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PREDICTING TIME SERIES TURNING POINTS WITH ARIMA MODELS

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Abstract

This paper reports an application to swine slaughter data of a proposed heuristic technique for use in conjunction with ARIMA models in predicting future turning points in time series. The technique involves the generation of empirical predictive distributions of future turning point indicators. Results of the application indicate that the technique yields results only marginally superior to conventional ARIMA forecasts.

PREDICTING TIME SERIES TURNING POINTS WITH ARIMA MODELS

Introduction

Several applications of the ARIMA (autoregressive-integrated-moving average) models popularized by Box and Jenkins have been reported in the recent agricultural economics literature (e.g., Schmitz and Watts; Bieri and Schmitz; Oliveira and Rausser; Gellatly; Bourke). The ARIMA models enjoy advantages relative to more conventional econometric forecasting models on several counts. The problems of obtaining observations on the exogenous variables in econometric forecasting models are not faced with ARIMA models. Owing to their simplicity, the ARIMA models may be less costly to develop and update.

In cases in which the mean square error of postsample forecasts of ARIMA models and more conventional econometric models have been compared, the former models have not always suffered from the comparison (e.g., see Bourke). However, as discussed by Wecker, forecasts from univariate ARIMA models which are based on a minimum mean square error criterion are devoid of information as to the subsequent time path of the series in question. A consequence is that ARIMA models frequently fail to satisfactorily predict turning points in time series. This may be a serious shortcoming. Gale has noted that both policy makers and business decision makers demand such answers as: (a) When will a series turn down (up)? and (b) How high (low) will a series go before turning down (up)?

As a solution to the problem of predicting turning points with time series models, Granger and Newbold (p. 316) have suggested the possibility of considering turning points as realizations of point processes and the building of single or multiple time series models to forecast such processes. Along these lines, Wecker has offered a heuristic solution to the turning point prediction problem for use in forecasting with ARIMA models. To the author's knowledge, the only reported application of this technique is Wecker's forecast of one GNP turning point.[1]

The objective of this paper is to further evaluate Wecker's technique by applying it to data from the swine sector. The plan of the paper is as follows. First, Wecker's technique is briefly sketched and some problems in identifying turning points are discussed. Next, the technique is applied to swine slaughter data. Finally, a summary and conclusions are offered.

Wecker's Technique

Let x_t, $t = 1,2,\ldots,n$, be the observed realization at time t of the time series in question. Also, define an indicator variable z_t such that

$$z_t = \begin{cases} 1 \text{ if a turning point occurs at } t, \\ 0 \text{ otherwise.} \end{cases}$$

Predictions of the next k values of x beyond the current time period n are required. The predictive distribution of these k values is given by

$$G_{n+1,\ldots,n+k}(x_{n+1},\ldots,x_{n+k}|x_n,x_{n-1},\ldots,x_1), \tag{1}$$

and may be estimated by the iterative ARIMA model building process described by Box and Jenkins.

Future turning points are determined, in part, by the unknown values of $x_{n+1},\ldots,x_{n+k}$, thus future values of z_t are uncertain. The predictive distribution of the z's is given by

$$F_{n+1,\ldots,n+k}(z_{n+1},\ldots,z_{n+k}|x_n,x_{n-1},\ldots,x_1) \ . \qquad (2)$$

The problem at hand is to estimate this distribution.

Wecker's solution to this problem is as follows. Let T represent the function relating the indicator variable z_t to the time series. It is assumed that z_t is a function of only $2\tau + 1$ time series values, $\tau < n$, so as to avoid the need to consider values of x_t which precede the first observation in the sampling interval. Thus, z_t can be written

$$z_t = T(x_{n-\tau},\ldots,x_{n+\tau}) \ . \qquad (3)$$

Rather than make an arbitrary choice as to the nature of T in determining the distribution of z, Wecker suggests that the following procedure be employed:

Step 1. estimate the predictive distribution of the x's by using the ARIMA modeling procedures of Box and Jenkins;

Step 2. by random number generation, obtain predicted values of $x_{n+1},\ldots,x_{n+\tau}$ from the distribution estimated in Step 1;

Step 3. compute values of z associated with the data generated in Step 2;

Step 4. replicate Steps 2 and 3 to obtain a convergent predictive distribution for z.

This procedure also may be used to predict other functions of the series of interest; e.g., the time until the next turning point, and minimum or maximum time series values associated with future turning points. The reader is referred to Wecker for a more complete description of the technique.

Prior to application of the technique, criteria for identifying relevant turning points in a time series must be ascertained. Bourke makes a distinction between statistical and economic turning points. A statistical turning point is said to occur when the movement of a time series changes direction, while an economic turning point is said to occur when a current trend (seasonal or cyclic) in the time series is reversed. This taxonomy presumes that the forecaster has an objective means of identifying movements of the time series which constitute a trend.[2] Whether the prediction of statistical or economic turning points is more important will depend upon the costs incurred (or benefits foregone) in the event of a turning point prediction error.

In the following analysis, attention is confined to forecasting statistical turning points. That is,

$$z_t = \begin{cases} 1 \text{ if either } x_t > x_{t-1} \text{ and } x_t > x_{t+1}, \text{ or} \\ \quad x_t < x_{t-1} \text{ and } x_t < x_{t+1} \\ \\ 0 \text{ otherwise.} \end{cases}$$

However, Wecker's technique may also be used to forecast economic turning points by appropriate specification of z_t.

An Application

In this section, Wecker's technique and a conventional ARIMA model are used to predict turning points in annual U.S. commercial hog slaughter, million head (HMKT) as displayed in Figure 1. Wallis has fitted an ARIMA model to this series using data for 1935-71 as follows:

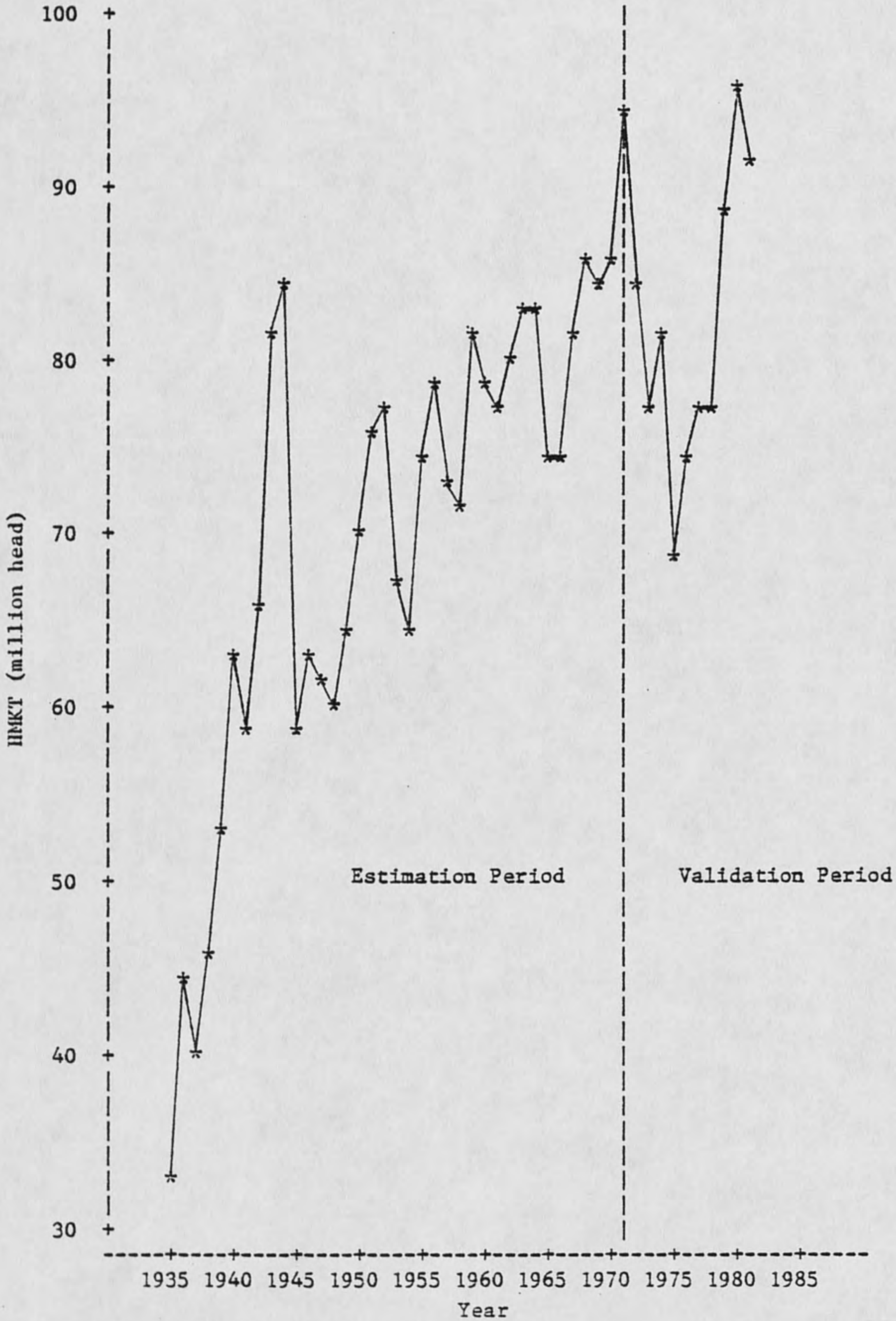

Figure 1. U.S. Commercial Hog Slaughter, 1935-1981.

$$(1 - 0.07\ L + 0.037\ L^2)\ \Delta\ \mathrm{HMKT} = 2.49 + \varepsilon_t, \quad \hat{\sigma}^2_\varepsilon = 51.36$$

where L and Δ are lag and difference operators, respectively.[3]

This ARIMA model was used to generate conventional ARIMA forecasts and as Step 1 in Wecker's technique in forecasting nearest future statistical turning points at forecast origins from 1971-1980. Actual statistical turning points in this series over this period occurred at 1971, 1973, 1974, 1975, and 1980. One hundred replications were used to generate the distributions of the z's at each forecast origin using Wecker's technique.

Empirical distributions of the turning point indicators ($z_t = 1$) from Wecker's technique are displayed in Figure 2, Panels A through J, for forecasts dating from 1971 through 1980. Summary statistics of these distributions and conventional ARIMA forecasts of future turning points, if any, are presented in Table 1 for each forecast origin. These results motivate the following comments.

First, the conventional ARIMA forecasts of nearest turning points were accurate for forecasts dating from 1972, 1973, 1979, and 1980. Using the mean of the empirical distribution of turning point indicators, forecasts from Wecker's technique also were accurate from four origins: 1972, 1973, 1978, and 1979. The mode of the empirical distributions of turning point indicators yielded accurate turning point forecasts at four origins: 1972, 1973, 1975, and 1980. The median of the empirical distribution of turning point indicators proved most accurate in forecasting nearest turning points, providing accurate turning point forecasts at five forecast origins: 1972, 1973, 1975, 1979, and 1980.

```
                                                                    FREQ  CUM.   PERCENT
Year                                                                      FREQ
      |
1971  |*****************                                              33    33    33.00
      |
1972  |*****************                                              33    66    33.00
      |
1973  |*********                                                      18    84    18.00
      |
1974  |****                                                            8     92     8.00
      |
1975  |**                                                             3     95     3.00
      |
1976  |**                                                             4     99     4.00
      |
1978  |*                                                              1    100     1.00
      |
      -----+----+----+----+----+----+----+----+----+----+
          10   20   30   40   50   60   70   80   90  100
                             Frequency
```

Panel A. 1971 Forecast Origin.

```
                                                                    FREQ  CUM.   PERCENT
Year                                                                      FREQ
      |
1972  |*******************                                            38    38    38.00
      |
1973  |***********************                                        45    83    45.00
      |
1974  |*******                                                        13    96    13.00
      |
1975  |**                                                             3     99     3.00
      |
1976  |*                                                              1    100     1.00
      |
      -----+----+----+----+----+----+----+----+----+----+
          10   20   30   40   50   60   70   80   90  100
                             Frequency
```

Panel B. 1972 Forecast Origin.

Figure 2. Empirical Distributions of Turning Point Indicators in U.S. Annual Commercial Hog Slaughter, 1971-1980.

```
                                                                 FREQ  CUM.   PERCENT
Year                                                                   FREQ
     |
1973 |**************************************                       76    76    76.00
     |
1974 |***********                                                  21    97    21.00
     |
1975 |**                                                            3   100     3.00
     |
     ----+----+----+----+----+----+----+----+----+----+
         10   20   30   40   50   60   70   80   90  100
                        Frequency
```

Panel C. 1973 Forecast Origin.

```
                                                                 FREQ  CUM.   PERCENT
Year                                                                   FREQ
     |
1974 |*******                                                      13    13    13.00
     |
1975 |********************                                         39    52    39.00
     |
1976 |**********                                                   20    72    20.00
     |
1977 |******                                                       11    83    11.00
     |
1978 |******                                                       12    95    12.00
     |
1979 |*                                                             2    97     2.00
     |
1980 |*                                                             1    98     1.00
     |
1982 |*                                                             2   100     2.00
     |
     ----+----+----+----+----+----+----+----+----+----+
         10   20   30   40   50   60   70   80   90  100
                        Frequency
```

Panel D. 1974 Forecast Origin.

Figure 2. (Continued)

```
                                                                FREQ  CUM.  PERCENT
Year                                                                  FREQ
       |
1975   |***************************                               53    53    53.00
       |
1976   |*********************                                     42    95    42.00
       |
1977   |**                                                         3    98     3.00
       |
1978   |*                                                          1    99     1.00
       |
1980   |*                                                          1   100     1.00
       |
       ----+----+----+----+----+----+----+----+----+----+
          10   20   30   40   50   60   70   80   90  100
                            Frequency
```

Panel E. 1975 Forecast Origin.

```
                                                                FREQ  CUM.  PERCENT
Year                                                                  FREQ
       |
1976   |*********                                                 18    18    18.00
       |
1977   |**************                                            27    45    27.00
       |
1978   |**************                                            27    72    27.00
       |
1979   |******                                                    12    84    12.00
       |
1980   |****                                                       8    92     8.00
       |
1981   |***                                                        5    97     5.00
       |
1982   |**                                                         3   100     3.00
       |
       ----+----+----+----+----+----+----+----+----+----+
          10   20   30   40   50   60   70   80   90  100
                            Frequency
```

Panel F. 1976 Forecast Origin.

Figure 2. (Continued)

```
                                                          FREQ   CUM.   PERCENT
Year                                                             FREQ
        |
1977    |**********************                             43     43     43.00
        |
1978    |*************                                      25     68     25.00
        |
1979    |*******                                            14     82     14.00
        |
1980    |****                                                7     89      7.00
        |
1981    |**                                                  4     93      4.00
        |
1982    |**                                                  4     97      4.00
        |
1983    |*                                                   1     98      1.00
        |
1984    |*                                                   2    100      2.00
        |
        ----+----+----+----+----+----+----+----+----+----+
           10   20   30   40   50   60   70   80   90  100
                             Frequency
```

Panel G. 1977 Forecast Origin.

```
                                                          FREQ   CUM.   PERCENT
Year                                                             FREQ
        |
1978    |********************                               40     40     40.00
        |
1979    |*********                                          17     57     17.00
        |
1980    |********                                           16     73     16.00
        |
1981    |*****                                              10     83     10.00
        |
1982    |*****                                              10     93     10.00
        |
1983    |*                                                   1     94      1.00
        |
1984    |**                                                  3     97      3.00
        |
1985    |*                                                   2     99      2.00
        |
1991    |*                                                   1    100      1.00
        |
        ----+----+----+----+----+----+----+----+----+----+
           10   20   30   40   50   60   70   80   90  100
                             Frequency
```

Panel H. 1978 Forecast Origin.

Figure 2. (Continued)

```
Year                                                                  FREQ   CUM.   PERCENT
                                                                             FREQ
       |
1979   |*******************                                             38     38     38.00
       |
1980   |*****************                                               34     72     34.00
       |
1981   |********                                                        16     88     16.00
       |
1982   |****                                                             7     95      7.00
       |
1983   |*                                                                2     97      2.00
       |
1984   |*                                                                2     99      2.00
       |
1985   |*                                                                1    100      1.00
       |
       ----+----+----+----+----+----+----+----+----+----+
          10   20   30   40   50   60   70   80   90  100
                          Frequency
```

Panel I. 1979 Forecast Origin.

```
Year                                                                  FREQ   CUM.   PERCENT
                                                                             FREQ
       |
1980   |*********************************                               66     66     66.00
       |
1981   |********                                                        15     81     15.00
       |
1982   |****                                                             8     89      8.00
       |
1983   |***                                                              5     94      5.00
       |
1984   |**                                                               4     98      4.00
       |
1985   |*                                                                1     99      1.00
       |
1987   |*                                                                1    100      1.00
       |
       ----+----+----+----+----+----+----+----+----+----+
          10   20   30   40   50   60   70   80   90  100
                          Frequency
```

Panel J. 1980 Forecast Origin.

Figure 2. (Continued)

Table 1. Summary of Turning Point Forecasts.

Forecast Origin	Nearest Actual Turning Point	Forecast of Nearest Turning Point			
			Wecker's Technique		
		ARIMA	Mean[a]	Mode	Median
---- year ----					
1971	1971	1972	1972	1971,1972	1972
1972	1973	1973	1973	1973	1973
1973	1973	1973	1973	1973	1973
1974	1974	∞[b]	1976	1975	1975
1975	1975	1976	1976	1975	1975
1976	1980	1978	1978	1977,1978	1978
1977	1980	∞[b]	1978	1977	1978
1978	1980	∞[b]	1980	1978	1979
1979	1980	1980	1980	1979	1980
1980	1980	1980	1981	1980	1980

a. Mean values are rounded to the nearest year.

b. ARIMA forecasts indicate a future free of turning points.

An alternative means of evaluating forecasting accuracy is to measure the incidence of errors of Type I (a turning point incorrectly predicted) and of Type II (a turning point not predicted when one actually occurs) (Bourke). Using the classification provided in Table 2, $f_1 = b/(a+b)$ and $f_2 = c/(c+d)$ measure the incidence of Type I and II errors, respectively.

Table 2. Types of Turning Point Errors.

	Prediction	
Actual	Turn	No Turn
Turn	a	c
No Turn	b	d

These measures were calculated in order to determine the relative accuracies of the conventional ARIMA model and Wecker's technique in forecasting whether the forecasts originating from 1971 to 1980 represented turning points. The results were as follows:

	f_1	f_2
ARIMA model	0.0	0.38
Wecker's technique:		
Mean	0.0	0.5
Mode	0.5	0.5
Median	0.0	0.29

Note that relative to the conventional ARIMA model, the median of the empirical distribution of turning point indicators provided marginally superior results, while the mean and mode of these distributions yielded inferior results.

On the whole, the median of the empirical distributions of indicator variables from Wecker's technique offered only a slight improvement in turning point forecasting accuracy over conventional ARIMA forecasts. The mean and mode of these distributions yielded no improvement in accuracy relative to ARIMA forecasts.

Summary and Conclusions

The objective of this paper was to evaluate a technique for predicting turning points with ARIMA models proposed by Wecker. This technique involves generation of estimated predictive distributions of turning points of the series to be forecasted. Application of this technique to annual U.S. commercial hog slaughter data revealed that use of the median of the estimated distribution of turning points provided only a slight improvement in forecasting turning points relative to conventional ARIMA forecasts.

Although Monte Carlo studies and applications of this technique to other time series would be needed to draw firm conclusions, the results presented here suggest that a solution to the turning point prediction problem with ARIMA models remains elusive.

Footnotes

1. Wecker uses an unusual definition of GNP turning points.

2. See Long for a discussion of alternative means of identifying cyclic turning points in macroeconomic time series.

3. The lag operator is defined so that $L^j \, HMKT_t = HMKT_{t-j}$. The difference operator is defined so that

$$\Delta HMKT_t = HMKT_t - HMKT_{t-1}.$$

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