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Limited arbitrage in international wheat markets: threshold and smooth transition cointegration†

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The strength of the adjustment towards arbitrage equilibrium can be expected to be somehow proportional to the extent of market price deviations from equilibrium. In this article, threshold and smooth transition cointegration models are applied to quarterly wheat prices of three major world suppliers over the period 1973–99. Results based on arranged autoregressions of the error term of a static regression do not prove to be robust. Although non-linear models relying on a multivariate system approach yield partly contradictory results, the main evidence from the latter suggests a weakening, rather than an outright inaction, of the adjustment process in the inner regime.

1. Introduction

The prices of identical or near-identical commodities traded in spatially separated markets tend to follow very similar movements and long-run trends, which are mainly determined by international arbitrage. However, the presence of transaction costs and arbitrage boundaries is likely to impinge upon the attainment of smooth and unilinear long-run equilibrium relationships. A discontinuous adjustment process to long-run equilibrium may lead to the co-existence of two or more separate regimes. Outside a given range, where price gaps can partly be removed through international trade, a mean-reverting equilibrium error can still ensure global stationarity. By contrast, whenever the price differences do not exceed the costs related to the above factors, the profit motive falls off and bounded random walk movements might characterise the error term, thus implying local non-stationarity. In the presence of non-stationary variables, a cointegrating relationship would then be activated only if the system is sufficiently out of

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equilibrium (in the upper and lower outer regimes), while long-run equilibrium agents are otherwise not effective (within an inner regime).

Pippinger and Goering (1993) and Balke and Fomby (1997) have studied the power of standard unit root and cointegration tests when the adjustment to long-run equilibrium is discontinuous. Both studies find that these tests are generally robust to non-linear threshold behaviour of the stochastic process, except in models with high persistence. High persistence can be induced by (i) high near-unitary autoregressive parameters in the outer regimes, which imply low speed of adjustment towards equilibrium, and/or (ii) the relatively wide range of the inner regime, namely, high threshold values relative to the residual variance. Under either of these conditions, the power of unit root tests tends to be reduced. This is due to (a) a weaker *signal* of reversion towards equilibrium (despite eventually a longer time spent outside the threshold boundaries if condition (ii) is not present) and/or (b) higher frequency of observations lying inside the threshold boundaries.

To account for this problem, some econometric studies relax the dichotomous zero-one order of integration assumption in the error correction term, by testing for fractional cointegration (Cheung and Lai 1993; Baille and Bollerslev 1994). Others propose an estimation procedure, defined as threshold cointegration, geared to distinguish different regimes and identify their stochastic properties (Balke and Fomby 1997). A third approach, which can be regarded as a generalisation of threshold cointegration, relies on a continuously varying strength of the attractor across these regimes. In terms of arbitrage equilibrium, relatively larger shocks in one market would be absorbed more quickly than ordinary-sized ones. The rationale for gradual, rather than sudden, regime shifts lies in the heterogeneous reaction of agents to changes in transaction costs, with each trader and investor responding differently, especially to changes occurring in the proximity of the thresholds. In this perspective, aggregate thresholds for an arbitrage process as a whole would become 'blurred', and this pattern would justify the estimation of smooth transition non-linear error-correction models (Granger and Hallman 1991; Granger and Teräsvirta 1997, p. 17, pp. 55–61; Anderson 1997).

This article builds on the latter two approaches, with an application to international wheat prices. In view of results of unit root tests for three major food grains,[1] wheat is taken as a case study. Main features of the international markets of food grain commodities are examined in the next section. This is followed by a short review of threshold and smooth transition

[1] Here a broad definition of food grain is adopted, thus including maize, besides wheat and rice. A distinction between food grains and feed grains depends on dietary habits and nutritional value of different varieties of individual grains (Atkin 1989, p. 123).

cointegration. The analysis is limited to aspects of a general nature and the choice of functional specifications, identification of key parameters and estimation techniques, which are of direct relevance for arbitrage modelling. Technical details are presented in the Appendix. The next section focuses on results partly based on a multivariate systems approach and, finally, conclusions are drawn.

2. International price formation in food grain markets

Compared to other primary commodities and overall economic activity, agricultural commodity prices tend to experience larger fluctuations, with these fluctuations being at times unrelated to the prevailing international business cycles (Labys and Pollak 1984, p. 15, p. 52; Atkin 1989, p. 75). Reasons on the supply side include periodic disruptions due to unforeseen weather conditions, and short- and medium-term constraints in supply adjustments to unexpected demand shortfalls or surpluses. For some of these commodities, such as staple goods, an additional source of price instability may originate from their supposed low price and income elasticities of demand (with the exception of the poorest countries in the case of income elasticities). This would hamper a partial absorption of supply-related price shocks, although it appears to be partly contradicted by empirical evidence on three staples (Davison and Arnade 1991, p. 20). Other possible determinants of price volatility in agricultural commodity markets arise from political crises, speculative movements, inflation and exchange rate realignments.

Major international commodity markets tend to be located close to the geographical nodes of production and, in some cases, consumption (Labys and Pollak 1984, p. 5). Apart from transaction costs,[2] non-price factors do influence the trade pattern and can be partly responsible for price differentials between international markets. These factors include (i) imperfect homogeneity of the products across countries of origin; (ii) a half-year asynchronism in harvests in the two hemispheres; and (iii) imposition of quotas or diversification of suppliers by importers (Thomson 1989, p. 51),

[2] The role of transaction costs is likely to be overstated if ordinary maritime freight rates are not adjusted to account for subsidised transport (e.g., as development aid) or similar export incentives. For wheat and other grains, some producers also apply export subsidies selectively, according to outlet markets, with competition among exporters taking place in both subsidised and non-subsidised markets (Sumner 1995). In coincidence with harvest periods, grain freight costs are also characterised by seasonality (Sewell 1992, p. 144). On the other hand, transaction costs can be understated due to a number of intangible costs not included in freight rates, such as risk and insurance premiums and market information gathering costs. These aspects limit the scope for a precise assessment of transaction costs.

or delivery lags by suppliers. These features may prevent a full adjustment of price levels in relatively less important international trading markets to the price movements in major trading centres (e.g., Rangoon versus Bangkok for rice,[3] or Buenos Aires versus New Orleans for wheat). Some of these factors, coupled with other government interventions and long-term trading arrangements, have also helped maintain substantial wedges between international and domestic prices for certain products.

Grains have long been subject to government measures affecting local consumer and producer prices, with this being particularly the case in the decade preceding World War II (Tracy 1993, p. 156). However, in contrast to other agricultural food products such as sugar, the 'world market in freely traded grain is large enough', so as not to be 'dismissed as a residual; it is extremely important to many of the world's grain farmers and producers' (Atkin 1989, p. 124). Wheat is characterised to some extent by differentiated markets, according to different climates and end-uses, namely the high protein *spring* type, and the hard and soft (lowest protein) red *winter* types. In the absence of significant crop disruptions, a higher protein content tends to be associated with a higher price. Similarly, wheat sells at a premium to maize, with this premium being partly determined by the higher nutritional value and weight of wheat per bushel compared to maize. Although this price spread tends to undergo substantial fluctuations, it barely shrinks to less than 12 per cent relative to per bushel maize prices, since this would encourage feed grain users to replace maize with low quality wheat (ibid., pp. 113, 138).[4]

Due to demand- and supply-specific characteristics of its market (e.g., climatic requirements for crops), world trade is more concentrated for maize than for wheat. This concerns both exports (with the United States and Argentina accounting for between two-thirds and three-quarters of world maize exports, as compared with nearly 40 per cent of wheat exports), and imports (Atkin 1989, pp. 129–30; FAO 1998 and 1999). For both cereals, world prices are mainly influenced by the US market, which sets a price floor to buffer against excess world supply through stockpiling or even to undercut competitors' subsidies, and vice versa in the presence of excess demand. In the case of wheat, since the early 1970s the market has experienced a gradual lessening of the duopolistic or triopolistic power by dominating producer

[3] Until the 1950s, Myanmar was the largest rice exporter, accounting for over a quarter of world trade. Ever since, this country has been replaced by other producers, among which Thailand holds by far the dominant position (Sewell 1992, p. 210).

[4] Over the 25-year period considered in the econometric analysis, the price spread in the US market fell short of this percentage premium only in two quarters in 1977 and one quarter in 1983.

countries, namely, the United States, Canada, and eventually Australia, and lower levels of carry-over stocks, with these stocks reaching a trough in 1972. This brought about a shift towards relatively greater competition (Alaouze *et al.* 1978), or even, according to an alternative view, an oligopsonistic market, influenced by a few major wheat importers (Carter and Schmitz 1979). Whereas previous empirical studies tend to attribute market price leadership to either the United States or Canada, recent econometric analysis highlights feedback effects in the international price setting among major producers, including Australia (Mohanty *et al.* 1999).

Wheat supply has also been relatively more sensitive to weather influences, particularly in two Southern hemisphere exporters, Argentina and Australia. In these two countries, unlike other export countries, the bi-directional association between land values and producer prices (and, to a lesser degree, export prices) is weak. Argentina is the only major wheat supplier not to have implemented a domestic price stabilisation scheme, and to have suffered from marked instability in terms-of-trade and exchange rate policies and domestic price volatility (Sutton and Webb 1988, pp. 178–81). The case of this country, as mainly a price-taker among world wheat suppliers, is examined later with a view to testing for threshold and smooth transition cointegration relative to export prices of two dominant producers in this market.

3. Threshold and smooth transition cointegration: problems and revised procedures

In a seminal contribution on the subject, Balke and Fomby (1997) propose a two-step approach to threshold cointegration. First, global (non)-stationarity in the equilibrium error z_t ($= y_t - \beta x_t$) is tested through standard unit root tests. In simulation experiments, the authors rely on the Engle-Granger single equation method. Second, if the variables appear to be cointegrated, local behaviour is investigated by applying non-linearity tests on an arranged autoregression of this error term. Based on these tests, key parameters for the threshold autoregressive (TAR) model tracing the stochastic behaviour of the residuals can be identified.

In a TAR($k; p, d$), k indicates the number of regimes (separated by thresholds ζ_i), p the order of the autoregressive process, and d a delay parameter, or threshold lag. In threshold cointegration, one can expect to have three regimes, with the two thresholds possibly being asymmetric relative to average location of the inner regime. Taking as an example a simple TAR(3; 1, 1) cointegration model, the process tends to converge to an equilibrium point (i.e. zero) when $|z_{t-1}| > \zeta$, while being locally unstable inside the inner regime. To distinguish it from more complex cointegration

TAR models, this is defined as *equilibrium* TAR by Balke and Fomby (1997). More realistically, a *band*-TAR model allows for an equilibrium band, rather than point (ibid., p. 631). The latter model looks more suited to the analysis of international arbitrage. This would especially be the case if the model were reformulated so as to allow a flexible convergence process to the target (or *attractor*) band, accounting for asymmetric responses and possible wedges between the borders of the band and the thresholds. Thus, the adjustment process towards the equilibrium band may systematically tend to either exceed or fall short of the distance between actual and threshold values (see the Appendix).

The techniques proposed to select the delay parameter d and the threshold values ζ_i are largely heuristic, even when there is evidence of superior statistical power. As a case of the latter, Tsay (1989) suggests choosing the delay parameter which maximises the rejection of the joint zero null hypothesis on the parameters of z_{t+d-j}, used as regressor for a non-linearity test on the recursive residuals of the TAR model ($j = 1, \ldots, p$) (provided that no residual autocorrelation is present in the regression).[5]

Relative to the preliminary selection of p, on the one hand, arranged autoregressions are less likely to be correctly specified if they are run on short-period *cases*, unless the original cross-case time sequence happens not to be substantially modified. Moreover, an under-parameterised model may not allow a clear distinction between omitted serial correlation and genuine breaks in linearity or non-linearity (Teräsvirta 1994; Granger and Teräsvirta 1997, p. 115). On the other hand, the higher the value of p, the greater the possible range of d. In practice, Tsay (1989, p. 235) finds that the value of the signalling lag may be insensitive to various alternative AR orders and generally low. However, in view of possibly high and low values of z_{t-d} within the same cases, there is as a consequence greater risk that the TAR model is unable to clearly distinguish between different regimes.

Furthermore, the identification of ζ_i may be constrained by insufficient observations in one or more regimes. Especially for relatively small samples, it may also not be consistent depending on whether the TAR$(k; p, d)$ is run on ascending or descending values of the variable z_{t-d}, as is observed in results reported later. If applied on both orderings, unequivocal results would be obtained only if different levels of statistical significance are associated with the respective F-test. Finally, if the TAR is used for cointegration analysis and the single equation approach is followed, the location of ζ_i may be affected by the bias in the Engle-Granger static long-

[5] The terms *standardised* and *recursive* are here used interchangeably. The advantage of using this measure of error, rather than other forms of residuals, is illustrated in du Toit *et al.* (1986, pp. 208–13).

run regression. This bias has been found to be more serious when residuals are highly autocorrelated, and to decline only slowly with larger sample sizes, despite the super-consistency property of the OLS estimator (Banerjee *et al.* 1986, pp. 260–1).

A statistically more powerful procedure, which tests for more than one cointegrating relationship with all variables assumed endogenous (with some eventually being weakly exogenous), is provided by Johansen maximum likelihood (ML) method (Johansen and Juselius 1990). Similarly to single equation cointegration, in the presence of different regimes, this method does not avoid misspecification problems under the threshold alternative, since parameter estimates of the linear VAR models would only reflect average values across regimes. However, maximum eigenvalue and trace statistics can still be relied on for an identification of the 'average' rank of the long-run matrix.

As a possible adaptation of the method to threshold cointegration in future research, Balke and Fomby (1997, p. 643) suggest a two-step approach, in which cointegration and threshold behaviour are tested separately. While the former is applied to a cointegrating VAR in I(1) space, the latter can be based on a (vector) error correction model in I(0) space, that is to a threshold ECM or VECM (henceforth TECM). To avoid possible misspecification at this stage, a more appropriate procedure ought to check for thresholds already in the first stage, so as to implicitly account for a possible band-TAR pattern in the equilibrium error as in (1) in the Appendix (thus estimating a threshold VAR, henceforth TVAR). Both procedures are applied here to international wheat prices, with results being discussed in the next section.

In either a single equation or a multivariate systems framework, piecewise cointegration based on TAR models assumes that the only alternative to a linear model is a discontinuous pattern of cointegration. Knife-edged regime switches are detected in I(1) and/or I(0) space, which respectively reflect changes in (i) level and/or direction; and (ii) strength of the long-term attractor. Even allowing for discrete behavioural changes by individuals, these changes are unlikely to take place simultaneously and with the same level of intensity for all individuals. For food grain arbitrage, the proportion of potential traders with sufficiently low transaction costs (so as to allow international trade to occur) may be assumed to increase gradually, rather than abruptly, with greater disequilibrium gaps. Hence, for these and other aggregate economic variables, non-linear smooth transition autoregressive (STAR) models can be better suited to capture these regime shifts.

STAR models, eventually adjusted to this purpose, can also account for non-symmetric responses to shocks around the attractor line(s). Escribano and Granger (1998, pp. 88–92) and Escribano and Aparicio (1999, p. 401) suggest the use of a cubic polynomial of the error correction term as a simple and flexible approximation to an unknown non-linear function. In more complex functional forms of smooth transition, the error correction term, with a given delay parameter d, can be used as a delay variable in inherently non-linear response functions H. Typical functions of this kind are used in exponential and logistic smooth transition regressions (ESTR and LSTR: see the Appendix). As in threshold regressions, estimated key parameters help locate, directly or indirectly, average transition levels between different regimes. As shown in the next section, smooth transition regressions can prove to be a sound alternative to TAR specifications.

4. Threshold and smooth transition cointegrating relationships in wheat prices

To test and examine the existence and kind of arbitrage boundaries in international food grain markets, this analysis has relied on statistical information on major producers' export prices of three relevant commodities of this category of goods, namely rice, maize and wheat. Seven quarterly price series in nominal terms were first collected from IMF (1999 and monthly issues in previous years), with samples ranging from the end of 1973 up to 1998/99 (list of variables in table 1). Two additional series, namely the US/New Orleans fob price of rice and the Australian wheat unit value, were not included because of data inconsistencies.[6] Simple and augmented Dickey-Fuller (DF and ADF) unit root tests tend to suggest stationarity in levels for the log-transformed variables in the case of rice and maize prices, and integration of order one for three wheat prices (table 1). Similar results are obtained if seasonal dummies are introduced in these regressions.

Relative to wheat prices, Granger-causality Wald tests have been applied on dynamic error correction models. These tests reject the null hypothesis of zero restrictions on the error corrections terms and the assumed exogenous log-differenced variables (with up to two possible lags) in all three cases, at a 5 per cent or lesser level of significance. This may indicate that all price variables have a role in international wheat price formation, by influencing each other's movements. However, a plot of wheat price fluctuations over the sample period reveals how in two peaks and two troughs the price levels

[6] In the absence of available information for rice in Myanmar in the third quarter of 1988, the mean value of price levels in the immediately preceding and following quarters has been taken. For possible extensions of this research, monthly data should be used: as pointed out by a referee, the arbitrage process can be expected to take place to a large extent within a quarter. However, substantial divergences across international wheat prices can be observed on a quarterly basis in terms of time lags and differences in levels (see later in the text). A comparison of results based on quarterly and monthly data would allow one to distinguish between different sources of incomplete price equalisation.

Table 1 Unit root tests on food grain prices

Variable	Myar	Thar	Tham	Usm	Argw	Ausw	Usw
$\tau\,[\ln(y)_{-1}]$	−3.71(0)*	−3.72(1)*	−4.00(0)*	−3.87(1)*	−2.66(0)	−2.49(1)	−3.17(1)
$\tau\,[\mathrm{d}\ln(y)_{-1}]$					−10.13(0)**	−8.20(0)**	−4.79(2)**

Notes: $\ln(y_{-1})$, $\mathrm{d}\ln(y)_{-1}$: variables in natural logarithms in level and first differenced form. DF and ADF regressions with intercept and linear trend, order of the test in parentheses (choice based on the highest lag in the autoregressive parameter with significant t-statistic).
Critical values: −3.46 (* 5 per cent significance level); −4.05 (** 1 per cent significance level)

List of variables

Rice (US$/tonne; 1973Q4–1998Q4)
myar Myanmar (unit value)
thar Thailand (fob Bangkok, white milled)

Maize (US$/bushel; 1973Q4–1999Q1)
tham Thailand (unit value)
usm United States (fob Gulf of Mexico ports, No. 2 yellow)

Wheat (US$/bushel; 1973Q4–1999Q1)
argw Argentina (unit value)
ausw Australia (export price, Sydney)
usw United States (fob Gulf of Mexico ports, hard red winter type)

of the dominant producers (the United States and Australia) anticipate the respective movements in the Argentine export market by one up to five quarters. Regarding the remaining three major turning points, the timing of these changes tends to coincide in all markets (figure 1).

Therefore, at first instance Argentina can be regarded as a price-taker with no feedback effects on the markets of the other two wheat producers. Hence, single equation threshold cointegration analysis can be applied. Unit roots tests on the equilibrium errors of the Engle-Granger static regression reject the non-stationarity hypothesis at a 1 per cent level of significance. The order of the autoregressive process in these residuals (z_t), as indicated by the autocorrelation function or ML information criteria, is 3 or 4, respectively. The Tsay F-test on an arranged autoregression of z_t, given $p = 3$, suggests the use of a delay parameter d equal to two. However, the Hansen parameter instability statistic identifies the third lag parameter as relatively more unstable, thus likely to be associated with a greater degree of non-linearity and possible regime shifts (Hansen 1992).[7]

Both results prove to be sensitive to slight variations in the sample period

[7] Unlike Chow tests, the Hansen instability statistic does not require *a priori* knowledge of breakpoints. Moreover, particularly relative to slope parameter regression estimates, this test is statistically more powerful than CUSUM and CUSUMSQ tests (Hansen 1992, p. 519).

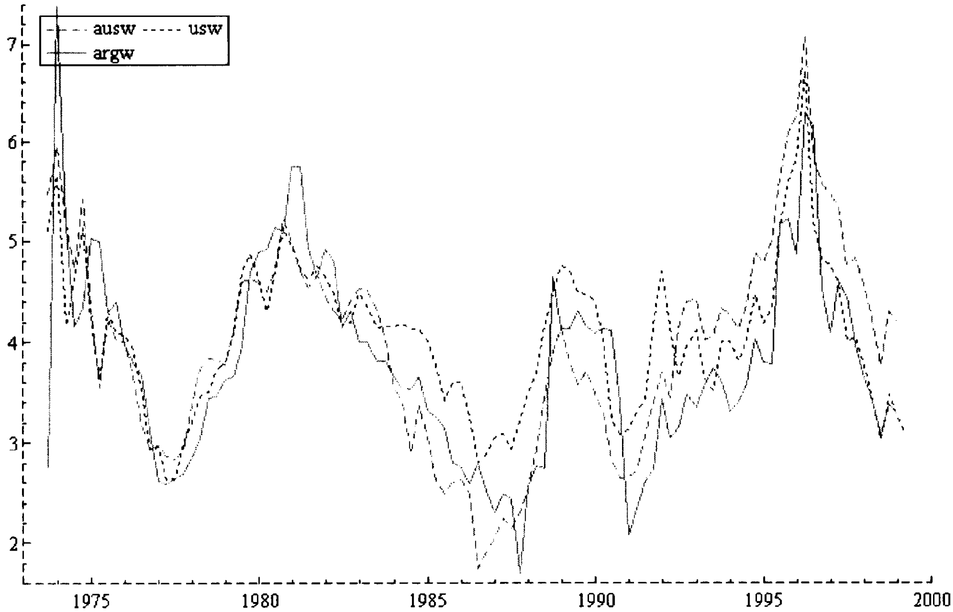

Figure 1 International wheat prices (US$/bushel, 1973–99)

ausw (short-dashed line), usw (dotted), argw (continuous): see table 1

and do not appear to be robust to reversal of the sequential order in the arranged autoregression.[8] Different non-linear patterns can be observed based on recursive estimated parameters. According to a TAR(3; 3, 3) model applied to varying samples and sequential orders, upper and lower thresholds are found to lie in the ranges (0.04, 0.11) and (−0.01, −0.07), respectively, with a mid-regime tending to cover nearly one-third of the sample period.[9]

[8] In one case, $d = 1$, which seems to be a more realistic adjustment lag to cross-country quarterly price disequilibria in wheat markets. A possibly longer delay may be explained by some mismatching of quarterly fluctuations and, in a few cases, remarkable lags in major turning points in the wheat price in Argentina relative to one or both of the other markets. Notice that, unlike the multivariate approach, residuals are obtained from a static regression. In this respect, TAR and STR models suffer from the inability to capture possible changes in key parameters such as delay and speed of adjustment, and threshold levels (see the Appendix).

[9] In view of the large variation in the results and for the sake of conciseness, no details are provided. One should also note that positive residual autocorrelation is present in the static Engle-Granger regression (DW = 1.23, with 102 observations). An alternative CUSUM test on the recursive residuals of the rearranged AR model has low power against the rejection of thresholds in a three-regime switching regression with similar stochastic processes in the outer regimes, and if a threshold is close to the end of the arranged series (Petruccelli and Davies 1986; Granger and Teräsvirta 1997, p. 36).

Since the middle bulk of the observations, as represented by the mid-spread, lies within the range of ± 0.075, even these minor discrepancies imply an uncertainty as to whether regime switches may occur close to or outside either of the two (upper and lower) quartiles. As a method avoiding some of these problems and relaxing the assumption of no feedback effects, the Johansen ML estimation technique has been preferred here.

Relative to wheat prices, ML (Schwarz and Hannan-Quinn)[10] information criteria for the order of the VAR, complemented with F-tests for sequential model reduction starting from a VAR(4) model, suggest the use of one lag. As reported in table 2, Johansen's tests on the reduced rank of the long-run adjustment matrix Π ($= \alpha\beta'$, with α being the speed of adjustment matrix) of the VAR(1) reject the null hypothesis of no cointegrating relationship at the 1 per cent level on Osterwald-Lenum critical values (Harris 1995, pp. 76–9; Doornik and Hendry 1997, pp. 60–1). However, other maximum eigenvalue and trace statistics are sufficiently lower than the 5 per cent critical values as to discard the hypothesis of more than one cointegrating vector. This is also suggested by plots of the linear combinations $\beta' x_t$ and respective recursive eigenvalues (graphs not reported for space reasons). Exclusively, the vector related to the Argentine wheat price appears stationary and associated with significant non-zero eigenvalues. The latter statistics appear to stabilise only towards the second half of the sample period, thus indicating a possible gradual change or intrinsic non-linearity (given also that recursive estimates are based on full-sample short-run dynamics).

Following normalisation of the cointegrating vector with respect to the Argentine price, large long-run effects are found to be exercised by the two dominant markets, particularly the United States. If two sub-periods are examined separately, based on the apparent structural change towards the mid-1980s, the influence of the US price has reduced in the last thirteen years, while the opposite tendency is evident for the Australian price. Wald χ^2 statistics on full-sample individual parameters reject the zero null hypothesis at the 5 per cent significance level (in relatively small samples this test has limited reliability, given its asymptotic property; Harris 1995, p. 116). The speed-of-adjustment coefficients suggest that disequilibrium shocks are almost fully recovered within each quarter, although some slowdown of this process is noticeable when passing from the first to the second half of the sample period (table 2: A–C).

[10] In contrast with these criteria, the AIC tends to select over-parameterised models, even in large samples (Mills 1990, p. 139). This turns out to be the case for these VAR models, also in view of the absence of residual autocorrelation in more parsimonious specifications (table 2). An opposite case is reported in Teräsvirta (1994, p. 211).

Table 2 Cointegrating VAR(1) and TVAR(1) models for wheat prices

Estimation period	73Q1–99Q1 A	73Q1–85Q4 B	86Q1–99Q1 C	73Q1–99Q1 D	73Q1–99Q1 E	73Q1–99Q1 F
Cointegration tests						
LR maximum eigenvalue	99.8 ($r=1$)	73.5 ($r=1$)	38.8 ($r=1$)	200.2 ($r=1$)	208.6 ($r=1$)	209.8 ($r=1$)
LR trace	116.3 ($r=1$)	87.9 ($r=1$)	56.2 ($r=1$)	222.2 ($r=1$)	228.9 ($r=1$)	228.8 ($r=1$)
Standardised β				(TVAR1)	(TVAR2)	(TVAR3)
Lnargw	1	1	1	1	1	1
Lnausw	0.41 [0.07]	0.27 [0.13]	0.54 [0.28]	0.34 [0.05]	0.35 [0.04]	0.35 [0.04]
Lnusw	0.78 [0.11]	1.01 [0.16]	0.59 [0.3]	0.81 [0.07]	0.80 [0.07]	0.82 [0.06]
Ln*y*pos (dpos in F)				0.16 [0.01]	0.16 [0.01]	0.22 [0.02]
Ln*y*neg (dneg in F)				−0.22 [0.03]	−0.20 [0.03]	−0.25 [0.03]
α (lnargw)	−0.90 [0.07]	−0.99 [0.09]	−0.81 [0.11]	−0.87 [0.03]	−0.89 [0.03]	−0.93 [0.03]
$-\ln\lvert\Omega\rvert$	15.0	15.8	15.1	16.0	16.0	16.1
AR(1–5) F-test	1.13 (5,91)	1.16 (4,39)	2.10 (4,44)	0.95 (5,89)	1.11 (5,89)	1.35 (5,89)
ARCH(4) F-test	1.11 (4,88)	1.44 (4,35)	0.91 (4,40)	0.88 (4,86)	0.91 (4,86)	1.65 (4,86)
Normality $\chi^2(2)$	9.11*	3.28	4.74	2.73	4.77	4.88
R^2(LM)	0.62	0.59	0.62	0.68	0.69	0.70

Notes: VAR models with unrestricted intercepts, a restricted linear trend, and two I(1) weakly exogenous variables (lnausw, lnusw: see table 1); standard errors in square brackets (for α and β parameters); threshold VAR models include restricted slope dummies, related to lnausw (TVAR1) or lnusw (TVAR2), or intercept dummies (TVAR3).

ln*y*pos	wheat price dummy for upper extreme unexplained observations (dpos · ln*y*)
ln*y*neg	wheat price dummy for lower extreme unexplained observations (dneg · ln*y*)
dpos	1 for observations in the upper end of the arranged equilibrium error from model A (see figure 2: $\zeta_u \sim 0.1$); 0 otherwise
dneg	1 for observations in the lower end of the arranged equilibrium error from model A (see figure 2: $\zeta_l \sim -0.175$); 0 otherwise
LR	log-likelihood ratio test (all reported statistics significant at the 1 per cent level)
β	cointegrating vector
α	speed of adjustment parameter
$\ln\lvert\Omega\rvert$	log-likelihood residual variance
AR(1–5)	residual autocorrelation of the fifth order (F-test, degrees of freedom in parentheses)
ARCH(4)	(residual) autoregressive conditional heteroscedasticity of the fourth order (F-test)
normality $\chi^2(2)$	Doornik-Hansen (residual) normality test (* 5 per cent significance level)
R^2(LM)	Lagrange Multiplier-based (Bartlett-Nanda-Pillai) R^2

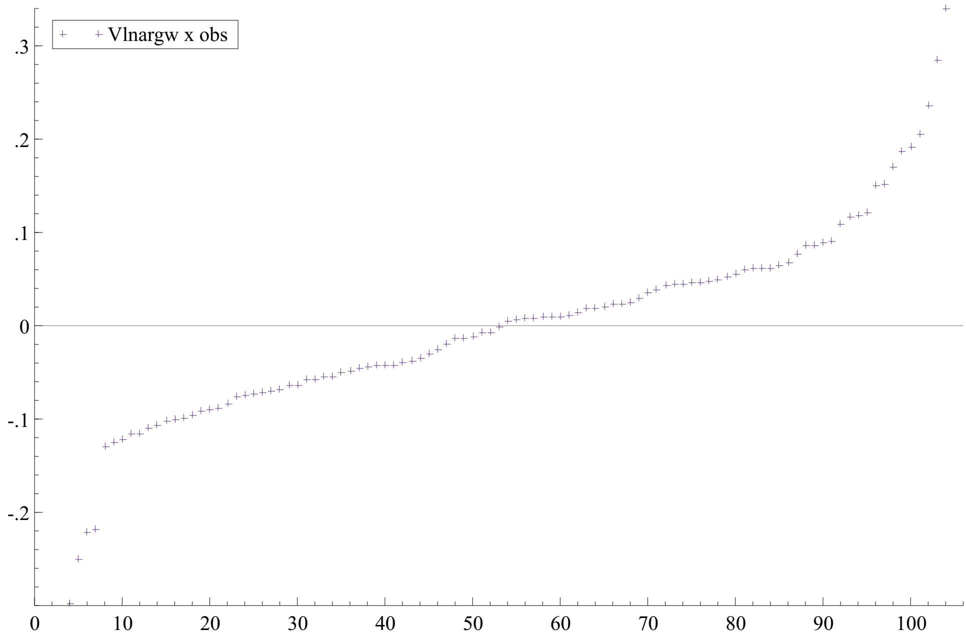

Figure 2 Rearranged equilibrium error of a cointegrating VAR(1) model for the Argentine wheat price (ascending order)

Vlnargw: cointegrating residual vector z_t from model A (table 2)

Alternative VAR specifications can be used to account for the non-normality of the cointegrating residual vector obtained for Argentina in the VAR(1) over the entire estimation period (table 2; the Doornik-Hansen test for normality reduces the small sample bias of the more common Jarque-Bera test: Doornik and Hendry 1997, pp. 216–17). If this equilibrium error is sorted in ascending order, while no remarkable breaks are visible in the region adjacent to zero, some degree of leptokurtosis is revealed by four negative outliers in the lower extreme and nearly fifteen observations with positive values straying away from the upper tail. The change appears to be more abrupt and pronounced in the lower end (corresponding to a 41 per cent increase in the value of the nearest outlying residual point), more gradual in the opposite extreme (with a 20 per cent increase if the gap between the 14th- and 13th-last observations is relied on) (figure 2).

Hence, the midpoints of these gaps can be relied on as approximate thresholds for TVAR models incorporating switches in level and/or direction of the cointegrating relationship (table 2: D–F). With reference to the financial literature, Escribano and Granger (1998) refer to these irregularities as 'bubble' periods. Partly due to the limited number of observations in the outer regimes, the use of both intercept and dummy variables leads to multicollinearity. Slope dummy parameter estimates based on either one of the two dominant markets are very similar, and long-run elasticities are stable across the three specifications. While the three regressions are equally acceptable according to goodness of fit and diagnostic tests, specifications E and F are obviously preferable in economic terms (note that the LM-based R^2 estimated in a multivariate system is not fully comparable with the standard R^2: Doornik and Hendry 1997, pp. 210–12). Results for TVAR2 (model E in table 2) indicate long-run elasticities *vis-à-vis* the US market of 0.96, 0.8 and 0.6, in the upper, mid, and lower regime, respectively.

To test for changing strength of the attractor depending on the relative distance from long-term arbitrage equilibrium, a standard ECM (table 3: A(1)) and three alternative TECM specifications have been applied (A(2)–(3) and E (4)). Model A(2) is a TECM with restricted dynamics in terms of parameters of short-run differenced variables (with the same dynamics across regimes) and zero restriction on the parameter of the inner regime z_{t-1}. A(3) consists in a TECM with unrestricted short-term dynamics, subsequently re-parameterised with zero restrictions on statistically insignificant (i.e. 10 per cent or higher level) coefficients in the original model. Finally, in E(4), a similarly unrestricted TECM is based on three distinct attractors, identified by a TVAR model capturing switches in direction driven by the US market (TVAR2).

As in the models in I(1) space, the speed-of-adjustment parameter is high, and declines in absolute value when a flexible specification is adopted. Obviously, this is particularly the case if the arbitrage process is assumed to converge to different attractors (table 3: E(4)). Once different short-run dynamics are accounted for, no statistically higher strength of the attractor is present in the outer regimes, except for an apparently over-compensating and unstable behaviour in the lower regime in model E(4). By contrast, a restricted TECM (table 3: A(2)) provides evidence of no cointegration in the inner regime, and mean reversion to long-run equilibrium in the outer regimes (with a quick adjustment especially in the upper one). However, these models turn out to yield serially correlated (except for A(3)) and non-normal residuals, and (except for A(2)) are possibly misspecified in terms of functional form or omitted variables.[11]

[11] See Kennedy (1998, p. 98) on the limitations of Ramsey's misspecification test. In the restricted TECM, statistically less significant results are obtained if the error correction term is represented by lagged residuals from equation (E) or (F), instead of (A) (tables 2–3). The exclusion of z_{t-1}, related to the inner regime, from model A(3) would lead to highly autocorrelated residuals. Similar results in terms of speed of adjustment parameters of the outer regimes and diagnostic testing are obtained if this exclusion restriction is applied to E(4). At the time of carrying out this analysis, the author was not aware of a study by Mohanty *et al.* (1999), which applies the Johansen method to monthly 1981–93 wheat prices

As an alternative approach, smooth transition ECMs have been applied, based on equilibrium errors from the VAR(1) for Argentina (model A). Among these models, the estimated adjustment parameters of a cubic function assume the expected signs of a smooth transition pattern, but the only statistically significant parameter is associated with the error correction term in level (table 4: A(5)). This outcome is unaltered even when statistically less significant lagged differenced variables are removed from the regression, thus indicating no evidence of non-linear cointegration. For the AESTR and ESTR estimations (see equations (2)–(4) in the Appendix), a non-linear least squares (NLS) procedure has been applied, based on mixed Gauss-Newton and quasi-Newton algorithms (Doornik and Hendry 1996, pp. 260–2; Cuthbertson *et al.* 1992, p. 64). The models are fully unrestricted dynamic ECMs, with and without the error correction term relative to the intermediate regime, and a delay parameter *d a priori* set equal to one (table 4: A(6)–(8)).[12] Compared with TECMs, higher explanatory power is reached, and, except for A(7), no residual autocorrelation is incurred.[13]

Due to slow convergence in the NLS estimation of the λ and ξ parameters, especially if the true parameters are large, the exponents in the AESTR and ESTR $H(z_{t-d})$ have been rescaled downwards, following similar empirical analysis (Appendix and table 4; Granger and Teräsvirta 1997, pp. 123–4). Additional problems arise from the tendency of the standard errors of these parameters to be large, for large values of λ and ξ (Teräsvirta 1994, p. 213;

of five world producers, including those examined here. In contrast to the results in A(1), their estimates of a VECM(1) for Argentina indicate a low speed of adjustment (given the use of monthly data) and no violation of the linearity hypothesis according to the Ramsey test, while the estimated equation has a relatively lower goodness of fit (ibid., table 3: adj. $R^2 = 0.38$).

[12] The same STR models were applied relying on response functions with $d > 1$. In most cases, there is no convergence in the NLS iterations (up to 500; for $d = 2$), or results are statistically less significant than those based on a one-quarter delay parameter (for $d = 3$). Among the latter, the speed of adjustment in the outer regimes is nearly -1 in all three cases. While rescaled λ parameters confirm a relatively slow transition ($\phi = -4.6, -6.1, -6.3$, for A(6), (7) and (8), respectively), the asymmetric pattern is not supported by estimated parameters ξ, c_l and c_u.

[13] The normality test still rejects the null hypothesis at a 1 per cent significance level. Following Chan and Tong (1986, p. 187), the normality assumption is not required for a smooth response function. However, Teräsvirta (1994) argues for the importance of checking for normality of residuals from STAR models, thus implying that, in the negative case, these models are unable to fully account for non-linear patterns. Similarly, Petruccelli (1990, pp. 34–5) and Teräsvirta and Anderson (1992) raise the issue of the difficulty of identifying true smooth transition relationships in real data, due to a possible confusion between the latter and apparent non-linear patterns determined by outliers.

Table 3 Threshold error correction models for the Argentine wheat price (1973Q1–1999Q1)

z_{-1} from model (table 2)	A (1) — ECM	A (2) — TECM (restricted dyn.)	A (3) — TECM (unrestricted dyn.) inner regime	A (3) — TECM (unrestricted dyn.) outer regimes	E (4) — TECM (distinct attractors; unrestricted dyn.) inner regime	E (4) — TECM (distinct attractors; unrestricted dyn.) outer regimes
Constant	−0.004 (−0.4)	0.01 (1.08)	−0.0.. (−0.01)		−0.0.. (−0.07)	
Dlnargw$_{-1}$	0.11 (1.40)	0.03 (0.32)	0.24 (2.68)**	–	−0.03 (−0.32)	–
Dlnausw	0.08 (0.52)	0.04 (0.23)	–	−9.93 (−3.35)**$_L$	–	−10.3 (−2.95)**$_L$
Dlnausw$_{-1}$	0.21 (1.32)	0.18 (1.04)	–	–	0.29 (1.62)	–
Dlnusw	0.25 (1.49)	0.23 (1.22)	–	5.89 (3.79)**$_L$	–	5.85 (3.16)**$_L$
Dlnusw$_{-1}$	0.61 (3.78)**	0.58 (3.24)**	0.79 (7.53)**	–	0.38 (2.0)*	–
z_{-1}	−0.86 (−7.30)**		−0.82 (−4.60)**		−0.64 (−3.05)**	
$z$pos$_{-1}$		−0.93 (−5.08)**		−0.04 (−0.15)		0.32 (0.54)
$z$neg$_{-1}$		−0.57 (−2.34)**		−0.15 (−0.61)		−1.74 (−1.82)†
AR(1–5) F	2.87* (5,88)	3.21* (5,57)	2.18 (5,87)		2.20† (5, 86)	
Norm. $\chi^2(2)$	20.9**	32.9**	20.2**		55.3**	
Reset F	4.45* (1,92)	1.15 (1,91)	4.21* (1,91)		5.15* (1, 90)	
R^2(adj)	0.55	0.46	0.57		0.42	

Notes: T-statistics in parentheses; ** 1 per cent significance level, * 5 per cent significance level, † 10 per cent significance level; L lower regime, U upper regime
z_{-1} error correction term (TECM: threshold ECM)
Reset F Ramsey misspecification test (degrees of freedom in parentheses)

Table 4 Smooth transition error correction models for the Argentine wheat price (1973Q1–1999Q1)

z_{-1} from model (table 2)	A (5) — CSTR-ECM		A (6) — AESTR-ECM		A (7) — ESTR-ECM		A (8) — LSTR-ECM	
	inner regime	outer regimes	inner regime	outer regimes	inner regime	outer regimes	inner regime	outer regimes
Constant	−0.005 (−0.5)		−0.009 (−0.98)		−0.002 (−0.25)		−0.02 (−1.61)	
dlnargw$_{-1}$	0.30 (3.45)**	–	0.08 (0.51)	0.21 (0.93)	0.04 (0.22)	0.13 (0.55)	−0.05 (−0.29)	0.44 (1.85)*
dlnausw	–	0.70 (3.15)**$_{U}$ −9.0 (−3.05)**$_{L}$	0.12 (0.41)	−0.36 (−0.80)	0.15 (0.40)	−0.22 (−0.43)	0.47 (1.02)	−0.67 (−1.20)
dlnausw$_{-1}$	–	1.33 (1.89)†$_{U}$	−0.14 (0.50)	1.03 (1.75)†	−0.34 (−0.85)	0.98 (1.57)	0.11 (0.32)	0.38 (0.70)
dlnusw	–	5.22 (3.22)**$_{L}$	0.20 (0.70)	0.42 (0.89)	0.25 (0.70)	0.17 (0.55)	−0.41 (−0.83)	1.18 (1.91)†
dlnusw$_{-1}$	0.74 (6.49)**	−1.22 (−1.87)†$_{U}$	0.46 (1.50)	0.23 (0.44)	0.76 (1.82)†	−0.27 (−0.47)	0.0.. (0.02)	0.87 (1.55)
z_{-1}	−0.74 (−4.75)**		[−0.44] (−1.4)	−1.39 (−4.94)**		−1.04 (−5.57)**		−1.49 (−5.94)**
z^2_{-1}	0.26 (0.46)							
z^3_{-1}	−4.88 (−1.45)							
$-\lambda$				82.1 [2.44]*		158.3 [1.35]		157.3 [1.74]†
$-\xi$				−51.7 [−1.01]				
$-c_l$								−0.03 (−1.0)
$-c_u$								0.17 (8.10)**
AR(1–5) F	1.50 (5,84)		1.71 (5,81)		3.02* (5,82)		0.57 (5,80)	
norm. $\chi^2(2)$	11.6**		31.1**		29.6**		16.3**	
Reset F	10.6** (1,88)		/		/		/	
R^2(adj)	0.62		0.60		0.57		0.65	

Notes: T-statistics in parentheses (in square brackets: t-statistics estimated for ϕ and ψ, whose parameter values are respectively weighted by the empirical variance and standard deviation of z_{-1}; for z_{-1} in A(6): estimated parameter for the inner regime if this variable were included in the AESTR-ECM); ** 1 per cent significance level, * 5 per cent significance level, † 10 per cent significance level; L lower regime, U upper regime.

z_{-1} error correction term (STR: smooth transition regression; functions: C cubic, AE asymmetric exponential, E exponential, L logistic)

λ steepness parameter of the response function ($= \phi/0.01074$)

ξ asymmetry parameter of the response function ($= \psi/0.10363$)

c_l, c_u mid-transition points (between different regimes)

Anderson 1997).[14] As in some TAR models (Granger and Lee 1989, p. 157), non-zero asymmetric coefficients are also frequently not identified as such, namely different from zero, by t statistics. In all three specifications, the rescaled λ ($= \phi$ in table 4) is not large (being equal to -0.88, -1.7 and -1.69 for A(6), (7) and (8), respectively), so that the respective original values are near the lower bound of the range of results reported in similar studies, quoted above (approximately (70, 3000) for λ, i.e. ϕ/σ^2). This implies a slow transition from one regime to another. The rescaled ξ parameter in A(6) ($= \psi$) is 5.4: apart from the limited reliability of the t-test (with a significance level of 30 per cent), its weighted (original) value indicates a relatively moderate asymmetry in the two responses.

In view of the sign of the asymmetry parameter and, similarly, the location of the mid-transition points, responses to equilibrium shocks do appear to be asymmetric in terms of strength and/or theoretical centrality of the attractor. The aggregate reaction to negative deviations in the Argentine market is relatively quicker, and may even take place over z_{-1} values around zero, before equilibrium is severely disturbed (table 4 and figure 3). Although not particularly accentuated, this pattern is evidenced not only by the better fit of the AESTR than the ESTR model, but also by the statistical insignificance of LSTR-ECM results if absolute values of the error correction term are used as a signalling variable in the respective $H(z_{t-1})$ function, with the related restriction of a unique mid-transition point c (results not shown). Unlike some of the TECM results, speed-of-adjustment parameters are very high in the outer regimes, and insignificantly different from zero if modelled for the inner regime. According to an analysis of correlogram (not shown), ML selection criteria and adjusted R^2, the LSTR-ECM is the most appropriate model. However, the distinction of inner, as opposed to outer, regime parameters is weakened by small non-zero intermediate values of the response function (figure 3). Hence, estimates reported in table 4, under A(8)-inner regime, are only approximately valid.

5. Conclusion

Original models of cointegration analysis assume an instantaneous process of readjustment, whereby cointegrating variables tend to move back to long-run equilibrium in every period. While this assumption seems unrealistic for

[14] While standard unit root tests concern *weak* stationarity, ergodicity and possibly complete (or *strong*) stationarity are theoretically required for non-linear stochastic processes (Chan and Tong 1986, p. 180; Granger and Teräsvirta 1997, p. 9). By contrast, relative to the delay parameter, the use of a non-stationary signalling variable is found to reduce NLS estimation problems, possibly due to super-consistency (Teräsvirta 1998, p. 512).

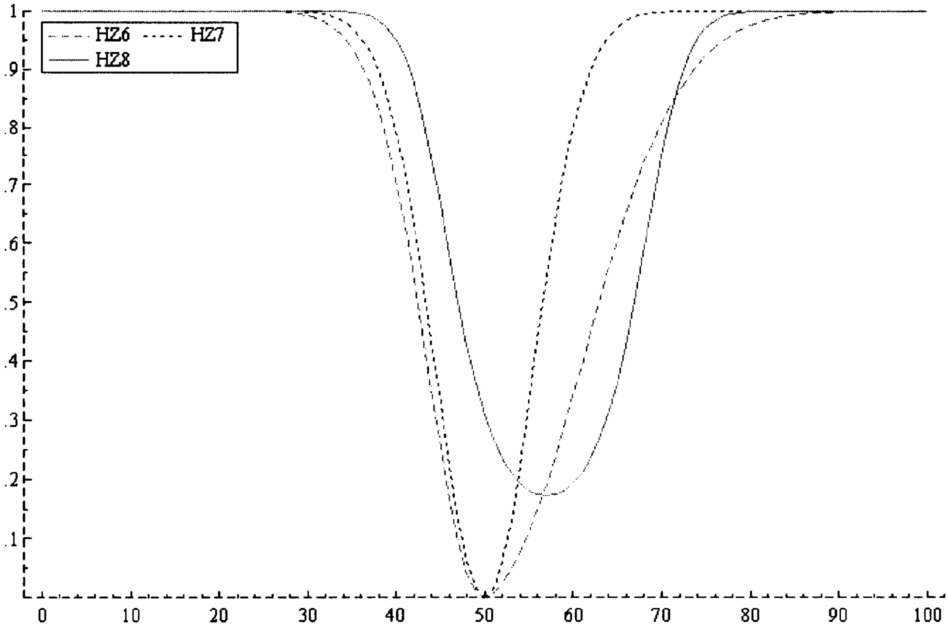

Figure 3 Estimated response functions of smooth transition models

HZ6 (short-dashed line), HZ7 (dotted), HZ8 (continuous): response functions of models A(6), A(7) and A(8) (see table 4). On the horizontal axis, the 50th observation corresponds to a zero value of z_{-1}. The error correction term is simulated over equidistant values within the range (−0.5, 0.5) (i.e. 0th and 100th obs., respectively).

several economic relationships, an alternative approach accounts for the possibility that the equilibrium error follows a threshold autoregression, thus reverting to the mean only if outside a given range. Threshold cointegration models have been developed by Balke and Fomby (1997), but have so far remained largely unutilised in applied econometric analysis (Maddala and Kim 1998, p. 190). This holds true for a more flexible non-linear extension of the concept of cointegration, based on smooth transition regressions. While encompassing linear and threshold cointegration, the general specification of these models implies a gradual transition between regimes, with each regime being characterised by different stochastic patterns.

Besides international arbitrage, many other cases of discontinuous or gradual adjustments to long-run equilibria among economic variables can be considered. Among them, issues of policy relevance concern the effects of discrete policy interventions on domestic or international interest rate spreads, on consumer demand (Moffitt 1990), or on portfolio distribution decisions between money and other assets (Granger and Teräsvirta 1997, p. 30). Other examples include relationships between actual (market) and target levels in industrial inventories (Granger and Lee 1989, p. 147) or in commodity price stabilisation schemes, among nominal exchange rates linked to arrangements with limited flexibility, such as typically target zones, and among real exchange rates in terms of purchasing power parity (Michael *et al.* 1997). Regarding the latter, since conditions of high persistence are likely to hold for many national price indices, standard cointegration analysis can be biased against the acceptance of the PPP hypothesis (Pippinger and Goering 1993, p. 479). This is in addition to the possible spurious rejection of cointegration if an insufficiently long span of the time series is used.

Relative to commodity arbitrage, the strength of the adjustment process towards equilibrium can be expected to be somehow proportional to the extent of market price deviations from equilibrium, with a higher speed of absorption for relatively larger deviations. Under homogenous conditions in terms of transaction costs and other disequilibrium determinants, transition from quick to slow adjustment would be abrupt. Ultimately, within an inner regime, arbitrage barriers would not be more than offset by economic advantages of trading.

This study has focused on wheat, as the only case, among three major food grains, with price variables following non-stationary homogeneous stochastic processes, as also found in studies based on linear cointegration (Goodwin 1992; Mohanty *et al.* 1999) or threshold cointegration (Michael *et al.* 1994). For three major wheat-producing countries, results of smooth transition functional specifications seem to favour a heterogeneous pattern of arbitrage, thus implying that the number of traders forgoing trade gradually increases with narrower price disequilibrium gaps. Moreover, asymmetric behaviour appears to prevail. Whereas virtually all traders tend to respond whenever the market price in Argentina settles below 20 per cent of the long-run equilibrium price, this proportion may vary from 75 to 80 per cent if equivalent upward price shocks occur, according to results of the logistic and the asymmetric exponential regression respectively. The respective proportions fall to shares of only 25–33 per cent (but still more than 60 per cent if a logistic regression is relied on) for 5 per cent negative shortfalls, and even less (11–18 per cent) in the opposite case (unchanged at 33 per cent for A(7)). Threshold and mid-transition points tend to have higher absolute values than the 2–3 per cent level assumed by Michael *et al.* (1994). However, comparability of results is limited, since this narrower band is largely due to the inclusion of transport costs and distinctions across various wheat varieties and markets of origin and destination.

Regarding the other food grains initially considered, log-transformed international market prices are found to be I(0) for maize and rice. Unbalanced cointegration regressions, i.e. with variables of different order of integration in the left- and right-hand side of the equation, need to meet specific conditions to be meaningful. The dependent variable should not be of a higher order than any one of the regressors, although no consensus exists on this subject (Charemza and Deadman 1992, p. 148; Baffes 1997, p. 70; Maddala and Kim 1998, pp. 251–2). If unbalanced cointegration problems are adequately treated, future research may examine whether possible long-term equilibrium relationships among prices of partly substitute grains are non-linear.

Provided reliable information is available, the study may also be deepened with a view to assessing the role of transaction costs as opposed to quality-related or institutional determinants of imperfect international arbitrage, as mentioned above (and in note 2). For the wheat price in the Argentine versus other major export markets, annual (July–June) average freight rates in US$/tonne charged by dry cargo vessels ready to load from three to four weeks ahead are reported in FAO (1999 and previous years). Relative to the period 1977–98 (freight statistics prior to 1976/77 are not consistent), the Spearman rank correlation coefficient between half-year lagged freight rates on the route River Plate–Rotterdam and absolute values of the (annualised) cointegrating residual vector z_t from model A is 0.34. This coefficient has the expected sign, but the respective t-test only rejects the null hypothesis at nearly 15 per cent level of significance. However, based on monthly 1978–89 data for three major exporter and two importer markets, Goodwin (1992) finds that a cointegrating equilibrium relationship among nominal wheat prices is present only once transport costs are accounted for. Similarly, relative to the same time series for six wheat varieties shipped from US ports to Rotterdam and Japan, Michael *et al.* (1994) obtain mixed results, with some cointegrating relationships being of a non-linear type and found after adjusting for transport costs in the case of Japan.

Non-linear cointegration models are found to have substantial explanatory power for international wheat prices. Nonetheless, both linear and non-linear models are unable to clearly distinguish among various possible determinants of non-linear patterns. In particular, non-linearity may arise from (i) changing market conditions, as the degree of monopoly power by dominant suppliers (B and C in table 2) changes; (ii) temporary switches in levels or size of long-run equilibrium parameters (D–F, table 2, and related threshold error correction models), or (iii) varying strength of the attractor and different short-run re-equilibrium-driving parameters, according to the relative disequilibrium gap (A, table 2, and related threshold and smooth transition error correction models). Regarding the last point, given values of speed-of-adjustment parameters close to or lower than -1, exponential and logistic smooth transition regression models even suggest a meta-stable or unstable oscillatory error correction mechanism in the extreme fringes of outer regimes.

Appendix: TAR and STR cointegration models and related tests

TAR models

In the arranged autoregression of the error term in Balke and Fomby's second-step estimation, the temporal sequence of the residuals is always maintained only within individual *cases* of data. In an AR(p) model, each of m cases is given by a $(p+1)$-dimensional vector of values of the stochastic process z_t, with the first term(s) of each vector representing a *signalling* lag value of z_{t-d}, that is a value providing a signal with a lag d for subsequent realisations of the process z_t. The cases in which this process is subdivided are rearranged according to ascending or descending order of the variable associated with the signalling lag or delay parameter d ($d \leq p$). If, for instance, the extreme s cases represent each of two outer regimes, the in-between $(m-2s)$ cases would correspond to the inner regime.

Alternative non-linearity tests on the standardised prediction errors from the arranged autoregression of z_t include the Tsay F-test (Tsay 1989), a CUSUM test by Petruccelli and Davies (1986), rolling or Chow tests for structural breaks, and direct Wald or LM-based χ^2 tests on the null hypothesis of no changes (Diebold and Chen 1996; Hansen 1996). If z_t is stationary and common heteroscedastic patterns are accounted for (usually by log-transforming the data), OLS is a consistent estimator and these non-linearity tests are statistically robust. As complementary tools of identification of the thresholds, Tsay suggests the use of two scatterplots, relating the above standardised one-step-ahead residuals and their recursive t-statistics on the potential delay variable z_{t-d}, respectively.

In a band-TAR(3; 1, 1) model, the equilibrium error in the outer regimes can be assumed to follow stationary ARMA(1, 1) processes in the outer regimes (with the upper and lower cases indicated with subscripts u and l, respectively), while behaving as a random walk with no drift in the inner regime. A general representation would then be formulated as follows:

$$
\begin{aligned}
z_t &= \delta_u + \rho_u z_{t-1} + \varepsilon_t - \theta_u \varepsilon_{t-1} && \text{if } z_{t-1} > \zeta_u \text{ and } \zeta_u \geq [\delta_u/(1-\rho_u)] > 0\,(0 < \rho_u < 1) \\
&= z_{t-1} + \varepsilon_t && \text{if } \zeta_l \leq z_{t-1} \leq \zeta_u \\
&= \delta_1 + \rho_1 z_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} && \text{if } z_{t-1} < \zeta_l \text{ and } \zeta_l \leq [\delta_1/(1-\rho_1)] < 0\ (0 < \rho_1 < 1)
\end{aligned}
\tag{1}
$$

Unlike in Balke and Fomby, the model so expressed does not presuppose symmetrical thresholds and functional forms above and below zero, except when particular linear restrictions are introduced ($\delta_u = -\delta_l$, $\rho_u = \rho_l$, $\zeta_u = \zeta_l$, and $\theta_u = \theta_l$). Moreover, unlike other threshold models which embody the scope for asymmetric responses as in Granger and Lee (1989), while maintaining the hypothesis of constant thresholds, this version relaxes the assumption that the borders of the long-run *attractor* band necessarily coincide with these thresholds. Given a strong effect of the signalling lag variable z_{t-1} or, in its absence, small differences between the means of the upper and lower-bound ARMA processes, i.e. μ_i $(= \delta_i/(1-\rho_i)$, with $i = l, u)$, and the respective threshold values in absolute terms, the two attractor lines may lie inside the inner regime. Rearranging the constraints, one can account for the opposite theoretical case of a weak signal and attractor lines located outside the inner regime. As noticed by a referee for this article, one could also further check for within-band behaviour of the error term.

As a further distinction, instead of a general ARMA representation, Balke and Fomby choose a stationary AR(1) model for the outer regimes: $z_t = \mu(1-\rho)+\rho z_{t-1}+\varepsilon_t$. If stability conditions are satisfied, both models ensure threshold cointegration. If $\varepsilon_t = (1-\rho B)\eta_t$ (with B being a backward shift operator) and $z_t = \mu + \eta_t$, the latter model would be equivalent to an imposition of COMFAC restrictions on the parameters of an ARMA(1, 1) model for z_t and η_t. Using the above standard symbols and ignoring the subscripts for asymmetric thresholds, the linear constraint would be the following (Charemza and Deadman 1992): $\rho - 1 = -(1-\Theta)$.

STR models

Given ergodic and stationary variables $w_t\ (=\Delta y_t)$ and $v_t\ (=\Delta x_t)$, two smooth transition error correction models are the following:

$$w_t = \delta + \sum_{i=1}^{p} \beta_0 w_{t-i} + \sum_{i=0}^{p} \gamma_0 v_{t-i} + \left(\sum_{i=1}^{p} \beta_1 w_{t-i} + \sum_{i=0}^{p} \gamma_1 v_{t-i} - \alpha z_{t-1} \right) H(z_{t-d}) + \varepsilon_t, \quad (2)$$

where:

$$H(z_{t-d}) = 1 - \exp\{-\lambda (z_{t-d})^2 [0.5 + (1+\exp(-\xi z_{t-d}))^{-1}]\} \qquad (\lambda > 0, \xi \neq 0) \quad (3)$$

or, alternatively:

$$H(z_{t-d}) = \{1 + \exp[-\lambda (z_{t-d} - c_l)(z_{t-d} - c_u)]\}^{-1} \qquad (\lambda > 0, c_l < c_u) \quad (4)$$

These models are defined as asymmetric exponential and logistic smooth transition regressions, i.e. AESTR and LSTR, relative to the response functions (3) and (4), respectively (AESTAR and LSTAR if only autoregressive terms were present) (Granger and Teräsvirta 1997; Anderson 1997; Teräsvirta 1998). In the AESTR, the monotonic asymmetry adjustment formula in square brackets can assume values in the range (0.5, 1.5) depending on the sign and absolute size of the parameter ξ, with the lower (higher) bound corresponding to a quicker reaction to negative (positive) deviations from equilibrium. This asymmetry formula equals unity whenever there is market equilibrium ($z_{t-d} = 0$) and/or $\xi = 0$, with the latter case defining a standard (symmetric) ESTR(3; p, d) (Anderson 1997, p. 473). In (4), the absence of an asymmetry formula is compensated by the allowance for different values of the half-way transition points c_l and c_u (even if the transition from/towards the outer regimes is identical). These switch-points can be expected to have opposite signs in an international arbitrage case, thus corresponding to blurred lower and upper thresholds, respectively.

In both functions, the larger the steepness parameter λ, the sharper is the transition from one regime to another. In one extreme, for $\lambda \to \infty$, (A)ESTR) $H(z_{t-d})$ becomes a heavy-sided function which assumes the values zero or one, depending as to whether $z_{t-d} < c$ or $z_{t-d} > c$ (absolute values). Similarly, for

$\lambda \to \infty$, depending on the sign of the polynomial terms in parentheses, LSTR $H(z_{t-d}) \to 0$ or $\to 1$, with values close to zero reflecting the inner regime in an arbitrage model. Therefore, equation (2) encompasses the scope of a TECM (as equation (1), suitably modified).

In the other extreme, for $\lambda \to 0$, in an AESTR or ESTR response function only short-run dynamic terms in first differences are present in equation (2) with no long-run elements, hence no cointegration. The LSTR-ECM boils down to a simple ECM with no thresholds. If the constant 0.5 is subtracted from the function, the LSTR-ECM also becomes an autoregressive distributed lag model with no long-run elements, but the boundaries of $H(z_{t-d})$ would shift to: $(-0.5, 0.5)$ (Jansen and Teräsvirta 1996*,* p. 739). Given the range (0, 1), both response functions can be regarded as a cumulative probability density for transaction cost thresholds varying across individuals (relative to the AESTR, see Anderson 1997, p. 471). If $\xi = c_l = c_u = 0$, the first derivative of $H(z_{t-d})$ is $2\lambda z_{t-d}[1 - H(z_{t-d})]$ and $2\lambda z_{t-d}\{H(z_{t-d})[1 - H(z_{t-d})]\}$, for (3) and (4) respectively. This highlights the relevance of the steepness parameter, among others.

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