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Lease allocation systems, risk aversion and the resource rent tax†

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This article examines the case of a risk-averse mining firm facing a resource rent tax in order both to incorporate the role of the risk-sharing quality of such a tax and to assess its implications given a government's lease allocation system. The model develops the conditions required for an investment-neutral RRT characterised by a threshold rate of return (r) and a rate of tax (t) and suggests that for an auction system of lease allocation, government revenue could be maximised by setting the tax rate below 100 per cent, but that for a discretionary system, it is in the government's interest to introduce an RRT which is effectively rate-of-return regulation.

1. Introduction

Mining leases are usually allocated either by auction or by a system based on discretionary considerations such as the size of the proposed work programme. For example, auctions are generally used to allocate offshore oil leases in the United States, whereas a form of the discretionary system is used for allocating such leases in the United Kingdom and Australia. Kretzer (1994) compared these two systems and demonstrated some of the merits of allocation by auction, particularly in the context of the impact on exploration activity.

The aim of this article is to make a further contribution to the assessment of these two systems by introducing an important practical consideration: the role of resource taxation. The particular form of resource taxation examined here is the recently developed resource rent tax (RRT) which has only been operating since 1987 in the context of offshore oil in Australia, and which has been advocated in the literature because of its potential not to distort a firm's investment behaviour (Garnaut and Clunies-Ross, 1975,

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1979; Fraser, 1993).[1] In addition, the article extends the analysis of Fraser (1993) to the situation of a risk-averse mining firm as this enables consideration of a central feature of the RRT as envisaged by Garnaut and Clunies-Ross (1975, 1979): its risk-sharing characteristics.[2] In so doing, it will investigate the potential for the government to choose the structure of the RRT so as to maximise expected government revenue from the allocation of a mining lease subject to the RRT, while at the same time leaving the firm's preferred level of investment unchanged. Finally, a comparison of the expected government revenue from the allocation of leases using each system will enable this aspect of the two systems in the context of the RRT to be illustrated.

The article is laid out as follows. Based on Fraser (1993) and Kretzer (1994) section 2 develops the model of a risk-averse firm making an investment decision about extracting an unknown quantity of resource stock over a finite lease period in the absence and presence of the RRT. This development enables the conditions which must be satisfied for the RRT to be investment-neutral to be specified. Unfortunately these conditions provide no unambiguous analytical insights into the structure of an investment-neutral RRT. In particular, the RRT features both a threshold rate of return (r) above which tax is payable and an associated rate of tax (t). If, as is expected, there is a set of (r, t) pairs which support investment-neutrality, the relationship between these (r, t) pairs is not clear from these conditions. In addition, the implications for maximising government revenue from allocating the lease subject to the RRT associated with such a set of (r, t) pairs are unclear. Consequently, in section 3 the model introduced in section 2 is subjected to a numerical analysis. This analysis not only confirms the expectation of a set of investment-neutral (r, t) pairs, but suggests that this set is characterised by increasing (r, t) values. That is, a higher threshold rate of return for tax to be payable must be balanced by a higher rate of tax applying above this level in order to achieve investment-neutrality. Moreover, this numerical analysis allows the government revenue implications of there being a set of investment-neutral (r, t) pairs to be clarified. In particular, it is shown that the choice of (r, t) pair which maximises potential government revenue from allocating the mining lease may depend both on whether the system of allocating such leases is by auction or by discretion and on the attitude to risk of the firm. Finally, it is shown that, in all cases, expected government revenue using an auction system of allocating leases exceeds that of the discretionary system, regardless

[1] Fraser (1991) also compares these two lease allocation systems, but limits the consideration of resource taxation to that of royalty payments.

[2] See also Emerson and Lloyd (1983).

of the specification of the RRT. The article ends with a brief summary and discussion of policy implications. Here a particular attempt is made to take account of the role of the attitude to risk of the government, as this will modify the extent to which its objective is simply maximising revenue from the RRT and, as a consequence, its perception of the most appropriate structure of the RRT.

2. The model

The model of optimal investment in Fraser (1993) and Kretzer (1994) specifies the mining firm's decision problem to be the choice of investment in capacity (k) to extract a resource deposit of unknown size in the context of a one-period lease.[3] Capacity has a constant per unit cost (c) and the price per unit of extracted resource (p) is assumed to be both constant and known with certainty. Because resource in excess of that which the firm has the capacity to extract is foregone, profit (π) in the absence of any resource taxation is given by:

$$\begin{aligned} \pi &= px - ck \quad \text{if} \quad x < k \\ \pi &= pk - ck \quad \text{if} \quad x \geq k. \end{aligned} \tag{1}$$

For the firm facing the RRT, it must pay tax at a given rate (t) on profits in excess of those consistent with the threshold rate of return on capital (r). Specifically, no tax is paid if:

$$(px - ck)/k < r$$

but if:

$$(px - ck)/k > r$$

then tax paid is given by:

$$t(px - (c + r)k)$$

up to a maximum of:

$$t(pk - (c + r)k)$$

if:

$$x \geq k.^{4}$$

[3] As suggested by a referee, this atemporal treatment of the investment decision is best suited to mining situations where the vast majority of investment occurs at the commencement of the project.

[4] Note that r must be such that $p > c + r$ or else no tax would be payable.

On this basis, profit in the presence of the RRT is given by:

$$\begin{aligned} \pi &= px - ck \quad \text{if} \quad x < \alpha k \\ \pi &= px - ck - t(px - (c+r)k) \quad \text{if} \quad \alpha k \le x < k \\ \pi &= pk - ck - t(pk - (c+r)k) \quad \text{if} \quad x \ge k \end{aligned} \tag{2}$$

where:

$$\alpha = (c+r)/p.$$

As shown in Fraser (1993), because equation 2 is a pay-off function with two kinks, it is simpler to re-define profit as a function of two random variables (x_1 and x_2) such that each of the variables captures the effect of one of the kinks:

$$\pi = (1-t)px_1 + tpx_2 - ck \tag{3}$$

where:

$$\begin{aligned} x_1 &= x \quad \text{if} \quad x < k \\ &= k \quad \text{if} \quad x \ge k \\ x_2 &= x \quad \text{if} \quad x < \alpha k \\ &= \alpha k \quad \text{if} \quad x \ge \alpha k. \end{aligned}$$

As a consequence, expected profit with the RRT ($E_t(\pi)$) is given by:

$$E_t(\pi) = (1-t)pE(x_1) + tpE(x_2) - ck \tag{4}$$

where:

$$E(x_1) = \int_0^k xf(x)\mathrm{d}x + \int_k^\infty kf(x)\mathrm{d}x$$

$$E(x_2) = \int_0^{\alpha k} xf(x)\mathrm{d}x + \int_{\alpha k}^\infty \alpha kf(x)\mathrm{d}x$$

$$f(x) = \text{probability distribution of resource size}$$

and the variance of profit with the RRT ($\mathrm{Var}_t(\pi)$) is given by:[5]

$$\begin{aligned} \mathrm{Var}_t(\pi) = (1-t)^2p^2\,\mathrm{Var}(x_1) + t^2p^2\,\mathrm{Var}(x_2) \\ + 2p^2(1-t)t\,\mathrm{Cov}(x_1, x_2) \end{aligned} \tag{5}$$

[5] Note that $\mathrm{var}(A = B) = \mathrm{Var}(A) + \mathrm{Var}(B) + 2\,\mathrm{Cov}(A, B)$ where A and B are random variables.

where:[6]

$$\mathrm{Var}(x_1) = F(k)\big(\mathrm{Var}(x|x < k) + (E(x|x < k))^2\big)$$
$$+ (1 - F(k))k^2 - (E(x_1))^2$$

$$\mathrm{Var}(x_2) = F(\alpha k)\big(\mathrm{Var}(x|x < \alpha k) + (E(x|x < \alpha k))^2\big)$$
$$+ (1 - F(\alpha k))\alpha^2 k^2 - (E(x_2))^2$$

$$\mathrm{Cov}(x_1, x_2) = F(\alpha k)\big(\mathrm{Var}(x|x < \alpha k) + (E(x|x < \alpha k))^2\big)$$
$$+ \alpha k(F(k) - F(\alpha k))E(x|\alpha k \le x < k)$$
$$+ (1 - F(k))\alpha k^2 - E(x_1)E(x_2)$$

$$F(k) = \text{cumulative probability of } k \text{ exceeding resource size}$$

$$F(\alpha k) = \text{cumulative probability of } \alpha k \text{ exceeding resource size}$$

For a risk-averse firm, joint concern over the expected level and variability of profit can be represented by the mean-variance framework:

$$E(U(\pi)) = U(E(\pi)) + \frac{1}{2}U''(E(\pi))\,\mathrm{Var}(\pi) \tag{6}$$

where:

$$U(\pi) = \text{utility of profit } (U'(\pi) > 0,\ U''(\pi) \le 0).$$

Note that Hanson and Ladd (1991) provide empirical support for the use of this framework even in the context of truncated probability distributions such as applies here.

With this framework, optimal investment can be found by substituting equations 4 and 5 into 6, differentiating equation 6 with respect to k and equating to zero:

$$\begin{aligned} U'(E_t(\pi)).\frac{\partial E_t(\pi)}{\partial k} + \frac{1}{2}U'''(E_t(\pi)).\frac{\partial E_t(\pi)}{\partial k}.\mathrm{Var}_t(\pi) \\ + \frac{1}{2}U''(E_t(\pi)).\frac{\partial\,\mathrm{Var}_t(\pi)}{\partial k} = 0 \end{aligned} \tag{7}$$

[6] These expressions for $\mathrm{Var}(x_1)$, $\mathrm{Var}(x_2)$ and $\mathrm{Cov}(x_1, x_2)$ are derived in the Appendix (part A).

where:

$$\frac{\partial E_t(\pi)}{\partial k} = (1-t)p\frac{\partial E(x_1)}{\partial k} + tp\frac{\partial E(x_2)}{\partial k} - c$$
$$= (1-t)p(1-F(k)) + tp\alpha(1-F(\alpha k)) - c \tag{8}$$

$$\frac{\partial \operatorname{Var}_t(\pi)}{\partial k} = (1-t)^2 p^2 \frac{\partial \operatorname{Var}(x_1)}{\partial k} + t^2 p^2 \frac{\partial \operatorname{Var}(x_2)}{\partial k}$$
$$+ 2p^2(1-t)t\frac{\partial \operatorname{Cov}(x_1, x_2)}{\partial k} \tag{9}$$

and where:[7]

$$\frac{\partial \operatorname{Var}(x_1)}{\partial k} = 2(1-F(k))(k-E(x_1))$$
$$\frac{\partial \operatorname{Var}(x_2)}{\partial k} = 2\alpha(1-F(\alpha k))(\alpha k - E(x_2))$$
$$\frac{\partial \operatorname{Cov}(x_1, x_2)}{\partial k} = \alpha(F(k)-F(\alpha k))E(x|\alpha k < x < k) + 2\alpha k(1-F(k))$$
$$+ (1-F(k))E(x_2) + \alpha(1-F(\alpha k))E(x_1).$$

Note that in the absence of the RRT, equations 8 and 9 simplify to:

$$\frac{\partial E(\pi)}{\partial k} = p(1-F(k)) - c \tag{10}$$

$$\frac{\partial \operatorname{Var}(\pi)}{\partial k} = 2p^2(1-F(k))(k-E(x_1)). \tag{11}$$

On this basis an investment-neutral RRT is represented by an (r, t) pair which equates equation 7 to zero using equations 4, 5, 8 and 9 for the value of k which also equates equation 7 to zero in the absence of the RRT (i.e. for $E(\pi) = pE(x_1) - ck$, $\operatorname{Var}(\pi) = p^2 \operatorname{Var}(x_1)$, and equations 10 and 11).

Note also that for the special case of a risk-neutral firm analysed in Fraser (1993), optimal investment is found by equating equation 8 to zero:[8]

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[7] The expressions for these derivatives are derived in the Appendix (part A).

[8] It follows from equation 7 that for:

$$\frac{\partial \operatorname{Var}_t(\pi)}{\partial k} > 0$$

and $U'''(E_t(\pi))$ positive, $\partial E_t(\pi)/\partial k$ must be positive at the optimal level of k for the risk-averse firm. Moreover, in this situation, since $\partial E_t(\pi)/\partial k$ is equal to zero for a risk-neutral firm, it follows that the optimal k for the risk-averse firm is smaller than that of the risk-neutral firm.

$$\frac{\partial E_t(\pi)}{\partial k} = (1-t)p(1-F(k)) + tp\alpha(1-F(\alpha k)) - c = 0. \tag{12}$$

This equation represents the finding of Kretzer (1994), that optimal investment is the level which just balances the costs of over- and under-investment, with these costs adjusted to allow for the presence of the RRT. In addition, it is shown in Fraser (1993) by comparing equations 10 and 12 that, in the case of a risk-neutral firm, the investment-neutral RRT depends only on the value of r.

However, it can be seen from equations 7, 8 and 9 that, with t both a linear and a non-linear feature of these equations, this analytical simplification is lost in the more general case of risk aversion. Moreover, an attempt to provide insights into the characteristics of an investment-neutral (r, t) pair by an examination of the above analytical expressions for the risk-averse firm is almost totally unproductive.

In addition, the absence of analytical results regarding the characteristics of investment-neutral (r, t) pairs also precludes any assessment of the government revenue implications of such pairs. For example, in the case of a discretionary system of allocating leases so that the only government revenue is from the RRT, although it is clear that an increase in t or a decrease in r increases expected tax revenue (ETR):

$$\mathrm{ETR} = pt(E(x_1) - E(x_2)) \tag{13}$$

so that:

$$\frac{\partial \mathrm{ETR}}{\partial t} = p(E(x_1) - E(x_2)) > 0 \tag{14}$$

$$\frac{\partial \mathrm{ETR}}{\partial r} = -tk(1 - F(\alpha k)) < 0, \tag{15}$$

without any information about the relative values of investment-neutral (r, t) pairs, no analytical conclusion can be reached regarding their relative contribution to government revenue. Similarly, in the case of an auction system, expected government revenue is provided not only by ETR but also by the successful bid for the lease, where the firm's maximum potential bid (BID) in this situation is given by the certainty equivalent of equation 6 using equations 4 and 5 to account for the impact of the RRT. However, the derivatives of equation 5 with respect to t and r are ambiguous.[9] Consequently, no analytical conclusion can be reached about the impact of only changing one of r or t on the firm's maximum potential bid, let alone

[9] Except in the special case of $t = \frac{1}{2}$ where $\frac{\partial \mathrm{Var}_t(\pi)}{\partial t} < 0$ because $\mathrm{Var}(x_1) > \mathrm{Var}(x_2)$.

assess the implications of different investment-neutral (r, t) pairs for the size of this bid.

Nevertheless, despite the absence of analytical conclusions, the model developed in this section does provide a basis for proceeding to a numerical assessment of the characteristics of an investment-neutral RRT, and of the associated implications for expected government revenue. Such an assessment is undertaken in the next section.

3. Numerical analysis

The numerical analysis of the model of section 2 requires a specification of the probability distribution of the uncertain resource size. Following Fraser (1993) it is assumed that this uncertainty can be approximated by the normal distribution.[10] For this situation the Appendix (part B) contains details of the distributional forms required to analyse numerically the model of section 2.

In addition, a functional form for the utility of profits must be specified. In what follows, the constant relative risk aversion form is used:[11]

$$U(\pi) = \frac{\pi^{1-R}}{1-R}$$

where:

$$\begin{aligned} R &= -U''(\pi)\pi/U'(\pi) \\ &= \text{coefficient of relative risk aversion.} \end{aligned}$$

Finally, for this analysis the parameter values of the model are set to:

$$\begin{aligned} p &= 10 \\ \overline{x} &= 100 \\ \sigma_x &= 40 \\ c &= 4.16. \end{aligned}$$

Table 1 contains details of the results of the numerical analysis for three levels of the coefficient of relative risk aversion (i.e. $R = 0.3$, 0.6 and 0.9).

[10] Fraser and Kingwell (1997) compare resource taxes using the alternative case of a log-normal distribution and find the pattern of results is similar to that for a normal distribution, particularly for moderate to small values of the coefficient of variation (i.e. less than or equal to 100 per cent).

[11] See Pope and Just (1991) for arguments supporting this choice of functional form.

Table 1 Results of the numerical analysis for three levels of risk aversion

	t			
	0	0.5	0.75	1
$R = 0.3$[a]				
r		0.72	0.78	0.93
$E_t(\pi)$	426.73	240.20	151.26	72.73
$\mathrm{Var}_t(\pi)$	59390	23510	12903	7276
ETR		186.54	275.47	354.00
BID	406.01	225.65	138.63	58.20
BID + ETR		412.19	414.10	412.20
$R = 0.6$[b]				
r		0.85	0.93	1.10
$E_t(\pi)$	423.48	243.86	159.39	85.83
$\mathrm{Var}_t(\pi)$	52476	21264	12057	7083
ETR		179.62	264.09	337.65
BID	387.28	218.54	137.66	63.17
BID + ETR		398.16	401.75	400.82
$R = 0.9$[c]				
r		0.90	1.01	1.16
$E_t(\pi)$	419.60	243.47	162.48	89.60
$\mathrm{Var}_t(\pi)$	47252	19356	11165	6639
ETR		176.13	257.12	330.00
BID	371.59	209.97	134.08	61.32
BID + ETR		386.10	391.20	391.32

Notes: [a] $k = 102.99$
[b] $k = 98.70$
[c] $k = 95.30$

The column headed $t = 0$ represents the situation in the absence of the RRT, while the remaining columns indicate the situation following the introduction of an investment-neutral RRT. As explained in section 2, this involves choosing an (r, t) pair which equates equation 7 to zero using equations 4, 5, 8 and 9 for the appropriate level of k. Table 1 contains details of three such (r, t) pairs, as well as details of the associated ETR and BID values. The results in table 1 not only confirm the expectation of there being a set of investment-neutral (r, t) pairs, but also indicate that a comparison of any two such pairs would feature one pair with both the threshold rate of return and the tax rate higher than the other pair. This finding suggests that increases in r and t have compensating impacts on the first-order condition for optimal investment in the presence of the RRT (i.e. equation 7). For example, if an increase in t represents an overall disincentive to investment through a reduction in its marginal expected profitability, then an increase in

r, which reduces the overall probability of tax being payable, compensates by increasing the marginal expected profitability of investment.[12]

Next, consider the implications of the results in table 1 in relation to government revenue from the allocation of the mining lease. These results indicate that higher (r, t) pairs feature higher expected tax revenue (ETR), which implies the extra tax revenue associated with the higher t more than offsets the reduction in tax revenue associated with the higher r (see equations 14 and 15). Consequently, if the government uses a discretionary system of lease allocation, then the investment-neutral RRT which maximises expected revenue features a tax rate of 100 per cent on profits in excess of those consistent with the threshold rate of return.[13] Note that this form of RRT is effectively one of rate-of-return regulation, and that this finding is consistent with that of Fraser (1993) for the special case of risk neutrality.

However, for a government which uses an auction system of lease allocation, a further finding of Fraser (1993) was that the rate of tax is irrelevant in determining total government revenue in the situation where a risk-neutral firm's auction bid is equal to its expected profit. Specifically, in this case payments made at an auction are perfect substitutes for tax payments. By contrast, the results for a risk-averse firm in table 1 indicate not only that the rate of tax is relevant in determining total government revenue even if the firm's auction bid is equal to the certainty equivalent of its expected utility (the BID value), but also that the revenue-maximising rate of tax depends on the attitude to risk of the firm. In particular, because the presence of the tax decreases both $E_t(\pi)$ and $\mathrm{Var}_t(\pi)$, there is no longer a one-to-one relationship between tax payments and auction bids. Moreover, this risk-sharing feature of the RRT means that, in the presence of the RRT, a risk-averse firm's BID + ETR value exceeds its BID value in the absence of the RRT. The results in table 1 show that the extent of this surplus is positively related to the level of the firm's risk aversion, with the surplus smallest in the case of $R = 0.3$. In addition, the size of the surplus is related to the rate of tax specified in the RRT. However, the pattern of influence of this rate is not straightforward, and as shown by the results in table 1, differs between the highest $(R = 0.9)$ and the lower levels of attitude to risk $(R = 0.3, 0.6)$.

[12] In fact, further numerical analysis verifies that for any given value of t, there exists an investment-neutral value of r, and that this value of r is monotonically increasing in the value of t.

[13] In this context note that Garnaut and Clunies-Ross (1975) suggest tax revenue is likely to be maximised for a tax rate of 100 per cent.

Table 2 Results of the numerical analysis for $R = 0.6$ and two levels of resource size uncertainty

	t			
	0	0.5	0.75	1
$\sigma_x = 0.4^{a}$				
ETR		179.62	264.09	337.65
BID	387.28	218.54	137.66	63.17
BID + ETR		398.16	401.75	400.82
$\sigma_x = 0.6^{b}$				
ETR		81.51	117.38	150.02
BID	250.57	173.65	138.55	105.53
BID + ETR		255.16	255.93	255.55

Notes: [a] $k = 98.70$
[b] $k = 91.90$

Nevertheless, this pattern can be explained by reference to the size of the trade-off between the expected level and variance of profit associated with each investment-neutral (r, t) pair. For example, the simplest pattern is that for the most risk-averse firm which, according to table 1, is willing to view each increase in t as providing a worthwhile trade-off of risk for return. However, it can also be seen from table 1 that the 'price' of each trade-off (in terms of the ratio of the change in expected profit to the change in the variance of profit) associated with successive increases in the investment-neutral (r, t) pairs is itself increasing. In this context, table 1 shows that for a firm with either of the lower levels of risk aversion, this 'price' is too high for the increase from $t = 0.75$ to $t = 1$ and so the firm's BID + ETR value is lower in the latter case.[14]

Consequently, for a government using an auction system of lease allocation the results in table 1 suggest that, if mining firms are believed to have low to moderate levels of risk aversion, then the RRT should feature a rate of tax less than 100 per cent. In this context, table 2 contains results for a higher level of resource size uncertainty, and it shows that higher levels of this uncertainty are similar to higher levels of risk aversion in weakening the argument for a rate of tax of less than 100 per cent. In particular, the extent to which the BID + ETR value for $t = 0.75$ exceeds that for $t = 1$ is proportionately less in the case of $\sigma_x = 0.6$ compared to $\sigma_x = 0.4$ (i.e. 0.15 per cent compared to 0.23 per cent).

[14] I am grateful to an anonymous referee for helping me to realise that because r and t are being adjusted to achieve investment-neutrality, the 'price' of the risk-return trade-off the firm is evaluating is not constant.

Finally, although this numerical analysis is only intended to be illustrative of patterns in the results featuring relative magnitudes, it appears from table 1 that the sum of BID + ETR for any investment-neutral (r, t) pair exceeds the value of ETR for $t = 1$.[15] Consequently, bearing in mind the qualification that the strength of competition at an auction of a mining lease will determine how close a firm's actual bid will be to the BID value, these results support the view that expected government revenue from an auction system of lease allocation typically exceeds that from a discretionary system, regardless of the specification of the RRT.

4. Conclusions

The focus of this article has been on further assessment of the relative merits of the auction and discretionary systems of allocating mining leases. Developing the analysis of Kretzer (1994), this article has incorporated an important practical consideration: the role of resource rent taxation as exemplified by the Petroleum Resource Rent Tax currently applying to offshore oil leases in Australia. In addition, it has broadened the scope of Fraser (1993) to allow for risk aversion on the part of the mining firm so as to capture the risk-sharing characteristic of the RRT.

Section 2 of the article developed the model of Fraser (1993) and Kretzer (1994) to allow for risk-averse behaviour in determining the conditions which must be satisfied for the RRT, characterised by a threshold rate of return (r) and a rate of tax (t), to be investment-neutral. However, it was shown that these conditions provided no unambiguous analytical insights. Consequently, section 3 of the article reported the results of a numerical analysis of this model. These results contained three robust features: a set of investment-neutral (r, t) pairs characterised by a monotonically increasing relationship between the associated values of r and t; the maximisation of expected tax revenue for a rate of tax of 100 per cent ($t = 1$) on profits in excess of those consistent with the associated threshold rate of return; and an excess of the sum of the certainty equivalent of the firm's after-tax valuation of the mining lease and expected tax revenue for any specification of the RRT (such as might be collected by a government using an auction system) over the expected tax revenue from a rate of tax of 100 per cent (such as might be collected by a government using a discretionary system).

[15] Note that although all the magnitudes of change presented in the numerical results are quite small, the parameter values of the analysis have been chosen simply for convenience. For example, for firms operating with smaller profit margins, the magnitudes will be larger.

A further feature of some of the results was the maximisation of the sum of expected tax revenue and the certainty equivalent of the firm's after-tax valuation of the mining lease for a rate of tax of less than 100 per cent. In addition, it was shown that this outcome was more likely for lower levels of risk aversion on the part of the firm (and, as a corollary, lower levels of resource size uncertainty). This feature suggests the importance of the attitude to risk of the firm in determining the characteristics of the investment-neutral RRT for a government which, because of its apparent advantage in terms of potential revenue, uses an auction system to allocate mining leases. In particular, if the government is confident that an auction of a mining lease will be sufficiently competitive for it to elicit bids from the firms involved which are close to their maximum potential valuations, that the mining lease in question is characterised by moderate uncertainty about resource size, and that the firms involved are only moderately risk averse, then expected government revenue may be maximised for a tax rate of less than 100 per cent. Moreover, bearing in mind that a government may also exhibit some degree of risk aversion in relation to uncertain tax revenues, the findings in this article relating to the use of an auction system for such a government serve to reinforce the case for a rate of tax of less than 100 per cent. However, in situations where resource size uncertainty is substantial, or the degree of risk aversion of the firms involved is believed to be relatively large, or for a government which, because of a lack of competition, uses a discretionary system of lease allocation, these findings are of no consequence. In these situations the policy implication here is that such a government, particularly if it has a diverse tax base, is best served by an investment-neutral RRT which features a tax rate of 100 per cent — effectively rate-of-return regulation.

Appendix

Part A: Expressions for Var(x_1), Var(x_2), Cov(x_3, x_2) and their derivatives with respect to k

$$\begin{aligned}\mathrm{Var}(x_1) &= \int_0^\infty (x_1 - E(x_1))^2 f(x)\mathrm{d}x \\ &= \int_0^k (x - E(x_1))^2 f(x)\mathrm{d}x + \int_k^\infty (k - E(x_1))^2 f(x)\mathrm{d}x.\end{aligned}$$

Rearranging and making the substitution:

$$\int_0^k x^2 f(x)\mathrm{d}x = F(k)\big(\mathrm{Var}(x|x < k) + (E(x|x < k))^2\big)$$

gives:

$$\mathrm{Var}(x_1) = F(k)\big(\mathrm{Var}(x|x < k) + (E(x|x < k))^2\big) \\ + (1 - F(k))k^2 - (E(x_1))^2. \tag{A1}$$

Taking the derivative of equation A1 with respect to k gives:

$$\frac{\partial \mathrm{Var}(x_1)}{\partial k} = 2(1 - F(k))k - 2(1 - F(k))E(x_1) \\ = 2(1 - F(k))(k - E(x_1)). \tag{A2}$$

Expressions A1 and A2 are reproduced in the main text.

$$\mathrm{Var}(x_2) = \int_0^{\infty} (x_2 - E(x_2))^2 f(x)\mathrm{d}x \\ = \int_0^{\alpha k} (x - E(x_2))^2 f(x)\mathrm{d}x + \int_{\alpha k}^{\infty} (\alpha k - E(x_2))^2 f(x)\mathrm{d}x.$$

Rearranging and making the substitution:

$$\int_0^{\alpha k} x^2 f(x)\mathrm{d}x = F(\alpha k)\big(\mathrm{Var}(x|x < \alpha k) + (E(x|x < \alpha k))^2\big)$$

gives:

$$\mathrm{Var}(x_2) = F(\alpha k)\big(\mathrm{Var}(x|x < \alpha k) + (E(x|x < \alpha k))^2\big) \\ + (1 - F(\alpha k))\alpha^2 k^2 - (E(x_2))^2. \tag{A3}$$

Taking the derivative of equation A3 with respect of k gives:

$$\frac{\partial \mathrm{Var}(x_2)}{\partial k} = 2(1 - F(\alpha k))\alpha^2 k - 2\alpha(1 - F(\alpha k))E(x_2) \\ = 2\alpha(1 - F(\alpha k))(\alpha k - E(x_2)). \tag{A4}$$

Expressions A3 and A4 are reproduced in the main text.

$$\mathrm{Cov}(x_1, x_2) = \int_0^{\infty} (x - E(x_1))(x - E(x_2)) f(x)\mathrm{d}x \\ = \int_0^{\alpha k} (x^2 - E(x_1)E(x_2)) f(x)\mathrm{d}x \\ + \int_{\alpha k}^{k} (x\alpha k - E(x_1)E(x_2)) f(x)\mathrm{d}x \\ + \int_{k}^{\infty} (xk^2 - E(x_1)E(x_2)) f(x)\mathrm{d}x.$$

Rearranging and substituting as above gives:

$$\begin{aligned}\mathrm{Cov}(x_1, x_2) = F(\alpha k)\big(&\mathrm{Var}(x|x < \alpha k) + (E(x|x < \alpha k))^2\big)\\ &+ \alpha k(F(k) - F(\alpha k))E(x|\alpha k \le x < k)\\ &+ (1 - F(k))\alpha k^2 - E(x_1)E(x_2)\end{aligned} \tag{A5}$$

Taking the derivative of equation A5 with respect to k gives:

$$\begin{aligned}\frac{\partial\,\mathrm{Cov}(x_1, x_2)}{\partial k} = {}& \alpha(F(k) - F(\alpha k))E(x|\alpha k \le x < k)\\ &+ 2(1 - F(k))\alpha k + (1 - F(k))E(x_2)\\ &+ \alpha(1 - F(\alpha k))E(x_1).\end{aligned} \tag{A6}$$

Expressions A5 and A6 are reproduced in the main text.

Part B: Distributional forms for the numerical analysis

Based on Fraser (1988) and Johnson and Kotz (1972), for the case of a normal distribution:

$$E(x_1) = F(k)(\overline{x} - \sigma_x Z(k)/F(k)) + (1 - F(k))k$$

$$E(x_2) = F(\alpha k)(\overline{x} - \sigma_x Z(\alpha k)/F(\alpha k)) + (1 - F(\alpha k))\alpha k$$

$$E(x|x < \alpha k) = \overline{x} - \sigma_x Z(\alpha k)/F(\alpha k)$$

$$E(x|\alpha k \le x < k) = \overline{x} + \frac{Z(\alpha k) - Z(k)}{F(k) - F(\alpha k)}.\sigma_x$$

$$\begin{aligned}\mathrm{Var}(x_1) = {}& F(k)\sigma_x^2\left(1 - (-Z(k)/F(k))^2 + \frac{(\overline{x} - k)}{\sigma_x}\cdot\frac{Z(k)}{F(k)}\right)\\ &+ (1 - F(k))F(k)(\overline{x} - \sigma_x Z(k)/F(k) - k)^2\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(x_2) = {}& F(\alpha k)\sigma_x^2\left(1 - (-Z(\alpha k)/F(\alpha k))^2 + \frac{(\overline{x} - \alpha k)}{\sigma_x}\cdot\frac{Z(\alpha k)}{F(\alpha k)}\right)\\ &+ (1 - F(\alpha k))F(\alpha k)(\overline{x} - \sigma_x Z(\alpha k)/F(\alpha k) - \alpha k)^2\end{aligned}$$

$$\mathrm{Var}(x|x < \alpha k) = \sigma_x^2\left(1 - (-Z(\alpha k)/F(\alpha k))^2 + \frac{\overline{x} - \alpha k}{\sigma_x}\cdot\frac{Z(\alpha k)}{F(\alpha k)}\right)$$

where:

$$\overline{x} = \text{mean of } x$$

$$\sigma_x = \text{standard deviation of } x$$

$$Z(k) = \text{ordinate of } x \text{ at } k$$

$$Z(\alpha k) = \text{ordinate of } x \text{ at } \alpha k$$

$$\sigma_x^2 = \text{variance of } x.$$

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