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Decisions and tradable production quota when yield is uncertain†

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This article analyses optimal decisions under regulation by tradable agricultural production/marketing quotas when production is stochastic. For risk-neutral and risk-averse producers the fraction of planned production that is covered by quota is separable from input decisions when yield randomness is additive. The role of quota in protecting against the risk of production shortfall is investigated. A producer is shown to benefit from being allowed to treat as one all tranches of production quota under his control. Production decisions are invariant to this amalgamation. But when production randomness is additive normal, the qualitative impact of amalgamation on quota positions depends upon whether the ratio of rental price to the price difference that is being protected exceeds one half.

1. Introduction

It is known that the existence of output uncertainty can give rise to curious incentive structures when output is quota-regulated. The incentives problem arises because the quota is a fixed number, while production is to some extent random and surplus production is treated differently from in-quota production. Thus, a risk-neutral decision-maker will care about uncertainty even if production is linear in the source of uncertainty.

The problem is important because many agricultural products are subject to production/marketing quotas, and agricultural production is fraught with uncertainty. Milk is subject to quota in the 15 European Union countries, Canada, Japan, Norway, and Switzerland.[1] Sugar beet is marketed under quota in the European Union, as are tobacco and peanuts in the United States. Canadian provinces regulate the marketing of chickens, turkeys,

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[1] See Burrell (1990) for a description of dairy quota policies in the OECD countries.

tobacco, and eggs by quota. In many cases these marketing rights can be leased or sold. Some of these production quotas are multi-tiered in that one type of quota right might entitle the producer to a high price while a second type of quota right might provide access to an intermediate marketing opportunity, and production surplus to the sum of these quotas can only be sold at a low price. Sugar beet production in the European Union has been subject to such a system of multiple production quotas.

A literature on the general problem is well established. With reference to Australian fresh milk quotas, Alston and Quilkey (1980) suggested that the social costs associated with the, presumably unintended, decision incentives arising from the interactions between uncertainty and the quota regulation could be mitigated by a well-functioning quota market. Fraser (1986) refined the argument by explicitly modelling the optimal decision of a risk-neutral producer. In his model he clearly showed that marginal revenue is discontinuous in the source of uncertainty. In analysing adjustments to rules governing the sale of Western Australian potatoes, Fraser (1995b) elaborated further on the topic. Borges and Thurman (1994) extended theoretical and empirical work by Babcock (1990) on production decisions for the quota-regulated US peanut crop.

However, none of the above models accommodate the ability to trade quota. Further, while Babcock (1990) formally poses the problem in a risk-aversion framework and develops empirical comparative statics, none of the models draw theoretical conclusions for the problem under risk aversion. Two objectives of this article are to extend previous analyses of decisions under stochastic production in these ways. In particular, we identify a separation between production and quota-holding activities for risk-neutral and risk-averse firms. A third objective is to demonstrate that quotas under yield uncertainty may provide an incentive for farms to amalgamate, or for one farm with two or more distinct tranches of quota to pool these tranches.

The first section of the article develops a model of the problem faced by risk-neutral and risk-averse producers who have some control over output and who are active in one or more quota rental markets. This is followed by a comparative statics analysis of optimal choices. In the fourth section, it is shown that the risk of not filling quota also may be managed by coinsuring risks through a consolidation of tranches of quota. The implications of this consolidation for other firm-level decisions are also studied. Simulations illustrate the main results, and a brief discussion concludes the article.

2. Model and separation issues

A risk-neutral decision-maker produces a partially random quota-regulated crop. The time 0 choice of planned output, Q, is mapped into time 1 stochastic output, $F(Q, \eta)$, where η follows distribution function $G(\eta)$ with continuous density function $g(\eta)$. It is assumed that $F(Q, \eta)$ is increasing in both arguments, $F_Q(Q, \eta) > 0$ and $F_\eta(Q, \eta) > 0$. The expectation operator over η is given by $E_{\{\eta\}}[\cdot]$. The quota regulation allows a price-quantity schedule that declines over n steps. The size of each of these steps depends upon the amount of ith-type quota, $i = 1, 2, \ldots, n-1$, owned by the firm in question. One unit of ith-type quota allows the firm to market one unit of output at price P_i. Without any loss of generality, we assume that $P_1 > P_2 > \ldots > P_{n-1}$. Further, we assume that the free market price is P_n where $P_{n-1} > P_n$. There are active markets for each of the $n-1$ different quota types, and the producer will take *ex ante* positions in each of these markets. Of course, given the pricing structure, *ex post* the firm will fill ith-type quota before filling $i+1$th-type quota. The firm faces rental price κ_i, $i = 1, 2, \ldots, n-1$ for ith-type quota, and must choose the stock $\overline{Q}_i$ of that quota type to hold. The cost of planned production is given by the increasing and convex function $C(Q)$.

Extending the model of Fraser (1986) and Babcock (1990) to the case of $n-1$ quota markets, the problem for the risk-neutral optimising firm with choice-conditioned stochastic profit $\pi(Q, \overline{Q}_1, \overline{Q}_2, \ldots, \overline{Q}_{n-1})$ is to

$$\max_{Q,\overline{Q}_1,\ldots,\overline{Q}_{n-1}} E_{\{\eta\}}[\pi(Q, \overline{Q}_1, \overline{Q}_2, \ldots, \overline{Q}_{n-1})] = \max_{Q,\overline{Q}_1,\ldots,\overline{Q}_{n-1}} P_1 E_{\{\eta\}}[F(Q, \eta)] - C(Q) - \sum_{i=1}^{n-1}(P_i - P_{i+1}) \int_{Z_i}^{\infty} [F(Q, \eta) - \Sigma_{k=1}^{i} \overline{Q}_k] dG(\eta) - \sum_{i=1}^{n-1} \kappa_i \overline{Q}_i, \tag{1}$$

where Z_i is the (unique) value of η that solves $F(Q, \eta) = \Sigma_{k=1}^{i} \overline{Q}_k$. It is easily seen that $Z_1 \leq Z_2 \leq \ldots \leq Z_{n-1}$. As presented in equation 1, it can be observed that the problem we will address has a yield option interpretation for quota usage under yield uncertainty. Specifically, in not owning quota of the ith-type, a producer forgoes a contingent claim to sell an undetermined amount of output surplus to production level $\Sigma_{k=1}^{i} \overline{Q}_k$ at price P_i rather than P_{i+1}. This contingent claim has expected value $(P_i - P_{i+1}) \int_{Z_i}^{\infty} [F(Q, \eta) - \Sigma_{k=1}^{i} \overline{Q}_k] dG(\eta)$. One additional unit of the ith-type quota provides partial claim on this amount. Further, it alters the values of claims due to positions in quota markets of the $i+1$th through $n-1$th types. The first-order conditions for equation 1 are

$$P_1 E_{\{\eta\}}[F_Q(Q, \eta)] - C_Q(Q) - \sum_{i=1}^{n-1}(P_i - P_{i+1}) \int_{Z_i}^{\infty} F_Q(Q, \eta) dG(\eta) = 0, \tag{2A}$$

$$\sum_{k=i}^{n-1}(P_k - P_{k+1})[1 - G(Z_k)] - \kappa_i = 0, \quad i = 1, \ldots, n-1, \tag{2B}$$

where Leibniz's rule for differentiating an integral bound is applied in equations 2A and 2B (Royden 1988, p. 107).

It is of interest to identify criteria under which the production decision is separable, in some sense, from the quota purchase decision. The criteria can be arrived at by studying how the conditions in equation 2B relate to condition 2A. Note that decisions Q and $\overline{Q}_i$ enter equation 2B only through the Z_i, the solutions to $F(Q, \eta) = \Sigma_{k=1}^{i}\overline{Q}_k$. It is not plausible for Q or any of the $\overline{Q}_i$ to vanish in the defining equation for Z_i, and so there cannot plausibly be complete separation. However, suppose that $F(Q, \eta) = Q + \eta$ where $E_{\{\eta\}}[\eta] \equiv 0$.[2] Then $Z_i = \Sigma_{k=1}^{i}\overline{Q}_k - Q$. If Z_i increases, then the probability that the sum of positions in the first through ith quota markets suffice to cover realised output increases. To abbreviate, we identify the first through ith quota market by $\{1, 2, \ldots, i\}$. Under the additive specification of production risk, the problem becomes

$$\max_{Q,\overline{Q}_1,\ldots,\overline{Q}_{n-1}} P_1 Q - C(Q) - \sum_{i=1}^{n-1}(P_i - P_{i+1})\int_{Z_i=\Sigma_{k=1}^{i}\overline{Q}_k - Q}^{\infty}[Q + \eta - \Sigma_{k=1}^{i}\overline{Q}_k]dG(\eta) - \sum_{i=1}^{n-1}\kappa_i\overline{Q}_i, \qquad (1')$$

and the first-order conditions are:

$$P_1 - C_Q(Q) - \sum_{i=1}^{n-1}(P_i - P_{i+1})[1 - G(Z_i)] = 0, \qquad (2A')$$

$$\sum_{k=i}^{n-1}(P_k - P_{k+1})[1 - G(Z_k)] - \kappa_i = 0, \quad i = 1, 2, \ldots, n-1. \qquad (2B')$$

The problem is solved if we can solve for $(Q, Z_1, \ldots, Z_{n-1})$ because $\overline{Q}_1 = Z_1 + Q$, $\overline{Q}_2 = Z_2 - Z_1$, and generally $\overline{Q}_i = Z_i - Z_{i-1}$, $i \geq 2$. The second-order conditions for problem (1′) are satisfied because $C_{QQ}(Q) \geq 0$, because $P_i \geq P_{i+1}$ for $i = 1, 2, \ldots, n-1$, because $d^2\pi(Q, \overline{Q}_1, \ldots, \overline{Q}_{n-1})/d\overline{Q}_i d\overline{Q}_j = 0$ for $i, j = 1, 2, \ldots, n-1$ and $i \neq j$, and finally because we will show shortly that $d^2\pi(Q, \overline{Q}_1, \ldots, \overline{Q}_{n-1})/dQd\overline{Q}_i = 0$ for $i = 1, 2, \ldots, n-1$.

We identify the maximising solutions to equations 2A′ and 2B′ as $(Q^*, Z_1^*, \ldots, Z_{n-1}^*)$. Because the Z_i presentation arises naturally in conditions 2A′ and 2B′, our equilibrium analysis will primarily study decisions as expressed in this form. But to provide intuition we will often translate

[2] Fraser (1995a), among others, holds that the multiplicative representation of yield uncertainty is more reflective of reality than the additive representation. Borges and Thurman (1994), in their study of North Carolina peanut production, found some evidence against the additive specification. We retain it because we will show that it carries with it strong and insightful implications concerning the nature of optimal decision-making.

analysis to solution vector $(Q^*, \overline{Q}_1^*, \ldots, \overline{Q}_{n-1}^*)$. For ease of interpretation, it might be best to re-write equation 2B′ as

$$P_i[1 - G(Z_i^*)] + \sum_{j=i+1}^{n-1} P_j[G(Z_{j-1}^*) - G(Z_j^*)] - P_n[1 - G(Z_{n-1}^*)] \\ - \kappa_i = 0, \quad i = 1, 2, \ldots, n-1. \tag{2B''}$$

Here, $P_i[1 - G(Z_i^*)]$ is an opportunity cost of not having an additional unit of ith-type quota because price P_i is foregone with probability $1 - G(Z_i^*)$. But this calculation overcounts because positions in quota markets $\{i+1, i+2, \ldots, n-1\}$ will recoup some of this loss in expected revenue as will sales in the open market for product not covered by quota. The terms involving $P_j, j = i+1, \ldots, n$ account for the recouped (in expectation) revenue.

At least two interesting observations can be made concerning equations 2A′ and 2B′. First, for $i = n-1$ we have $(P_{n-1} - P_n)[1 - G(Z_{n-1}^*)] = \kappa_{n-1}$. Consequently, for $i = n-2$ a substitution yields $(P_{n-2} - P_{n-1})[1 - G(Z_{n-2}^*)] = \kappa_{n-2} - \kappa_{n-1}$. By repeated substitutions, we have

$$(P_i - P_{i+1})[1 - G(Z_i^*)] = \kappa_i - \kappa_{i+1}, \quad i = 1, 2, \ldots, n-1. \tag{3}$$

Thus, the output price differential arising from a marginal unit of ith-type quota places an upper bound on the rental differential between quota of the ith and $i+1$th-types. Equation 3 also reveals that the size of Z_i^* increases with output price differential $P_i - P_{i+1}$ because there is added incentive to protect against the probability that output is not covered by positions in quota markets $\{1, \ldots, i\}$. And Z_i^* decreases with an increase in rental price differential $\kappa_i - \kappa_{i+1}$ because this differential reflects incremental costs of protecting against the probability that output is not covered by quota markets $\{1, \ldots, i\}$.

The second, and perhaps more surprising, observation is that when $i = 1$ in condition 2B′, we may substitute back into condition 2A′ to obtain $P_1 - C_Q(Q^*) = \kappa_1$.[3] This means that planned production is separable from positions taken in quota markets. Thus, as in the case of deterministic production, marginal cost is set equal to the in-quota marginal revenue less the rental price. It is the role of the tradable quota decision to speculate on production risk whereas Q is chosen without regard to risk. Thus, similar to the separation result in the analysis of futures hedging under risk aversion (see, e.g. Holthausen (1979)), the decision environment is sufficiently rich that the production decision is separated from the speculation decision. The

[3] And so $d^2\pi(Q, \overline{Q}_1, \ldots, \overline{Q}_{n-1})/dQd\overline{Q}_i = 0$ for $i = 1, 2, \ldots, n-1$.

separability is a consequence of the additive nature of production risk in our model.

But the separability result is quite robust to other aspects of the decision environment. Notwithstanding the non-linearities in our monetary pay-off function, separation carries through to decisions under risk aversion. Suppose that the rational decision-maker has an increasing and concave von Neumann and Morgenstern utility of pay-off function, $U[\pi]$. The problem is then to

$$\max_{Q,\overline{Q}_1,\ldots,\overline{Q}_{n-1}} E_{\{\eta\}} U\left[(Q+\eta)P_1 - C(Q) - \sum_{i=1}^{n-1}(P_i - P_{i+1}) \max[Q+\eta-\Sigma_{k=1}^{i}\overline{Q}_k, 0] - \sum_{i=1}^{n-1}\kappa_i \overline{Q}_i \right], \tag{4}$$

and the first-order conditions are:

$$E_{\{\eta\}}\left[U_\pi(\pi)\left\{ P_1 - C_Q(Q^*) - \sum_{i=1}^{n-1}(P_i - P_{i+1}) I_{\{\eta \geq Z_i^*\}} \right\}\right] = 0, \tag{5A}$$

$$E_{\{\eta\}}\left[U_\pi(\pi)\left\{ \sum_{k=i}^{n-1}(P_k - P_{k+1}) I_{\{\eta \geq Z_k^*\}} - \kappa_i \right\}\right] = 0, \quad i = 1, 2, \ldots, n-1. \tag{5B}$$

Here, $I_{\{\eta \geq Z_i\}}$ is the indicator function that equals unity when $\eta \geq Z_i$ and equals zero otherwise. Adding condition 5B when $i = 1$ to condition 5A, we have $E_{\{\eta\}}[U_\pi(\pi)\{P_1 - C_Q(Q^*) - \kappa_1\}] = 0$, or $P_1 - C_Q(Q^*) - \kappa_1 = 0$. And so separability continues to adhere.[4] Consequently, the optimised production decision Q is invariant to risk aversion, risk distribution or any wealth considerations. This independence is driven by the linear nature of the randomness, and would not hold if, for example, $F(Q, \eta) = Q\eta$. But the optimum choices of the $\overline{Q}_i$, i.e. the Z_i^*, depend upon these factors even when production risk is additive. In the next section, we will analyse how exogenous parameters affect optimal choices under risk neutrality and, to some extent, under risk aversion.[5]

[4] Concerning second-order sufficient conditions, it is readily shown that they are satisfied when $n = 2$ if $C_{QQ}(Q) \geq 0$ and $U_\pi(\pi) \geq 0 \geq U_{\pi\pi}(\pi)$. However, concavity for larger values of n could not be affirmed.

[5] A large literature now exists concerning the applicability of separability when positions can be taken in price-contingent markets. The interested reader is referred to Lapan and Moschini (1994) and Moschini and Lapan (1995).

3. Comparative statics

The comparative statics of Q^* are the same under risk neutrality and risk aversion. They are readily arrived at, and we will not dwell upon them. We have $dQ^*/dP_1 = -dQ^*/d\kappa_1 = 1/[C_{QQ}(Q^*)] \geq 0$, while $dQ^*/dP_i = dQ^*/d\kappa_i = 0$, $i = 2, 3, \ldots, n$. The effect, if any, of P_i, $i = 2, 3, \ldots, n$ on optimum planned production comes through equilibrium effects on κ_1. This is because the role of the quota rental markets is to manage risk exposure to the per-unit penalty for excess production. The smaller the size of the P_i, $i = 2, 3, \ldots, n-1$, the greater will be the demand for the stock of quota that provides access to price P_1.

Availing of the fact that choice Q^* is independent of choices Z_i^*, we can investigate the effects of the κ_i and the P_i on optimal Z_i by differentiation of condition 2B′, for risk neutrality. We will not present the comparative statics of Z_i^* under risk aversion because our analysis did not yield noteworthy insights. Under risk-neutrality:

$$\frac{dZ_i^*}{d\kappa_i} = -\frac{1}{(P_i - P_{i+1})g(Z_i^*)} < 0, \tag{6}$$

and so, as one would expect, cumulative position in quota markets $\{1, 2, \ldots, i\}$, i.e. $\Sigma_{k=1}^{i}\overline{Q}_k$, is decreasing in the rental premium for the ith type quota. The reason is that when an increase in κ_i occurs, then the ith-type quota market becomes less attractive as a means of protecting against the price consequences of a good harvest. Use of quotas markets $\{1, 2, \ldots, i\}$ actually increases in response to an increase in κ_i, i.e. from equation 3, when lagged,

$$\frac{dZ_{i-1}^*}{d\kappa_i} = \frac{1}{(P_{i-1} - P_i)g(Z_{i-1}^*)} > 0. \tag{7}$$

Impact 7 may be interpreted as a substitution effect whereby if quota that secures a relatively low output price comes at a larger rental rate, then it is best to increase the fraction of the yield distribution that is covered by quota which carry more favourable output prices. Considering equations 6 and 7 together, the interval of η that is covered by positions in the ith quota market, i.e. of length $\overline{Q}_i$, contracts from both end points in response to the increase in κ_i.

The impact of an increase in P_i on the cumulative positions in markets $\{1, 2, \ldots, i\}$ is given by

$$\frac{dZ_i^*}{dP_i} = \frac{1 - G(Z_i^*)}{(P_i - P_{i+1})g(Z_i^*)} > 0, \tag{8}$$

and so the incentive to protect against not filling quota at P_i increases with an increase in this higher price. Similarly,

$$\frac{dZ_i^*}{dP_{i+1}} = -\frac{1 - G(Z_i^*)}{(P_i - P_{i+1})g(Z_i^*)} < 0, \tag{9}$$

so that $dZ_i^*/dP_i = -dZ_i^*/dP_{i+1}$. The invariance to a price shift that preserves $P_i - P_{i+1}$ arises because the role of quota is to protect the price differential, and if the differential does not change on moving from the ith to the $i+1$th-type quota, then there is no incentive to change cumulative coverage of production through positions in quota markets $\{1, 2, \ldots, i\}$. While a change in P_i will induce a change in Z_{i-1}^*, this will be offset by a change in $\overline{Q}_i^*$. And the change in $\overline{Q}_i^*$ will be exactly that required to preserve Z_i^* because P_{i+1} changes to preserve the value of $P_i - P_{i+1}$.

A first-degree stochastically dominating (FSD) shift in η decreases $G(Z_i)$ at any Z_i, and so increases $1 - G(Z_i^*)$. Therefore, optimum Z_i must increase to restore the risk-neutral equilibrium. This result can be seen from representing equation 2B$'$ in the form of equation 3. If distribution $G(\eta)$ undergoes a stochastic shift to $H(\eta)$ such that $H(Z_i^*) < G(Z_i^*)$, then the value of Z_i^* must increase to restore equilibrium. Thus, quantities $\overline{Q}_1^*$, $\overline{Q}_1^* + \overline{Q}_2^*$, and generally $\Sigma_{k=1}^{i}\overline{Q}_k^*$ increase. But the impact on each $\overline{Q}_i^*$, $i = 2, 3, \ldots, n-1$ is not clear. The effect on Z_i^* of a mean-preserving contraction in the manner of Rothschild and Stiglitz (1970) is ambiguous because the expression $1 - G(Z_i^*)$ may either rise or fall under it.

4. Firm amalgamations

We turn next to the incentives implications that production quota might have for firm amalgamations. We address two issues. First, we will show that it is always optimal for risk-neutral firms to amalgamate their production quota because amalgamation reduces the size of the loss that arises when quota is exceeded. Second, we will identify what determines the impact of such an amalgamation on optimal decisions.

4.1 Amalgamation decision

We assume that two firms, 1 and 2, each face additive production risk so that their production technologies are given by $F(Q_k, \eta_k) = Q_k + \eta_k$, $k = 1, 2$. Consequently, equation 2A$'$ and equation 2B$'$ apply. The firms are alike in all ways except that the production risk distributions they are exposed to differ.[6] For firm $k \in \{1, 2\}$, the *ex post* random profit is

[6] The conclusions in subsection 4.1 continue to hold if the firm cost functions and the κ_i are allowed to differ.

$$(Q_k + \eta_k)P_1 - C(Q_k) - \sum_{i=1}^{n-1}(P_i - P_{i+1})\max[Q_k + \eta_k - \Sigma_{j=1}^{i}\overline{Q}_{k,j}, 0] - \sum_{j=1}^{n-1}\kappa_j\overline{Q}_{k,j}, \tag{10}$$

where $\overline{Q}_{k,j}$ is the position of firm k in quota market j. Assume that equation 10 is evaluated at the *ex ante* optimum for the expected profit-maximising firms when they are separate. If the firms amalgamate, then the sum of the *ex post* random profits, when evaluated at the *ex ante* optimal choices for separated firms, is

$$\begin{aligned} &P_1\sum_{k=1}^{2}(Q_k + \eta_k) - \sum_{k=1}^{2}C(Q_k) \\ &- \sum_{i=1}^{n-1}(P_i - P_{i+1})\max[\Sigma_{k=1}^{2}(Q_k + \eta_k) - \Sigma_{k=1}^{2}\Sigma_{j=1}^{i}\overline{Q}_{k,j}, 0] - \sum_{k=1}^{2}\sum_{j=1}^{n-1}\kappa_j\overline{Q}_{k,j}. \end{aligned} \tag{11}$$

Comparing equation 10, summed over k, with equation 11, the latter, when evaluated at the *ex ante* optimal choices for separated firms, is at least as large as the former if for $i = 1, 2, \ldots, n-1$ we have $\Sigma_{k=1}^{2}\max[Q_k + \eta_k - \Sigma_{j=1}^{i}\overline{Q}_j, 0] \geq \max[\Sigma_{k=1}^{2}(Q_k + \eta_k) - \Sigma_{k=1}^{2}\Sigma_{j=1}^{i}\overline{Q}_{k,j}, 0]$. But this inequality follows immediately from a state-space analysis of bankruptcy presented in Scott (1977). To see why, note that it is possible for $Q_1 + \eta_1 - \Sigma_{j=1}^{i}\overline{Q}_{1,j} > 0 > Q_2 + \eta_2 - \Sigma_{j=1}^{i}\overline{Q}_{2,j}$. The amalgamated firm is free to re-optimise the levels of choice variables. Thus, regardless of the state of nature, it is always best to amalgamate. This risk management incentive to consolidate has arisen elsewhere. Hennessy *et al.* (1997) came across it when comparing whole-farm with crop-specific insurance policies, while Hennessy and Roosen (1999) studied it in the context of pollution permit management. Scott (1977) and also MacMinn and Brockett (1995) identified converse implications in that a firm facing liabilities might benefit from partitioning firm assets into separate limited liability companies in order to maximise the expected future value to shareholders at the expense of debt holders.

4.2 Impact of amalgamation on decisions

We now turn to the implications of this incentive to merge for optimal choices. We continue to assume that the firms are identical in all ways except for the distribution of the firm's η. Regardless of whether the firms amalgamate and regardless of whether decision-makers are risk averse, from first-order conditions 2A′–2B′ and 5A–5B it is clear that Q_k^* solves $P_1 - C_Q(Q_k^*) = \kappa_1$, $k = 1, 2$. The solution is independent of k, and so we drop the subscript on Q_k^*. To facilitate an identification of the critical issues, we also assume that there is just one quota market, i.e. that $P_i = 0 \; \forall \; i \geq 3$. If the firms are separate and the marginal distributions are common, then for risk-neutral firms the optimum $\overline{Q}_k$ solves $(P_1 - P_2)\text{Prob}[\eta_k \geq Z^s] = \kappa_1$ where $Z^s = \overline{Q}_k^s - Q^*$ and where the superscripted s denotes the optimal solution under firm separation and where the value of Q^* is known from the separation result. Because the marginals are common, the value of $\overline{Q}_k^s$ does not depend on the firm in question and so we write it as $\overline{Q}^s$.

If the firms amalgamate, then the optimising $\overline{Q}$, averaged over the two identical firms, solves $(P_1 - P_2)\text{Prob}[\frac{1}{2}\eta_1 + \frac{1}{2}\eta_2 \geq Z^a] = \kappa_1$ where $Z^a = \overline{Q}^a - Q^*$ and where the superscripted a identifies amalgamation. We will confine ourselves, as does Fraser (1995a), to a study of a bivariate normal distribution for (η_1, η_2). Just and Weninger (1999) have argued that the hypothesis of normal agricultural yield distributions has not yet been refuted in statistical analyses. We have already required that the means of (η_1, η_2) equal zero. Denote the common variance of the marginals by σ^2 and the correlation coefficient by ρ. Then the variance of $\frac{1}{2}\eta_1 + \frac{1}{2}\eta_2$ is $\frac{1}{2}(1 + \rho)\sigma^2$ with least upper bound σ^2. We can now compare the survival functions of the cumulative normal distributions for the identically distributed η_k, $k \in \{1, 2\}$ with the cumulative normal distribution of $\frac{1}{2}\eta_1 + \frac{1}{2}\eta_2$. Both pass through the point $(0, \frac{1}{2})$ and the single crossing is at that point. On the half-line $(-\infty, 0)$ the survival function of the η_k, $k \in \{1, 2\}$ is below that of $\frac{1}{2}\eta_1 + \frac{1}{2}\eta_2$, whereas on the half-line $(0, \infty)$ the common survival function for the separated risks is above that of $\frac{1}{2}\eta_1 + \frac{1}{2}\eta_2$. The situation is depicted in figure 1. From the first-order conditions, we can conclude that if $\frac{1}{2} \leq \kappa_1/[P_1 - P_2]$, then $Z^a \geq Z^s$ and so $\overline{Q}^a \geq \overline{Q}^s$ because quota is expensive to buy. Consequently, both the amalgamated firm and the separated firms will expect not to need to sell any product at price P_2. The diversification, and the consequent variance reduction, associated with amalgamation will make it more likely that the amalgamated firm will overproduce relative to the separated firms. From equation 2B′, it is the probability of exceeding quota that determines the amount of rented quota, and so the amalgamated firm has a larger position in rented quota. The reverse is true when $\frac{1}{2} \geq \kappa_1/[P_1 - P_2]$. In this case, the amalgamated firm and also the separated firms will expect to exceed quota. Diversification then makes the probability of excess production smaller for the amalgamated firm, and so $\overline{Q}^a \leq \overline{Q}^s$.

5. Simulation analysis

Table 1 reports simulated impacts of price parameters on optimal choices. In it, we have assumed that $P_1 > P_2 > 0$ and that $P_i = 0 \; \forall \; i \geq 3$. Motivated by equation 2A′ and equation 2B′ under these circumstances, we abbreviate $\psi = P_1 - \kappa_1$ and $\omega = \kappa_1/[P_1 - P_2]$. We choose a quadratic cost function,

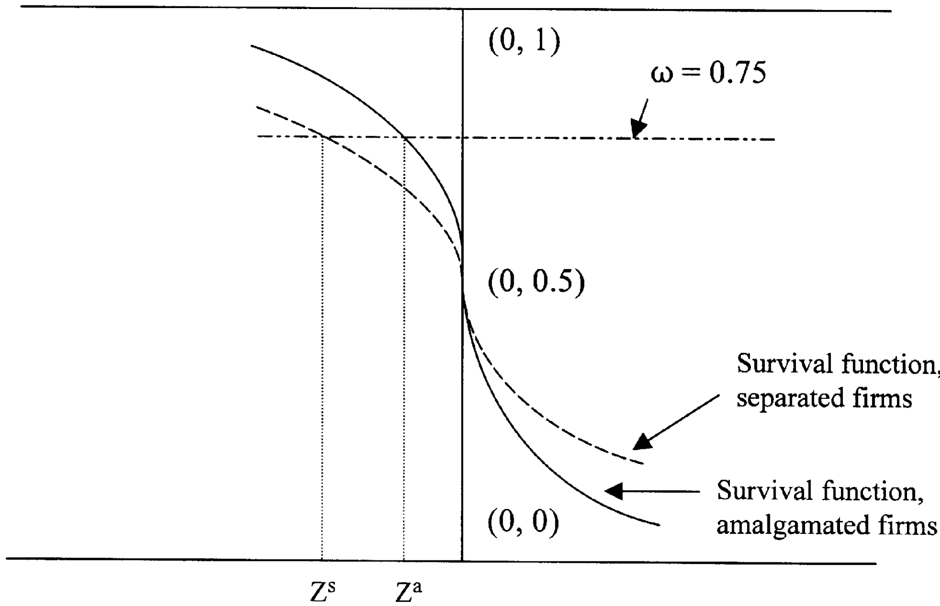

Figure 1 Effect of amalgamation on quota rental position

Table 1 Optimum single firm decisions under different pricing and distribution parameters

	$\psi = 5, \sigma = 1$	$\psi = 10, \sigma = 1$	$\psi = 5, \sigma = 2$	$\psi = 10, \sigma = 2$
$\omega = 0.1$	$Q^* = 5$	$Q^* = 10$	$Q^* = 5$	$Q^* = 10$
	$\overline{Q}_1^* = 6.28$	$\overline{Q}_1^* = 11.28$	$\overline{Q}_1^* = 7.57$	$\overline{Q}_1^* = 12.57$
$\omega = 0.5$	$Q^* = 5$	$Q^* = 10$	$Q^* = 5$	$Q^* = 10$
	$\overline{Q}_1^* = 5$	$\overline{Q}_1^* = 10$	$\overline{Q}_1^* = 5$	$\overline{Q}_1^* = 10$
$\omega = 0.9$	$Q^* = 5$	$Q^* = 10$	$Q^* = 5$	$Q^* = 10$
	$\overline{Q}_1^* = 3.72$	$\overline{Q}_1^* = 8.72$	$\overline{Q}_1^* = 2.43$	$\overline{Q}_1^* = 7.43$

$C(Q) = c_0 + c_1 Q + \frac{1}{2}c_2 Q^2$ where the c_j, $j = 0, 1, 2$ are parameters. We let $c_2 = 1$, while parameter c_1 is set equal to zero because it is arbitrary in that it is subsumed in P_1 in equation 2A$'$. The distribution of η is held to be normal. Values for ψ in table 1 are set at 5 and 10. Values of ω are allowed to range on $[0.1, 0.9]$. The two levels of standard deviation that we consider in table 1 are $\sigma = 1$ and $\sigma = 2$. These are sufficiently small that the draws where $Q^* + \eta < 0$ are rare. Choices conform to our analysis in that there is separation between planned production and quota usage. Note that the level of $\overline{Q}_1^* - Q^*$ decreases with σ if $\omega = 0.9$, but increases with σ if $\omega = 0.1$.

Table 2 Expected percentage gain in profit from amalgamation under different bivariate distribution parameters

	$\rho = 0$	$\rho = 1$
$\sigma = 1$	19.4	0
$\sigma = 2$	29.2	0

Table 2 provides an assessment of the change (increase) in expected profit arising from the amalgamation of two firms. When firms are identical in all ways including common marginal distributions, and when there is only one quota market, then expected profit for each of the separated firms is

$$Q^s P_1 - C(Q^s) - (P_1 - P_2)E_{\{\eta\}}\{\max[Q^s + \eta - \overline{Q}_1^s, 0]\} - \kappa_1 \overline{Q}^s, \qquad (12)$$

where subscripts identifying the firm in question have been removed because they are not relevant. If the firms amalgamate, then half the *ex ante* expected value of amalgamated profit, when evaluated at the *ex ante* optimal choices for separated firms, is

$$Q^s P_1 - C(Q^s) - (P_1 - P_2)E_{\{\eta\}}\{\max[\tfrac{1}{2}\eta_1 + \tfrac{1}{2}\eta_2 + Q^s - \overline{Q}_1^s, 0]\} - \kappa_1 \overline{Q}_1^s. \qquad (13)$$

Under the assumption of normality, we will calculate the expected value of the max functions. From Theorem 21.2 in Greene (1990), we have that:

$$E_{\{\eta\}}\{\max[Q^s + \eta - \overline{Q}_1^s, 0]\} = (Q^s - \overline{Q}_1^s)\text{Prob}[\eta \geq \overline{Q}_1^s - Q^s] + \sigma\phi(\alpha), \qquad (14)$$

where $\alpha = (Q^s - \overline{Q}^s)/\sigma$ and $\phi(\cdot)$ is the density function of the standard normal distribution. In table 2, we set $P_1 = 12$, $P_2 = 4$, and $\kappa_1 = 4$ so that $\psi = 8$ and $\omega = 0.5$. It can then be shown that $\overline{Q}_1^* = Q^* = 8$. Setting $c_0 = 24$, the percentage impact of amalgamation on firm expected profits is presented in table 2. When the marginals are common and $\rho = 1$, then there are no gains from amalgamation. But, under common marginals, if $\rho \neq 1$ then there are gains from amalgamation. Further, if the marginals are not common, differing by location and scale parameters, then there would be gains from amalgamation even if $\rho = 1$. For farm operations that are geographically proximate, the value of ρ is likely closer to 1 than 0. It should be noted that while, for business entities contemplating an amalgamation, data such as those in table 2 are important in deciding whether the gains from the consolidation offset costs associated with consolidation, our figures are arbitrary. In a low margin industry, combination to better manage revenue risks due to not filling quota will be important. But in many industry circumstances it may be a minor determinant of firm size.

Table 3 Effects of amalgamation on quota rental decisions; change in production units covered by quota [a]

	$\rho = 0, \sigma = 1$	$\rho = 0.5, \sigma = 1$	$\rho = 0, \sigma = 2$	$\rho = 0.5, \sigma = 2$
$\omega = 0.1$	−0.375	−0.172	−0.750	−0.344
$\omega = 0.3$	−0.154	−0.070	−0.308	−0.141
$\omega = 0.7$	+0.154	+0.070	+0.308	+0.141
$\omega = 0.9$	+0.375	+0.172	+0.750	+0.344

[a] The changes reported pertain to pre-merger business units. When the reference point is post-merger business units, then the changes are twice as large.

Table 3 presents the impact of amalgamation on decisions when the assumptions made are as in table 2. We compare $\rho = 0$ with $\rho = 0.5$ at different levels of σ. Due to the separation result, there is no impact on the optimum level of production. And so we only report the impact on the quota rental decision when the unit of analysis is the pre-merged firm. For example, when $\rho = 0$, $\sigma = 1$, and $\omega = 0.1$, then the post-merger output level will be twice the pre-merger level less 0.75 production units. Due to the symmetry attributes made concerning firms and distributions, the impact of amalgamation on quota rental positions is negative when $\omega < 0.5$ and is positive when $\omega > 0.5$. The impacts in table 3 involve changes amounting to 2–10 per cent of the quota held per unit of planned production.

6. Conclusion

Production and marketing quotas are important agricultural policy instruments, and production uncertainty pervades the sector. For a risky production technology, the phenomenon that we considered was the discontinuity in marginal revenue to which quotas give rise. Other policy instruments that may have similar consequences include grading in grain and horticultural markets and bulk discounts under uncertain demand. Each situation has distinctive attributes, and so each policy may have to be analysed in its distinct context. Incentives to merge also may arise under these policy environments.

Many of the results obtained in this article are stronger than those identified in models of non-tradable production quotas when output is stochastic. For example, it is not possible to ascertain the effects of a mean preserving contraction on input choices in the models of Fraser (1986, 1995a). Yet it is shown here that a mean preserving contraction has no effect on production. It may seem strange that the more involved (i.e. two decisions) model renders results more readily. Upon reflection, however, this should be no surprise. The producer is not in equilibrium when quota is not tradable. Freedom to choose both factors expands the set of choices but places more restrictions upon the structure of the set of optimal choices. The stronger conclusions are a consequence of the more structured set of choices.

A related issue that warrants some attention is that of the interactions between risk and dynamics when an optimiser seeks to allocate quota over the time period for which the quota pertains. It is not clear what conditions will encourage the firm to save quota early in the time interval just in case production is abundant later, and what conditions will encourage the firm to use quota early just in case the marginal cost of filling quota later turns out to be high. If quota markets are active throughout the production period, then the problem is more involved. Rather than rent early, the producer could wait until the size of the producer's harvest is more certain. This just-in-time strategy will likely involve paying a higher price for quota if the producer's harvest is highly correlated with aggregate harvest in the quota trading area. The problem of optimal quota allocation among time periods and under non-random production does not appear to have been tackled yet, and so it might be best to await this analysis before introducing dynamics into the stochastic model.

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