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The state-contingent approach to production under uncertainty*

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The central claim of this paper is that the state-contingent approach provides the best way to think about all problems in the economics of uncertainty, including problems of consumer choice, the theory of the firm, and principal–agent relationships. This claim is illustrated by recent developments in, and applications of, the state-contingent approach.

Key words: risk, state-contingent production, uncertainty.

1. Introduction

Production is an uncertain business, and agricultural production more so than most. The problem of uncertainty is a primary concern in relation to policy issues ranging from the marketing of agricultural commodities to the allocation of water in irrigation systems.

Two very different approaches have been taken to the analysis of production under uncertainty. The general equilibrium theory, along with offshoots such as modern finance theory, has been dominated by the state-contingent approach pioneered by Arrow (1953) and Debreu (1952) (see also Arrow and Debreu 1954). In microeconomic analysis, the dominant approach has been based on stochastic production functions, or the formally equivalent case of a non-stochastic production function for a producer facing stochastic prices. The seminal analysis of the latter case was put forward by Sandmo (1971) and developed in the context of production uncertainty by Just and Pope (1978) and a large number of subsequent writers.

The crucial insight of Arrow and Debreu was that, if uncertainty is represented by a set of possible states of nature, and uncertain outputs by vectors of state-contingent commodities, production under uncertainty can be represented as a

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multi-output technology, formally identical to a non-stochastic technology. Hence, the necessary and sufficient conditions for the existence and optimality of equilibrium are not affected by the introduction of uncertainty. On the other hand, the empirical plausibility of the relevant necessary and sufficient conditions is significantly reduced by consideration of uncertainty. In the absence of uncertainty, the requirement that, for each commodity, there should exist a market seems relatively innocuous. On the other hand, the more general requirement that a market should exist for each commodity in each possible state of nature is clearly not satisfied, even as an approximation. Interest is therefore focused on the case when markets are incomplete.

The approach inaugurated by Sandmo (1971) and Just and Pope (1978) was seemingly much simpler. The basic analytical approach was to derive first-order conditions for optimisation, then use the implicit function theorem to characterise comparative static responses to changes in parameters such as the mean price level. However, this approach is often intractable when applied to multi-output production or to the case of non-linear incentives such as those arising in principal–agent models.

Chambers and Quiggin (2000, p. i) claim that 'the state-contingent approach provides the best way to think about all problems in the economics of uncertainty, including problems of consumer choice, the theory of the firm and principal–agent relationships.' The purpose of this paper is to restate this claim, and to defend it in the light of recent developments in, and applications of, the state-contingent approach.

2. The state-contingent model

The standard approach to the representation of production under uncertainty is based on the concept of a stochastic production function, most commonly represented in the form

$$z = f(\boldsymbol{x}, \varepsilon), \tag{1}$$

where z is a scalar output, $\boldsymbol{x}$ is a vector of inputs, and ε is a scalar random shock, which may be conceived of as an input from nature, such as rainfall. The stochastic production function model is presented in Gravelle and Rees (2004, Chapter 19), with a summary of the main comparative static results.

Chambers and Quiggin (2000) argue that this representation is inflexible and, in important respects, unrealistic, and argue for the use of an alternative model, based on the notion of state-contingent production. This model originated from Arrow (1953) and Debreu (1952) in the context of general equilibrium theory. Gravelle and Rees (2004, Chapter 21) provide an accessible summary of some of the properties of exchange and production economies with state-contingent commodities.

In the general state-contingent model, there are M distinct outputs, N distinct inputs, and S possible states of nature.[1] Inputs $\boldsymbol{x} \in \Re_+^N$ are committed *ex ante* and fixed *ex post*. State-contingent outputs $\boldsymbol{z} \in \Re_+^{S \times M}$ are chosen *ex ante* but produced *ex post*. That is, if state s is realised, and the *ex ante* output choice is the matrix $\boldsymbol{z}$, the observed output is $\boldsymbol{z}_s \in \Re_+^M$, which corresponds to the M outputs produced in state s. Inputs that are variable *ex post* may be regarded as negative state-contingent outputs, in which case we generalise to allow $\boldsymbol{z}_s \in \Re^M$. We denote by $1_S \in \Re^S$ the unit vector with all entries equal to 1.

The formal structure may be considered as a two-period game with nature, with periods denoted 0 and 1. In period 0, the producer commits inputs $\boldsymbol{x} \in \Re_+^N$. When nature reveals the state s, the individual produces the output $\boldsymbol{z}_s$. The technology of production determines the feasible strategies $(\boldsymbol{x}, \boldsymbol{z})$.

Chambers and Quiggin (2000) show that a state-contingent technology may be summarised in terms of the input correspondence, which maps state-contingent output vectors into sets of inputs that can produce that state-contingent output matrix. Formally, it is defined by $X: \Re_+^{M \times S} \to \Re_+^N$

$$X(\boldsymbol{z}) = \{\mathbf{x} \in \Re_+^N : \mathbf{x} \text{ can produce } \boldsymbol{z} \in \Re_+^{M \times S}\}.$$

Intuitively, one can think of this input correspondence as yielding all input combinations that are on or above the production isoquant for the state-contingent output matrix $\boldsymbol{z}$.

Conversely, we can consider an output correspondence

$$Z(\boldsymbol{x}) = \{\boldsymbol{Z} \in \Re_+^{M \times S} : x \in X(\boldsymbol{z})\},$$

which, in a sense, is the inverse of the input correspondence. Intuitively, one can think of it as giving the state-contingent output matrices that are on or below a state-contingent transformation curve. In what follows, we routinely restrict attention to the case of a single stochastic output, so that $M = 1$.

Technology may also be represented using the distance and benefit functions commonly used in comparisons of productive efficiency. The distance function $O(\boldsymbol{z}, \boldsymbol{x})$ is defined by:

$$O(\boldsymbol{z}, \boldsymbol{x}) = \inf\{\theta > 0 : z/\theta \in Z(\boldsymbol{x})\},$$

if $(z/\theta) \in Z(\boldsymbol{x})$ for some θ, and ∞ otherwise. The benefit function $\mathrm{B}(\mathbf{x}, \mathbf{z})$ is defined by:

$$B(\boldsymbol{x}, \boldsymbol{z}) = \max\{\beta \in \Re : \boldsymbol{z} - \beta 1 \in Z(\boldsymbol{x})\},$$

if $\boldsymbol{z} - \beta 1 \in Z(\boldsymbol{x})$ for some β, and $-\infty$ otherwise.

[1] It is straightforward to extend the model presented here to the case of an infinite state space (Chambers 2005; Chambers and Quiggin 2005; Racionero and Quiggin 2006). Marginal cost may then be characterised by either Fréchet or Gateaux derivatives.

2.1 Objectives

The welfare of producers depends on state-contingent consumption, which will be denoted by $\boldsymbol{y}$, and on inputs $\boldsymbol{x}$. In general, producers are assumed to have an objective of the form $W(\boldsymbol{x}, \boldsymbol{y})$ decreasing in the first argument and increasing in the second. Consumption takes the general form $\boldsymbol{y} = \boldsymbol{a} + \boldsymbol{r}$, where $\boldsymbol{a}$ is returns from asset holdings and $\boldsymbol{r}$ is revenue. In most cases, outputs are assumed to be sold in competitive markets so $\boldsymbol{r} = \boldsymbol{pz}$, where $\boldsymbol{p} \in \mathcal{R}^S_{++}$ is a state-contingent output price vector, and $\boldsymbol{pz}$ is an element-wise product.

The cost of effort is summed up by an effort cost function $g(\boldsymbol{x})$. In the case where all inputs are purchased in competitive markets, we have $g(\boldsymbol{x}) = \boldsymbol{wx}$, where $w \in \mathcal{R}^N_{++}$ is an input price vector. Two special cases are of particular interest. The first is the separable effort case

$$W(\boldsymbol{x}, \boldsymbol{y}) = w(\boldsymbol{y}, \pi) - g(\boldsymbol{x})$$

where w is a concave utility function and π is a subjective probability distribution. The second is the net returns case

$$W(\boldsymbol{x}, \boldsymbol{y}) = w(\boldsymbol{y} - \boldsymbol{wx}, \pi).$$

The most popular form for w is the expected-utility functional

$$\begin{aligned} w(\boldsymbol{y}, \pi) &= \mathrm{E}[u(\boldsymbol{y})] \\ &= \Sigma_s \pi_s u(y_s) \end{aligned}$$

where u is a von Neumann–Morgenstern utility function. It is important to emphasise, however, that the use of the state-contingent approach does not require the assumption of expected-utility preferences, or even of well-defined subjective probabilities π. In fact, the symmetry between technology and preferences has allowed conditions developed in the context of state-contingent production under uncertainty to be translated into corresponding conditions on preferences under uncertainty, and, as a result, to new models of preferences under uncertainty. This point is discussed further below.

2.2 Cost functions

The state-contingent representation of production under uncertainty is formally identical to the standard representation of a multiproduct technology. As a result, it is possible to apply the toolkit developed in production theory since the work of Shephard (1953, 1970), including cost and profit functions, distance functions, and all the associated duality theory. The most useful single tool has proved to be the cost function. The general form of the cost function is

$$c(\boldsymbol{z}) = \inf\{g(\boldsymbol{x}):\boldsymbol{x} \in X(\boldsymbol{z})\}.$$

If all inputs are purchased in competitive markets, a cost function may be written as

$$c(\boldsymbol{w}, \boldsymbol{z}) = \inf\{\boldsymbol{wx}:\boldsymbol{x} \in X(\boldsymbol{z})\}.$$

If, in addition, outputs are sold in competitive markets, we may define the revenue–cost function

$$C(\boldsymbol{w}, \boldsymbol{r}, \boldsymbol{p}) = \inf\{\boldsymbol{wx}:\boldsymbol{x} \in X(\boldsymbol{z}), \boldsymbol{pz} \geq \boldsymbol{r}\}.$$

Noting that consumption is, in general, given by a combination of revenue from productive activity and income from asset holdings, so that $\boldsymbol{y} = \boldsymbol{a} + \boldsymbol{r}$, it is possible to extend these cost functions to take account of the interaction between production and finance decisions, as in Chambers and Quiggin (2004).

3. The case for the state-contingent approach

The representation of production in terms of state-contingent production theory is a natural generalisation of the standard modern theory of production and therefore allows production under uncertainty to be treated in the same way as production under certainty. It might be argued, however, that this general representation is, in the words of Tobin's (1969) criticism of state-preference theory, 'graceful but empty'. In this section, it will be argued that there are numerous practical advantages to be realised by adopting the state-contingent approach.

3.1 Stochastic production functions as a special case

The standard approach to the representation of production under uncertainty is based on the concept of a stochastic production function, most commonly represented in the form

$$z = f(\boldsymbol{x}, \varepsilon), \tag{2}$$

where z is a scalar output, x is a vector of inputs, and ε is a scalar random shock, which may be conceived of as an input from nature, such as rainfall. Chambers and Quiggin (1998) show that this is a special polar case of the general state-contingent technology, with some highly restrictive properties. Using the state-contingent representation of the stochastic production function technology, it is possible to analyse the properties of the technology and the extent to which results in the existing published work on production technology can be extended to more general technologies.

To represent the polar case of stochastic production function in (2), set $M = 1$ (a scalar output) and suppose that ε is a discrete random variable, which may be represented as a real-valued mapping from the state space $\Omega = \{1, \ldots S\}$ to $\mathfrak{R}$, or, equivalently, as a vector in $\mathfrak{R}^S$. Given free disposal of outputs, the technology may be represented by the constraints

$$z_s \leq f(\boldsymbol{x}, \varepsilon_s),\ s \in \Omega. \tag{3}$$

The state-contingent input correspondence associated with (3) is

$$\begin{aligned} X(\boldsymbol{z}) &= \{\boldsymbol{x}: z_s \leq f(\boldsymbol{x}, \varepsilon_s),\ s \in \Omega\} \\ &= \cap_{s\in\Omega} \{\boldsymbol{x}: z_s \leq f(\boldsymbol{x}, \varepsilon_s)\} \\ &= \cap_{s\in\Omega} \bar{x}\ (z_s; \varepsilon_s), \end{aligned}$$

where $\bar{x}(z_s; \varepsilon_s)$ may be interpreted as the *ex post* input set associated with the production function for a given realisation of the random variable.

The dual cost structure for the stochastic production function specification defined as,

$$c(\boldsymbol{w}, \boldsymbol{z}) = \text{Min}\{\boldsymbol{wx}: \boldsymbol{x} \in \cap_{s\in\Omega}\ \bar{x}(z_s; \varepsilon_s)\},$$

satisfies

$$c(\boldsymbol{w}, \boldsymbol{z}) \geq \text{Max}\{\bar{c}(\boldsymbol{w}, z_1; \varepsilon_1), \ldots,\ \bar{c}(\boldsymbol{w}, z_s; \varepsilon_s)\}, \tag{4}$$

where $\bar{c}(\boldsymbol{w}, z_s; \varepsilon_s)$ is the *ex post* cost function dual to $\bar{x}(z_s; \varepsilon_s)$. There can exist instances where the inequality in (4) is strict (Chambers and Quiggin 1998, 2000).

The limitations of the stochastic production function approach may be seen by illustrating the output isoquants for the cases $S = 2$ and $S = 3$ (Figures 1 and 2). It can be seen from Figure 1 that the resulting technology is analogous to the Leontief or fixed-output-proportions technology that is a special case of standard multi-output technologies. Figure 2 motivates the characterisation of the stochastic production function technology as 'output cubical' (Chambers and Quiggin 2000).

As Figures 1 and 2 suggest, if the technology takes the form (3), the cost function $c(\boldsymbol{w}, \boldsymbol{z})$ will not, in general, be differentiable as a function of $\boldsymbol{z}$. As a result, corner solutions will commonly arise. A necessary condition for differentiability is that the number of inputs N should be at least as great as the number of states S.

One interpretation of the condition $N \geq S$ may be derived from consideration of state-allocable inputs. The state-contingent properties of stochastic production functions with multiple inputs are considered further by Rasmussen (2003). Rasmussen distinguishes between state-allocable and state-specific inputs and

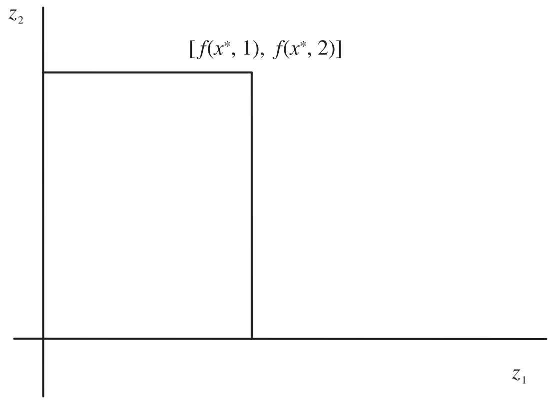

Figure 1 Stochastic production function: $S = 2$.

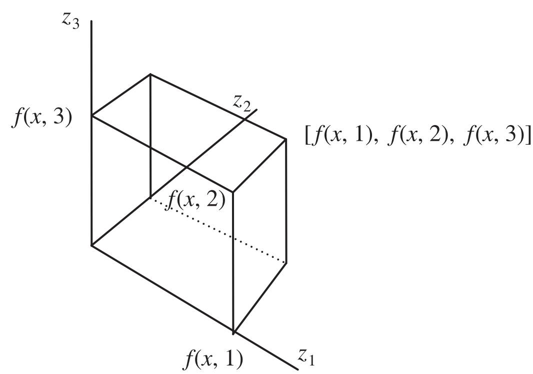

Figure 2 Stochastic production $S = 3$. Output cubical output set.

derives conditions for optimal input allocations. In effect, a state-allocable input may be regarded as a primary input from which S distinct inputs to the stochastic production may be derived according to the allocation made by the producer. Hence, in Rasmussen's analysis, the existence of at least one state-allocable input guarantees that $N \geq S$, and this is normally sufficient for differentiability of the cost function.

In the case of production under uncertainty, there is a large and complex published work on principal–agent relationships. Much of the complexity results from the implicit assumption that the agent has a stochastic production function technology with a single scalar input (effort). The fixed-output-proportions property of this technology means that, if the principal can control output in one state of nature (say, the worst), that principal can control the agent's effort, and therefore the output in all states of nature. This implausible property allows for the construction of theoretically optimal incentive structures that can achieve first-best outcomes in situations of asymmetric information. As shown by Quiggin and Chambers (1998), this result does not apply for general state-contingent technologies.[2]

3.2 Consistency with general equilibrium and finance theory

The idea of state-contingent production was originally developed in the context of general equilibrium theory, and the state-contingent representation is the standard approach in models of general equilibrium under uncertainty. It follows that, in any problem concerning the implications of production uncertainty for the existence, stability and optimality of general equilibrium, it is appropriate to use the state-contingent representation.

In the modern context, a more significant, but closely related, advantage is the fact that state-contingent production models have a structure that is logically consistent with that of modern finance theory. The same set of states of nature used to model production under uncertainty can be used to describe the spanning properties of securities structures.

In the absence of a well-developed state-contingent theory of production, most financial modellers have focused on the case of an endowment economy. Attempts to introduce production uncertainty through stochastic production functions have, not surprisingly, proved problematic.

In discussing the relationship between state-contingent production theory and finance theory, it is necessary to address the (apparently widely held) misconception that the applicability of state-contingent models depends on the existence of a complete set of state-contingent markets, as claimed, for example, by Shogren and Crocker (1999).

Chambers and Quiggin (2000) begin the analysis of state-contingent production by examining the case when there are no financial markets, and therefore no state-contingent claims. In subsequent chapters, a variety of financial instruments, including forward and futures markets, crop insurance, and sharecropping agreements, are examined. In none of these cases is the existence of a complete set of state-contingent claims assumed or implied.

The unrealistic case of complete state-contingent markets is of little interest in itself. However, the properties of the Pareto-optimal equilibrium that arise in this case are of interest as a benchmark for analysis. In particular, in the presence of complete state-contingent markets, preferences and production decisions are separate. That is, each producer will choose the state-contingent output vector that maximises net returns at the uniquely given state-contingent prices, and will then use those returns to purchase the vector

[2] The unsatisfactory properties of the standard principal–agent model have increased the popularity of the approach favoured by Holmstrom and Milgrom (1987). In this approach, the agent can vary inputs continuously over time, leading to an infinite-dimensional technology. As noted above, a stochastic production is smooth (that is, continuously differentiable) if the dimensionality of the input set is greater than or equal to that of the state space.

of state-contingent consumption claims that maximises utility, given the producer's preferences.

Chambers and Quiggin (2003a) have examined conditions under which this separation applies even in the absence of complete state-contingent claims. It is well known, for example, that, with a nonstochastic production technology, the existence of a forward market (with no transactions costs and unrestricted short selling) is sufficient to ensure that all producers choose the output that maximises expected profit at the given forward price. Chambers and Quiggin (2003b) generalise this result to encompass technologies that are not inherently risky, in the sense that a non-stochastic output vector minimises costs for a given level of expected output.[3] Building on this insight, conditions under which partial price stabilisation is, and is not, beneficial are derived.

3.3 Applicability of duality theory and modern production theory

Arguably the single most important development in the theory of cost and production was Shephard's (1953, 1970) discovery of the dual correspondence between the production structure and the cost function. This discovery has had important consequences at both an empirical and theoretical level. Yet economists have struggled with attempts to extend duality theory to the analysis of production under uncertainty.

State-contingent production under uncertainty, like production of commodities differentiated in time and space, is merely a special case of a general multiple-input, multiple-output technology. Hence, as we demonstrate above, the duality tools developed for the latter automatically apply to the former. This proposition stated in this way appears self-evident, but the issue of whether duality methods are applicable under uncertainty has remained shrouded in confusion and conflicting claims. As argued above, and in more detail by Chambers and Quiggin (1998, 2003c), provided input sets are closed and non-empty, a well-behaved cost function can be derived from any stochastic production or revenue function. The resulting cost function, in turn, is always dual to a stochastic production structure exhibiting convexity of input sets and free disposability of inputs. Hence, any stochastic production structure possessing closed and non-empty input sets will be observationally equivalent to a stochastic production structure possessing closed, convex, and input-disposable input sets.

More generally, the state-contingent approach permits the application of the entire panoply of techniques developed in modern published works on production theory, including distance and benefit functions, generalised concepts of homotheticity and separability, and so on. An example of the interaction between the two approaches is Chambers *et al.* (2004).

[3] The expectation here is calculated with respect to the same set of subjective probabilities used to determine that the forward price is unbiased.

3.4 Structural forms and reduced forms

Any given choice generates a state-contingent vector of outcomes that may be described by a random variable.[4] Many researchers have therefore chosen to disregard the underlying state space, and analyse problems purely in terms of choices over random variables. Working in these terms is what we are referring to when we talk about the parameterised distribution approach.

The relationship between the parameterised distribution formulation and the state-space representation of the problem is analogous to that between reduced forms and structural models in econometrics. The state-space representation, which contains all the relevant information about the possible states of nature, the input choices of the producers, and the possible range of outcomes, corresponds to the structural form. The parameterised distribution formulation, which confounds all these relationships into a simple relationship between possible outcomes and inputs, corresponds to the reduced form.

As with the identification problem in econometrics, it is always possible, in general, to derive a parameterised distribution formulation from any state-space representation, but the reverse does not apply. In particular, as a general rule, most parameterised distribution formulations may correspond to several different state-space representations. This certainly achieves some economy in representation and analysis just as reduced-form estimation achieves some economy in econometric estimation. Unfortunately, the economy generally is a false one because it is purchased at the cost of confounding causal factors for any economic phenomena that may emerge in such models. As a result, only limited comparative-static analysis can be undertaken in the parameterised distribution formulation.

Other problems emerge with the parameterised distribution formulation as well. For example, when specified in state-contingent terms, an uncertain production technology may have reasonable properties, but when captured in its reduced form, it may have unreasonable properties. For example, as shown by Chambers and Quiggin (2000), a convex state-contingent production technology may give rise to non-convex sets of feasible random variables.

In economics, as elsewhere, an inappropriate choice of problem representation usually leads to a complex and confusing analysis. By diverting attention from the underlying state space and the richer information structure available therein, the parameterised distribution formulation has been an obstacle to progress. In particular, in problems involving production under uncertainty, it has further widened the gap between the theory of asymmetric information and the general equilibrium tradition going back to Arrow and Debreu.

[4] This statement depends on the existence of well-defined subjective probabilities, and is therefore valid subject to some relatively weak assumptions about preferences.

3.5 Analogy between state-contingent production and choice under uncertainty

As has already been observed in relation to finance theory, there is a natural symmetry between models of choice and models of production when both are expressed in state-contingent terms. Hence, it is not surprising that tools used to analyse production under uncertainty can be used to analyse choice under uncertainty and vice versa. Quiggin and Chambers (1998) show that a wide range of standard tools for the analysis of economic problems involving uncertainty, including risk premiums, certainty equivalents, and the notions of absolute and relative risk aversion, can be developed and applied without making specific assumptions on functional form beyond the basic requirements of monotonicity, transitivity, continuity, and the presumption that individuals prefer certainty to risk.

The approach relies on the distance and benefit functions, described above in relation to production under uncertainty, to characterise preferences relative to a given state-contingent vector of outcomes, and then derives results directly from the properties of these functions. The distance and benefit functions are then used to derive absolute and relative risk premiums and to derive conditions under which preferences display constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). An immediate by-product of this discussion is a result characterising preferences displaying both CARA and CRRA. This result is then used to suggest several flexible functional specifications of preferences satisfying both properties.

The analysis may be extended further using the concept of invariant risk attitudes (Quiggin and Chambers 2004a). Invariant risk attitudes may be represented by a function of two parameters, the mean and an index of riskiness that is both linear and translation-invariant (that is, unaffected by the addition of riskless wealth). The paradigmatic instance of such an index is the standard deviation. Using the results previously derived, Quiggin and Chambers (2004a) show how many of the attractive properties of mean–variance (or, more properly, mean–standard deviation) preferences may be generalised to a large class of preference structures, which can be neatly characterised in dual terms.

4. Policy applications

Because the state-contingent model of production under uncertainty is more general, and, for most problems, more realistic than the stochastic production function model, it is a superior tool for the analysis of policy problems involving uncertainty. Two main approaches have proved useful. The first is to exploit the fact that the stochastic production function model is a special case, and examine the question of whether results derived using that model can be generalised to a larger class of production technologies, or whether they depend crucially on the fixed-output-proportions property of the model. As economic actors will seek to exploit arbitrage and substitution opportunities in response to changes in policy, policy prescriptions based on the assumption of fixed output proportions are unlikely to prove robust. A second approach is to examine problems that cannot easily be addressed (or at least have not been addressed) using the standard tools of the stochastic production function model. Results for a variety of special cases can then be derived.

A wide range of policy issues have been addressed using the state-contingent model, including non-point-source pollution (Chambers and Quiggin 1997), crop insurance (Chambers and Quiggin 2002), and social security reform (Grant and Quiggin 2002). In this section, attention will be focused on a selection of policy topics that illustrate a variety of features of the state-contingent model.

Quiggin and Chambers (2004b) examine drought policy in Australia. It is increasingly recognised that, under Australian conditions, drought should not be thought of as an unpredictable natural disaster. Rather, any rational assessment of the states of nature under which farmers produce must allocate a significant probability to low-rainfall states, including lengthy droughts. The key issue in drought policy has been the problem of assisting farmers to deal with the consequences of drought, while maintaining incentives to prepare appropriately for drought conditions. A popular policy that fails this test is the provision of fodder subsidies to producers who have insufficient pasture to feed their livestock. *Ex post*, such policies relieve suffering, but *ex ante*, they encourage overstocking and discourage measures to prepare against drought, such as the purchase and storage of fodder at favourable prices.

Discussion of these problems has not, until recently, been assisted by formal models of production under uncertainty. In the stochastic production function approach, there is no way to model the idea that producers might take action to increase their net returns in low-rainfall states of nature, at the cost of lower returns in high-output states. Rather, an increase in scalar effort raises output in every state of nature. Quiggin and Chambers (2004b) show how the risk-reducing or risk-increasing properties of a range of drought policies may be analysed in a state-contingent framework.

The problem of contract design is a central concern of modern economics. However, the standard approach has produced complex models that tend to yield implausible results. For a stochastic production function, output in every state is degenerately determined by the effort level. Hence, once output in one state is known, output in every other state is known. This does not seem like a plausible description of most situations in which incentive schemes are offered. If the agent is told that he will be severely punished for falling below some minimum target, but will receive no reward for performance above the target level, it is natural to suppose that he will devote all his efforts to meeting the minimum target. In the general state-contingent production framework, this requires reallocating resources towards the least favourable state of nature and away from all of the others. There have been a variety of attempts to overcome this problem, but the appropriate response is to reformulate the assumptions regarding the agent's production technology. State-contingent models of contract design have been applied to problems of point-source pollution (Quiggin and Chambers 1998), bioprospecting (Smith and Kumar 2002), and banking regulation (Suwandi 1995).

The problem of price stabilisation has given rise to a large and complex published works. These published works began with an air of paradox, generated by Oi's (1961) finding that price instability is beneficial to producers. This finding mirrored an earlier, previously neglected result of Waugh (1944) for consumers. Samuelson (1972) responded to Oi by arguing that, in general, a feasible buffer-stock mechanism would not stabilise prices at the mean, but would yield benefits to consumers. A voluminous published work has sprouted from this beginning.

A large number of papers analysed the implications of buffer-stock stabilisation under a wide range of assumptions. Much of these earlier published works were superseded by Newbery and Stiglitz (1981). However, problems remained. In particular, although much of the published work relied on the traditional apparatus of supply and demand curves, and the associated surplus measures, there was no clear understanding of what was required for stochastic supply and demand to be represented by a curve. Chambers and Quiggin (2003c) revisited the original Oi (1961) result and showed that a risk-neutral firm possesses a state-independent supply curve if and only if the cost function is both additively separable and not inherently risky, in the sense that increasing the riskiness of output, maintaining mean output fixed, always leads to an increase in costs. This result leads to a more general characterisation of the conditions under which partial price stabilisation benefits producers.

5. Empirical applications

Empirical application of the state-contingent approach has proved challenging. This is not because the approach is intractable, but because consideration of the state-contingent representation reveals the difficulty of estimating production technology in the face of endogenous producer responses to uncertainty. To apply standard methods for the analysis of multi-output production technologies to the problem set out above, we would require a dataset with observations of the form $(\mathbf{x}, \mathbf{z})$. However, given that each observation is associated with the realisation of some particular state s, observed data points are of the form $(\mathbf{x}, \mathbf{z}s)$. Most of the data that would be required for the application of standard methods are unavailable, lost in the potentiality of unrealised states of the world.

This problem can be assumed away with a standard stochastic production function technology, with a single scalar input or with inputs separable from state-contingent outputs. It has long been recognised, however, that such a representation is inadequate for a serious empirical treatment of the problems of risk-averse producers facing an uncertain production technology. The preferred approach has been the moment-based model of Just and Pope (1978) in which input choices determine the mean and variance of output, with one function determining the mean and a second determining the variance. More generally, one might consider third and higher moments, depending on the availability of data.

Chambers and Quiggin (2002) show that the Just–Pope technology, like any technology satisfying the minimal requirements of free disposability and convexity, can be represented in state-contingent terms. If the number of moments is less than the number of states of nature, the Just–Pope technology will give rise to a non-differentiable cost function. Conversely, any estimated Just–Pope technology can be interpreted as arising from a state-contingent model with two states of nature.

For a completely accurate representation of a problem involving uncertainty, both the number of states of nature and the number of moments must be infinite (or at least larger than the degrees of freedom granted by any feasible data set). Hence, in empirical work, the number of states (or moments) included in the model involves a trade-off between parsimony and goodness-of-fit, just as is the case with lag structures. There is no obvious reason to suppose that the trade-off will differ for moments-based and state-based representations. This suggests that the number of states used in state-contingent analysis should typically be fairly small, say two or three, as is usually the case with moments-based analysis.

Regardless of the trade-off that is made in choosing the number of states of nature, the problem that only one state of nature is realised for any given observation remains. Griffiths and O'Donnell (2004) use a maximum-likelihood approach to resolve this problem, in the context of a frontier production model, in which output may fall short of the technical optimum because of firm-specific inefficiency. The underlying technology is state contingent. Observations are assigned to one of three states of nature based on a maximum-likelihood criterion.

A notable feature of this approach is that, compared with standard frontier models, it yields a significant reduction in estimated levels of inefficiency. Observations that might fall inside the frontier in a standard approach may instead be modelled as arising from an unfavourable state of nature. This feature of the model addresses the concern that frontier models detect inefficiencies when the enterprises concerned may simply have suffered an adverse shock (O'Donnell *et al.* 2006).

This result has significant policy implications. Findings that a large number of firms lie inside the efficiency frontier are commonly taken to imply that there exists potential for beneficial policy interventions, or alternatively, that existing interventions are contributing to inefficiency. A state-contingent analysis would suggest that such inferences should be drawn with more care.

There are a wide range of potential enhancements to the approach pioneered by Griffiths and O'Donnell (2004). For example, the number of states of nature could be determined exogenously, using a likelihood ratio test approach. In addition, instrumental variables such as observations on rainfall could be used in the assignment of observations to states of nature.

Chambers (2004) uses the state-contingent approach to define stochastic productivity indicators, which are applied to data for post-war US agriculture.

Comparison with existing non-stochastic productivity indicators suggests that properly accounting for the stochastic environment in which firms operate could have important empirical implications for measuring productivity growth.

Chambers and Quiggin (2005) consider the implications of state-contingent production for asset pricing. The crucial tool is a generalisation of the cost function, measuring the cost of generating a given revenue through a combination of production decisions and financial transactions. Using this cost function, an equilibrium asset pricing relationship may be derived, in a manner that is symmetrical with the usual consumption-based approach. The model is estimated using annual US macroeconomic data on aggregate production (gross domestic product) and its price, aggregate investment (gross private domestic investment), unit labour cost, unit non-labour cost, stock price returns (returns on the Standard & Poor's 500), and returns on commercial paper for the period 1929–1995. The implied technology shows increasing costs in both the mean and variance of output, suggesting that demand variation is the dominant source of fluctuations in aggregate output.

The state-contingent approach is also being applied in the context of simulation. The central problems in management of the Murray–Darling river system relate to the variability and unpredictability of aggregate rainfall and to the way in which policy institutions act to allocate and manage risk. The evolution of these policy institutions is, itself, the subject of considerable uncertainty. Adamson *et al*. (2005) describe the development of a simulation model aimed at representing these uncertainties in a state-contingent framework.

6. Concluding comments

Chambers and Quiggin (2000, p. 357) concluded that 'we have only scratched the surface of what can be achieved using state-contingent production models.' Developments since then have made the scratches a little deeper, and have resolved some of the problems seen as obstacles to widespread application of the state-contingent approach. In particular, some of the difficulties surrounding empirical application of the model have been resolved, notably by the work of Griffiths and O'Donnell (2004). The relationship between the general state-contingent approach and the special case of technology derived from a stochastic production function has been clarified. Substantial progress has been made on integrating finance and production theory.

Despite this progress, there is room for more work than has been carried out so far. There are a huge range of issues, from monopoly pricing under uncertainty to financial intermediation that could profitably be explored using the state-contingent approach. With improved tools, the range of empirical applications that could be undertaken is almost limitless. Almost every problem in economics involves uncertainty and, in almost every case, uncertainty is best interpreted in a state-contingent framework.

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