The Australian Journal of Agricultural and Resource Economics, 44:2, pp. 289–298

The role of expected protein levels in determining the impact of protein premiums and discounts: a note†

Elizabeth Petersen and Rob Fraser*

Fraser (1997) considered the impact of protein premiums and discounts on a grower's income stream and willingness-to-pay for a forward contract where the protein premium and discount system is centred on a grower's existing expected protein level. This article extends these results to consider the impact of a protein premium and discount system which is not centred on a grower's existing expected protein level. The article suggests that the grower's existing expected protein level plays a crucial role in determining the impact of the system.

1. Introduction

Fraser (1997) showed that, due to the negative correlation between wheat price and yield, the effect of a protein premium and discount system centred on a grower's existing expected protein level is a decrease both in the expected level and variance of income. Additionally, Fraser (1997) found that protein premiums and discounts increase a grower's willingness-to-pay for a forward contract, and that this relationship is positively related to both the size of the premium or discount and the grower's level of seasonal variability.

This article extends these results to consider the role of a grower's existing expected protein level relative to the central point of the system in determining the impact of protein premiums and discounts. This role is examined over a realistic range of expected protein levels for wheat growers (9 per cent–13 per cent) and it is shown that, depending on this level, both the expected level and variance of income effects can be negative, or the former positive and the latter negative. This analysis determines not only the

† The authors gratefully acknowledge the helpful comments of anonymous reviewers and the Editor. They also acknowledge the financial support of the Grains Research and Development Corporation (GRDC).

* Elizabeth Petersen and Rob Fraser, Agricultural and Resource Economics, University of Western Australia, Nedlands, Western Australia.

expected protein level at which these effects are in conflict, but also which effect dominates in determining the overall impact on grower utility. In addition, it is suggested that the impact of the protein premium and discount system on a grower's willingness-to-pay for a forward contract is dependent on the expected protein level.

The structure of the article is as follows. The second section develops the model of the impact of protein premiums and discounts which allows for a range of levels of expected protein. The third section reports the results of a numerical analysis of this model in relation to a grower's income stream and willingness-to-pay for a forward contract. The article concludes with a brief summary.

2. Modelling the impact of the protein premium and discount system

The model is based on that developed in Fraser (1997), with two main modifications. The first is a simplification of the specified relationship between yield and protein. In Fraser (1997) this relationship was represented by a hyperbolic form:

$$y = \gamma / r \tag{1}$$

where:

r = uncertain protein level;
γ = parameter relating the joint variability of yield and protein;
y = uncertain yield per hectare.

In what follows this relationship is simplified to a linear form:

$$y = a - br \tag{2}$$

where:

a = notional maximum yield;
b = parameter relating the joint variability of yield and protein.

Note that while the parameter a is simply a scaling factor, the parameter b represents the biological trade-off between yield and protein level in the development of the wheat crop. As pointed out in Fraser (1997), the accuracy with which this linear form can be substituted for the hyperbolic form depends on the extent of seasonal variation. For example, a coefficient of yield variation of 20 per cent means that 70 per cent of the probability distribution lies within one standard deviation of the mean. Consequently, while such a substitution in the context of the relatively stable-yielding wheat-growing regions of Western Australia (WA) appears to be acceptable, its application to other less-stable wheat-growing regions in Australia is questionable.[1] Nevertheless, in these situations sensitivity analysis can clarify the significance of the simplification in question, and in what follows the sensitivity analysis verifies the robustness of the pattern of results to changes in the slope of the linear relationship. Moreover, since any hyperbolic form can be closely approximated by a series of linear segments, our view is that this simplification does not significantly weaken the applicability of our analysis.

The second modification is a generalisation of the relationship between price and protein. In Fraser (1997) this relationship was restricted to only three discrete grades of wheat: 'high, medium and low protein' (ibid., p. 142). It has subsequently become apparent that a weakness of this specification is that it substantially inhibits analysis of the role of differing protein levels in determining the impact of protein premiums and discounts on the wheat grower's income stream. Consequently, in what follows, the relationship between price and protein is specified to represent more accurately the Australian Wheat Board's (AWB) existing protein premium and discount scale. Since this scale is based on protein premium and discount increments for each 0.1 per cent of protein (AWB 1998), the (uncertain) price the grower receives (p) can be represented by:

$$p = p_B + (r - 0.1)x \tag{3}$$

where:

p_B = uncertain base price per tonne for wheat;
x = premium or discount per unit of protein above or below 10 per cent.[2]

With this specification, the grower's uncertain income per hectare in the absence of protein premiums and discounts (I_0) is given by:

$$\begin{aligned} I_0 &= py \\ &= p_B(a - br) \end{aligned} \tag{4}$$

so that expected income ($E(I_0)$) and the variance of income ($Var(I_0)$) are given by:[3]

$$E(I_0) = \overline{p}_B(a - b\overline{r}) \tag{5}$$

[1] We are grateful to an anonymous referee for pointing this out to us.

[2] Note that this specification implies a symmetrical impact of the protein premium and discount system on expected price. Because an asymmetrical system has clear implications for its impact on expected price, and therefore on expected income, this complication is not considered here.

[3] See Mood, Graybill and Boes (1974), p. 180.

$$Var(I_0) = (a - b\bar{r})^2 Var(p_B) + \bar{p}_B^2 b^2 Var(r) + b^2 Var(p_B) Var(r) \tag{6}$$

where:

$\bar{p}_B$ = expected base price;
$\bar{r}$ = expected protein level;
$Var(p_B)$ = variance of base price;
$Var(r)$ = variance of protein level.

Note that, as in Fraser (1997), the grower's uncertain base price and protein level (as determined by seasonal uncertainty) have been assumed to be independent.

In addition, the grower's uncertain income per hectare in the presence of protein premiums and discounts (I_1) is given by:

$$\begin{aligned} I_1 &= py \\ &= (p_B + (r - 0.1)x)(a - br) \end{aligned} \tag{7}$$

so that expected income ($E(I_1)$) and the variance of income ($Var(I_1)$) can be represented by:[4]

$$\begin{aligned} E(I_1) &= \bar{p}_B(a - b\bar{r}) + E((a - br)(r - 0.1)x) \\ &= \bar{p}_B(a - b\bar{r}) + a(\bar{r} - 0.1)x + b0.1x\bar{r} - bx(Var(r) + \bar{r}^2) \end{aligned} \tag{8}$$

$$\begin{aligned} Var(I_1) \approx{} & (a - b\bar{r})^2(Var(p_B) + x^2 Var(r)) + (\bar{p}_B + (\bar{r} - 0.1)x)^2 b^2 Var(r) \\ & - 2(\bar{p}_B + (\bar{r} - 0.1)x)(a - b\bar{r}) bx Var(r) \end{aligned} \tag{9}$$

---

[4] Note that: $E(X^2) = Var(X) + (E(X))^2$, and
$$Var(XY) \approx \bar{Y}^2 Var(X) + \bar{X}^2 Var(Y) + 2\bar{X}\bar{Y} Cov(X, Y)$$
(see Mood *et al.* 1974, p. 181).

Therefore,

$$\begin{aligned} Var(I_1) \approx{} & (a - b\bar{r})^2 Var(p_B + (r - 0.1)x) + (\bar{p}_B + (\bar{r} - 0.1)x)^2 b^2 Var(r) \\ & + 2(\bar{p}_B + (\bar{r} - 0.1)x)(a - b\bar{r}) Cov((p_B + (r - 0.1)x), (a - br)) \end{aligned} \tag{A}$$

Since:

$$\begin{aligned} & Var(p_B + (r - 0.1)x) = Var(p_B) + x^2 Var(r), \text{ and} \\ & Cov((p_B + (r - 0.1)x), (a - br)) \\ & \quad = E((p_B + (r - 0.1)x) - (\bar{p}_B + (\bar{r} - 0.1)x)((a - br) - (a - b\bar{r})) \\ & \quad = -bx Var(r) \qquad \text{(see Mood } et\ al. \text{ 1974, p. 178)} \end{aligned}$$

it follows that (A) can be rewritten as:

$$\begin{aligned} Var(I_1) \approx{} & (a - b\bar{r})^2(Var(p_B) + x^2 Var(r)) + (\bar{p}_B + (\bar{r} - 0.1)x)^2 b^2 Var(r) \\ & - 2(\bar{p}_B + (\bar{r} - 0.1)x)(a - b\bar{r}) bx Var(r) \end{aligned}$$

This equation is reproduced as equation 9 in the main text.

On this basis, the impact of the protein premium and discount system on a grower's income stream is represented by the differences between the pairs of equations 5 and 8, and 6 and 9. For $\bar{r} = 0.1$, these differences can be shown to reproduce the special cases of Fraser (1997): a decrease in both the expected level and variability of income (for 'small' values of x). However, for other values of $\bar{r}$ these differences are analytically ambiguous. Therefore, the numerical analysis of the next section will explore the role of the level of $\bar{r}$ in determining the impact of the protein premium and discount system. In addition, this analysis will extend that of Fraser (1997) by considering the role of the expected protein level in determining the impact of a protein premium and discount system on a grower's willingness-to-pay for a forward contract.

3. Numerical analysis of the impact of protein premiums and discounts on a grower's income stream

In order to undertake the numerical analysis it is necessary to specify a functional form for the grower's utility of income. As in Fraser (1997), it is assumed that this utility function is given by the constant relative risk aversion form:

$$U(I) = \frac{I^{1-R}}{1-R}$$

where R = (constant) coefficient of relative risk aversion.

Note that Pope and Just (1991) provide empirical evidence to support this specification. In addition, Fraser (1991) provides evidence of the robustness of numerical results with respect to different forms of the utility function.

The following parameter values have been chosen for a base case:

$\overline{p}_B = 150$;
CVp_B = coefficient of variation of base price = 0.2;
$a = 3.5$;
$b = 15$;
CVr = coefficient of variation of protein = 0.2;
$x = 500$ (\$A5/ per cent protein);
$R = 0.5$.

Note that $\overline{p}_B$, a and b have been chosen to approximate actual values. In particular, the findings of Robinson (1995) have been used to empirically inform the value chosen for b to represent the biological trade-off between yield and protein level in the wheat crop. In addition, Anderson *et al.* (1988) provide supporting estimates of seasonal variability for WA wheat growers, and Bardsley and Harris (1987) provide supporting estimates of attitudes to risk in the wheatbelt of Australia. Finally, x is based on the current premium and discount level (AWB 1998).

Figures 1, 2 and 3 present values for $E(I)$, $Var(I)$ and Expected Utility (EU) in the presence and absence of protein premiums and discounts for a range of values of $\bar{r}$. They show that, for $\bar{r}$ less than 10.19 per cent, the effects on $E(I)$ and $Var(I)$ with the introduction of protein premiums and discounts are both negative. However, the overall negative impact on EU shows that the effect on $E(I)$ is dominating. Whereas for $\bar{r}$ between 10.19 per cent and 10.33 per cent, the effects on $E(I)$ and $Var(I)$ are both negative, but the positive impact on EU shows that the effect on $Var(I)$ is dominating. For $\bar{r}$ greater than 10.33 per cent, $E(I)$ is increased and $Var(I)$ is decreased, so both effects are contributing to a positive impact on the grower's EU.

These results both confirm those of Fraser (1997) that a protein premium and discount system centred on the grower's average protein level (i.e. 10 per cent) reduces $E(I)$ and $Var(I)$, and generalise them for growers with other values of $\bar{r}$. In particular, although for growers with $\bar{r}$ greater than 10.33 per cent $E(I)$ increases, allowing for the fact that the effect on $E(I)$ is not always the dominating effect, it can be seen that growers with $\bar{r}$ less than 10.19 per cent are disadvantaged by the scheme while growers with $\bar{r}$ greater than this level are advantaged.

Table 1 contains details of a sensitivity analysis investigating the influence of changes in b, CVr, and x on this pattern of results.[5] In column (1) values of $\bar{r}$ are presented where the overall impact on EU is equal to 0. For values of $\bar{r}$ less than those presented in this column the effects on $E(I)$ and $Var(I)$ are both negative but the $E(I)$ effect is dominating, resulting in a negative impact on EU. For values of $\bar{r}$ greater than those presented in column (2), the effect on $E(I)$ is positive while that on $Var(I)$ remains negative, and so both effects are contributing to a positive impact on EU. It can be seen from table 1 that the overall impact of protein premiums and discounts changes little under each scenario.[6]

Finally, table 2 presents estimates of a grower's willingness-to-pay for a forward contract in the presence and absence of protein premiums and

[5] This article does not report the effect of changes in a grower's attitude to risk. However, unreported numerical analysis shows that different levels of risk aversion do not significantly modify the findings of this analysis.

[6] For CVr values of 0.30 and 0.35, which are representative of farming systems in the eastern states of Australia (Anderson *et al.* 1988 estimate these values for CVy, however, given the linear approximation of yield, values of CVy and CVr are interchangeable), the protein window increases in size and level to 10.46 per cent–10.85 per cent and 10.63 per cent–11.30 per cent, respectively. Hence, where CVr is very high, the impact of protein premiums and discounts on a grower's income stream is such that a grower will only be better off if their expected protein level is substantially above 10 per cent.

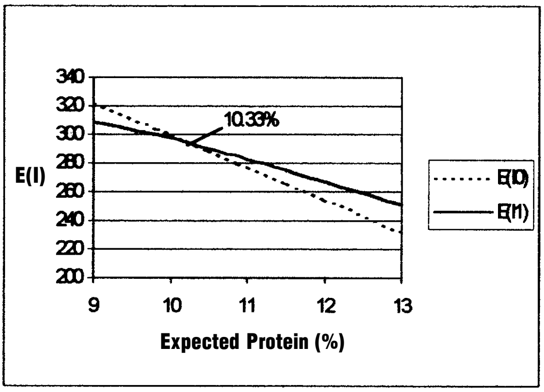

Figure 1 Effect of protein premiums and discounts on $E(I)$

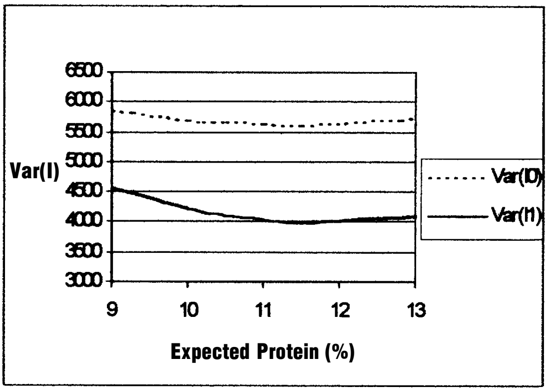

Figure 2 Effect of protein premiums and discounts on $Var(I)$

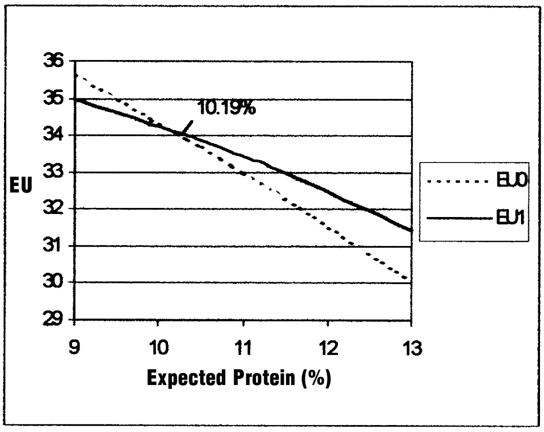

Figure 3 Impact of protein premiums and discounts on EU

Table 1 Sensitivity analysis of the impact of protein premiums and discounts

Scenario		(1) $EU_1 - EU_0 = 0$	(2) $E(I_1) - E(I_0) = 0$
Standard solution	$b = 15$ $CVr = 0.20$ $x = 500$	10.19	10.33
1	$b = 10$ $CVr = 0.20$ $x = 500$	10.12	10.17
2	$b = 20$ $CVr = 0.20$ $x = 500$	10.33	10.67
3	$b = 15$ $CVr = 0.25$ $x = 500$	10.31	10.54
4	$b = 15$ $CVr = 0.20$ $x = 1000$	10.23	10.33

Table 2 Percentage change in Certainty Equivalent of income (CE) derived from a forward contract in the absence and presence of protein premiums and discounts

r (%)	% Δ in CE_0	% Δ in CE_1
9.00	1.026	1.097
9.94	1.033	1.033
10.00	1.033	1.029
11.00	1.044	0.969
13.00	1.081	0.869

discounts in terms of the certainty equivalent of income using the same technique outlined in Fraser (1997). In general terms, the results in table 2 extend the findings of Fraser (1997) by showing that this willingness-to-pay only increases when the decrease in $E(I)$ associated with the introduction of protein premiums and discounts is dominant.[7] However, in other situations where the risk benefits of protein premiums and discounts are relatively important, or where $E(I)$ increases, it is clear that willingness-to-pay for a forward contract decreases. This latter finding reflects the associated perception of a reduced relative value from the risk benefits of a forward contract.

4. Conclusion

This article extends the analysis of Fraser (1997) to consider the role of expected protein levels in determining the impact of protein premiums and discounts on a grower's income stream and willingness-to-pay for a forward contract. Fraser's results that protein premiums and discounts cause

[7] Note that although the switching point in table 2 of 9.94 per cent is slightly below the value of 10 per cent on which the findings of Fraser (1997) were based, further sensitivity analysis shows that this is likely to be due to the re-specification of the protein-yield relationship. For example, with $b = 10$, this switching point occurs at $\bar{r} = 10.06$ per cent.

expected income and variance of income to decrease are generalised for expected protein levels less than 10.33 per cent. Moreover, it was shown that for expected protein levels less than 10.19 per cent the effect on expected income is the dominant effect and expected utility decreases, while for protein levels between 10.19 per cent and 10.33 per cent the variance effect is dominant and expected utility increases. However, for protein levels greater than 10.33 per cent, expected income increases while the variance of income effect remains negative, and so both effects contribute to a positive impact on expected utility.

A sensitivity analysis conducted on key parameter values did not significantly modify outcomes, except where levels of seasonal variability are extremely high (in these cases, such as applies to the eastern states of Australia, a grower's level of expected income will only increase if their expected protein level is substantially above 10 per cent).

When considering the role of expected protein levels in determining the effect of protein premiums and discounts on a grower's willingness-to-pay for a forward contract, the finding of Fraser (1997) that a grower is willing-to-pay more for a forward contract in the presence of protein premiums and discounts was shown to hold for levels of expected protein less than approximately 10 per cent. However, at higher levels this finding was not supported, and growers were shown to be willing-to-pay less for a forward contract. This perception of a reduced value for price risk management is typically a consequence of lower levels of income risk, but also in some cases is because of higher expected levels of income following the introduction of protein premiums and discounts.

Overall, it may be concluded that a grower's existing expected protein level plays a crucial role in determining whether the instrument of protein premiums and discounts is viewed favourably or unfavourably.

References

Anderson, J.R., Dillon, J.L., Hazell, P.B.R., Cowie, A.J. and Wan, G.H. 1988, 'Changing variability in cereal production in Australia', *Review of Marketing and Agricultural Economics,* vol. 56, pp. 270–86.

AWB 1998, *The Growers Survival Guide to the AWB (WA)*, Australian Wheat Board, Belmont, Western Australia.

Bardsley, P. and Harris, M. 1987, 'An approach to the econometric estimation of attitudes to risk in agriculture', *Australian Journal of Agricultural Economics*, vol. 31, pp. 112–26.

Fraser, R.W. 1991, ' "Nice work if you can get it": an analysis of optional set-aside', *Oxford Agrarian Studies*, vol. 19, pp. 61–9.

Fraser, R.W. 1997, 'Seasonal variability, land values and willingness-to-pay for a forward wheat contract with protein premiums and discounts', *The Australian Journal of Agricultural and Resource Economics*, vol. 41, pp. 139–55.

Mood, A.M., Graybill, F.A. and Boes, D.C. 1974, *Introduction to the Theory of Statistics*, 3rd edn, McGraw-Hill, Kogakusha.
Pope, R.D. and Just, R.E. 1991, 'On testing the structure of risk preferences in agricultural supply analysis', *American Journal of Agricultural Economics*, vol. 73, pp. 743–8.
Robinson, S.D. 1995, 'Selecting wheat varieties in a stochastic environment', paper presented to the 39th Annual Conference of the Australian Agricultural Economics Society, University of Western Australia, Perth, 14–16 February.