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The dynamics of phase farming in dryland salinity abatement

Ute Mueller, Steven Schilizzi and Tuyêt Tran*

In Australia, soil salinisation has become a major concern. One way to deal with the problem is for farmers to plant trees or regenerate native bush. However, doing so raises several questions which involve optimal switching times, when switching involves a cost in the form of up-front investments. Optimality conditions are derived for the three-stage problem, and applied to dryland salinity control in Western Australia. Optimal management practices are found to be very sensitive to farmers' discount rates and to the speed at which the watertable rises or falls.

1. Introduction

The problem addressed here is best introduced by the following example. Consider an area of land dedicated to annual cropping. Over the years, cropping degrades the land, thereby reducing yields. Although falling yields may be offset by increased farm inputs, this raises costs and ultimately reduces profits. At some stage, the land is in such poor condition that cropping must be stopped and, unless the land is abandoned, some form of land rehabilitation must be initiated. This assumes no land use other than farming is profitable. During the rehabilitation phase, the land and its productive potential are gradually restored, but at the cost of forgone profits from lost cropping opportunities. Thus, during rehabilitation, two opposing forces are at work. On the one hand, increasing land quality implies increasing future yields and profits, while, on the other hand, forgone profits from lost cropping accumulate. At some point, when the land has recovered enough of its productive potential, it becomes preferable again to revert to cropping. And the cycle starts over again.

This description is not complete, however. There are costs associated with switching from cropping to rehabilitation, typically in the form of an up-

* Ute Mueller and Tuyêt Tran, Department of Mathematics, Edith Cowan University, Perth. Steven Schilizzi, Agricultural and Resource Economics, The University of Western Australia.

front investment. Likewise, another up-front investment is associated with the switch from rehabilitation back to cropping.

The problem then is to determine the optimal switching times from one activity to the other such that some performance function of the farm is maximised over a given time horizon. For instance, this function could be the sum of discounted profits over the whole period, or the value of the land at terminal time. The problem may have a finite horizon, relating to the farmer's time of retirement, or an infinite one, relating to the sustainability of farming practices for future generations.

Many real-world examples correspond to this type of problem. One of importance in Western Australia relates to soil salinity in the wheatbelt areas (300 to 600 mm annual rainfall, mostly in winter). Clearing of the original vegetation, deep-rooted woody perennials, and replacement by shallow-rooted annual crops and pastures, have destroyed the balance of the water cycle. Deep-rooted shrubs and trees tapped rainwater down to some depth and their high level of evapotranspiration, due to the hot and dry climate, kept the level of saline groundwater from rising. With the trees and shrubs gone, and additional rainfall being added to the subsoil every year, the saline water table rose closer to the surface, eventually in some places hitting crop root systems. Where this has happened, yields or profits per hectare have fallen (Williams 1989; Hall and Hyberg 1991; Ferdowsian *et al.* 1996; Australian Government 1996).

One solution CALM (Conservation And Land Management) in Western Australia is looking into to reverse the process in the wheatbelt is the planting of oil mallee trees (Bartle *et al.* 1996). This tree species has been viewed with some hope because not only should it reverse the salinisation process, it should also produce revenues to the farmer by yielding essential oils for the solvent industry.

In this case, the cost of switching from cropping to rehabilitation by trees mainly involves the planting of the trees and their protection in early years. With the reverse switch, a cost may be associated with removing the stumps, either mechanically or otherwise. However, if new technologies and new markets allow the wood to fetch a high enough price, the cost associated with this second switch may be negative, meaning a positive net benefit.

Similar problems may involve restoration of land fertility in the form of organic matter content, soil structure (due to soil compaction, hard pans, or waterlogging) and water retention capacity. The idea here is different from the more classical remedies, where restoration of some aspect of land quality takes the form of inputs. Nutrient deficiency can be restored by fertiliser applications; water deficiencies, where possible, by irrigation; acidity, by liming. In the present case, restoration demands the exclusion of the initial most profitable activity. However, this setting is neither general nor always appropriate.

The system described above refers to dense tree plantations alternating with cropping on the same land area. An alternative scheme is an agroforestry system that replaces time sequences with space sequences. The model developed here could easily be adapted and interpreted as a combined, alternating agroforestry scheme, with alley-cropping between tree bands alternating over the long run with periods of pure cropping or pasture. Another possibility, not explicit in the model, is the choice of strategic paddocks where trees would be planted on a permanent basis, allowing crops to thrive in adjacent paddocks. Other configurations are possible, depending on local topography and hydrogeology.

Farmland salinity usually extends beyond the farm gate and involves a whole water catchment, comprising a number of farms. Salinity management usually cannot ignore this wider dimension, where both the extent of the underground aquifer and farmer collective organisation are determining factors (see, for example, Greiner 1997). Here, we shall only investigate a specific type of action a farmer may consider on his or her own farm, leaving other aspects for further research.

The article is organised as follows. Section 2 reviews previous work. Section 3 provides a general formulation of the three-stage problem and provides first-order conditions for optimality. Section 4 investigates an application to dryland salinity control using phase farming strategies. Section 5 presents results from numerical simulations. Section 6 offers some concluding comments.

2. Previous investigations

Problems of optimal switching times seem to have been first encountered in resource management where switches to backstop technologies in extractive industries were involved (Hoel 1978; Dasgupta *et al.* 1982). The problem investigated is that of exercising some control over the time at which such technologies become profitable. In other contexts, problems of this kind have involved delivery lags associated with the acquisition of new capital goods, when some control over such lags is sought (Rossana 1985). A general characterisation has been given by Tomiyama (1985) for two-stage optimal control processes. It was extended by Tomiyama and Rossana (1989) in the case where the performance function explicitly depends on the switching time. The introduction of non-zero switching costs was introduced by Amit (1986) in a key application that involved the switching from primary to secondary petroleum recovery. Kamien and Schwartz, in the second edition of their book (1991), point out that such problems are analogous to the existence of discrete jumps in the state variables of the system. An extension to multi-stage switching problems is carried out by Babad (1995), using multiprocess theory.

Hertzler (1990) considered a similar problem in an agricultural setting. However, in his formulation there were no costs associated with the switch from one farming practice to another. The formulation was applied to the optimal management of topsoil/subsoil acidity. The problem was to determine when to lime and when not to lime, given dynamic interactions between topsoil and subsoil acidity (Hertzler and Tierney 1995). In that formulation, the problem was concave in time and general solutions could be identified. In our case, as will appear, this is no longer true. Also, Hertzler solves for the steady-state, assuming the system has got there. In our case, the world may have changed considerably before a steady-state is ever reached. The important aspect is the first stages of the transition.

Other work has tackled the economics of land rehabilitation and more particularly salinity abatement. Examples are Gomboso and Hertzler (1991) and Hertzler and Barton (1992). However, these approaches focused on the management of the rehabilitation phase itself. The switching problem was not considered. Similarly, Wang and Lindner (1990) examined the rehabilitation of degraded rangelands with stochastic effects on range re-growth, but again with no switching problem.

Work has also been carried out at the catchment level, rather than from the point of view of a single farm. Greiner, for instance, used dynamic simulation models (Greiner 1994, 1996; Greiner and Parton 1995) and multi-period mathematical programming (Greiner and Hall 1995) to explore catchment-wide management options. Here, we focus on the management of the single farm and use optimal control theory. A good introduction to this approach from an economics perspective is Kamien and Schwartz (1991). For a single farm, optimal control is the preferred method, given the few state and control variables involved and its power in computing farm strategies that are both profitable and sustainable in the long run.

3. Problem formulation

The basic formulation of the optimal control problem as described above is modified from Amit (1986). It covers only the first three periods and involves two switches. We also assume that the second and third integrand explicitly depend on the switching time. The problem can be written as follows:

Maximise

$$J = \int_{t_0}^{t_1} D(t, x(t), u(t))dt + \int_{t_1}^{t_2} F(t, t_1, x(t), u(t)dt + \int_{t_2}^{t_3} G(t, t_2, x(t), u(t)dt$$
$$- \Phi_1(t_1, x(t_1), u(t_1)) - \Phi_2(t_2, x(t_2), u(t_2)) \tag{I}$$

Subject to:

$$x_i' = \begin{cases} d(t, x(t), u(t)) & t_0 \le t \le t_1 \\ f(t, t_1, x(t), u(t)) & t_1 \le t \le t_2 \\ g(t, t_2, x(t), u(t)) & t_2 \le t \le t_3 \end{cases} \tag{II}$$

where $t_0, x(t_0) = x_0$ are fixed, and $t_1, x(t_1)$, t_2, $x(t_2)$, t_3, $x(t_3)$ are free (III). The notation x' denotes the time derivative dx/dt.

In this formulation, J measures the net present value of the land. The function $x(t)$ is the state variable and represents land quality or, in our example, salinity measured as the depth to the saline watertable. The function u is the control function of the problem. In our case it may represent the cropping intensity or the tree density during rehabilitation. Both functions, x and u may be vector-valued. The objective functions D and G are associated with profits from cropping. The function F is the profit function of the rehabilitation phase. They depend on time (t), the cropping intensity or tree density (u), and the land quality (x). The functions Φ_1 and Φ_2 represent the cost of switching from cropping to rehabilitation and from rehabilitation to cropping, respectively. The switching times are t_1 and t_2. They separate the first cropping period $[t_0, t_1]$ from the rehabilitation period $[t_1, t_2]$ and the rehabilitation period from the second cropping phase $[t_2, t_3]$. In this formulation, we restrict ourselves to a deterministic problem; uncertainty over costs and prices, or over the effects of u, is not considered.

The constraints are given by differential equations, the equations of motion of the system. They describe the dynamics of land quality depending on what phase is active, cropping or rehabilitation. In our example, the first equation, given by the functions d and g, would describe a decreasing saline watertable depth. The second equation would describe an increasing watertable depth as a function of time, the current level and tree density.

This problem seeks to determine the values of t_1 and t_2 (when to switch) and the value of u, such that J is maximised over the entire period spanning from initial t_0 to final t_3.

We first derive necessary conditions for the general optimisation problem with three stages and two costly switches.

We define the Hamiltonian functions H_1, H_2, H_3 by:

$$H_1 = D + \sum_{i=1}^{n} \lambda_{1i} d_i = D + \lambda_1 d \quad t_0 \leq t \leq t_1$$

$$H_2 = F + \sum_{i=1}^{n} \lambda_{2i} f_i = F + \lambda_2 f \quad t_1 \leq t \leq t_2$$

$$H_3 = G + \sum_{i=1}^{n} \lambda_{3i} g_i = G + \lambda_3 g \quad t_2 \leq t \leq t_3$$

the interpretation of which is in terms of dynamic annual profits, meaning the current profits affected by all discounted future losses and gains, or net user costs. The functions λ_1, λ_2 and λ_3 are Lagrange multiplier functions (or the costate variables) and measure the marginal value of land quality (the state variable), given d, f and g. A lengthy, but standard argument (Tran 1997) using the calculus of variations results in the following necessary conditions for optimality of (I) subject to (II) and (III):

$$\lambda_1' = -\nabla_x H_1^* \quad t_0 \leq t \leq t_1 \tag{1}$$

$$\lambda_2' = -\nabla_x H_2^* \quad t_1 \leq t \leq t_2 \tag{2}$$

$$\lambda_3' = -\nabla_x H_1^* \quad t_2 \leq t \leq t_3 \tag{3}$$

There are three standard optimal control conditions for optimality, namely:

$$\nabla_u H_1^* = 0 \quad t_0 \leq t \leq t_1 \tag{4}$$

$$\nabla_u H_2^* = 0 \quad t_1 \leq t \leq t_2 \tag{5}$$

$$\nabla_u H_3^* = 0 \quad t_2 \leq t \leq t_3 \tag{6}$$

and those relating to the switching functions:

$$\nabla_u \Phi_1(t_1) = 0 \tag{7}$$

$$\nabla_u \Phi_2(t_2) = 0 \tag{8}$$

In addition there are matching conditions involving the switch functions:

$$\lambda_1(t_1^-) + \nabla_x \Phi_1(t_1^-) = \lambda_2(t_1^+) \tag{9}$$

$$\lambda_2(t_2^-) + \nabla_x \Phi_2(t_2^-) = \lambda_3(t_2^+) \tag{10}$$

with the usual terminal conditions written as:

$$G^*(t_3) + \lambda_3(t_3) g^*(t_3) = 0 \tag{11}$$

$$\lambda_3(t_3) = 0 \tag{12}$$

The conditions relating to the switching times are given in the Appendix.

The reader may be interested in knowing how the general multiphase problem might be formulated. For future reference, it may be given as follows:

$$\max J = \sum_{k=0}^{n} \Big\{ \int_{t_{2k}}^{t_{2k+1}} D[t, x(t), u(t)]dt + \int_{t_{2k+1}}^{t_{2k+2}} F[t, x(t), u(t)]dt \\ - \Phi_{2k}[t, x(t_{2k+1}), u(t_{2k+1})] \\ - \Phi_{2k+1}[t, x(t_{2k+2}), u(t_{2k+2})] \Big\}$$

subject to

$$x'(t) = \begin{cases} d[t, x(t), u(t)] \text{ for } & t_{2k} \leq t \leq t_{2k+1} \\ & \\ f[t, x(t), u(t)] \text{ for } & t_{2k+1} \leq t \leq t_{2k+2} \end{cases} \quad \text{and } k = (0, 1, \ldots, n)$$

$$x(t_0) = x_0; \quad t_{2k+1}, x(t_{2k+1}), t_{2k+2}, x(t_{2k+2}) \text{ free}$$

where now a succession of cropping phases (D and d functions) and rehabilitation phases (F and f functions) follow each other.

4. Application to dryland salinity control

As a specific case we consider the problem of determining optimal switching times for a switch from cropping wheat at a predetermined intensity to rehabilitation using oil mallee and a switch from rehabilitation back to cropping wheat (at the same intensity). Since the intensity for cropping wheat is predetermined and fixed, there is no control variable in the first and third phase.

The annual profit calculated in today's value in the first cropping phase is

$$D(t, x) = \left(p_1 Y_{01} \frac{x}{m} - c_1 \right) e^{-rt}. \tag{13}$$

where p_1 is the price of the wheat crop per ton, c_1 is the yearly cropping cost, assumed constant, Y_{01} is the maximum yield (tons/hectare) when there is no salinity, m is the maximum water depth under the tree stand, and r is the discount rate.

Similarly, the profit function for the second cropping phase is:

$$G(t, x) = \left(p_3 Y_{03} \frac{x}{m} - c_3 \right) e^{-rt}. \tag{14}$$

We will assume that in the rehabilitation phase, trees are planted which will contribute to profit via the sales of oil extracted from the leaves. The profit function is given by

$$F(t, u) = R_2 u \underline{L} (1 - e^{-l(t - t_1)}) e^{-rt} \tag{15}$$

where R_2 is the profit obtained by selling oil extracted from one ton of leaves after subtracting the cost, and $\underline{L}$ measures the maximum canopy leaf mass harvestable per tree. t_1 is the switching time to rehabilitation, and l represents the yearly growth rate of the trees. The function u is the tree density function. It denotes the number of trees planted per hectare in the rehabilitation phase and is constrained by

$$u \leq D_{\max}, \tag{16}$$

where $D_{\max}$ is the maximum number of trees that can be planted per hectare. This maximum is given by forest ecology and reflects a competition threshold between trees. Here u is a control variable for the rehabilitation phase.

The state equations are given by

$$x' = \begin{cases} d(t, x(t)) = -\alpha x & 0 \leq t < t_1 \\ f(t, x(t), u) = \beta x u\left(1 - \dfrac{x}{m}\right) & t_1 \leq t < t_2 \\ g(t, x(t)) = -\gamma x & t_2 \leq t < t_3 \end{cases} \tag{17}$$

Here, m denotes the maximum depth to which the ground water level can be lowered, while α and γ are intrinsic rates for the increase of the water level with time; β is the intrinsic rate for the decrease of the water level per tree during the rehabilitation phase (Schilizzi and Mueller 1997). Moreover, we will require the function x to be continuous at the switching times.

The cost for switching from phase 1 to phase 2 consists of the fixed cost per hectare and the cost for planting young trees and caring for them during their initial growth stages. The fixed cost is an establishment cost such as cost for preparing the land or for fencing. The equation for the switching cost from phase 1 to phase 2 is:

$$\Phi_1(t, u) = (Sw_1 + c_2 \times u) \times e^{-rt} \tag{18}$$

where Sw_1 denotes the fixed cost, c_2 is the cost of buying and planting a tree.

For switching back to cropping, we will assume that the only cost involved is that for clearing the land. This may be offset by the sale of the wood resulting from the clearing. Thus

$$\Phi_2(t) = (Sw_2) \times e^{-rt} \tag{19}$$

where the establishment cost Sw_2 may be either negative or positive.

In this simplified model, there is no control in the cropping phases and the control in the rehabilitation phase is discrete. This means that the conditions relating to the optimality of the control do not apply in this setting. We moreover required a fixed time horizon of 100 years. This leaves the following necessary conditions for optimality: (1), (2), (3), (10), (11), and (1b) of the Appendix.

$$\lambda_1' = -p_1 Y_{01} e^{-rt}/m + \alpha\lambda_1 \tag{20a}$$
$$\lambda_2' = \beta\lambda_2(2x/m - 1) \tag{20b}$$
$$\lambda_3' = \gamma\lambda_3 - p_3 Y_{03} e^{-rt}/m \tag{20c}$$
$$\lambda_2(t_2^-) = \lambda_3(t_2^+) \tag{20d}$$
$$G(t_3) + \lambda_3(t_3)g(t_3) = 0 \tag{20e}$$

These conditions on their own do not suffice to determine t_1, t_2 and t_3.

We further require that

$$D(t_1, x(t_1)) \geq 0 \tag{21a}$$

and

$$G(t_3, x(t_3)) \geq 0. \tag{21b}$$

These two conditions require that an unprofitable farming practice be abandoned.

5. Numerical application and simulation results

The equations of motion were then solved, expressions for the above conditions were calculated explicitly and then entered into an Excel workbook. Table 1 summarises the constants used in the model. The constants k_1, k_2 and k_3 are constants of integration arising in the solution of the equations of motion. Their values are such that the state variable x is continuous at the switching times.

The necessary conditions together with the Excel tool Solver were then used to determine the optimal values for t_1, t_2 and t_3 given specific values of tree density. Note that since the objective function is not concave, the necessary conditions are not sufficient, so that there may be more than one optimum point, if the algorithm gets caught in a local optimum. Careful manipulations were necessary to handle this difficulty.

Numerical simulations were carried out to assess how the rate at which the water level changes and the discount rate impact on the optimal switching times. To do so one parameter was varied at a time. The parameters α and γ (the annual rise in water level during cropping phases) were assumed to be equal.

The value of the discount rate largely affected the shape of the profit curve (figure 1): the greater the discount rate, the smoother the objective function, and the smaller the indentations caused by the switch of phase. An increase in the discount rate increased the optimal duration of the cropping phase and decreased the optimal duration of the rehabilitation phase. Raising the annual discount rate from 4 per cent to 10 per cent lengthened the first

Table 1 Parameter values used in the model

Name	Value	Unit	Description or comment
Price 1	200	Dollars/tonne	Price of crop planted in phase 1
Price 2 (Revenue 2)	20	Dollars/tonne	Revenue obtained by growing trees in phase 2
Price 3	200	Dollars/tonne	Price of crop planted in phase 3
Cost 1	80	Dollars/ha	Cropping cost for phase 1
SwCost 1	1000	Dollars/ha	Fixed cost for switching from phase 1 to phase 2
Cost 2	0.5	Dollars/tree	Cost planting 1 tree/plant in phase 2
SwCost 2	80	Dollars/ha	Fixed cost for switching from phase 2 to phase 3
Cost 3	80	Dollars/ha	Cropping cost for phase 3
Discount Rate	$0.01 \leq r \leq 0.1$		Farmer discount rate
Alpha	$0.02 \leq \alpha \leq 0.05$	≈ 4–14 cm/year	Rise in water level in cropping — phase 1
Beta	$0.001 \leq \beta \leq 0.0025$	≈ 2–14 cm/year	Drop in water level in rehab — phase 2
Gamma	$0.02 \leq \gamma \leq 0.05$	≈ 4–14 cm/year	Rise in water level in cropping — phase 3
m	6	metres	Maximum water depth under tree stand
Y_01	1.5	ton/ha	Max crop yield with no salinity for phase 1
Y_03	1.5	ton/ha	Max crop yield with no salinity for phase 3
L_bar	0.05	tonne/tree	Maximum canopy mass harvested per tree
l	0.04	m/year	Growth rate of tree (width)
Dmax	1600	trees/ha	Maximum number of trees per hectare
u	1200	trees/ha	Density of tree/ha (= Control variable)
X1_0	4	metres	Initial depth of saline water (the first phase)
K_1	4.000000	—	Scaling parameter for integrating logistic function
K_2	0.000099	—	Scaling parameter for integrating logistic function
K_3	13.025418	—	Scaling parameter for integrating logistic function
Leaf yield	5	tonnes/ha	Leaf productivity
Oil content	40	kg/tonne fresh	Oil content in oil mallee leaf
Oil price	2	Dollars/kg oil	⇒ Gross annual revenue of $400/ha
Harvest cost	60	Dollars/tonne leaf	⇒ Net annual revenue of $100/ha or $20/tonne
Establishment cost	1000	Dollars/ha	Up-front cost of planting trees

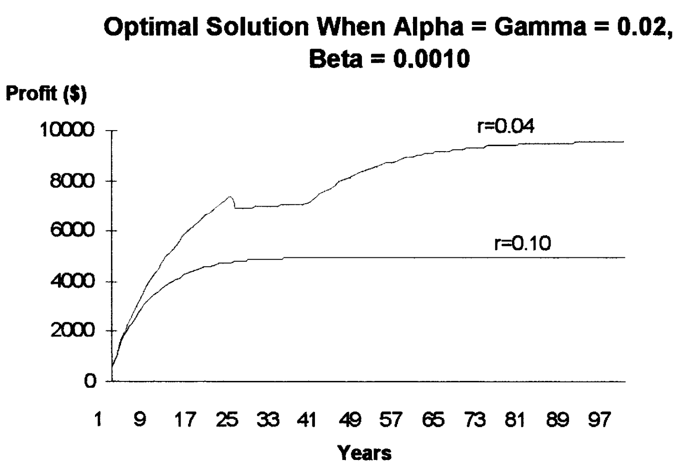

Figure 1 Optimal cumulative profit for two different values of the discount rate

Notes: r = farmer's discount rate
α = intrinsic rate of change of watertable level in phase 1 (cropping)
β = intrinsic rate of change of watertable level in phase 2 (trees)
γ = intrinsic rate of change of watertable level in phase 3 (cropping)

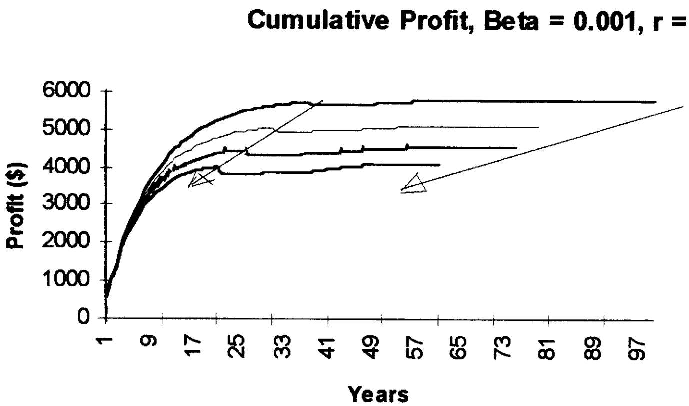

Figure 2 Optimal cumulative profit for $.02 \leq \alpha = \gamma \leq 0.05$

Note: Top curve is for $\alpha = \gamma = 0.2$ and bottom curve is for $\alpha = \gamma = 0.5$ (0.3 and 0.4 in between).
r = farmer's discount rate
α = intrinsic rate of change of watertable level in phase 1 (cropping)
β = intrinsic rate of change of watertable level in phase 2 (trees)
γ = intrinsic rate of change of watertable level in phase 3 (cropping)

cropping phase from 25 to 64 years, shortened the rehabilitation phase from 15 to 9 years, and shortened the second cropping phase from 60 to 27 years. Total discounted profits just about halved. Thus, higher discount rates push back the time to rehabilitation and make this phase shorter, a result that was to be expected, but ultimately drive down profits. This result highlights the price farmers must pay for preferring short-term to long-term profits. The reason is clear: high discount rates make it profitable in the short run to let salinity drive down farm productivity, the cost of which, however, ends up being higher than the perceived short-term benefits.

Figure 2 shows that an increase in the rate at which the water level rises in cropping phases 1 and 3 decreases the optimal duration of the cropping phase (from 37 years to 21), increases the optimal duration of the rehabilitation phase (from 11 to 17 years) and decreases that of the second cropping phase (from 52 to 21 years). The total duration of the three phases was optimally reduced from 100 to 59 years. In other words, higher salinisation rates from cropping bring forward the time to rehabilitation and make this phase last longer, leading, not surprisingly, to lower farm profits. In our case, an increase in salinisation rates from 4 cm to 14 cm per year decreased total profits by about a third.

With the lowering of the water level during the rehabilitation phase (figure 3), an increase in the draw-down rate (from 2 to 4 cm/year) leads to a decrease in the optimal duration of the first cropping phase (from 29 to 17 years), a decrease in the optimal duration of the rehabilitation phase (from 12 to 7 years) and an increase in the optimal duration of the second cropping phase (from 59 to 76 years). When the draw-down capacity of trees is stronger, both the initial cropping phase and the rehabilitation phase are made shorter, but the second cropping phase lasts longer. As expected, higher water use efficiency by trees does affect total profits positively. However, with the values used in this calculation (see table 1), its effect appears small compared to that of higher salinisation rates under cropping: +8 per cent instead of − 33 per cent. This result reflects the differences in the relative magnitudes of the parameter changes: 2 to 4 cm/year for draw-down rates against 4 to 14 cm/year in water rise rates. Nevertheless, considering these relative changes as measures of realistic technical efforts, this result suggests, interestingly, that reducing salinity encroachment is more important than focusing on highly water-efficient trees. Put otherwise, the scale of action, in terms of the area of trees planted, appears more important than the choice of particular tree species, provided that they are adapted to and survive local conditions. Insofar as salinity abatement reflects area planted to deep-rooted perennials, this result would seem to suggest that it is worth more helping farmers actually plant acceptable trees, rather than choosing

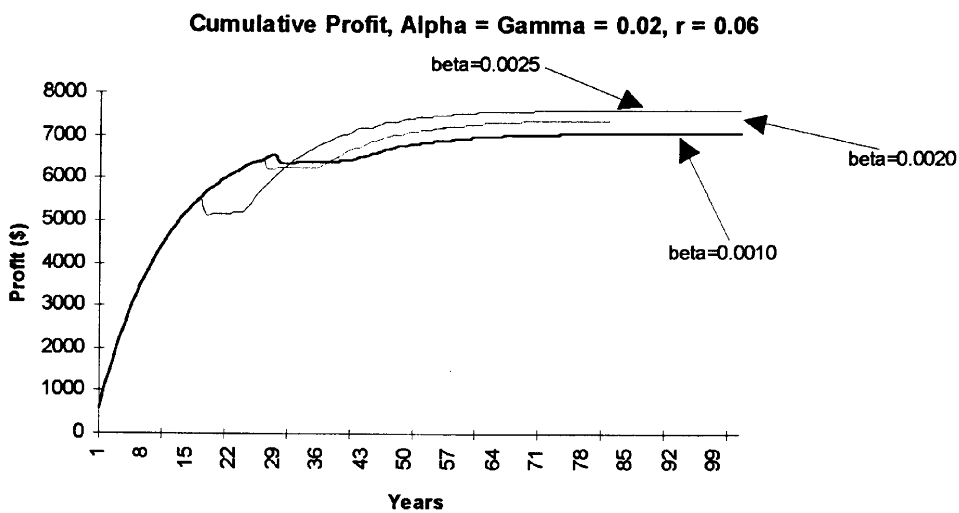

Figure 3 Optimal cumulative profit for .001 ≤ β ≤ 0.0025 (= 2 to 4 cm/yr)

Notes: r = farmer's discount rate
α = intrinsic rate of change of watertable level in phase 1 (cropping)
β = intrinsic rate of change of watertable level in phase 2 (trees)
γ = intrinsic rate of change of watertable level in phase 3 (cropping)

the species that will do the best hydrological job. This conclusion does not extend, however, to the economic value of trees, which will depend on the end-use of their products and on market developments.

These results together suggest that it should pay for government to help farmers use lower discount rates, for example by promoting 'green' (that is, lower) interest rates on loans for rehabilitation, and thus encourage farmers to reduce salinity encroachment on their farms. Further encouragement can be promoted by increasing the value of trees relative to cropping, through further processing and value-adding of their products (like mallee oil for the solvent industry), or through market developments based on new uses.

6. Conclusion

The type of problem examined in this article is a difficult one. No general solution has been attempted as yet. Rather, a rigorous formulation and a characterisation of optimality conditions have been sought as a preliminary step, followed by an investigation of a three-stage version of the problem. Although the qualitative results are not surprising, our objective was to be able to derive numerical magnitudes of potential practical use. Insofar as the numerical parameters in table 1 are acceptable and representative of typical situations, our numerical results may be taken seriously, that is, be subject to criticism.

The problem investigated departs from standard optimal control problems in several ways. It considers more than one stage, with a switch from one stage to the other. The profit function as of the second stage explicitly depends on the timing of the switch. Costs are associated with switches, analogous to discrete jumps in the state variables. Phases have different durations and so do stages of each phase, at least until a steady state is reached, if ever. Due to the multi-period nature of each phase (several years), the transition to a steady state becomes a more important problem than the rather academic exercise of characterising the steady state itself.

Notwithstanding, this problem is a highly simplified one. Prices and costs have been considered constant, as have revenues from rehabilitation, when they should be variable over time and stochastic. System dynamics, particularly of watertable movements, and the equation for tree growth are also simplified. No interactions have been considered, and no uncertainties assumed in effects on crop yields or rehabilitation functions.

Technological innovations, government policies and a structurally changing economy make very long-term strategies somewhat irrelevant. It is not clear how this conundrum may be solved. Possibly, limited phase farming, such as ensuring a profitable return to stage three, as considered in this article, may be sufficient. A strategy like minimising the cost of any future switch may be an option preserving reversibility and ensuring flexibility. Switching-cost minimisation strategies, subject to uncertain benefits over time, are as yet an unexplored problem.

Behind the general idea of sustainability, which serves as a battle banner rather than as an operational concept (Pannell and Schilizzi 1999), very different and often extremely complex management issues are involved. More often than not, these issues include long time horizons, time management, uncertainties and stochastic processes, irreversibilities, and highly nonlinear, often logistic dynamics, all at once. If problems are to be tackled in a tractable manner, they have to be broken up into sub-problems, the cost of which, however, means restricted applicability leading to a burgeoning of specific applications. This article illustrates the matter.

General qualitative results, such as those using implicit functional forms only defined by their mathematical properties, will become increasingly irrelevant, as the outcomes of not only the magnitude, but the direction of changes become parameter and number specific. For instance, optimal management rules valid for salinity abatement may not be valid for abatement of soil erosion; those valid for saline watertable abatement in monotonously sloping land may not be valid in undulating regions; or those valid for wind erosion abatement may not be valid for water erosion abatement, even if the general optimising functional always relates to farm profits. As dynamics are taken more seriously, optimal management strategies become process-specific.

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Appendix

The optimality conditions related to each of the switching times are the following (Tran 1997):

For $t_0 < t_1 < t_2 < t_3$,

$$H_1^*(t_1^-) + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt = H_2^*(t_1^+) + \frac{\partial \Phi_1^*}{\partial t} \tag{1a}$$

$$H_2^*(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = H_3^*(t_2^+) + \frac{\partial \Phi_2^*}{\partial t} \tag{1b}$$

For $t_0 = t_1 < t_2 < t_3$,

$$H_1^*(t_1^-) + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \leq H_2^*(t_1^+) + \frac{\partial \Phi_1^*}{\partial t} \quad (1c)$$

$$H_2^*(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = H_3^*(t_2^+) + \frac{\partial \Phi_2^*}{\partial t} \quad (1d)$$

For $t_0 < t_1 = t_2 < t_3$,

$$H_1^*(t_1^-) + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \geq H_2^*(t_1^+) + \frac{\partial \Phi_1^*}{\partial t} \quad (1e)$$

$$H_2^*(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = H_3^*(t_2^+) + \frac{\partial \Phi_2^*}{\partial t} \quad (1f)$$

For $t_0 < t_1 < t_2 = t_3$,

$$H_1^*(t_1^-) + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt = H_2^*(t_1^+) + \frac{\partial \Phi_1^*}{\partial t} \quad (1g)$$

$$H_2^*(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \geq H_3^*(t_2^+) + \frac{\partial \Phi_2^*}{\partial t} \quad (1h)$$

For $t_0 < t_1 = t_2 = t_3$,

$$H_1^*(t_1^-) + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \geq H_2^*(t_1^+) + \frac{\partial \Phi_1^*}{\partial t} \quad (1i)$$

$$H_2^*(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \geq H_3^*(t_2^+) + \frac{\partial \Phi_2^*}{\partial t} \quad (1j)$$

For $t_0 = t_1 = t_2 < t_3$,

$$H_1^*(t_1^-) + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \leq H_2^*(t_1^+) + \frac{\partial \Phi_1^*}{\partial t} \quad (1k)$$

$$H_2^*(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \leq H_3^*(t_2^+) + \frac{\partial \Phi_2^*}{\partial t} \quad (1l)$$

For $t_0 = t_1 < t_2 = t_3$,

$$H_1^*(t_1^-) + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \leq H_2^*(t_1^+) + \frac{\partial \Phi_1^*}{\partial t} \quad (1m)$$

$$H_2^*(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \geq H_3^*(t_2^+) + \frac{\partial \Phi_2^*}{\partial t} \quad (1n)$$

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