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Drought policy: a graphical analysis*

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The standard approach to modelling production under uncertainty has relied on the concept of the stochastic production function. In the present paper, it is argued that a state-contingent production model is more flexible and realistic. The model is applied to the problem of drought policy.

1. Introduction

Risk and uncertainty are crucial features of agricultural production. Even writers whose main concerns are with issues unrelated to agriculture, such as the role of stock markets and the design of labour contracts, frequently illustrate their analysis with agricultural examples, the most notable of which is the occurrence or non-occurrence of rain (Diamond 1967).

Like all producers, agricultural producers are subject to uncertainty regarding the demand for their products. Assuming competition in supply, this demand uncertainty takes the form of price uncertainty. To a greater extent than many others, agricultural producers must also deal with uncertainty related to production. In Australia, the most important source of uncertainty is the unpredictability of rainfall and, in particular, the occurrence of droughts.

The standard tool for modelling production uncertainty is the stochastic production function, that is, a production function in which one or more exogenous random variables enter as inputs. The stochastic production function model of uncertainty has yielded a wide range of significant insights. Nevertheless, in a number of contributions over recent years, most notably Chambers and Quiggin (2000), we have argued that the model is insufficiently flexible to capture a number of critical issues in relation to producer behaviour under uncertainty.

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Most importantly, in the standard stochastic production function model there is no state-contingent supply response. Producers can increase or reduce their total productive effort and, in some cases, change their input mix, but they cannot reallocate so as to substitute output in one state of nature for output in another. By contrast, in standard multi-output models of production under certainty, producers can allocate effort among different outputs in response to technological shocks and changes in relative input and output prices.

Arguably, an even more critical weakness of the stochastic production function approach lies in the fact that it has not proved amenable to diagrammatic representation of the kind that remains the main source of intuition for most economists. This lack of diagrammatic facility is typically reflected in analyses that emphasise computation to the near exclusion of intuition. Given that state-contingent production is, in essence, a special case of multi-output production, it ought to admit the same diagrammatic representation.

In the present paper, we will develop these points in detail. Our intent is to show that the crucial issues regarding production under uncertainty can be represented using the graphical representations familiar from standard production theory and standard international trade theory. Viewed in this light, the benefits of risk reduction through insurance are simply a special case of gains from trade. The analysis will be illustrated with reference to the debate over drought policy and rainfall insurance in Australia, and the related debate over crop insurance in the USA. Both topics have recently been analysed in a general algebraic framework (Chambers and Quiggin 2004), encompassing a broad range of financial and technological instruments for risk management. The graphical analysis presented here complements the algebraic approach.

2. Debate over drought policy

Until 1992, the Australian approach to drought policy was centred on the notion of 'drought declaration' of districts, normally at the discretion of State governments. A variety of relief measures, which varied over time and between states, was made available to farmers in 'drought declared' areas. Examples included subsidies for the purchases of fodder, low-interest loans and cash grants. The implicit policy model was that of an unpredictable natural disaster, like an earthquake. Policy was focused on the provision of assistance to farmers who had suffered, or who were exposed to, losses as a result of drought.

This policy was criticised by economists including Freebairn (1983), who argued that it undermined incentives to prepare appropriately for drought and encouraged practices such as overstocking. Studies of the implementation of drought relief in the 1980s reinforced Freebairn's arguments and raised new concerns. Only a minority of eligible producers received any relief. In Queensland, 36 per cent of the state had been drought declared every one in three years, and over the period 1984–1985 to 1988–1989, 40 per cent of relief had gone to 5 per cent of the claimants (Smith *et al.* 1992).

Economists also debated a range of market-based measures aimed at providing a more coherent and less costly response to climatic uncertainty. A natural starting point for the debate was the multiple-risk crop insurance system then in use in the USA (Gardner and Kramer 1986). Most Australian commentators saw this system, in which producers are reimbursed for yield shortfalls arising from a range of climate events, as being even more exposed to moral hazard problems than the Australian system of ad hoc relief. Attention was therefore focused on schemes where insurance payments were conditioned on events exogenous to individual farmers, such as rainfall (Bardsley *et al.* 1984; Quiggin 1986) or yield in a given district (Industries Assistance Commission 1978).

The main outcome of the Australian debate was the adoption of the National Drought Policy in 1992. O'Meagher (2003) summarises the key features of the policy. Its stated rationale is that 'Drought is one of several sources of uncertainty affecting farm businesses and is part of the farmer's normal operating environment. Its effects can be reduced through risk management practices which take all situations into account, including drought and commodity price downturns.' The key policy implication is that 'farmers will have to assume greater responsibility for managing the risks arising from climatic variability. This will require the integration of financial and business management with production and resource management to ensure that financial and physical resources of farm businesses are used efficiently.'

3. Model

The application of a state-contingent approach to drought policy is a natural one. As many writers have observed (Yaari 1969; Cochrane 2001), the two-dimensional case state-contingent model lends itself naturally to a diagrammatic representation, exactly analogous to that representing choices between two commodities, or between consumption bundles at two different dates.

The implications of this illustrative example for real-world problems involving drought have never been properly developed. In the present paper, these implications will be analysed in the context of a formal state-contingent model.

In the general form of the model, there are M distinct outputs, N distinct inputs and S possible states of nature. Inputs $\boldsymbol{x} \in \Re_+^N$ are committed *ex ante*, and fixed *ex post*. State-contingent outputs $\boldsymbol{z} \in \Re_+^{S\times M}$ are chosen *ex ante* but produced *ex post*. That is, if state s is realised, and the *ex ante* output choice is the matrix $\boldsymbol{z}$, the observed output is $\boldsymbol{z}_s \in \Re_+^M$, which corresponds to the M outputs produced in state s. Inputs that are variable *ex post* might be regarded as negative state-contingent outputs, in which case we generalise to allow $\boldsymbol{z}_s \in \Re^M$. We denote by $\mathbf{1}_S \in \Re^S$ the unit vector with all entries equal to 1.

The formal structure may be considered as a two-period game between the producer (with commitment on part of the producer) and Nature, with periods denoted 0 and 1. In period 0, the producer commits fixed inputs $\boldsymbol{x} \in \Re_+^N$. When Nature reveals the state s, the individual produces the output $\boldsymbol{z}_s$. The technology of production determines the feasible strategies $(\boldsymbol{x}, \boldsymbol{z})$.

Chambers and Quiggin (2000) show that a state-contingent technology may be summarised in terms of the input correspondence, which maps state-contingent output vectors into sets of inputs that can produce that state-contingent output matrix. Formally, it is defined by $X: \Re_+^{M\times S} \to \Re_+^N$

$$X(\boldsymbol{z}) = \{\boldsymbol{x} \in \Re_+^N : \boldsymbol{x} \quad \text{can produce} \quad \boldsymbol{z} \in \Re_+^{M\times S}\}. \tag{1}$$

Intuitively, one can think of this input correspondence as yielding all input combinations that are on or above the production isoquant for the state-contingent output matrix $\boldsymbol{z}$. Moreover, upon imposing suitable curvature conditions on the state-contingent technology, the lower boundary to this set of inputs (the production isoquant) will have the familiar convex-to-the-origin shape from standard microeconomic theory.

Conversely, we can consider an output correspondence

$$Z(\boldsymbol{x}) = \{\boldsymbol{Z} \in \Re_+^{M\times S} : \boldsymbol{x} \in X(\boldsymbol{z})\}, \tag{2}$$

which, in a sense, is the inverse of the input correspondence. Intuitively, one can think of it as giving the state-contingent output matrices that are on or below a state-contingent transformation curve. In what follows, we routinely restrict attention to the case of a single stochastic output, so that $M = 1$. That is, there is only one output but S possible realisations of the output corresponding to each state of Nature.

A farmer's welfare depends only on his state-contingent consumption, which will be denoted by $\boldsymbol{y}$. In this model, consumption is equal to income net of all financial market transactions. For any given cost level, farmers seek to maximise an objective function, $W(\boldsymbol{y})$, where $W: \Re^S \to \Re$. For the moment we will assume only that W is monotone-increasing, quasi-concave and continuous. We will assume that, *ex post*, farmers are concerned with net returns, given by

$$y_s = p_s z_s - \boldsymbol{wx}, \quad s = 1, 2, \ldots, S. \tag{3}$$

It is useful to introduce three analytical tools, the properties of which are discussed in detail by Chambers and Quiggin (2000). The first is the cost function

$$c(\boldsymbol{w}, \boldsymbol{z}) = \inf\{\boldsymbol{wx} : \boldsymbol{x} \in X(\boldsymbol{z})\}. \tag{4}$$

Assuming an objective function of the general form $W(\boldsymbol{y})$, producers will seek to minimise costs for given output $\boldsymbol{z}$, so that net returns, without loss of generality, may be rewritten as

$$y_s = p_s z_s - c(\boldsymbol{w}, \boldsymbol{z}). \tag{5}$$

The second is the benefit function

$$B(\boldsymbol{y}, w) = \sup\{b : \boldsymbol{W}(\boldsymbol{y} - \boldsymbol{b1}) \geq w\}. \tag{6}$$

The third and most important analytical tool is the idea of risk-substituting and risk-complementary inputs. For any given output vector $\boldsymbol{z} \in \Re^S_+$, let the associated input demand be

$$\boldsymbol{x}(\boldsymbol{w}, \boldsymbol{z}) = \arg\min\{\boldsymbol{wx} : \boldsymbol{x} \in X(\boldsymbol{z})\}. \tag{7}$$

Input n is said to be a risk-substitute at prices $\boldsymbol{w}$ if, whenever $\boldsymbol{z}'$ is riskier than $\boldsymbol{z}$, $x_n(\boldsymbol{w}, \boldsymbol{z}') \leq x_n(\boldsymbol{w}, \boldsymbol{z})$, and a risk-complement if whenever $\boldsymbol{z}'$ is riskier than $\boldsymbol{z}$, $x_n(\boldsymbol{w}, \boldsymbol{z}') \geq x_n(\boldsymbol{w}, \boldsymbol{z})$.

In models based on a stochastic production function technology, the notion that input n is a risk-substitute might be intuitively rephrased by saying that input n is risk-reducing, because there is a deterministic relationship between inputs and outputs. In the more general case modelled here, input choices arise from cost minimisation at given factor prices. A change in factor prices might change the risk-complementarity properties of an input. For example, if risk arises from pest infestation, labour might be a risk substitute when labour-intensive methods of pest control are optimal, but might become a risk complement if, for example, the price of an easily applied pesticide declines.

The definition of risk substitutes and risk complements proposed here requires that we formalise the statement $\boldsymbol{z}'$ is riskier than $\boldsymbol{z}$. There are a great many different ways in which the riskiness of vectors may be compared. The simplest, analysed by Sandmo (1971) is that of a multiplicative spread about the mean. A more general definition, widely used in the published literature is that of Rothschild and Stiglitz (1970), also analysed by Hadar and Russell (1969) and Hanoch and Levy (1969). The concept of monotone spreads (also known as deterministic transformations) is intermediate between these two and has proved tractable in comparative static analysis (Meyer and Ormiston 1989; Quiggin 1991). For the case $S = 2$, and monotonic vectors, all these definitions, and most others that have been considered in the published literature, coincide. For the case $S = 2$, and assuming $z_1 \leq z_2$ we, therefore, say $\boldsymbol{z}'$ is riskier than $\boldsymbol{z}$ if

$$z_1' \leq z_1 \leq z_2 \leq z_2'. \tag{8}$$

4. Diagrammatic representations

Most of the key issues in drought policy can be analysed in a very simple version of the model, in which $S = 2$, $M = 1$. In this simple model, there are two states of nature, with state 1 corresponding to 'drought' and state 2 corresponding to 'normal', a single input, and a single state-contingent output in each state of nature. To permit graphical exposition, we will simplify further by assuming for the moment that the input vector $\boldsymbol{x}$ is exogenously given.

Before considering the implications of the state-contingent production model with a single output and S states of nature, it is useful to observe that exactly the same representation is applicable for a standard production problem with $M = S$ outputs, or for an intertemporal production problem with $T = S$ time periods. In this section, we will develop the diagrammatic representation of these three cases in turn, with $M = T = S = 2$.

4.1 Production, consumption and trade with two goods

For a standard two-output technology, and any given input level $\bar{\boldsymbol{x}}$, the set $Z(\bar{\boldsymbol{x}})$ may be represented diagrammatically by the familiar concave-to-the-origin product transformation curve represented in figure 1(a). As in Ricardo's classic exposition of comparative advantage, the commodities are named 'Cloth' and 'Wine'. Thus, the output of cloth and wine is represented by a vector $\boldsymbol{z} \in \Re^2$, where z_1 denotes the output of wine and z_2 the output of cloth. Consistent with the notation set out for the state-contingent case we will denote consumption by $\boldsymbol{y} \in \Re^2$ and assume that individuals seek to maximise an objective function, $W(\boldsymbol{y})$.

Consider first the case of autarky, where there is a single representative individual with no opportunities for trade. Hence, $\boldsymbol{z} = \boldsymbol{y}$. The optimal output $\boldsymbol{z}^0$, represented in figure 1(b), is the point of tangency between the individual's indifference curves (level sets of W) and the production possibility set $Z(\bar{\boldsymbol{x}})$.

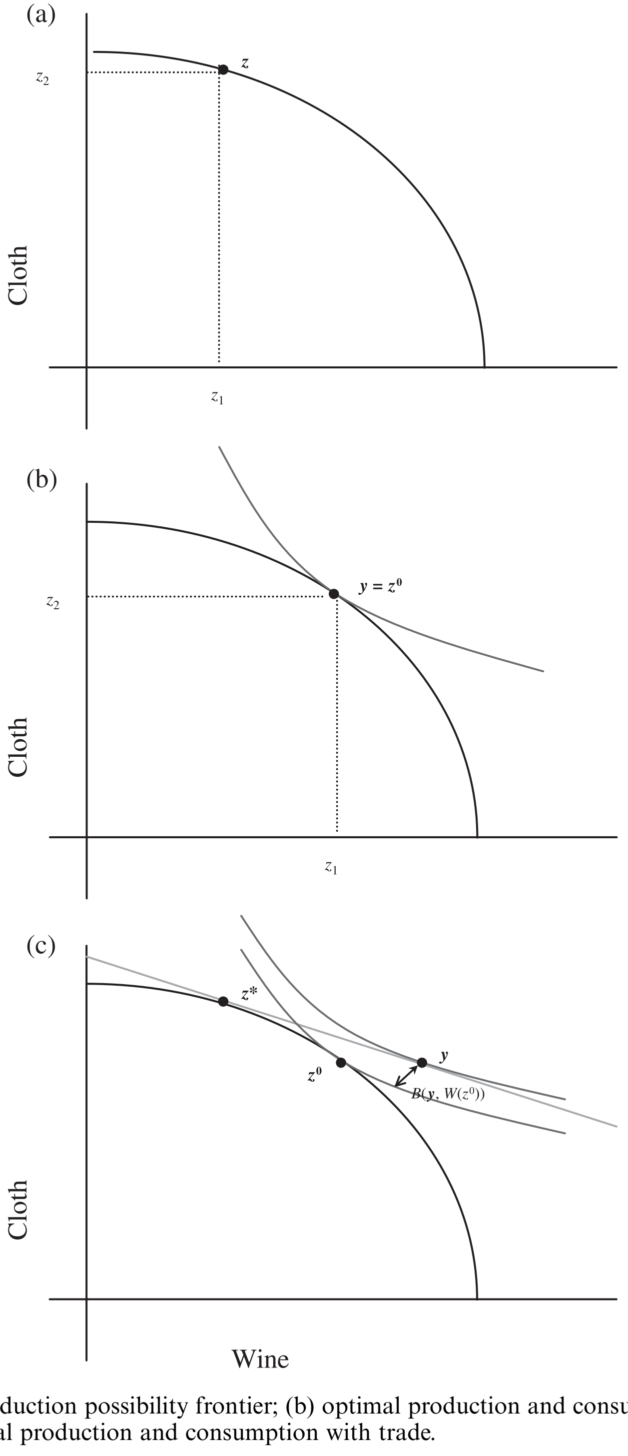

Figure 1 (a) Production possibility frontier; (b) optimal production and consumption without trade; (c) optimal production and consumption with trade.

Now suppose that trade is possible and assume, for simplicity that a small-country assumption applies, so that the budget set is determined by a single relative price ρ. We will take commodity 1, 'Wine', to be the numeraire. Hence, given output $\boldsymbol{z}$, the budget set is

$$Y(\boldsymbol{z}) = \{y : y_1 + \rho y_2 \leq z_1 + \rho z_2\} \tag{9}$$

or, for a general price vector $\boldsymbol{p}$,

$$Y(\boldsymbol{z}) = \{\boldsymbol{y} : \boldsymbol{py} \leq \boldsymbol{pz}\}. \tag{10}$$

As is illustrated in figure 1(c), the budget set through $\boldsymbol{z}$ is represented by a line with slope $-\rho$. The optimal solution is to choose $\boldsymbol{z}^*$ to maximise

$$\boldsymbol{pz} = z_1 + \rho z_2 \tag{11}$$

and then to choose $\boldsymbol{y}$ to maximise $W(\boldsymbol{y})$ subject to the budget constraint $\boldsymbol{py} \leq \boldsymbol{pz}$. Gains from trade may be measured, using the benefit function, as $B(\boldsymbol{y}, W(\boldsymbol{z}^0))$.

Note that the optimal output is independent of the preferences of the representative individual and depends only on world prices. This 'separation' property is a standard feature of models with complete markets, that is, models in which individual actors can trade all goods at prices which they take to be exogenous. As drawn in figure 1(c), separation leads the producer to specialise in cloth, but not completely. Conversely, we might say that in the absence of trade, the production bundle is more diversified.

4.2 Intertemporal production, consumption and borrowing

Although Ricardo solved the problem of comparative advantage, he, like all economists of the 19th century, had considerable difficulty with the concept of interest, and the relationship between time and production. Fisher (1930) was the first to show that the problem of intertemporal production and choice was not, in its essentials, any different from the standard production–consumption problem solved by Ricardo.

In figure 2(a), the commodities 'Cloth' and 'Wine' have been replaced with quantities of a single good (say, wheat) produced at two different dates, Today and Tomorrow. The more wheat is saved as seed Today, the less the final output today and the greater the final output Tomorrow. As in the case of a two-good production technology, the opportunity for intertemporal substitution (producing less Today saves resources, which can be devoted to producing more Tomorrow) gives rise to a transformation curve,

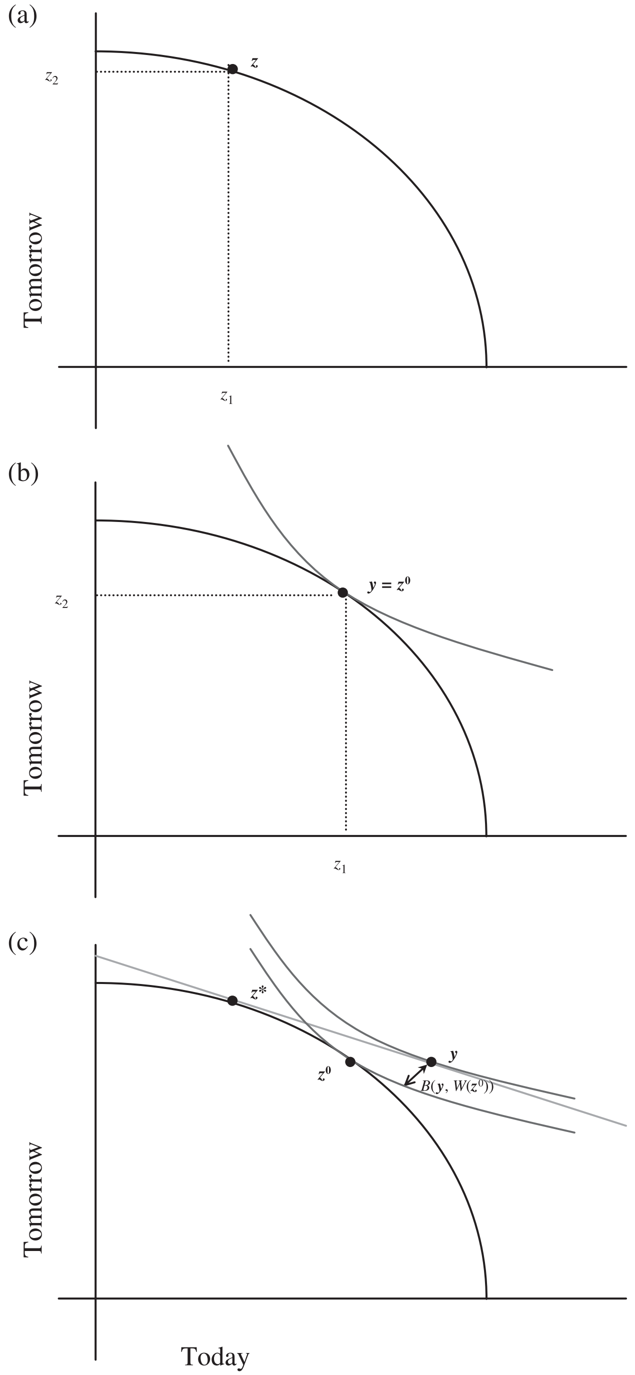

Figure 2 (a) Production possibility frontier; (b) optimal production and consumption without borrowing; (c) optimal production and consumption with borrowing.

concave to the origin, representing the boundary of the production possibility set.

In the absence of opportunities for trade, $\boldsymbol{z} = \boldsymbol{y}$ as before (figure 2b). The tangency point $\boldsymbol{z}^0$ might now be interpreted as the point of equality between the marginal rates of substitution in intertemporal consumption and production.

In the intertemporal setting, trade takes the form of borrowing and lending transactions. In figure 2(c), it is assumed that the individual can borrow or lend as much as is desired at a rate of interest r, giving rise to a budget line with slope $-(1 + r)$. Thus, with period 1 consumption as numeraire, the price of period 2 consumption is $1/(1 + r)$.

As in figure 1(c), the gain from the availability of borrowing and lending presents the opportunities for gains from trade, measured from $B(\boldsymbol{y}, W(\boldsymbol{z}^0))$. The gain from trade in this case is sometimes referred to as a dynamic efficiency gain.[1] As the figures are drawn, the producer at the initial production–consumption point in figure 2(b) might be thought of as having an investment opportunity that has a positive net present value at the interest rate r but that is not attractive at the individual's own, higher marginal discount rate. The availability of borrowing allows the individual to undertake the investment while increasing consumption in period 1, as shown in figure 2(c).

4.3 Production under uncertainty and insurance

Fisher's solution of the problem of intertemporal choice applied only under certainty. The problem of choice under uncertainty remained a complex mystery. The contributions of Keynes (1920) and Knight (1921), while insightful, served more to complicate the issue than to resolve it. It was left to Arrow (1953) and Debreu (1952, 1959) to point out that Fisher's idea of time-dated commodities could easily be extended to provide a tractable model of production and choice under uncertainty. The idea is illustrated in figure 3(a), where the axes are now labelled 'Drought' and 'Rain'.[2] These denote quantities of a given commodity produced in two different states of nature.

[1] Note that this usage of the term 'dynamic efficiency' relates to efficient intertemporal allocation of resources. It bears no relationship to the notion, commonplace in Australian policy discussion, that exposure to competition will produce 'dynamic efficiency gains' distinct from those measured in standard static analysis based on comparative advantage. As can be seen from a comparison of figures 1 and 2, the only difference between dynamic and static efficiency gains relates to the kind of commodities that are under discussion.

[2] Drought will normally be the less favourable state of nature. The precise meaning of 'less favourable' will be discussed in following text.

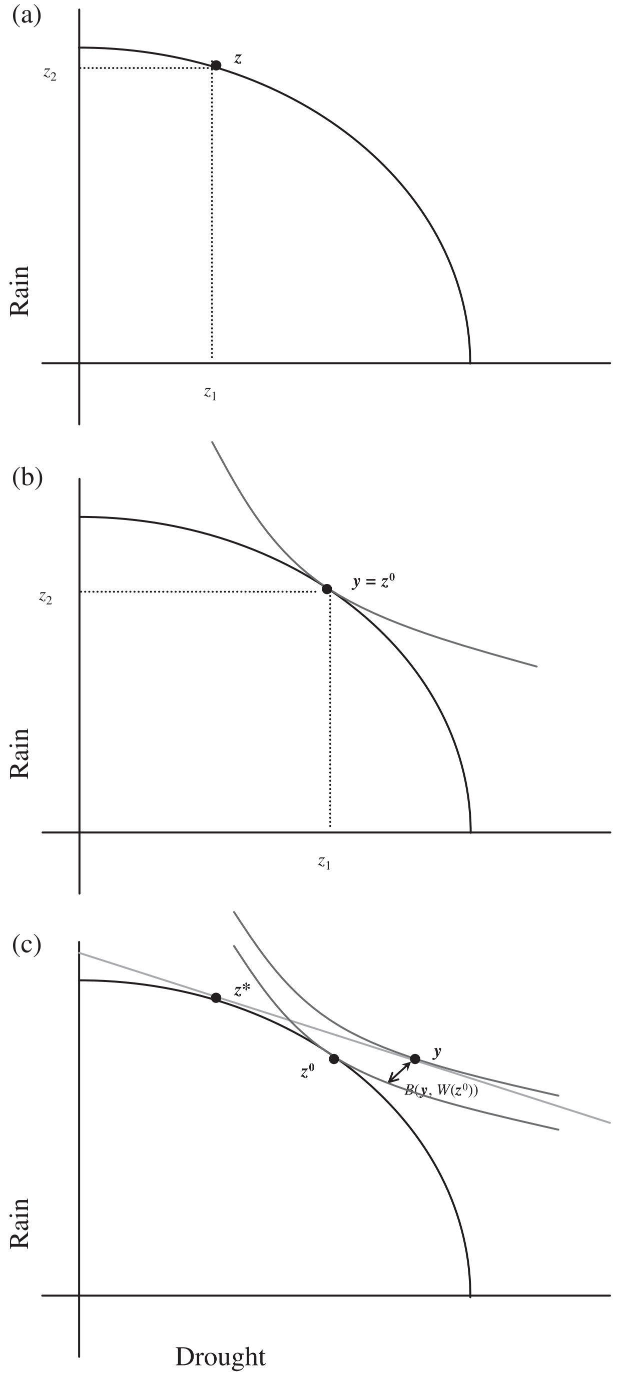

Figure 3 (a) Production possibility frontier; (b) optimal production and consumption without insurance; (c) optimal production and consumption with insurance.

The transformation curve drawn there shows the combinations of state-contingent outputs feasible for an exogenously given input level. It corresponds to the boundary of $Z(\boldsymbol{x})$ defined earlier. For clarity's sake, it is important to emphasise that figure 3(a) is drawn holding fixed the bundle of inputs used by the producer. We do this for two reasons.

First, we want to emphasise the role that substitutability between state-contingent outputs plays in producer decisionmaking under conditions of production uncertainty. Models based on stochastic production functions exclude substitutability between state-contingent outputs by assumption (please see next section and figure 4(b) in particular).

Second, analysing state-contingent output–consumption choice, while holding inputs fixed, crystallises the intuitive connection between producer decisionmaking under uncertainty and autarkic behaviour in traditional Heckscher–Ohlin general-equilibrium models, which treat input endowments as fixed at the country level. It is easily seen, however, that the output equilibration processes described below must always pertain at the producer's optimal input choice. If they did not, then by simply reallocating state-contingent outputs for a given input choice, the producer could always improve welfare.

In the case of wheat production, inputs might be allocated between activities such as expanding the area under dryland cultivation, which will yield an increased output in the presence of adequate rain, and expanding irrigation, which will yield an increased output if there is no rain. Chambers and Quiggin (2000) show how allocation of effort between activities, for each of which state-contingent output is a linear function of effort, gives rise to a production possibility set like that depicted in figure 3(a). The derivation is identical in all important respects to the derivation of a production possibility frontier in the standard two-good, two-factor Heckscher–Ohlin model in the absence of production externalities (jointness between the two outputs).

The problem of production under uncertainty in the absence of financial markets is illustrated in figure 3(b). As before, $\boldsymbol{y} = \boldsymbol{z}$, so producers can only consume what they have produced in any state of nature, just as, in the case of autarky, a nation can only consume what it produces. The optimal choice, represented on the diagram, is $\boldsymbol{y} = \boldsymbol{z}^0$.

We now consider the introduction of insurance. Recall again that output prices have been normalised to unity. Suppose that an insurer is willing to offer insurance against the occurrence of Drought. A standard way of representing such a contract is to suppose that for each dollar of premiums, the insured receives an indemnity $i > 1$ if Drought occurs. However, to see the analogy with trade and with borrowing and lending, it is more useful to think of the producer trading Rain state-contingent income for Drought state-contingent income. For each dollar of Rain state-contingent income paid in premiums, the producer receives a net payout of $(i-1)$ in the Drought state. Hence, setting Drought state-contingent income as the numeraire, the price of Rain state-contingent income is $(i-1)$. We illustrate this in figure 3(c). Note that we assume that the producer can trade freely at the relative price $(i-1)$ and can, if desired, take a position that yields a positive payout in state 2. This means that the producer can 'short' the insurance market and provide insurance to other producers if he or she desires. However, with preferences as drawn, the producer insures against Drought.

The existence of gains from trade is illustrated, as before, by the benefit function $B(\boldsymbol{y}, W(\boldsymbol{z}^0))$. Comparing figure 3(b) and 3(c), the availability of drought insurance leads the producer to increase the value of output at the state-contingent claim-price vector $\boldsymbol{p} = (1, i-1)$ and also to optimise consumption at these prices. Conversely, in the absence of insurance, the producer engages in both self-protection, by choosing the more diversified output $\boldsymbol{z}^0$ rather than the more specialised $\boldsymbol{z}^*$ and in self-insurance, by consuming $\boldsymbol{y} = \boldsymbol{z}^0$ rather than the more diversified $\boldsymbol{y}^*$.[3] The concepts of self-protection and self-insurance, first analysed by Ehrlich and Becker (1972) are discussed in a state-contingent setting by Quiggin (2002).

Notice, in particular, that the self-protecting and self-insuring activities of producers in the absence of an insurance market leaves them at a position that is exactly analogous to the autarkic solution in the production and trade example considered in preceding text.

4.4 Stochastic production function

It might be thought that the state-contingent approach is a new and unnecessarily complex alternative to analysis based on the simple idea of a stochastic production function. The truth is quite the opposite. The state-contingent approach predates, by two decades (early 1950s *vs* 1970s), the representation of production uncertainty in terms of stochastic production functions. More importantly, although superficially simple, the stochastic production function model, when viewed in state-contingent terms is a complex and unnecessarily intractable special case of the general state-contingent technology. So, in reality, the stochastic production is the newer and more complicated alternative to the state-contingent approach.

This is scarcely surprising as exactly the same relationship holds for non-stochastic production. The Arrow–Debreu approach based on production

[3] Note that, as in standard portfolio theory, the adoption of a more diversified production plan will lead to a less risky output vector.

sets incorporates, as a special case, the approach based on production functions. The important difference is that production economics discarded the production function decades ago, because of its implausibility and intractability, and replaced it with an axiomatic version of the Arrow–Debreu approach.[4]

The stochastic production function approach works most naturally with a single output, related to input by a function of the form

$$z = f(\boldsymbol{x}, \varepsilon) \tag{12}$$

where $\varepsilon \in \Re^S$ is a random variable such as rainfall. The idea is to take the standard production function model and extend it by allowing for a stochastic input ε supplied by nature. In the case where there are only two states, that is, two possible values for ε, the behaviour of the production function may be illustrated by two curves as in figure 4(a).

For a given input x, the outputs

$$z_1 = f(x, 1) \tag{13}$$

$$z_2 = f(x, 2) \tag{14}$$

may be read off this graph. Allowing for free disposability, we may draw the corresponding transformation curve as in figure 4(b). This rectangular form is immediately recognisable as the production possibility set for a Leontief-in-outputs, or fixed-output-proportions, technology. While useful as a polar case, everywhere except in the published literature on production uncertainty, this technology is viewed as analytically intractable (because of its points of nondifferentiability) and unrealistic because it denies that producers have the ability to substitute state-contingent outputs.

5. Similarities and differences

The exposition above shows that there are no fundamental analytical differences between the standard model of production and consumption

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[4] Interestingly, this displacement took place at almost the same time that agricultural economists began to focus on stochastic production functions. This bifurcation in the two approaches has often led agricultural economists interested in production uncertainty to question basic conceptual points long since settled elsewhere. A good example is given by the debate over whether producers facing a stochastic technology minimise cost (Chambers and Quiggin 1998). The conclusion to be drawn from the published literature based on the stochastic production function is 'apparently not'. The conclusion from the more general state-contingent approach is 'obviously and trivially yes'.

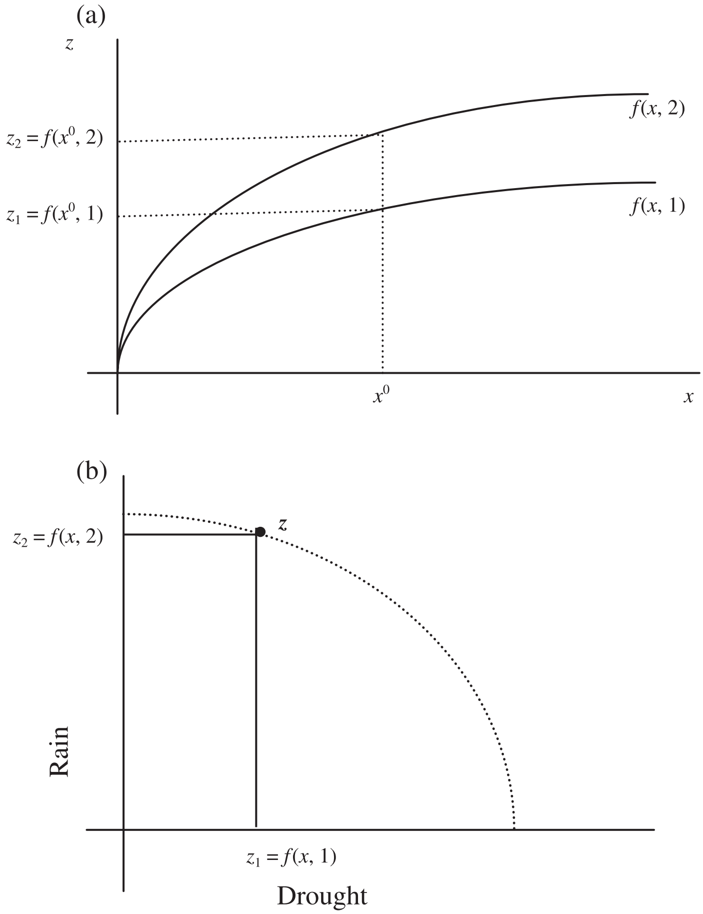

Figure 4 (a) Stochastic production function; (b) production possibility frontier for a stochastic production function.

under certainty, the Fisher model of intertemporal production and consumption and the Arrow–Debreu model of state-contingent production. In all three cases, the theory of optimisation is identical. In all three cases, the Arrow–Debreu existence theorems and the first and second welfare theorems are valid. In all three cases, the concept of 'gains from trade' forms the basis of the economic analysis of market transactions. And in all three cases, the tools of convex analysis and duality theory are applicable.

There are, however, important differences. In some cases, these enhance the power of economic analysis. The special characteristics of time and uncertainty allow for additional structure that is not available for general choice and production problems. For example, the fact that time flows sequentially allows, with some plausible assumptions about the associated preferences, the application of differential and difference equation methods and the derivation of the associated Euler equations.

In the case of uncertainty, the concepts of subjective probability and various forms of expected utility arise naturally from the structure of the choice problem and from basic intuitions about relative likelihood. These points are developed in detail in the next section.

There are also important features of the intertemporal and uncertainty problems that limit the applicability of basic results, such as the fundamental welfare theorems. In the standard production model the assumption that there exist competitive markets for each commodity seems a plausible starting point for analysis. Violations of this assumption are commonly described as 'imperfections' reflecting the intuition that, while important, they represent modifications to reasonable *prima facie* assumptions about the optimality of market outcomes.

By contrast, in intertemporal choice and even more in the case of uncertainty, missing markets are the rule rather than the exception. Agricultural producers, for example, can hedge against variations in output prices, but output prices represent only a few of the many uncertain variables that might affect the profitability of agricultural enterprises.

In these circumstances, results derived under the assumption of complete markets, as the results developed in preceding text have been, must be seen as a polar case useful in analysis rather than as a starting point for a realistic assessment of policy issues. Policy analysis will almost invariably take place in a second-best context where, for example, the creation of a new market will not necessarily enhance welfare, as the number of missing markets is large.

A final, more transitory, distinction between the economics of uncertainty and the economics of certainty arises from the persistent impact of analytical confusions regarding uncertainty, reflected, for example, in the prevalence of stochastic production function models. Distinguishing the significant positive contributions made in the published literature on production under uncertainty from the errors associated with inappropriate approaches to modelling is a complex task that will take a long time to complete.

6. Probabilities

The concept of probability plays a crucial role in most discussions of uncertainty. Thus far, we have not made any use of probability concepts, focusing on those aspects of the problem of production under uncertainty that are also relevant for general multi-output production. Before considering probability, it is useful to focus on the notion of certainty. In figure 5(a), the

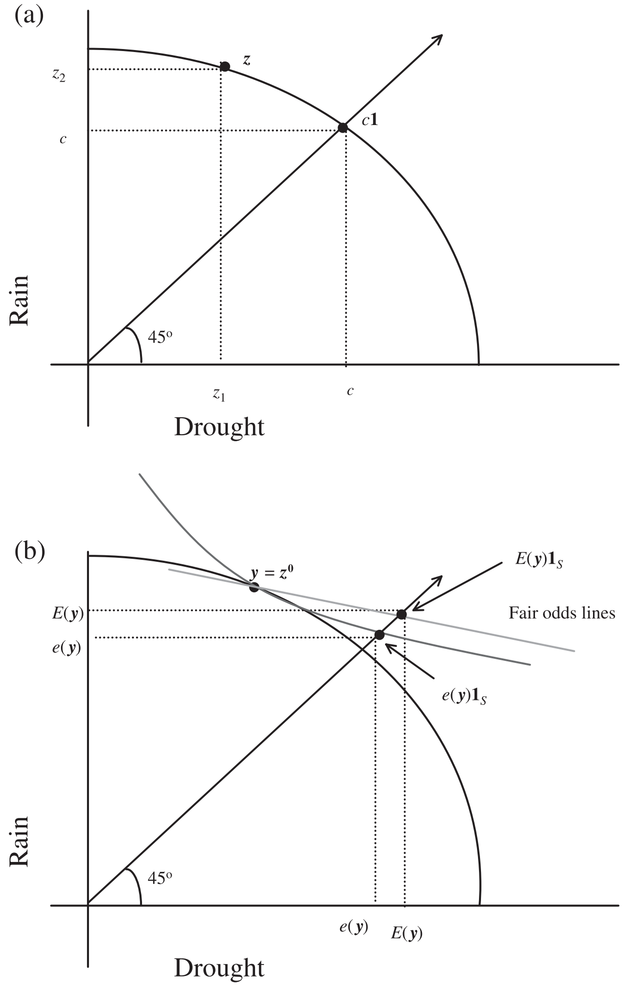

Figure 5 (a) Stochastic and nonstochastic outputs; (b) fair odds.

production possibility diagram of figure 3(a) has been augmented by the inclusion of a diagonal line through the origin, referred to by Chambers and Quiggin (2000) as the 'bisector', as it bisects the positive quadrant and, for that matter, the negative quadrant.

Points on the bisector, of the general form $c_{\mathbf{1}}$, satisfy $z_1 = z_2 = c$ and therefore correspond to non-stochastic outputs. A crucial point observed by Chambers and Quiggin (2000) is that, in the state-contingent model, output uncertainty is the result of producer choices. Producers, if they choose, can stabilise output completely, though they may incur (potentially very large) costs in doing so. This is true even in the case of the stochastic production function. However, in this case, output stabilisation can be achieved only by throwing away the 'surplus' output in all but the worst state of nature.

In figure 5(b), we consider the relationship between preferences and probabilities. Drawing the indifference curve through $\boldsymbol{z}^0$, and noting that, in the absence of insurance, $\boldsymbol{y}^0 = \boldsymbol{z}^0$, we include a line tangential to the indifference curve at the bisector. Suppose that the probabilities of the two states are π_1 and π_2, with

$$\pi_1 + \pi_2 = 1 \tag{15}$$

and that the individual is risk-averse in the sense that for any $\boldsymbol{y}$

$$W(\boldsymbol{y}) \leq W(E(\boldsymbol{y})\mathbf{1}_S), \tag{16}$$

where

$$E(\boldsymbol{y}) = \boldsymbol{\pi}_1 y_1 + \boldsymbol{\pi}_2 y_2. \tag{17}$$

Considering $\boldsymbol{y}$ on the marked indifference curve and close to the bisector, it is easy to see that risk-aversion holds if and only if the slope of the tangent line is $-\pi_1/\pi_2$ at every point on the bisector.[5] A line with this slope is referred to as a 'fair-odds line'.

Moreover, consideration of the intersection between the indifference curve and the bisector gives rise to a natural canonical representation of preferences under uncertainty. For any $\boldsymbol{y}$, the certainty equivalent $e(\boldsymbol{y})$ is defined as:

$$e(\boldsymbol{y}) = \min\{t : W(t\mathbf{1}_S) \geq W(\boldsymbol{y})\}. \tag{18}$$

The risk premium is given by

$$\rho(\boldsymbol{y}) = E(\boldsymbol{y}) - e(\boldsymbol{y}), \tag{19}$$

and risk-aversion is equivalent to the requirement that $\rho(\boldsymbol{y}) \geq 0$, for all $\boldsymbol{y}$. The risk premium is illustrated as the (vertical or horizontal) distance from $E(\boldsymbol{y})$, given by the intersection with the bisector of the fair-odds line through $\boldsymbol{y}$, to $e(\boldsymbol{y})$, given by the intersection with the bisector of the indifference curve through $\boldsymbol{y}$.

[5] The precise statement of this result depends on the assumption that W is differentiable in a neighbourhood of the bisector. In the absence of differentiability, the tangent must be replaced by a subdifferential, allowing for risk-aversion with respect to multiple probability distributions.

Next, consider the fair-odds line through z^0 in figure 5(b). As output is shifted from Rain to Drought, the fair-odds line cuts the production possibility curve from below. By convexity, the same is true at any point on the production possibility curve between z^0 and the bisector. That is, if state-contingent outputs are initially equal, a reallocation of effort from Drought state-contingent output to Rain state-contingent output will result in an increase in expected output. In this sense, the Rain state is the more favorable state of nature. A producer concerned to maximise expected output must produce more in the Rain state than in the Drought state.

7. Drought policy

Drought policies can be divided into three classes: (i) drought relief policies designed to offset losses incurred by farmers as a result of drought; (ii) insurance, including multiple-risk and area-yield crop insurance programs of the kind operated in the USA (Chambers and Quiggin 2002) and rainfall insurance programs, such as those discussed by Bardsley (1984) and Quiggin (1986); and (iii) consumption smoothing policies aimed at enabling farmers to smooth their consumption over drought and non-drought states of nature through saving and borrowing.

The idea of drought relief is simple. Farmers who have incurred losses as a result of drought receive partial compensation from government. A drought-relief program might be modelled as a state-contingent payment $q_s(\boldsymbol{x}, z_s)$ such that $q_s \geq 0$ if s is a drought state and $q_s \leq 0$ if s is not a drought state. The payment q_s is normally increasing in inputs $\boldsymbol{x}$ and decreasing in output z_s.

The idea that drought relief can involve negative payments $q_s < 0$ in non-drought states might seem inconsistent with observed experience. Historically, various kinds of assistance to farmers have been justified as 'tariff compensation' or as part of a system of 'protection all round'. In this view, payments to farmers in drought conditions might be seen as compensation for negative rates of effective protection for farmers that lead to the extraction of rent from farmers under normal conditions.

Much of the analysis of drought relief also applies to multiple-risk and area-yield crop insurance of the type that has prevailed in the USA. The two main differences are that payments are not conditional on the occurrence of a particular state of nature and that participation is voluntary and requires payment of a premium. Voluntary participation implies the existence of an adverse selection problem as discussed by Just *et al.* (1999). The payment of a premium might involve some wealth effects. These issues will not be addressed in the present paper. Conditional on participation, and assuming wealth effects are not important, the producer's problem in the presence of multiple-risk crop insurance is similar to that in the presence of drought relief programs.

Drought insurance schemes involve a payment to the producer q_s that is purely state-contingent, such that $q_s \geq 0$ if s is a drought state and $q_s \leq 0$ if s is not a drought state. An actuarially fair drought insurance scheme involves a payment q_s satisfying

$$\sum_s \pi_s q_s = 0. \tag{20}$$

Schemes designed to facilitate consumption smoothing have many of the same characteristics as drought insurance schemes. In particular, consider an idealised consumption-smoothing scheme in which farmers can deposit into and withdraw or borrow from a 'buffer fund', subject only to a requirement of long-run solvency that, for each farmer, average deposits should equal average withdrawals. Suppose farmers follow a fixed policy, in which the amount of the deposit or withdrawal q_s, depends only on the state of nature s. Then the average solvency requirement is that

$$\sum_s \pi_s q_s = 0, \tag{21}$$

which is the same as (20). To distinguish between the models of choice under uncertainty over time, it is necessary to impose more structure than is present in the model presented here.

For the case $S = 2$, $q_1 = i - 1$, $q_2 = -1$, and actuarial fairness requires

$$\frac{1}{i-1} = \frac{\pi_1}{\pi_2}. \tag{22}$$

Equivalently, the insurer's expected profit, at the probabilities (π_1, π_2) is

$$1 - i\pi_1 = 0. \tag{23}$$

8. Effects of drought policy

We are now ready to consider the resource allocation and welfare effects of a range of polices that might be considered as a response to drought. A large class of such policies may be represented by output–payment plans $(\boldsymbol{z}, \boldsymbol{y})$.

We assume that government is risk-neutral. Hence, any first-best allocation must involve the production of the risk-neutral optimal output, that is, the solution $\boldsymbol{z}^{FB}$ to the problem:

$$\boldsymbol{z}^{FB} = \arg\max_{z} \{E[\boldsymbol{z}] - c(\boldsymbol{w}, \boldsymbol{z})\}, \tag{24}$$

and the receipt by farmers of a non-stochastic income vector of the form $y^{FB}\mathbf{1}_S = E[z]$, where

$$\boldsymbol{z} = \arg\max_{z}\{E[z] - c(\boldsymbol{w}, \boldsymbol{z})\}. \tag{25}$$

By contrast, in the absence of intervention, we have the 'self-insurance' solution, $\boldsymbol{y}^0 = \boldsymbol{z}^0$, where

$$\boldsymbol{z}^0 = \arg\max_{z}\{e(\boldsymbol{z} - c(\boldsymbol{w}, \boldsymbol{z}))\}. \tag{26}$$

We will assume that the first-best solution involves a Pareto improvement on the self-insurance solution. That is, assume that $y^{FB} \geq e(\boldsymbol{y}^0)$ so that farmers are better off, and that $y^{FB} \leq E[\boldsymbol{z}^{FB}]$ so that the government has a non-negative expected return.[6] The question of the precise distribution of gains is not crucial to the analysis. To avoid wealth effects, and simplify the diagrammatic exposition, we will assume that producer preferences display constant absolute risk aversion and that the first-best solution is that derived from actuarially fair insurance, so that $y^{FB} = E[\boldsymbol{z}^{FB}]$.

In general terms, we expect that the first-best solution $(\boldsymbol{z}^{FB}, y^{FB}\mathbf{1}_S)$ will involve a riskier output plan than the solution $(\boldsymbol{z}^0, \boldsymbol{y}^0 = \boldsymbol{z}^0)$ chosen by a risk-averse producer. Note that, by definition $E[\boldsymbol{z}^{FB}] - c(\boldsymbol{w}, \boldsymbol{z}^{FB}) \geq E[z] - c(\boldsymbol{w}, \boldsymbol{z})$ for any feasible $\boldsymbol{z} \neq \boldsymbol{z}^{FB}$, including $\boldsymbol{z}^0$. For the case $S = 2$, we can make this more precise. Let the plan $(\boldsymbol{z}^{FB}, y^{FB}\mathbf{1}_S)$ be a Pareto-improvement on $(\boldsymbol{z}^0, \boldsymbol{y}^0)$, where $\boldsymbol{y}^0 = \boldsymbol{z}^0$. Then

$$z_1^{FB} - c(\boldsymbol{w}, \boldsymbol{z}^{FB}) \leq z_1^0 - c(\boldsymbol{w}, \boldsymbol{z}^0) \leq z_2^0 - c(\boldsymbol{w}, \boldsymbol{z}^0) \leq z_2^{FB} - c(\boldsymbol{w}, \boldsymbol{z}^{FB}). \tag{27}$$

That is, returns net of costs are lower in the bad state and higher in the good state for the first-best solution than for that chosen by a risk-averse producer, as shown in figure 6(a). Having access to fair insurance, the producer does not need to engage in costly self-insurance.

It remains to examine the welfare effects of alternative drought policies relative to the first-best. For simplicity, we will focus on the case where policies completely stabilise net farm income, as in the first-best, assuming optimal production responses. It is straightforward to show that, under the assumptions stated above, drought insurance that completely stabilises income will lead to a Pareto-optimal outcome. Choose (q_1, q_2) such that

$$q_1 - q_2 = z_2^{FB} - z_1^{FB}. \tag{28}$$

[6] This means that the market is 'insurable' in the sense used by Chambers (1989). More simply, there are potential gains from trade.

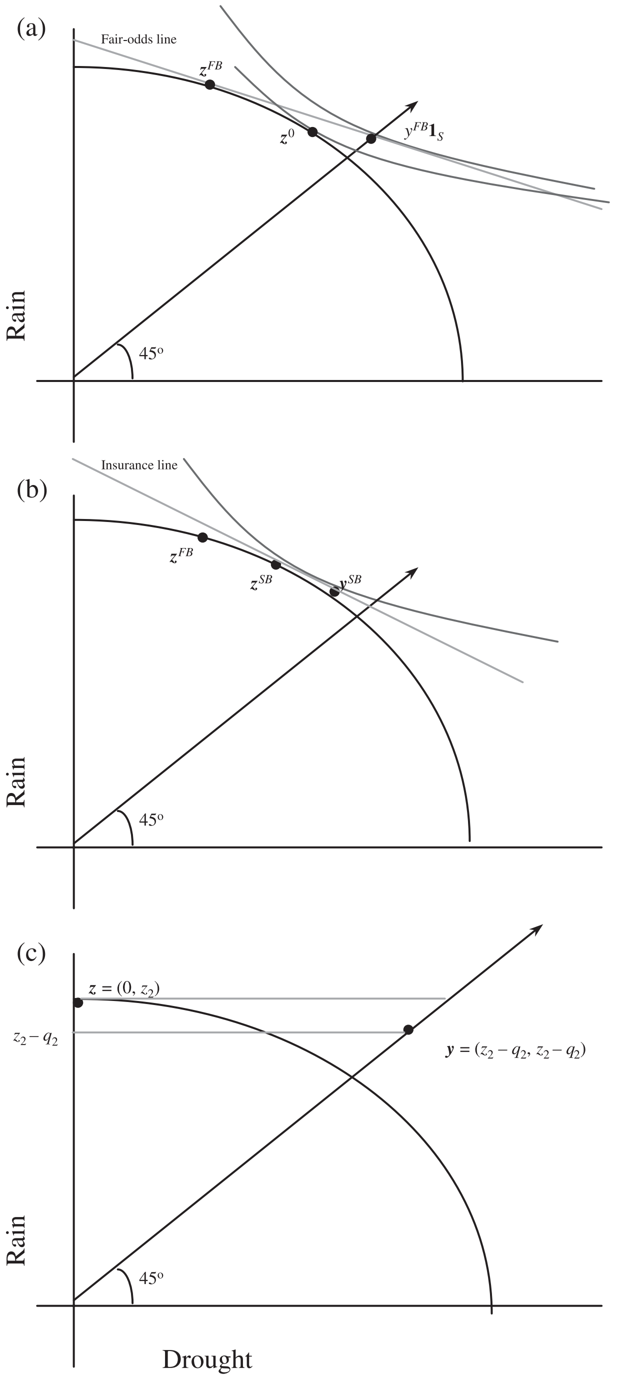

Figure 6 (a) Optimal production and consumption with fair insurance; (b) optimal production and consumption with unfair insurance; (c) drought relief and moral hazard.

Then the consumption vector $\boldsymbol{y} = \boldsymbol{z}^{FB}$, obtained by producing the first-best output, is a non-stochastic vector of the form $y^{FB}\mathbf{1}_S$, where

$$y^{FB} = q_2 + z_2^{FB}. \tag{29}$$

Now consider any $\boldsymbol{z}' \neq \boldsymbol{z}^{FB}$. Two possible cases arise. If $\boldsymbol{y}' = \boldsymbol{z}' + \boldsymbol{q}$ is non-stochastic, it is equal to $y'\mathbf{1}_S$ for some $y' \leq y^{FB}$, and is therefore statewise dominated by $y^{FB}\mathbf{1}_S$. If $\boldsymbol{y}'$ is stochastic it is riskier than the non-stochastic $y^{FB}\mathbf{1}_S$ and has a lower mean. Hence, all risk-averse decision makers will prefer $y^{FB}\mathbf{1}_S$.

This point may be illustrated by consideration of figure 6(a). Because the fair-odds line is tangent to the production possibility frontier at $\boldsymbol{z}^{FB}$, it lies above the fair-odds line through any other $\boldsymbol{z}$. Risk aversion implies that the most preferred point on the fair-odds line is $y^{FB}\mathbf{1}_S$.

Figure 6(b) shows the case where drought insurance is offered at actuarially unfair prices. The insurance line is steeper than under actuarially fair insurance, implying that

$$\frac{-q_2}{q_1} \geq \frac{\pi_1}{\pi_2}. \tag{30}$$

As a result, the optimal output $\boldsymbol{z}^{SB}$ is less risky and has a lower mean than the first-best $\boldsymbol{z}^{FB}$. Moreover, full insurance is no longer optimal. The preferred consumption vector $\boldsymbol{y}^{SB}$ leaves the producer bearing some risk.[7]

Finally, in figure 6(c), we consider the implications of drought relief, modelled as an *ex post* payment with the property

$$q_1(\boldsymbol{z}) - q_2 = z_2 - z_1, \tag{31}$$

so that income is completely stabilised. Note that, unlike the case of drought insurance, the payment made in the 'drought state' depends on the producer's choice of state-contingent output $\boldsymbol{z}$. The effect of drought relief is that the producer can pick the 'rain' output z_2, receiving net income $z_2 - q_2$, and is guaranteed the same net income if a drought occurs. In these circumstances, there is no incentive to allocate any resources to preparation for drought.

[7] The optimality of partial insurance depends on the assumption of smooth preferences, which is implicit in the way the figure has been drawn.

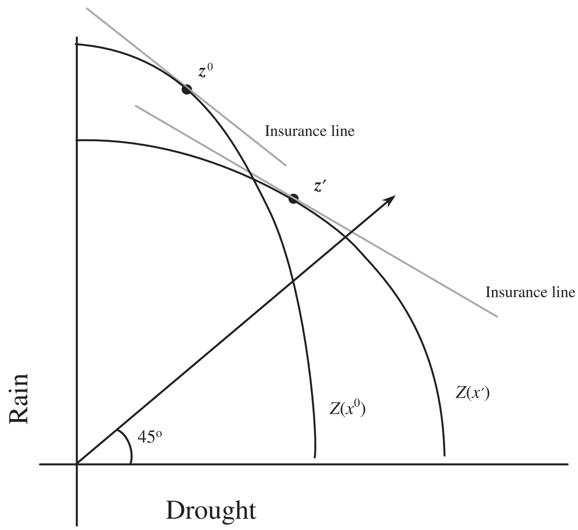

Figure 7 Endogenous input choices.

8.1 Drought policy and input choices

To permit a complete analysis, we drop the simplifying assumption that we have been maintaining since our discussion of figure 3(a, b) that input levels are fixed,[8] and now briefly consider equilibrium input choices. However, we retain the simplifying assumption of a single output commodity $M = 1$, with non-stochastic price normalised to 1, and focus on the case $S = 2$.

Consider a choice between two input vectors $\boldsymbol{x}^0$ and $\boldsymbol{x}'$, and assume, for simplicity, that $\boldsymbol{wx}^0 = \boldsymbol{wx}'$. As shown in figure 7, each gives rise to a separate output set, $Z(\boldsymbol{x}^0)$ and $Z(\boldsymbol{x}')$ respectively, each with its own production possibility frontier. For any given set of state-contingent prices, the optimal output $\boldsymbol{z}^0$, conditional on inputs $\boldsymbol{x}^0$, is riskier than the optimal output $\boldsymbol{z}'$, conditional on inputs $\boldsymbol{x}'$. This implies, that $\boldsymbol{x}^0$ contains a higher level of risk-complementary inputs and a lower level of risk-substituting inputs than $\boldsymbol{x}'$.

Now consider the insurance lines through $\boldsymbol{z}^0$ and $\boldsymbol{z}'$, corresponding respectively to actuarially fair insurance and to unfair insurance with $-q_2/q_1 \geq \pi_1/\pi_2$. With actuarially fair insurance the more risk-complementary input vector $\boldsymbol{x}^0$ yields a higher return, but the reverse is true for unfair insurance. Thus, under unfair insurance the producer will engage in 'self-protection' through the choice of a risk-substituting input bundle.

[8] As pointed out earlier, this is tantamount to examining optimal state-contingent output choices given optimally chosen input levels.

Conversely, if the effect of drought policy is to provide subsidised insurance, the preferred input bundle will be one that is more risk-complementary than the cost-minimising first-best choice. As critics of Australian drought policy have long pointed out, the provision of *ex post* relief based on observed losses penalises producers who have engaged in prudent and efficient self-protection.

The extreme case is that of the drought relief policy considerered above, in which producers receive a payment $z_2 - z_1$. In effect, the insurance line is horizontal. For any given level of input cost, the optimal state-contingent production vector is that which maximises z_2. Since this output is riskier than the first-best, it will be associated with higher use of risk-complementary inputs and lower use of risk-substituting inputs.

9. Concluding comments

The idea that production and choice under uncertainty can be represented in terms of commodities contingent on the occurrence of a state of nature is an old one. It traces back to the pioneering general equilibrium models of Arrow (1953) and Debreu (1952). Arrow and Debreu used convex analysis to demonstrate the existence and Pareto-optimality of equilibrium, but neither they nor subsequent writers in the published general equilibrium literature presented much analysis of comparative statics and similar issues of concern to policy economists. At about the same time, Shephard (1953) was developing the first systematic applications of duality theory in economics. It has taken nearly fifty years for these two streams of thought to merge. Chambers and Quiggin (2000) give the first systematic application of modern tools of convex analysis and duality theory to the problems of production and choice under uncertainty.

A noteworthy feature of the analysis presented in Chambers and Quiggin (2000) is that the expected-utility hypothesis is not required, except as an illustrative special case. At this point, we call the reader's attention to the fact that we have nowhere invoked the expected-utility hypothesis in our discussion. Thus, the same is true of the graphical analysis presented in the present paper. In some particular applications, the additive functional form associated with the expected-utility model proves useful as a simplifying assumption, but for most purposes the assumption of risk-aversion is sufficient to permit a simple and informative analysis.

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