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An analysis of the Western Australian gold royalty†

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This article analyses the modified form of ad valorem royalty recently announced by the WA government in relation to gold production, which features a threshold price below which there is no tax liability and compares this royalty with a profit-based royalty. The level at which the threshold price is set plays an important role in determining the performance of the royalty in relation to its impact on production and the expected level and variability of tax revenue. It is argued that the higher this price is set, the stronger the grounds for preferring a profit-based royalty, even taking into account the reliability of each form for generating tax revenue.

1. Introduction

In its 1997 Budget the Western Australian (WA) government introduced a royalty on gold to take effect in 1998. This royalty is of the ad valorem form (i.e. based on revenue) rather than of the resource rent form (i.e. based on profits). Subsequent developments in the gold market have resulted in the WA government modifying its original proposal to delay the introduction of the royalty and to make its collection contingent on the price of gold exceeding a threshold level. In so doing, the WA government has responded to the expressed concerns of the industry in relation to the detrimental impact of the royalty on profits, especially in times of relatively low prices. Moreover, at the same time it has created a novel form of resource taxation, where the revenue base of the tax is modified to take account of periods of unusually low profits, and where the price of gold is treated as a simple proxy for the level of profits.

For imperfectly competitive industries there are a number of second-best arguments which support the use of ad valorem taxes (Conrad and Hool 1981; Kay and Keen 1983; Fane and Smith 1986). However, economists have

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for some time argued the merits of profit-based resource taxation in competitive mining industries, of which gold is perhaps an example (e.g. Dowell 1978; Leland 1978; Garnaut and Clunies-Ross 1975, 1979; Emerson and Lloyd, 1983; Fraser, 1993, 1998). And so it is interesting to observe a government struggling to modify its preferred form of taxation (i.e. ad valorem) to provide a feature which is inherent in the structure of a profit-based tax. Moreover, it seems worthwhile to analyse the WA government's novel form of resource taxation in order to assess the extent to which it provides the acknowledged advantages of a purely profit-based alternative. Such an analysis is the aim of this article.

The structure of the article is as follows. Section 2 sets out a simple model of both forms of resource taxation which provides the basis for their comparison using the concept of tax revenue neutrality. Section 3 undertakes a numerical analysis of the two taxes in relation to the level of the threshold price and the expected level and variability of market prices for gold. In addition, this section considers the two taxes in relation to the benefits from improvements in production technology. The article ends with a brief summary of the analysis, and concludes that the modification of an ad valorem royalty to include a threshold price for tax liability diminishes the two key advantages of such a tax relative to a profit-based tax: its low rate and its low revenue variability. This latter finding is potentially a concern given the view of the WA Department of Minerals and Energy (DOME 1994) that, because of the current significance of royalty receipts as a proportion of total government income (around 10 per cent), revenue stability is a key characteristic of the state's royalty system.

2. The model

The mining company is assumed to be a price-taker and that this price (p) is uncertain at the time of the production decision. In the absence of any resource taxation, the company's objective is to maximise expected profits ($E(\pi)$) by the optimal choice of production level (y):

$$E(\pi) = E(py - c(y)) \tag{1}$$

where: $c(y)$ = known cost of producing y

$$(c'(y) > 0, c''(y) > 0).$$

In this case optimal production will occur where expected price is equal to marginal cost:

$$\bar{p} = c'(y) \tag{2}$$

where: $\bar{p}$ = expected price.

The WA gold royalty is specified to be at a rate (v) on revenue if the actual price exceeds a threshold level ($\hat{p}$), so that the profit function in the presence of the royalty can be written as:[1]

$$\begin{aligned} \pi_v &= py - c(y) \quad \text{if} \quad p \leq \hat{p} \\ \pi_v &= py - c(y) - vpy \quad \text{if} \quad p > \hat{p}. \end{aligned} \tag{3}$$

On this basis expected profit in the presence of the royalty ($E(\pi_v)$) is given by:

$$E(\pi_v) = \int_o^{\hat{p}} pyf(p)dp + \int_{\hat{p}}^{\infty} (py - vpy)f(p)dp - c(y) \tag{4}$$

where: $f(p) =$ probability distribution governing price.

And optimal production is given by:

$$\bar{p} - v\int_{\hat{p}}^{\infty} pf(p)dp = c'(y) \tag{5}$$

which may be rearranged to give:

$$\bar{p} - vE(p|p > \hat{p})(1 - F(\hat{p})) = c'(y) \tag{6}$$

where: $F(\hat{p}) =$ cumulative probability of $\hat{p}$ exceeding p, $E(p|p > \hat{p}) =$ expected price given p exceeds $\hat{p}$.

A comparison of equations 5 and 2 shows clearly that the ad valorem royalty reduces the expected revenue from extra production. As a consequence, production is lower in the presence of this type of royalty than in its absence. In addition, the derivative of the left-hand side of equation 5 with respect to $\hat{p}$ is given by:

$$v\hat{p}f(\hat{p}) > 0.$$

Consequently, the lower is $\hat{p}$ relative to $\bar{p}$, the stronger is the negative impact of this royalty on production.[2] Finally, it is clear that the level of uncertainty of price also affects the strength of this impact. In particular, an increase in uncertainty will affect both the overall level and the spread of the probability mass in the second term on the left-hand side of equation 5, and therefore has an ambiguous analytical impact on the value of this term. Note that this ambiguity will be clarified during the numerical analysis of the next section.

[1] The royalty also only applies for production in excess of a threshold level. The presumption here is that this production threshold is irrelevant to the company in question.

[2] I am grateful to an anonymous referee for clarifying this result.

The expected tax revenue from the ad valorem royalty $E(TR_v)$ is given by:

$$E(TR_v) = vE(p|p > \hat{p})y(1 - F(\hat{p})) \tag{7}$$

while the variance of tax revenue ($\mathrm{Var}(TR_v)$) is shown in the appendix to be given by:

$$\begin{aligned}\mathrm{Var}(TR_v) = {} & (E(TR_v))^2 - 2E(TR_v)vE(p|p > \hat{p})y(1 - F(\hat{p})) \\ & + v^2y^2[(1 - F(\hat{p}))\mathrm{Var}(p|p > \hat{p}) - (1 - F(\hat{p}))\bar{p}^2 \\ & + 2\bar{p}E(p|p > \hat{p})(1 - F(\hat{p}))]\end{aligned} \tag{8}$$

where: $\mathrm{Var}(p|p > \hat{p})$ = variance of p given p exceeds $\hat{p}$.

Reflecting the impact of $\hat{p}$ on the magnitude of the tax-related term in equation 5, it can be seen from equation 7 that $E(TR_v)$ decreases with increases in $\hat{p}$. However, the derivative of equation 8 with respect to $\hat{p}$ gives:[3]

$$\frac{\partial \mathrm{Var}(TR_v)}{\partial \hat{p}} = \hat{p}vyf(\hat{p})(2E(TR_v) - vy\hat{p}). \tag{9}$$

While for $\hat{p}$ close to zero equation 9 shows that increases in $\hat{p}$ increase $\mathrm{Var}(TR_v)$, for larger $\hat{p}$ the impact of increases in $\hat{p}$ on equation 8 is unclear. This impact will also be clarified in the numerical analysis of the next section.

As an alternative to the WA gold royalty, next consider an overtly profit-based royalty. Such royalties also typically have a threshold feature, with the company paying no resource taxation if profits are inadequate in relation to the threshold, but tax being payable on excess profits. In the resource rent tax literature this threshold is usually specified in relation to a rate of return on capital.[4] However, in order to simplify the analysis, the threshold specified here is an actual level of profits, with the level of capital invested in production and the associated rate of return on this capital being suppressed. In particular, profits in the presence of a profit-based royalty (π_t) are given by:

$$\begin{aligned}\pi_t &= py - c(y) \quad \text{if} \quad \pi \leq b \\ \pi_t &= py - c(y) - t(py - c(y) - b) \quad \text{if} \quad \pi > b\end{aligned} \tag{10}$$

where: b = threshold level of profits, t = rate of tax on surplus profits.

[3] I am grateful to an anonymous referee for providing this derivative.

[4] See for example, Fraser (1993).

Note that tax is payable if:

$$\pi > b$$

which may be rearranged to give:

$$p > \frac{b + c(y)}{y} \tag{11}$$

On this basis, expected profit ($E(\pi_t)$) is given by:

$$E(\pi_t) = \int_o^{\frac{b+c(y)}{y}} (py - c(y))f(p)dp + \int_{\frac{b+c(y)}{y}}^{\infty} (py - c(y) - t(py - c(y) - b))f(p)dp \tag{12}$$

and optimal production is given by:

$$\bar{p} - t\left(E\left(p|p > \frac{b + c(y)}{y}\right) - c'(y)\right)\left(1 - F\left(\frac{b + c(y)}{y}\right)\right) = c'(y) \tag{13}$$

As with the ad valorem royalty, it can be seen from the left-hand side of equation 13 that optimal production will be less than in the absence of the profit-based royalty. This distorting effect arises because, while the government shares relatively large positive profit states (i.e. in excess of b) with the company, it does not share relatively low positive and all negative profit states. Consequently, on balance the company is benefiting less from increases in production in the presence of the royalty than in its absence.[5] However, a comparison of equations 5 and 13 does not indicate which of the two royalty systems is relatively more distorting to production in achieving equivalent expected tax revenues. Specifically, although the profit-based royalty is payable only on marginal expected profit (for π in excess of b) as opposed to marginal expected revenue (for p in excess of $\hat{p}$) in the case of the ad valorem royalty, the rate of tax on excess profit (t) required to achieve expected tax revenue neutrality with the ad valorem royalty will exceed the rate for this royalty (v). Consequently, the second terms of each of the left-hand sides of equations 5 and 13 are of ambiguous relative magnitude. This ambiguity will be clarified in the numerical analysis of the next section.

[5] I am grateful to an anonymous referee for correcting equation 13 and justifying this result.

In the case of the profit-based royalty expected tax revenue $E(TR_t)$ is given by:

$$E(TR_t) = t\left(E\left(p|p > \frac{b+c(y)}{y}\right)y - c(y) - b\right)\left(1 - F\left(\frac{b+c(y)}{y}\right)\right) \tag{14}$$

while the variance of tax revenue ($\mathrm{Var}(TR_t)$) is shown in the appendix to be given by:

$$\begin{aligned}\mathrm{Var}(TR_t) = {} & (E(TR_t))^2 - 2E(TR_t)t\left(E\left(p|p > \frac{b+c(y)}{y}\right)y - c(y) - b\right) \\ & \times \left(1 - F\left(\frac{b+c(y)}{y}\right)\right) + t^2y^2\left[\left(1 - F\left(\frac{b+c(y)}{y}\right)\right)\right. \\ & \times \mathrm{Var}\left(p|p > \frac{b+c(y)}{y}\right) - \left(1 - F\left(\frac{b+c(y)}{y}\right)\right)\bar{p}^2 \\ & \left. + 2\left(1 - F\left(\frac{b+c(y)}{y}\right)\right)\bar{p}E\left(p|p > \frac{b+c(y)}{y}\right)\right] \\ & + t^2(b+c(y))^2\left(1 - F\left(\frac{b+c(y)}{y}\right)\right) \\ & - 2t^2y(b+c(y))E\left(p|p > \frac{b+c(y)}{y}\right)\left(1 - F\left(\frac{b+c(y)}{y}\right)\right).\end{aligned} \tag{15}$$

Note that, even with t chosen to achieve:

$$E(TR_v) = E(TR_t) \tag{16}$$

a comparison of equations 8 and 15 reveals no unambiguous ranking of the relative size of $\mathrm{Var}(TR_v)$ and $\mathrm{Var}(TR_t)$. Once again, this ambiguity will be clarified by the numerical analysis of the next section.

3. Numerical analysis

The algebraic analysis of the previous section revealed ambiguities regarding both the impact of changes in key parameters and the relative size of particular expressions. Often, numerical analysis can clarify such ambiguities and, with the choice of plausible parameter values, can provide a useful feel for the central relationships of a model. Consequently, a numerical analysis is undertaken in this section with the aim of illustrating and enhancing the findings of the algebraic analysis.

In order to undertake a numerical analysis of the tax systems outlined in the previous section, it is necessary to specify a probability distribution of price, as well as a form of the cost function and values for the other parameters of the model. In what follows it is assumed that the probability distribution of prices is normal, in which case:[6]

$$E(p|p > \hat{p}) = \bar{p} + \sigma_p Z(\hat{p})/(1 - F(\hat{p})) \tag{17}$$

$$\mathrm{Var}(p|p > \hat{p}) = \sigma_p^2 \left[1 + \frac{\hat{p} - \bar{p}}{\sigma_p} \cdot \frac{Z(\hat{p})}{(1 - F(\hat{p}))} - \left(\frac{Z(\hat{p})}{(1 - F(\hat{p}))} \right)^2 \right]. \tag{18}$$

Note that similar forms apply for:

$$E\left(p|p > \frac{b + c(y)}{y} \right) \quad \text{and} \quad \mathrm{Var}\left(p|p > \frac{b + c(y)}{y} \right).$$

In addition, the cost function is specified as:

$$c(y) = a + dy^c. \tag{19}$$

Finally, parameter values have been chosen as follows:

$$\begin{aligned} \bar{p} &= 500 \\ \sigma_p &= 150 \quad (CV_p = 30\%) \\ a &= 0 \\ d &= 2.5 \\ c &= 2. \end{aligned}$$

Note that the expected price has been chosen to approximate prevailing Australian dollar gold prices, while the coefficient of variation of price is typical of the level of price variability of commodities traded on world markets (Hazell, Jaramillo and Williamson 1990). Note also that unreported numerical analysis indicates the overall pattern of results is unrelated both to the specification of the cost function and to the magnitude of its parameters.

On this basis, in the absence of any form of resource taxation:

$$\begin{aligned} y &= 100 \\ E(\pi) &= 25000. \end{aligned}$$

The ad valorem royalty is specified at the actual rate which is to apply in the case of WA gold:

$$v = 0.025$$

[6] See Fraser (1988) for details. Note that the parameter values chosen below for the probability distribution of prices mean that more than 99.95 per cent of the total probability mass exceeds zero. Consequently, values calculated using these formulae are virtually exact.

Table 1 Numerical analysis of ad valorem and profit-based royalties

	y	$E(\pi)$	Tax Rate (%)	$E(TR)$	Var(TR)
No taxation	100	25000	–	–	–
$\hat{p} = 0$					
Ad valorem	97.5	23765.1	2.5	1219.3	132991
Profit-based[a]	99.2	23779.2	8.6	1219.3	975820
$\hat{p} = 350$					
Ad valorem	97.7	23870.7	2.5	1116.3	296838
Profit-based[a]	99.3	23882.5	7.9	1116.3	818286
$\hat{p} = 450$					
Ad valorem	98.1	24078.9	2.5	912.4	494954
Profit-based[a]	99.4	24086.8	6.4	912.4	547145
$\hat{p} = 500$					
Ad valorem	98.5	24231.4	2.5	762.6	562937
Profit-based	99.5	24236.8	5.4	762.6	382437

Note: [a] $b = 12500$

over a range of values of the threshold price ($\hat{p}$). The profit-based royalty is specified such that the threshold level of profits is equal to half the total cost of production in the absence of resource taxation:

$$b = 0.5 \;\; c(y) = 12500.$$

On this basis the company's income must exceed 150 per cent of its costs before any resource taxation is payable. Finally, the rate of tax on excess profits (t) is specified such that:

$$E(TR_t) = E(TR_v)$$

for each threshold price.[7]

Table 1 provides details of this numerical analysis. Focusing initially on the results for the ad valorem royalty, it can be seen that, consistent with the finding in section 1, the 'pure' ad valorem royalty (i.e. $\hat{p} = 0$) yields the greatest expected tax revenue but also has the largest distorting effect on production. Moreover, the variance of tax revenue is lowest in this case. Higher levels of the threshold price are beneficial to the company's expected profits and in association with this are less distorting of production. But

[7] Note that the relationship between b and t with this expected revenue neutrality requirement is quite straightforward. In particular, a lower value of b means that resource taxation becomes payable at a lower level of profits and therefore the value of t required to achieve expected revenue neutrality is also lower.

there is a cost to the government both in terms of the expected level and variance of tax revenue. Note that this increase in the variance of tax revenue at higher levels of $\hat{p}$ follows from the positive impact of an increase in $\hat{p}$ on the probability of zero tax revenue. This increase in the likelihood of an extreme outcome (i.e. zero) both reduces expected tax revenue and increases its variance. Finally, recalling the algebraic ambiguity in section 2 of equation 9 ($\partial \mathrm{Var}(TR_v)/\partial \hat{p}$), note that table 1 clarifies the sign of this expression for $\hat{p}$ up to $\bar{p}$.

Turning to the profit-based royalty, it can be seen that for each $\hat{p}$ the optimal level of production is higher than in the case of the ad valorem royalty. This feature of the results clarifies the ambiguity identified in section 2 regarding whether expected tax revenue neutrality could be achieved by the profit-based royalty with less distortion to the optimal level of production than that of the ad valorem royalty.

In addition, it can be seen that the rate of tax on excess profits required to achieve tax revenue-neutrality is dependent on the level of $\hat{p}$. For example, matching the expected tax revenue of the 'pure' ad valorem royalty requires a rate of 8.6 per cent on excess profits compared with the ad valorem rate of 2.5 per cent revenue.[8] In addition, for this case revenue from the profit-based royalty is considerably more variable than from the ad valorem royalty. These results are consistent with the findings of Fraser and Kingwell (1997) that in general the tax rate on profit needs to be higher than that on revenue to generate the same amount of tax, and that the revenue from a profits tax is generally more variable than that from a pure revenue tax. However, the results in table 1 also show that, if the form of the ad valorem royalty is modified by the introduction of a threshold price for tax liability, then the relationship between the two forms of taxation is also modified. In particular, the tax-revenue neutral rate of tax on excess profits decreases with increases in the threshold price. For example, table 1 shows that for a threshold price of 500 (equal to $\bar{p}$) the rate of tax on excess profits is only slightly more than double that on revenue (5.4 per cent compared with 2.5 per cent). Moreover, because of this decrease in the rate of tax on excess profits, the associated variance of tax revenue is also decreased (see equation 15). Table 1 shows that, combined with the positive impact of a higher threshold price on the variance of tax revenue from the ad valorem royalty, the situation occurs for $\hat{p} = 500$ that the variance of tax revenue from the ad valorem royalty exceeds that of the profit-based royalty.

[8] Note that the less distorting impact on production of the profit-based tax means that expected tax revenue can be equated with a small surplus of expected profit for the company.

Table 2 Impact of increased price uncertainty

	$\sigma_p = 150$			$\sigma_p = 200$		
	y	$E(\pi)$	$E(TR)$	y	$E(\pi)$	$E(TR)$
$\hat{p} = 350$						
Ad valorem	97.7	23870.8	1116.3	97.8	23895.2	1092.3
Profit-based	99.3	23882.5	1116.3	98.9	23762.8	1234.2
$\hat{p} = 450$						
Ad valorem[a]	98.1	24079.0	912.4	98.1	24067.2	924.0
Profit-based	99.4	24086.8	912.4	99.1	23988.8	1009.3

Note: [a] At two decimal places production declines from 98.14 to 98.12.

It may be concluded that the modification of an ad valorem royalty to include a threshold price for tax liability diminishes the two key advantages of such a tax relative to a profit-based tax: its low rate and its low revenue variability.

Next, consider the performance of the modified ad valorem royalty relative to the profit-based royalty in the situation of changes in market conditions, in particular in the expected level and variability of market prices. In section 2 it was demonstrated that in the case of the modified ad valorem royalty an increase in the uncertainty of prices had an analytically ambiguous impact both on optimal production and, in association with this, on expected tax revenue. The results in table 2 illustrate this ambiguity by showing that the impact of increased price uncertainty on optimal production and expected tax revenue is positive or negative depending on the level of the threshold price for tax liability. These results show that for the lower threshold price ($\hat{p} = 350$) the negative impact of increased uncertainty on the probability mass $(1 - F(\hat{p}))$ dominates the positive impact on $E(p|p > \hat{p})$ and $E(TR_v)$ declines overall, with an associated increase in optimal production. However, for the higher threshold price ($\hat{p} = 450$) the reverse applies and overall $E(TR_v)$ increases (with an associated decline in optimal production from 98.14 to 98.12). Note that this reversal of impact is in contrast to that for the profit-based royalty where the overall impact in each case is an increase in expected tax revenue and a decline in optimal production. Consequently, it may be concluded that in the case of the modified ad valorem royalty the level at which the threshold price for tax liability is set determines whether the impact of an increase in price uncertainty on optimal production and the associated expected tax revenue is positive or negative.

A second aspect of market conditions is the expected price level, and table 3 contains details of the impact of fluctuations in this level from the initial setting of $\bar{p} = 500$ on expected tax revenue for each tax system and for

Table 3 Impact of fluctuations in the expected price level on expected tax revenue

	$\bar{p}$		
	450	500	550
	$E(TR)$	$E(TR)$	$E(TR)$
$\hat{p} = 350$			
Ad valorem[a]	846.2 (– 24.2%)	1116.3	1407.8 (+26.1%)
Profit-based[a]	794.2 (– 28.9%)	1116.3	1490.5 (+33.5%)
$\hat{p} = 450$			
Ad valorem[b]	630.7 (– 30.9%)	912.4	1236.1 (+35.5%)
Profit-based[b]	649.1 (– 28.9%)	912.4	1217.9 (+33.5%)

Notes: [a] Changes in $E(\pi)$ are:
$E_v(\pi)$: +20.8% ; – 18.8%
$E_t(\pi)$: +20.4% ; – 18.6%

[b] Changes in $E(\pi)$ are:
$E_v(\pi)$: +20.4% ; – 18.5%
$E_t(\pi)$: +20.5% ; – 18.6%

two settings of the price threshold of the ad valorem tax. The results in table 3 show that at the lower threshold price the volatility of expected tax revenue in relation to changes in the expected price level is less in the case of the ad valorem royalty than in the case of the profit-based royalty. However, at the higher threshold price the reverse situation applies with the ad valorem royalty exhibiting greater volatility of expected tax revenue in relation to fluctuations in the expected price level. This feature of the results supports the previous finding in relation to the level of price uncertainty that the setting of the threshold price for the ad valorem tax is an important determinant of its performance. In this situation it is suggested that the closer the threshold price is set to the level of the expected price, the more volatile are expected tax revenues with respect to fluctuations in this level. Moreover, in association with the results in table 1, this finding further undermines the view that a profit-based royalty is a relatively unreliable form of generating tax revenue when compared with this modified ad valorem royalty.[9]

Finally, the numerical version of the model can be used to consider the relative performance of the two forms of taxation in the context of improvements in production technology. Table 4 contains details of the impacts where such an improvement has been represented by a decline in the

[9] Note that, unlike the situation for expected tax revenues, table 3 shows the associated fluctuations in expected profits to be similar for both types of tax in both situations.

Table 4 Impact of an improvement in production technology

	$d = 2.5$			$d = 2.25$		
	y	$E(\pi)$	$E(TR)$	y	$E(\pi)$	$E(TR)$
$\hat{p} = 350$						
Ad valorem	97.7	23870.7	1116.3	108.6 (+11.1%)	26523.0 (+11.1%)	1240.3 (+11.1%)
Profit-based	99.3	23882.5	1116.3	110.4 (+11.2%)	26447.3 (+10.7%)	1329.3 (+19.1%)
$\hat{p} = 450$						
Ad valorem	98.1	24079.0	912.4	109.0 (+11.1%)	26754.4 (+11.1%)	1013.8 (+11.1%)
Profit-based	99.4	24086.8	912.4	110.5 (+11.2%)	26690.6 (+10.8%)	1086.4 (+19.1%)

slope of the marginal cost function (d decreased from 2.5 to 2.25). The results in this table show that, unlike the previous situations, the level of the threshold price plays no role in determining the impact of this change in the case of the ad valorem royalty. However, there is also a clear difference in the performance of the two tax systems in this situation. In particular, while the increase in the company's expected profits is similar for the two tax systems, the increase in expected tax revenue is considerable larger (70 per cent) in the case of the profit-based royalty than for the ad valorem royalty. This divergence reflects the different base of the two taxes and emphasises the point that improvements in production technology primarily benefit profits as distinct from revenues. In addition, this finding highlights an important advantage of a profit-based royalty over an ad valorem royalty in that it better enables society to share with the company in the benefits of technological advances.

4. Conclusion

This article has analysed the modified form of ad valorem royalty recently announced by the WA government in relation to gold production. The key feature of this modification is a threshold price for gold below which there is no tax liability.

Section 2 of the article developed a simple analytical model which enabled a comparison of this modified ad valorem royalty with a profit-based royalty. This model used the concept of tax revenue neutrality to establish the appropriate benchmark for comparing the two tax systems. In particular, this concept was used to determine the rate of tax on profit required to just balance its expected revenue with that of the ad valorem royalty.

Development of this model revealed a number of ambiguities in the performance of the modified ad valorem royalty relative to the profit-based royalty, which prompted the development of a numerical version of the model in section 3. A key finding of this numerical analysis was the important role of the level of the threshold price in determining the characteristics of the modified ad valorem royalty. Specifically, it was concluded that the threshold feature diminishes the two main advantages of an ad valorem royalty relative to a profit-based tax: its low rate and its low revenue variability. In addition, it was shown that the level at which this threshold price is set determines not only whether the impact of an increase in price uncertainty on optimal production and the associated expected tax revenue is positive or negative, but also the relative volatility of expected tax revenue from the two tax systems in the context of fluctuations in expected price.

On this basis it can be argued that the performance of the modified ad valorem royalty relative to a profit-based royalty is extremely sensitive to the setting of the threshold price for tax liability. Moreover, the higher this price is set, the stronger the argument for preferring a profit-based royalty, even taking into account the reliability of each form for generating tax revenue. Add to this the final demonstration in section 3 of the superiority of the profit-based royalty in the context of enabling society to share in the benefits of improvements in production technology, and the grounds for preferring the modified ad valorem royalty to a profit-based royalty are considerably eroded, at least in the context of price-taking companies analysed in this article.

Appendix: Derivation of Var(TR_v) and Var(TR_t)

1. Var(TR_v)

$$\mathrm{Var}(TR_v) = \int_o^{\hat{p}} (0 - E(TR_v))^2 f(p)dp + \int_{\hat{p}}^{\infty} (vpy - E(TR_v))^2 f(p)dp \tag{A1}$$

Rearranging gives:

$$\mathrm{Var}(TR_v) = (E(TR_v))^2 + \int_{\hat{p}}^{\infty} v^2p^2y^2 f(p)dp - 2\int_{\hat{p}}^{\infty} E(TR_v)vpyf(p)dp \tag{A2}$$

Since:

$$\begin{aligned}\int_{\hat{p}}^{\infty} v^2p^2y^2 f(p)dp = &\int_{\hat{p}}^{\infty} v^2y^2(p-\bar{p})^2 f(p)dp \\ &+ \int_{\hat{p}}^{\infty} 2v^2y^2\bar{p}pf(p)dp - \int_{\hat{p}}^{\infty} v^2y^2\bar{p}^2 f(p)dp\end{aligned} \tag{A3}$$

Substituting (A3) into (A2) gives:

$$\begin{aligned}\mathrm{Var}(TR_v) = (E(TR_v))^2 - 2E(TR_v)vyE(p|p > \hat{p})(1 - F(\hat{p}))\\ + v^2y^2[(1 - F(\hat{p}))\mathrm{Var}(p|p > \hat{p}) - (1 - F(\hat{p}))\bar{p}^2\\ + 2\bar{p}E(p|p > \hat{p})(1 - F(\hat{p}))]\end{aligned} \tag{A4}$$

which is reproduced as equation 8 in the main text.

2. Var(TR_t)

$$\mathrm{Var}(TR_t) = \int_o^{\frac{b+c(y)}{y}} (0 - E(TR_t))^2 f(p)dp + \int_{\frac{b+c(y)}{y}}^{\infty} (t(py - c(y) - b) - E(TR_t))^2 f(p)dp \tag{A5}$$

Rearranging gives:

$$\begin{aligned}\mathrm{Var}(TR_t) = (E(TR_t))^2 + \int_{\frac{b+c(y)}{y}}^{\infty} t^2(py - c(y) - b)^2 f(p)dp\\ - 2\int_{\frac{b+c(y)}{y}}^{\infty} E(TR_t)t(py - c(y) - b)f(p)dp\end{aligned} \tag{A6}$$

Since:

$$\begin{aligned}\int_{\frac{b+c(y)}{y}}^{\infty} t^2(py - c(y) - b)^2 f(p)dp = \int_{\frac{b+c(y)}{y}}^{\infty} t^2y^2p^2 f(p)dp\\ - \int_{\frac{b+c(y)}{y}}^{\infty} 2t^2(c(y) + b)pyf(p)dp\\ + \int_{\frac{b+c(y)}{y}}^{\infty} t^2(c(y) + b)^2 f(p)dp\end{aligned} \tag{A7}$$

and since:

$$\begin{aligned}\int_{\frac{b+c(y)}{y}}^{\infty} t^2y^2p^2 f(p)dp = \int_{\frac{b+c(y)}{y}}^{\infty} t^2y^2(p - \bar{p})^2 f(p)dp\\ + 2\int_{\frac{b+c(y)}{y}}^{\infty} t^2y^2\bar{p}pf(p)dp - \int_{\frac{b+c(y)}{y}}^{\infty} t^2y^2\bar{p}^2 f(p)dp.\end{aligned} \tag{A8}$$

Substituting (A8) into (A7) and (A7) into (A6) gives:

$$\mathrm{Var}(TR_t) = (E(TR_t))^2 - 2E(TR_t)t\left(E\left(p|p > \frac{b+c(y)}{y}\right)y - c(y) - b\right)\left(1 - F\left(\frac{b+c(y)}{y}\right)\right)$$
$$+ t^2y^2\left[\left(1 - F\left(\frac{b+c(y)}{y}\right)\right)\mathrm{Var}\left(p|p > \frac{b+c(y)}{y}\right) - \left(1 - F\left(\frac{b+c(y)}{y}\right)\right)\bar{p}^2\right.$$
$$\left. + 2\left(1 - F\left(\frac{b+c(y)}{y}\right)\right)\bar{p}E\left(p|p > \frac{b+c(y)}{y}\right)\right]$$
$$+ t^2(b + c(y))^2\left(1 - F\left(\frac{b+c(y)}{y}\right)\right)$$
$$- 2t^2y(b + c(y))E\left(p|p > \frac{b+c(y)}{y}\right)\left(1 - F\left(\frac{b+c(y)}{y}\right)\right) \quad \text{(A9)}$$

which is reproduced as equation 15 in the main text.

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