested (see Appendix A), initially with all variables included. However, all socioeconomic variables appeared to be highly insignificant. Therefore, a redundant variable test was performed to test whether all socioeconomic variables have zero coefficients; and might, therefore, be deleted from the analysis. From the test, it could be concluded that age, household income and education do not seem to explain the variation of VI about its mean, so it was decided to proceed with a model that only includes travel cost as an independent variable. The reduced model was:

$$VI_j = F(p_j^r). \quad (10)$$

Systematic testing revealed that function F is best approximated by a reciprocal functional form (see Appendix A). Hence, the chosen regression equation is

$$VI_j = a + \frac{b}{p_j^r} + \varepsilon_j. \quad (11)$$

In the zonal TCM, demand is usually measured in relative terms by the visitation rate (VI), as in our regression equation (11). Indeed, it seems reasonable to assume that the travel cost (p_j) influences the relative propensity to visit the site. However, it is the absolute number of visitors (VIS) that determines the CS. Inspecting the OLS results, we found that the predicted total number of visitors from all zones far exceeded the observed number of

visitors. The source of the problem becomes evident if we multiply the regression equation (11) by the population variable (POP) to obtain:

$$VIS_j = a \cdot POP_j + b \cdot \frac{1}{p_j} \cdot POP_j + \varepsilon_j \cdot POP_j. \quad (12)$$

Note that the error term is multiplied by the population variable. Inspecting the residuals, we found that zones with a high population indeed tended to be associated with negative errors (i.e., predicted visitation exceeds observed visitation) while low-population zones typically exhibited positive errors (i.e., predicted visitation is less than observed visitation). This would overestimate CS in highly populated zones and underestimate it in low-population zones, leading to an overestimation of the total CS.

To obtain unbiased demand and CS estimates, the parameters a and b of equation (11) were estimated using a variant of the weighted least squares (WLS) approach. Specifically, whereas OLS minimises the unweighted sum of squares of error terms, we minimise the weighted sum of squares using the proportions of the zonal population to the total population as the weights.⁵ Applying such weighting, the impact of each zone to parameter estimates is proportional to its population. This is consistent with our ultimate purpose of estimating the total CS where the contribution of each zone is also proportional to its population. In practice, the WLS demand curve will provide better fit for highly populated zones than the OLS curve. In the WLS model, the predicted number of visitors per zone will sum to the observed total number of visitors and, therefore, the problem of the OLS estimates noted above is avoided.

Table 1 reports the summary statistics for the extreme value (minimum and maximum) approach, and compares the results to those obtained by treating MDT as single destination trips (i.e., ignoring MDT treatment).

Considering the empirical fit (table 1), the best results were obtained by ignoring the MDT. From the econometric point of view this is nothing surprising, recalling that our MDT treatment decreases the variance of prices (travel costs) in the sample. In the Mmax case the fit is still very good, but the Mmin case is quite disappointing.⁶ In all three cases, the parameter

⁵ In fact, our weighted least squares (WLS) model is equivalent to the ordinary least squares (OLS) model with regression equation:

$$VIS_j / \sqrt{POP_j} = a \cdot \sqrt{POP_j} + b \cdot (1/p_j) \cdot \sqrt{POP_j} + \varepsilon_j$$

⁶ Zones for which the estimated average minimum expected value was equal to zero were omitted from the regressions. Therefore, only 13 zones could be used for regressions in the minimum expected value case.

Table 1 Summary statistics of the WLS regressions using different MDT approaches

	Mmin	Mmax	O
R^2	0.279	0.887	0.945
F statistic	1.305	8.330**	17.197**
Intercept \hat{a}	-0.388	-0.555	-0.283
Standard error	5 678 577	256 241	87 043
Slope \hat{b}	204.43**	549.43**	623.72**
Standard error	0.180	0.012	0.004

*Significant at 95 per cent confidence level; **significant at 99 per cent confidence level. MDT, multi-destination trips; WLS, weighted least squares.

Table 2 CS estimated from WLS demand curve*

	Mmin	Mmax	O
Choke price (\$A)	527	990	2204
CS per visit per person (\$A)	137	773	645

CS, consumer surplus; WLS, weighted least squares.

estimates for the slope b are statistically significant at very high confidence levels, while the estimates of the intercept a have large standard errors and, hence, fail the significance test in all cases.

The estimated equations have been used to calculate total predicted visitation, at increasing entrance fees, and corresponding CS. By taking the definite integral over the zonal inverse demand functions and summing over all zones, we obtain the CS as:

$$CS_{WLS} = \sum_j [(\hat{b} \cdot \ln(k) - \hat{a} \cdot k) - (\hat{b} \cdot \ln(p_j^r) - \hat{a} \cdot p_j^r)] \cdot P_j \quad (13)$$

where k is the choke price at which demand equals zero, and P_j is the population of zone j . Table 2 reports the estimated CS per visit and person for each of the three MDT scenarios.

The results show that using the Mmin value for travel cost yields very different results compared to using the MDT-omitted or Mmax value. The difference is more than 470 per cent. The difference between MDT-omitted and the Mmax approaches is relatively small. These large differences in CS strengthen the argument to pay specific attention to MDT in applied TCM with a high fraction of MDT visitors. Still, the differences in CS are statistically insignificant at the 10 per cent level (see table 3).⁷

⁷ The standard error of the estimated CS was calculated using the approach suggested by Adamowicz *et al.* (1989).

Table 3 Testing statistical significance of the CS differences

	Mmin versus Mmax	Mmin versus O	Mmax versus O
<i>t</i> -test statistic	1.652	0.724	0.132
<i>p</i> -value	0.117	0.479	0.896

CS, consumer surplus.

Table 4 Decomposition of the MDT-effect: the Mmin and the Mmax models*

	Mmin	Mmax
CS^M (\$A per visit)	137	773
DE (\$A per visit)	(-) 113.4	(-) 713.1
IE (\$A per visit)	(+) 622.1	(+) 585.0
$=CS^O$ (\$A per visit)	645	645

CS, consumer surplus; DE , direct effect; IE , indirect effect; MDT, multideestination trips.

Consider next the magnitudes of the direct effect, DE , and the indirect effect, IE . The magnitudes of these effects for the min and the max cases were calculated using (3) and (5), and are reported in table 4. Starting from the MDT-corrected (min or max) CS estimate, the subtraction of the direct effect and the addition of the indirect effect gives us the MDT-omitted CS estimate. In the Mmin-case the indirect effect dominates, and hence the MDT-omitted estimate is greater. Conversely, in the Mmax-case the direct effect dominates, and hence ignoring MDT leads to a lower CS estimate.

It is important to note the relatively high levels of these two offsetting components compared to the levels of the CS estimates themselves. (Note that inequalities $DE \leq CS^M$ and $IE \leq CS^O$ must always hold.) Consequently, the CS estimates appear highly sensitive to estimation error in the sense that a minor change in either effect, the indirect effect in particular, can have a major impact on the CS estimates. This is aptly illustrated by our results: even when applying the same estimation method and the same data, the way of allocating the travel costs of MDT visitors leads to dramatic differences in the relative magnitudes of the direct and indirect effect; in the Mmin-case the indirect effect is the most important determinant, whereas in the Mmax-case the direct effect dominates.

As a final remark, we suspect that the high levels of the direct and indirect effect compared to the CS estimates are at least partly a result of the curvature of the estimated demand function. According to our econometric model, the own price elasticity of recreational demand for the present nature park is relatively high; that is, demand drops rapidly as the price increases. Still, there are visitors who are observed to pay considerable sums of money

(i.e., travel across the continent) to visit the park. Consequently, the estimated demand functions are highly nonlinear (convex). If the demand functions were linear, as in figure 1, then these two offsetting effects would tend to be smaller compared to the CS estimates.

4.4 Results of non-parametric demand analysis

Next, the demand function was estimated using non-parametric methods to obtain a conservative estimate of the CS. Because of the small number of zones, we pooled all zones together for this exercise. In contrast to the OLS regressions where zone-specific demand functions were estimated, we focus here on the overall (aggregate) demand curve using the actual numbers of visitors, and the travel costs estimated for each zone. Because of the limited sample size, no demographic variables were considered.

Our approach builds on the following two assumptions: (1) every visitor is willing to pay any price less than or equal to the observed price; and (2) no visitor is willing to pay a higher price. Under these two intuitive assumptions, we obtain the non-parametric estimates of the demand functions and the CS. The computational procedures are described in more detail in Appendix B.

Figure 2 illustrates these piecewise linear demand functions for all three cases considered (plotting the inverse demand like in figure 1). In all three cases, demand is very sensitive to price changes at low price levels. Still, the

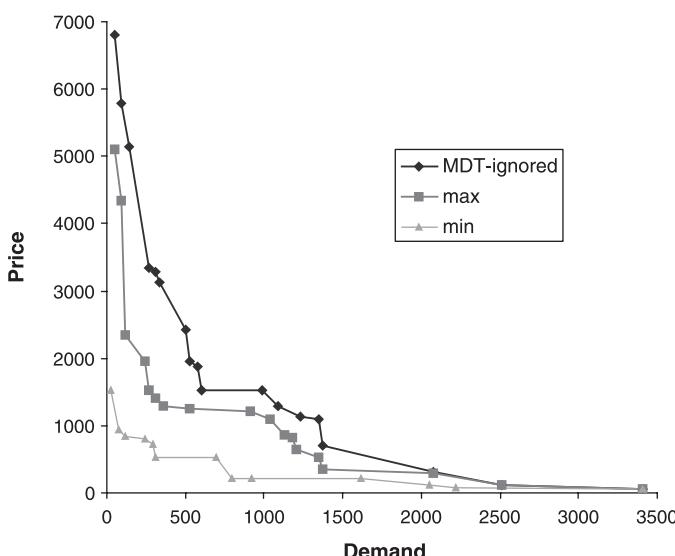


Figure 2 Piecewise linear inverse demand curves estimated in non-parametric fashion.

Table 5 Non-parametric CS estimates and their decomposition

	Mmin	Mmax
CS^M (\$A per visit)	100	343
DE (\$A per visit)	(-) 93.3	(-) 116.3
IE (\$A per visit)	(+) 572.3	(+) 352.6
$=CS^O$ (\$A per visit)	579	579

CS , consumer surplus; DE , direct effect; IE , indirect effect; MDT, multideestination trips.

Mmax-case and especially the MDT-omitted scenario suggest that small but persistent demand exists even at very high price levels.

Table 5 reports the CS estimates for the Mmin- and Mmax-cases, and their decomposition into the direct and indirect effects. Starting from the MDT-corrected estimate, subtracting the direct effect and adding the indirect effect results in the MDT-omitted estimate. Overall, our non-parametric CS estimates are relatively close to our parametric WLS estimates, which reassures us that both estimates are reasonable. However, in contrast to the WLS case, the indirect effect is found to strongly dominate both in the Mmin and Mmax scenarios. In other words, ignoring the MDT seems to lead to a substantial overestimation in this case. The levels of the two effects are very high in the Mmin scenario, but the Mmax case seems much more robust to these effects.

Unfortunately, there is no tractable analytical method for testing hypotheses and deriving confidence intervals within this non-parametric framework. Therefore, we resorted to the bootstrapping approach (Efron 1979), the standard technique used in the published non-parametric literature. Assuming a uniform density over the observed price range, we drew 2000 pseudo-samples of size 18 observations (like the original sample) from the empirical piecewise linear demand curve for each of the three cases. We subsequently applied the same non-parametric method to fit the piecewise linear demand curve to each pseudo-sample, and calculated the CS. The distribution of CS values in the set of these 2000 pseudo-samples should, hence, give us an idea of magnitudes of the sampling bias and standard error in the original estimation.

Table 6 reports the key statistics from the bootstrapping analysis. The bootstrapping results suggest a relatively large standard error in the results, from 8.65 per cent (MDT-omitted) up to 35.9 per cent (Mmin) of the mean. It also revealed a significant downward bias in the estimates, from 5.2 per cent (MDT-omitted) all the way to 61.9 per cent (Mmin). That is, the mean CS value of the bootstrap pseudo-samples was, in all cases, significantly lower than the original CS estimate. Therefore, if our pseudo-sampling procedure reasonably mimics the actual sampling procedure, we may expect our

Table 6 Bootstrapping analysis

	Mmin	Mmax	O
Point estimate:			
CS per visit	100	343	579
Sampling bias	(+) 62.0	(+) 25.0	(+) 29.7
=Bias corrected CS estimate	162	368	609
Std. error	13.67	39.95	47.56
95% confidence interval	(42–213)	(275–416)	(475–662)

CS, consumer surplus.

original estimate to be similarly downward biased. Consequently, we adjusted our CS estimates upwards by the measured bias factor.

We can also derive confidence intervals directly from the simulated error distribution. Comparing the confidence intervals, we find that the differences between the CS estimates in the three scenarios are statistically significant at the 95 per cent level. Note that the error distribution need not be symmetric or conform to normality and, hence, our point estimates do not generally coincide with the median value of the confidence interval.

5. Conclusions

Our analysis of data from the Bellenden Ker National Park reveals that the treatment of MDT as SDT in the TCM does not involve any systematic upward or downward bias; both underestimation and overestimation of CS are possible. The magnitude of the error depends on the slope and curvature properties of the demand function. While excluding MDT observations from the data tends to lower the CS as a result of the direct effect, treating MDT as SDT typically overestimates the value of the site because of the indirect effect. Our empirical analysis also highlights the sensitivity of CS estimates to the model specification and estimation technique, not to mention potential data errors and sampling and non-response biases.

Since the true price perceptions for visiting the site are not known for the MDT travellers, minimum and maximum bounds for the subjective price were derived based on respondents preference ranking of sites visited in the MDT package. While we recognise the general problems in any stated-preference techniques, we find the extreme bound approach quite useful. The proposed survey method is convenient for respondents as they are only asked to provide ordinal rankings of a small number of alternatives. In contrast to other arbitrary techniques, our approach builds on the fundamental duality relationship between expenditure and utility functions. To avoid strong assumptions about consumer preferences, the minimum and maximum bounds

for cost shares are derived from the ordinal preference rankings. Given the sensitivity of CS estimates to model specification and estimation technique, resorting to extreme bounds rather than sharp point estimates of the subjective price can improve the robustness of inferences. Our empirical analysis shows that the treatment of MDT can make a considerable difference in CS estimation of recreational sites even if we consider the most extreme scenarios for cost shares.

The take-home message for policy makers and national park managers is that using minimum weights for MDT-visitors results in a safe minimum value of the CS. If the costs for managing the park can be justified using those results, park managers and policy makers can be quite confident that the money is well-spent.

References

Adamowicz, W.L., Fletcher, J.J. and Graham-Tomasi, T. 1989, 'Functional form and the statistical properties of welfare measures', *American Journal of Agricultural Economics*, vol. 71, pp. 414–421.

Beal, D.J. 1995, 'A travel cost analysis of the value of Carnarvon Gorge national park for recreational use', *Review of Marketing and Agricultural Economics*, vol. 63, pp. 292–303.

Bennett, J. 1995, 'Economic value of recreational use: Gibraltar Range and Dorrigo National Parks', New South Wales Parks and Wildlife Service, Sydney.

Bojö, J. 1985, 'A cost-benefit analysis of forestry in mountainous areas: the case of Valadelen', Stockholm School of Economics, Stockholm.

Brouwer, R. and Slangen, L.H.G. 1998, 'Contingent valuation of the public benefits of agricultural wildlife management: the case of Dutch peat meadow land', *European Review of Agricultural Economics*, vol. 25, pp. 53–72.

Cooper, J.C. 2000, 'Nonparametric and semi-nonparametric Recreational Demand Analysis', *American Journal of Agricultural Economics*, vol. 85, pp. 451–462.

Driml, S.M. 1996, 'Towards sustainable tourism in the Wet Tropics World Heritage Area', PhD Thesis, Australian National University, Canberra.

Efron, B. 1979, 'Bootstrap methods. Another look at the jackknife', *Annals of Statistics*, vol. 7, pp. 1–26.

Hajkowicz, S.A., McDonald, G.T. and Smith, P.N. 2000, 'An evaluation of multiple objective decision support weighting techniques in natural resource management', *Journal of Environmental Planning and Management*, vol. 43, pp. 505–518.

Hanley, N.D. and Ruffell, R. 1992, 'The valuation of forest characteristics', working paper no. 849, Institute for Economic Research, Queen's University, Kingston, Ontario.

Haspel, A.E. and Johnson, F.R. 1982, 'Multiple destination trip bias in recreation benefit estimation', *Land Economics*, vol. 58, pp. 364–372.

Kmietowicz, Z.W. and Pearman, A.D. 1981, 'Decision theory and incomplete knowledge', Gower, Aldershot.

Knapman, B. and Stanley, O. 1991, 'A travel cost analysis of the recreation use value of Kakadu National Park', Resource Assessment Commission Inquiry, AGPS, Canberra.

Loomis, J.B. and Walsh, R.G. 1997, 'Recreation economic decisions: comparing benefits and costs', Venture Publishing, State College, PA.

Loomis, J.B., Yorizane, S. and Larson, D. 2000, 'Testing significance of multi-destination and multi-purpose trips effects in a travel cost method demand model for whale watching trips', *Agricultural and Resource Economics Review*, vol. 29, pp. 183–191.

Mendelsohn, R., Hof, J., Peterson, G. and Johnson, R. 1992, 'Measuring recreation values with multiple destination trips', *American Journal of Agricultural Economics*, vol. 74, pp. 926–933.

Nillesen, E., Wesseler, J. and Cook, A. 2003, 'Correcting for multiple destination trips in recreational use values using a mean-value approach', Mansholt working papers, Social Science Department, Wageningen University, Wageningen, the Netherlands.

Rietveld, P. 1989, 'Using ordinal information in decision making under uncertainty', *Systems Analysis, Modeling, Simulation*, vol. 6, pp. 659–672.

Stoeckl, N. 1993, 'A travel cost analysis of Hinchinbrook National Park', Masters Thesis, Department of Economics, James Cook University of North Queensland, Townsville.

Ward, F.A. and Beal, D. 2000, *Valuing Nature with Travel Cost Models. A Manual*, Edward Elgar, Cheltenham.

Willis, K.G. and Garrod, G.D. 1991, 'An individual travel cost method of evaluating forest recreation', *Journal of Agricultural Economics*, vol. 42, pp. 33–42.

Appendix A

Six different function forms of the trip demand function have been tested, including and excluding socioeconomic variables. Results for all three cases (Mmin, Mmax, total) have been displayed in tables 7–9, respectively.

The model with inverse travel cost outperformed all other models in terms of log-likelihood, adjusted R^2 and F - and t -values, for Mmax, Mmin and total travel costs. Therefore, we have chosen to proceed, for all three cases, with a reciprocal form. Furthermore, because the socioeconomic variables appeared to be insignificant as explanatory variables, they were discarded from the analysis.

Appendix B

The non-parametric estimates of the demand functions are obtained as follows. First, rank the zones in ascending order according to the observed travel costs. Let the travel costs be denoted by $p_1 \leq p_2 \leq \dots \leq p_{18}$, and the corresponding visitor volumes by x_1, x_2, \dots, x_{18} . Construct a cumulative index of the number of visitors as $X_1 \geq X_2 \geq \dots \geq X_{18}$, where $X_j \equiv \sum_{i=1}^j x_i$. Value X_j indicates the actual number of visitors who have paid the price (travel cost) less than or equal to p_j and, hence, it is reasonable to assume $\hat{x}(p_j) = X_j$. Using the trapezoidal-rule (see, e.g., Cooper 2000, p. 453, for further details), we obtain non-parametric, piecewise linear demand functions as:

$$\hat{x}(p) = \max_{j: p_j \leq p} X_j + \left[\frac{p - \max_{p_j \leq p} p_j}{\min_{p_j \geq p} p_j - \max_{p_j \leq p} p_j} \right] \left(\min_{j: p_j \geq p} X_j - \max_{j: p_j \leq p} X_j \right) \quad (14)$$

Table 7 Testing of six function forms for Mmax values

Function form	Variables included	LL	Adjusted R^2	F-value	t-value C	t-value p	t-value E	t-value I	t-value A	t-value p^2
Linear	all	-39.26	0.04	1.2	1.56	-1.61	-1.18	-0.2	-0.37	-
Linear	p	-40.59	0.09	2.88	2.66	-1.70	-	-	-	-
$\log p$	all	-32.18	0.57	6.51	3.91	-4.61	-0.88	-0.85	0.01	-
$\log p$	p	-33.17	0.61	27.09	5.64	-5.21	-	-	-	-
$\log VI$	all	-32.78	-0.01	0.97	0.78	-1.11	-1.4	0.13	-0.13	-
$\log VI$	p	-34.24	0.03	1.66	-1.56	-1.29	-	-	-	-
$\log p/\log VI$	all	-30.54	0.21	2.16	1.68	-2.29	-1.14	0.33	-0.27	-
$\log p/\log VI$	p	-31.45	0.29	8.08	2.12	-2.84	-	-	-	-
p^2	all	-34.49	0.39	3.18	1.94	-3.33	-1.62	-0.88	1.07	2.9
p^2	p	-36.81	0.37	5.97	4.19	-3.26	-	-	-	2.8
$1/p$	all	-18.14	0.91	43.23	-0.76	12.25	0.35	0.04	0.64	-
$1/p$	p	-18.47	0.92	204.44	-1.43	14.30	-	-	-	-

A, age; C, constant; E, education level; I, income; LL, log-likelihood; p, travel costs; VI, visitation rate.; -, not applicable.

Table 8 Testing of six function forms for Mmin values

Function form	Variables included	LL	Adjusted R^2	F-value	t-value C	t-value p	t-value E	t-value I	t-value A	t-value p^2
Linear	all	-28.39	0.14	1.48	0.86	-1.76	-1.41	0.51	0.67	-
Linear	p	-30.18	0.18	3.56	2.84	-1.88	-	-	-	-
$\log p$	all	-26.20	0.39	2.88	2.17	-2.75	-1.36	0.61	0.72	-
$\log p$	p	-28.09	0.40	9.05	3.39	-3.01	-	-	-	-
$\log VI$	all	-18.92	0.42	3.2	0.47	-1.79	-2.31	0.83	1.47	-
$\log VI$	p	-23.74	0.12	2.62	-0.05	-1.62	-	-	-	-
$\log p/\log VI$	all	-16.70	0.59	5.32	1.78	-2.79	-2.41	1.86	0.81	-
$\log p/\log VI$	p	-22.36	0.29	5.86	2.00	-2.42	-	-	-	-
p^2	all	-26.15	0.30	2.04	1.22	-2.22	-1.28	0.12	0.99	1.70
p^2	p	-28.80	0.27	3.18	3.26	-2.10	-	-	-	1.54
$1/p$	all	-24.09	0.56	4.75	0.11	3.68	-1.05	0.84	0.35	-
$1/p$	p	-25.66	0.59	18.18	-0.82	4.26	-	-	-	-

A, age; C, constant; E, education level; I, income; LL, log-likelihood; p, travel costs; VI, visitation rate.; -, not applicable.

Table 9 Testing of six function forms for total travel costs

Function form	Variables included	LL	Adjusted R^2	F-value	t-value C	t-value p	t-value E	t-value I	t-value A	t-value p^2
Linear	all	-38.49	0.12	1.59	1.59	-2	-1.34	-0.28	0.07	-
Linear	p	-39.87	0.17	4.47	3	-2.11	-	-	-	-
$\log p$	all	-29.19	0.69	10.37	4.52	-5.89	-1.08	0.37	-0.52	-
$\log p$	P	-30.09	0.72	44.67	7.21	-6.68	-	-	-	-
$\log VI$	all	-31.86	0.09	1.42	0.86	-1.66	-1.54	0.31	-0.12	-
$\log VI$	p	-33.47	0.12	3.22	-1.03	-1.8	-	-	-	-
$\log p/\log VI$	all	-27.65	0.43	4.21	2.2	-3.49	-1.24	-0.16	0.55	-
$\log p/\log VI$	p	-28.66	0.48	16.81	3.2	-4.09	-	-	-	-
p^2	all	-32.69	0.5	4.41	1.91	-3.89	-1.86	-0.34	1.37	3.3
p^2	p	-35.27	0.47	8.49	4.89	-3.79	-	-	-	3.16
$1/p$	all	-13.21	0.95	77.15	-1.01	16.4	0.55	1.33	-0.13	-
$1/p$	p	-14.46	0.95	328.45	-0.25	18.12	-	-	-	-

A, age; C, constant; E, education level; I, income; LL, log-likelihood; p, travel costs; VI, visitation rate; -, not applicable.

Let \bar{p} denote the weighted average of the zonal travel costs, using the number of visitors per zone as the weights. To calculate the consumer surplus (CS) estimate, we use \bar{p} for the prevailing price level. By basic geometry, the non-parametric CS estimate is then given by:

$$CS = \sum_{j:p_j \geq \bar{p}} [X_{j+1}(p_{j+1} - p_j) + \frac{1}{2}(X_{j+1} - X_j)(p_{j+1} - p_j)] \\ + \min_{j:p_j \geq \bar{p}} X_j \left((\min_{p_j \geq \bar{p}} p_j) - \bar{p} \right) + \frac{1}{2} \left((\min_{p_j \geq \bar{p}} X_j) - \hat{x}(\bar{p}) \right) \left((\min_{p_j \geq \bar{p}} p_j) - \bar{p} \right) \quad (15)$$

The decomposition is obtained using the following formulas:

$$DE_{NP} = CS^M - \sum_{j:p_j^M \geq \bar{p}^O} \left[X_{j+1}^M(p_{j+1}^M - p_j^M) + \frac{1}{2}(X_{j+1}^M - X_j^M)(p_{j+1}^M - p_j^M) \right] \\ - \left[\left(\min_{j:p_j^M \geq \bar{p}^O} X_j^M \right) \cdot \left(\left(\min_{p_j^M \geq \bar{p}} p_j^M \right) - \bar{p}^O \right) + \right. \\ \left. - \left[\frac{1}{2} \left(\left(\min_{j:p_j^M \geq \bar{p}^O} X_j^M \right) - \hat{x}^M(\bar{p}^O) \right) \cdot \left(\left(\min_{p_j^M \geq \bar{p}^O} p_j^M \right) - \bar{p}^O \right) \right] \right] \quad (16)$$

and

$$IE_{NP} = CS^O - \sum_{j:p_j^M \geq \bar{p}^O} \left[X_{j+1}^M(p_{j+1}^M - p_j^M) + \frac{1}{2}(X_{j+1}^M - X_j^M)(p_{j+1}^M - p_j^M) \right] \\ - \left[\left(\min_{j:p_j^M \geq \bar{p}^O} X_j^M \right) \cdot \left(\left(\min_{p_j^M \geq \bar{p}} p_j^M \right) - \bar{p}^O \right) + \right. \\ \left. - \left[\frac{1}{2} \left(\left(\min_{j:p_j^M \geq \bar{p}^O} X_j^M \right) - \hat{x}^M(\bar{p}^O) \right) \cdot \left(\left(\min_{p_j^M \geq \bar{p}^O} p_j^M \right) - \bar{p}^O \right) \right] \right] \quad (17)$$