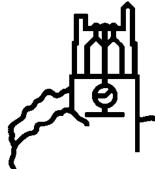# *Staff Paper*

Convergence of the G-7:
A Cointegration Approach

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CONVERGENCE OF THE G-7:
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Abstract

Income convergence among the G-7 countries was demonstrated using Theil's inequality (entropy) index. G-7 convergence was also found for three potential factors of influence on economic growth: government expenditure, investment expenditure, and industrial employment. Pairwise cointegration tests indicated that income inequality was cointegrated with the other three inequality measures for the time period of 1950-88. Finally, Johansen's I(2) multi-cointegration tests indicated that three of the four inequality measures (i.e. income, investment expenditure, and industrial employment) were cointegrated suggesting that there exists a long-run equilibrium between the inequality in income, investment expenditure, and industrial employment.

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Introduction

Whether countries are becoming more similar (convergence) in terms of per capita income and the identification of the factors that contribute to income convergence are important issues in the economic growth literature. One way for convergence to occur is for relatively poor countries to grow faster than relatively rich ones (Barro 1991). The empirical literature has taken two main approaches in studying convergence issues: the construction of inequality measures (e.g., Wright 1978; Ahlualia et al. 1979; Ram 1988 and 1989; Theil 1989; Berry et al. 1991; and Gao et al. 1992), and regression analysis (e.g., Branco and Williamson 1988; Ram 1988; Grier and Tullock 1989; Barro 1991; Barro and Sali-i-Martin 1992; and Baradaran-Shoraka 1992). The evidence supports the idea that high-income countries are converging; however, the reasons why are less clear (Grier and Tullock 1989 and Goa et al. 1992). Unlike these former studies which have focused on the short-run or on static models, this study focuses on the long-run by determining a method of measuring convergence, testing convergence on a group of countries, and determining the long-run relationships among selected macroeconomic variables.

Specifically, two questions are posed. The first is whether the G-7 are converging in terms of income, government expenditure, investment expenditure, and industrial employment.[1] Theil's inequality measure is used to answer this question. The second is whether the inequality of income has a long-run relationship with the three other inequalities. This question is explained using both pairwise cointegration analysis and Johansen's multiple cointegration technique.

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[1]The G-7 countries are Canada, W. Germany, Italy, Japan, the U.K., the U.S., and France.

DATA

This study focuses on the G-7 countries because of their increased importance in recent years, and the fact that the data for these countries are readily available and relatively accurate. The data used in this study are from two sources: Summers and Heston 1991 data set, and the OECD (1963, 1969, 1989, 1991a, and 1991b). The Summers and Heston data were constructed based on purchasing power parity.[2] The variables in this study from the Summers and Heston data were: income per capita, government expenditure per capita, investment expenditure per capita, and population.[3] The data that came from the OECD were used for the industrial employment variable. The criteria for choosing these variables are based on macroeconomic relationships (Romer 1986; Lucas 1989; De Long 1992; Grossman and Helpman 1991), results from empirical investigations (Wright 1978; Adams 1990; De Long and Summers 1991; Zind 1991; Barro 1991; Barro and Sala-i-Martin 1992; Glomm and Ravkumar 1992) and the availability of data.[4]

Measuring Convergence

To derive a consistent index of inequality over time, Theil's inequality index was chosen as several other international comparison studies have elected to do (e.g., Ahlualia et al. 1979; Ram 1989; Theil 1989; Berry et al. 1991; and Gao et al. 1992). A major strength of Theil's inequality index, is that it

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[2]Purchasing power parity (PPP) is the number of currency units required to buy goods equivalent to what can be bought with one unit of the currency of the base country (Kravis et al. 1982, p. 383). Cross-country comparisons based on PPPs are generally thought to be superior to those based on official exchange rates (Kravis et al., 1978).

[3]Summers and Heston defined government expenditure as public consumption and investment expenditure as private and public expenditure.

[4]We had hoped to include a variable measuring human capital investment but the necessary data were not available.

meets all four criteria for an inequality index yields a consistent index, and is additively decomposable.[5,6] The derivation of Theil's inequality index can be found in Appendix A.

Inspection of the inequality measures reported in columns 2, 6, 10, and 14 of Table 1 clearly indicates that total inequality for all four variables has decreased considerably.[7] Income Inequality (column 2) declined from 0.22 in 1950 to 0.02 in 1988, or more than 90% while the inequality of government expenditure decreased from 0.14 to 0.05, a 65% decrease (column 6) during the same time period. Total investment inequality (column 10) decreased by 96% (0.26 to 0.01) while the inequality in industrial employment (column 14) decreased by 54% (0.02 to 0.01). Thus the G-7 countries became much more affluent on average and more similar. These countries are converging in terms of income, government expenditure, investment expenditure, and industrial employment.

The decomposability of Theil's index can be used to determine whether the driving force behind the strong tendency toward convergence among the G-7 is due to changes in regional inequality or changes in within-region inequality. Accordingly, the seven countries were grouped into two regions, R_1 for North America (USA and Canada), and R_2 for the Other 5 (Japan, the U.K., W. Germany, France, and Italy). The regional inequalities, J_g, are displayed in columns 3, 7, 11, and 15 of Table 1.

The Other 5 succeeded in substantially narrowing the income gap between themselves and North America (86%, column 3). The macroeconomic variable that may have contributed the most to this

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[5]The four requirements for an index are symmetry, mean-independence, population homogeneity, and the Pigou-Dalton condition (Bourguignon 1979; Osberg 1991).

[6]Bourguignon (1979) defines additive decomposability as a measure that the total inequality of a population can be broken down into a weighted average of the inequality existing within subgroups of the population and the inequality existing among them.

[7]The inequality measure has a lower bound of 0 but no upper bound. Zero represents the case where no inequality exists.

change was investment expenditure with inequality between the two regions declining 99% (column 11).

Over the same time period the regional inequality in government expenditure declined 56% (column 7),

and contradictorily the inequality in industrial employment increased 94% from 1950 to 1988

(column 15).

Dividing J by J_R yields the percentage of total inequality due to regional inequality. In the 1950's,

regional income inequality accounted for 67% of total income inequality and increased to about 76% in

the 1960's, 90% in the 1970's, and up to 96% in the 1980's. Hence, the reduction in total income

inequality was primarily due to the reduction in regional income inequality (between the two regions).

Concerning the other inequality indexes, governmental regional inequality accounts for 50% to

55% of total governmental inequality on average. The majority of the reduction in total investment

inequality was due to the average within-region inequality which accounts for the following percentages

by decade: 64% in the 1950's, 76% in the 1960's, 91% in the 1970's, and 78% in the 1980's. Lastly,

industrial employment was equally influenced by regional and average within-region inequalities.

It was determined by Gao et al. (1992) that Japan was mainly responsible for the reduction in total

income inequality. By analyzing the within-region inequality and the income per capita data, it can be

stated that Germany and Italy also grew at a faster rate than the other countries which also encouraged

convergence of the G-7 (columns 4 and 5).[8]

In terms of government inequality it was determined that government inequality was equally

divided between regional and within-region influences. Column 8 contains the North American

government within-region inequality and column 9 contains that of the Other 5. The inequality between

Canada and the U.S. has decreased 86% since 1950. The reason for this large decrease was due to

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[8]Within-region inequality, J_g, calculates the inequality among the countries within a given region.

Canada's increased government expenditure per capita (three times their 1950 value by 1988, while the U.S. increased only 1.5 times). The Other 5 countries reduced their within-region inequality by 30% from 1950 to 1988. The rates of increase in government expenditure for the Other 5 were: Japan, 3.3; W. Germany, 3.4; France, 2.3; Italy, 3.6; and the U.K., 1.7. Therefore, the convergence in terms of government expenditure appears to be due to the slow growth of the U.S. and the U.K. and the faster growth of the other countries.

Column 11 shows that regional investment inequality decreased by 99%. The North American within-region investment inequality was insignificant (Table 1, Column 12). Therefore, the reduction in total inequality must have been due to the within-region inequality of the Other 5 (Table 1, column 13). The Other 5 within-region inequality reduced 92% from 1950 to 1988. To determine which countries were responsible for this decrease consider Table 2 which illustrates each countries initial (1950) and final (1988) per capita investment expenditure and their rate of increase in investment.[9]

The initial value for Japan was significantly lower than the other G-7 countries. However, Japan's increase in the rate of expenditure on investment per capita (21 times the initial value) has boosted them to being one of the top countries in terms of investment expenditure per capita. Hence, Japan's increase in investment expenditure was part of the reason for the large decrease in the total inequality of investment expenditure. The relatively fast rate of increase in investment in the U.K. also helped influence convergence. In addition, the relatively slow rate of increase in investment in the U.S. and Canada allowed the Other 5, all of which had a faster rate of increase, to catch up.

The inequality in industrial employment was the only complex category. As stated before, the inequality between the two regions in industrial employment increased 94% while total inequality

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[9]The rate of increase in investment was defined as dividing the final expenditure by the initial expenditure.

decreased by 54%. It was also determined that industrial employment was equally influenced by regional and average within-region inequalities. The within-region inequalities were insignificant for North America and small for the Other 5 (columns 16 and 17). Although the within-region inequality was small for the Other 5, it was the largest inequality for the industrial employment category. It is apparent that the regional inequality and the average within inequality are going in opposite directions. In the 1950's regional inequality accounted for 18% of total inequality, 57% in the 1960's, 68% in the 1970's, and 46% in the 1980's; regional inequality became more important up until the 1970's then declining in the 1980's. Thus, it seems that the convergence of industrial employment was due to the reduction in inequality in the Other 5. The industrial employment rates of increase for each country are: Canada, 1.7; U.S., 1.5; Japan, 2.5; U.K., .77; W. Germany, 1.2; France, 1.1; and Italy, 1.3. Once again, Japan was largely responsible for the decrease in the inequality in industrial employment.

In summary, convergence was supported for all four variables and generally occurred at a faster rate in the income and investment variables. However, there was little inequality among the countries in terms of industrial employment. One of the strongest driving forces behind the decrease in all of the inequalities was Japan's behavior.

Cointegration

Several studies have combined inequality measures with multiple regression techniques (Braun 1988; Ram 1989b; Ram 1992; McGillivray 1991; and Amos 1991). This study, however, attempts to establish the co-movement of these inequality indices over time in order to analyze the long-run equilibrium relationships between income inequality and factors that influence growth. To accomplish this, cointegration was used in this analysis to determine the long-run relationships among the four inequality indices for the G-7.

There are three basic differences between standard regression analysis and cointegration analysis. First, regression analysis establishes a linear or nonlinear combination of the dependent variable and independent variables that must equal white noise.[10] Cointegration analysis only requires that slow or trending movements in the dependent variable equal linear combinations of similar movements in the independent variable. The cointegrating relationship does not have to be purely random, it can be a stationary process. Second, there is no need to designate a variable as exogenous. If the two series are found to be cointegrated, then the relationship is symmetric (i.e. if y and x are cointegrated, then x and y are cointegrated)(Engle and Yoo, 1991). Lastly, cointegration analysis can be used to determine the long-run trends in data series while regular regression analysis produces spurious results if it is used on variables that have trends in them (Maddala, 1992).

Testing for cointegration involves analyzing the residuals from a cointegrating regression for stationarity. If the cointegrating equation is stationary, then the variables are cointegrated (Maddala, 1992; and Moss, 1992).[11]

A necessary prerequisite of cointegration analysis is that the variables under consideration be integrated of the same order. A time-series variable is integrated of order d if the dth difference of x_t is stationary and is denoted I(d). Maddala (1992) suggests examining graphs as well as using unit root tests to determine if a time series is stationary. The graphs of the inequality of income, government, investment, and industrial employment for the G-7 countries were examined and confirmed to be I(2)(not shown).

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[10]White noise in time series is a sequence of uncorrelated random variables with zero mean and identical finite variances (Judge et al., 1980).

[11]Stationarity refers to a time series having a constant mean and a bounded variance over time.

Unit root tests can also be used to determine the order of integration of a time-series. The two unit root tests used in this study were the augmented Dickey-Fuller (ADF) and the Phillips tests (Fuller 1976; Evans and Savin 1981; Engle and Granger 1991; Phillips 1987; and Maddala 1992). The results, presented in Table 3, indicate that the inequalities in income, investment expenditure, industrial employment, and government expenditure with a 99% confidence level are I(2) which supports the interpretation of the graphs.[12]

Given that all of the variables were I(2), pairwise cointegration tests were conducted. Engle and Granger (1987) state that two I(d) variables, x_t and y_t, are cointegrated of order (d,b) if there exist a constant $B \neq 0$ such that $u_t = y_t - \alpha - Bx_t$ is integrated of order (d - b), b > 0. If these restrictions are satisfied then x_t and y_t are cointegrated which is written as CI(d,b). In this example, α is a constant, and u_t is the residual vector.

The two pairwise cointegration tests used were the Durbin Watson (DW) test and the Augmented Dickey-Fuller (CADF) test (Maddala, 1992). These tests were used on the following sets of inequality measures: income - investment, income - government expenditures, and income - industrial employment.

The results from the two types of pairwise tests for the G-7 were somewhat inconsistent. Table 4 shows that the Durbin Watson test statistic was significantly different from 0 at the 99% confidence level for income - investment expenditure, and 90% for income - government expenditure. However, the income - industrial employment relationship was not significantly different from zero. These results do not reject cointegration for income - investment, possibly for income - government, but do so for income - industrial employment.

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[12]Increasing the number of lags (autocovariance terms for Phillips tests) in the model had no effect on the significance level for the I(2) series.

The CADF tests did not reject cointegration for any of the three pairs. The income - government regression did not reject cointegration at the 99% confidence level, while the income - industrial employment regression did not reject cointegration at the 95% level. The income - investment regression did not reject cointegration at the 90% level.

Conflicting results from different time-series tests are fairly common (Maddala, 1992). The distribution of the Durbin Watson has not yet been fully investigated. The general rule is that the smaller the statistic, the greater the chance that the null hypothesis of non-cointegration is not rejected. Engle and Yoo (1987) conclude that the Durbin Watson is not useful for testing cointegration. In general, the test results indicated that income inequality was cointegrated with the investment expenditure, government expenditure, and perhaps industrial inequality. This suggests that there exists a long-run equilibrium among income inequality and the inequality in government expenditure, investment expenditure, and industrial employment.

Multiple Cointegration

The results from the pairwise cointegration tests were somewhat inconclusive. Those results only suggest that certain pairs of the inequalities appeared to be cointegrated. As a final step, this study analyzed whether multiple cointegration exists among the four variables. Due to the complication of having four I(2) variables, Johansen's I(2) procedure was chosen because of its maximum likelihood properties (Johansen 1992a and b). An overview of the procedure is presented in Appendix B.

The first step of the Johansen's (Johansen, 1992a and b) test is to solve an eigenvalue problem (eq. B.12). The solution to this problem provides eigenvalues and their associated eigenvectors which are presented in Table 5. Let p be the number of variables and r the number of significant eigenvalues. The value of r can be determined by reading from top to bottom the Q_r (column 4 of Table 6) and

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comparing the observed values with the 95% critical value (C_{p-r}) for p-r degrees of freedom found in column 5 of Table 6. Conditional on r, the value of s, the number of common I(1) trends, can be chosen by reading the row associated with the selected r value in the $Q_{r,s}$ rows and comparing the observed values with the critical values at the bottom of the table (C_{p-r-s}).

The trace statistic Q_r clearly rejected r = 0, since the test statistic was 80.87 and the 95% calculated quantile was 49.09. The hypothesis H_1 of r ≤ 1 was also rejected with the statistic being 44.24 and the quantile being 31.62. The hypothesis H_2 of r ≤ 2 was a borderline case and could not be rejected since the statistic 16.57 corresponded approximately to the 95% quantile (17.65) in the asymptotic distribution. Based on r = 2, the two estimated cointegrating vectors (B) were given by the first two columns of the eigenvectors in Table 5.

To determine the value of s, the row equal to r = 2 in Table 6 was read. The hypothesis $H_{2,0}$ that r = 2 and s = 0 was rejected based on the test statistic 37.00 and the quantile of 17.65. The next test $H_{2,1}$ that r = 2 and s ≤ 1 cannot be rejected. This was determined by comparing $Q_{2,1}$ = 2.3 with the quantile of 8.11. Therefore, the number of common I(2) trends in the data series was p-r-s = 1 and the number of common I(1) trends was s = 1.

There was one common I(2) trend that drives all of the variables. The vector $B'X_t$ in this case is just one linear combination, and it is I(1) (not stationary). However, this representation was made stationary by including the differences, that is $B'X_t + \kappa B_\perp^{2'}\Delta X_t$ (Johansen, 1991a and b), where the κ coefficient is equal to $(\alpha'\alpha)^{-1}\alpha'\Gamma B_\perp^2(B_\perp^{2'}B_\perp^2)^{-1} = \alpha_\perp^{1'}\Gamma B_\perp^2(B_\perp^{2'}B_\perp^2)^{-1}$,

$\alpha_\perp^1 = \alpha(\alpha'\alpha)^{-1}$, and $B_\perp^2 = B_\perp \eta_\perp$. The two normalized stationary relationships were

(1)   INC - .12 GOV$_t$ -.59 INV$_t$ + 2.29 IND$_t$ + 144.93 △INC

          + 78.33 △GOV$_t$ + 202.67 △INV$_t$ - 7.31 △IND$_t$

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and

(2)   INC - .13 GOV$_t$ -.71 INV$_t$ - 1.13 IND$_t$ - 70.17 $\triangle$INC

         - 37.93 $\triangle$GOV$_t$ - 98.13 $\triangle$INV$_t$ + 3.54 $\triangle$IND$_t$

The B vectors used to determine (1) and (2) are the first two columns of the eigenvector in

Table 5, with the exception of being normalized by the income coefficient. These two equations

represent the long-run equilibrium among the four inequality indices for the G-7. Given that there were

two stationary relationships, the equilibrium can be thought of as a plane instead of a line in hyperspace.[13]

The next step was to use all the information from this estimation to determine which variable or

variables were determining this equilibrium. The two cointegrating vectors normalized by their respective

income coefficient have approximately the following relationships (1,*,-1,*) and (1,*,-1,-1). The

interpretation of the first vector was that income inequality and the inequality in investment were

stationary and the other two variables do not affect the relationship. The interpretation of the second

vector was that the inequality in income, investment, and industrial employment form a stationary long-

run relationship. The inequality in government expenditure has no effect on this equilibrium although it is

in equilibrium with the other inequalities.

Determining the number of significant s's identifies how many common I(1) processes there are in

the model. It was found that there was one common I(1) process. In addition, there was only one

common I(2) trend that drives all of the variables. To determine the common I(1) trend is difficult.

Therefore, the determination of the common I(2) trend is addressed.

The coefficient $B_{\perp}^2$ in Table 7 shows which variables are actually I(2). The variable that has a

coefficient closest to one or negative one is the common I(2) trend. $\alpha_{\perp}^2$ represents the average speed of

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[13]Hyperspace in this case refers to a four dimensional space with two stationary relationships forming an
equilibrium within this space. Since there are two relationships, the equilibrium is a plane.

adjustment towards the estimated equilibrium and is interpreted as the linear combination that describes the common I(2) trend. A small coefficient indicates a slow adjustment and a large coefficient represents a fast adjustment.

The $B_{\perp}^2$ vector in Table 7 indicates that the inequality in industrial employment is the I(2) variable. The $\alpha_{\perp}^2$ vector indicates that the heaviest weights are given to industrial employment and income. Hence, the I(2)-ness of the model is ascribed to the inequality of industrial employment. This means that when an innovation occurs causing the inequalities to be out of equilibrium, the inequality in industrial employment adjusts the most and the quickest to restore the equilibrium.[14]

In summary, the stationary equilibrium was dependent on two stationary relationships. The first stationary relationship for the G-7 was described as the inequality in income and investment expenditure. The second relationship was the inequality in income, investment expenditure, and industrial employment. These two stationary relationships form a long-run equilibrium and can be described as a plane in four dimensional space which acts as an attractor every time the four inequities deviate from this equilibrium. It was also determined that the inequality in government expenditure had no effect on the equilibrium among the four inequalities. The inequality in industrial employment was determined to be the common I(2) trend. Whenever an innovation occurs in one of the inequalities and a deviation from the long-run equilibrium exists, industrial employment adjusts first to return the economy to the long-run equilibrium.

Summary and Conclusion

Data of the G-7 countries for four variables, (income, government expenditure, investment expenditure, and industrial employment) over the period of 1950 - 1988 were used to construct Theil's

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[14]That is, when a shock to one of the inequalities occurs which forces the inequalities out of equilibrium, the main force to restore the equilibrium comes from the inequality in industrial employment.

inequality indices. It was determined that the inequality in all four variables for the G-7 countries has declined over this period. This suggests that the G-7 countries are becoming more equal in terms of income, government expenditure, investment expenditure, and industrial employment and that convergence is occurring.

These four inequality indices were then tested for long-run relationships using cointegration analysis. Pairwise cointegration tests suggested that income inequality was cointegrated with the other three inequalities. Results of Johansen's multiple cointegration I(2) test supported the hypothesis that a long-run equilibrium existed among the inequalities of income, investment expenditure, and industrial employment for the G-7. The inequality of government expenditure was in equilibrium, but had no effect on the equilibrium. Industrial employment appeared to be the driving force in returning the G-7 economies to their long-run equilibrium.

These results support the idea that the G-7 are converging and that there are key factors in the economies that influence convergence. This study specifically illustrates the importance of investment expenditure, and industrial employment for economic growth. However, government expenditure does not appear to be as important in the process of making these particular economies more equal.

The implications of these results are potentially important when considering the economic growth in middle to high income countries. If these countries mimic the G-7 by devoting human resources to industrial employment in terms of percentage of population and approach the G-7 countries per capita expenditure on investment, they may begin to converge with the G-7 countries in terms of income per capita. The problem may be that the middle and some high-income countries rely too heavily on government expenditure to improve their economic growth. These interesting issues should be pursued as data become available or when econometric methods are better able to handle small samples.

Appendix A: Theil's Inequality Index

Theil's income inequality index (Theil 1979, and 1989) when applied to n countries can be written as

$$\text{(A.1)} \qquad J = \sum_{i=1}^{n} p_i \log(p_i/y_i),$$

where p_i is the population share of country i and y_i is its income share relative to total population and total income respectively.[15]

This measure can be decomposed additively to measure inter   and within inequality. Let $R_1,...,R_g$ represent regions such that each country is in only one region. Let P_g and Y_g be the population and income shares of region R_g:$P_g=\sum_i p_i$ and $Y_g=\sum_i y_i$, where the summations are over $i\in R_g$ (g=1,...,G). Then the extension of (A.1) to regions is

$$\text{(A.2)} \qquad J_R = \sum_{g=1}^{G} P_g \log(P_g/Y_g),$$

which measures inter-regional inequality among G regions, while

$$\text{(A.3)} \qquad J_g = \sum_{i\in R_g} (p_i/P_g) \log[(p_i/P_g)/(y_i/Y_g)]$$

measures the inequality among the countries of region R_g. It is then easily verified that

$$\text{(A.4)} \qquad J = J_R + J^* \quad \text{where} \quad J^* = \sum_{g=1}^{n} P_g J_g,$$

which is an additive decomposition expressing total inequality J among the n countries as the sum of regional inequality J_R, and the average within-region inequality J^*. This average is a weighted average with weights proportional to the populations.

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[15]All logarithms in this paper are natural logarithms.

Appendix B

Johansen's cointegration test is a general vector autoregressive (VAR) model with p variables and k lags. The time series are collected in a vector such as $X_t' = [x_{1t}, x_{2t}, ..., x_{pt}]$ where all the series are assumed to be integrated of the same order.

Johansen (1988) and Johansen and Juselius (1990) begin by defining a general polynomial distributed lag model for X_t as

$$(\text{B.1}) \qquad X_t = \mu + \sum_{i=1}^{k} \Pi_i X_{t-i} + \epsilon_t \qquad t=1,...,T$$

where μ is a constant and ϵ_t is an independently identically distributed p dimensional vector with zero mean and variance matrix Λ.[16] Given this framework the cointegrating matrix is

$$(\text{B.2}) \qquad I - \Pi_1 - \Pi_2 ... - \Pi_k = \Pi.$$

The π matrix is therefore a p x p matrix. The number of cointegrating relationships among the variables in X are r, where r is the rank of π.

The estimation of (B.1) when the variables are I(1) has been well documented (Johansen, 1988; Johansen and Juselius, 1990; and Johansen, 1991d). The development of the I(2) processes, tests, and interpretations are fairly new. For the derivation of the I(2) process the following documentation should be consulted: Johansen, 1990; Johansen, 1991a; Johansen, 1991b; Johansen, 1992b; and Weatherspoon, 1993.

To implement, (B.1) must be reparameterized into an error correction model

$$(\text{B.3}) \qquad \Delta^2 X_t = \sum_{i=1}^{k-2} \Gamma_i \Delta X_{t-i} + \Gamma \Delta X_{t-1} + \Pi \Delta^2 X_{t-1} + \epsilon_t.$$

The parameter restrictions for the I(2) model are

$$(\text{B.4}) \qquad \Pi = \alpha B'$$

$$(\text{B.5}) \qquad \alpha_{\perp}^{1}{}' \Gamma B_{\perp}^{1} = \phi \eta'$$

where α and B are p x r matrices of rank r, and ϕ and η are (p-r) x s matrices. $S = 0,1,...,p-r$, and $\alpha_{\perp}$ and $B_{\perp}$ are orthogonal to α and B. The general definition of 1 as a superscript is that $\alpha^{1}{}' = (\alpha'\alpha)^{-1}\alpha$, $\alpha'\alpha^1 = I$, and $\alpha'\alpha_{\perp} = 0$.

The first step in solving this system as an eigenvalue problem is to construct (B.3) in a maximum likelihood format. The parameters $\Gamma_1,...,\Gamma_{k-1}$ can then be eliminated by partially maximizing this function with respect to Γ_i. The result is

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[16]In Johansen's 1988 paper, μ is assumed to equal zero. The same assumption is made here.

$$(B.6) \quad \Gamma_i = \sum_{t=1}^{T} \sum_{i=1}^{k-2} [(\Delta^2 X_{t-i}' \Delta^2 X_t)(\Delta^2 X_{t-i}' \Delta^2 X_{t-i})^{-1}$$

$$- (\Delta^2 X_{t-i}' \Delta^2 X_{t-i})^{-1}(\Delta^2 X_{t-i}' \Delta X_{t-1}\Gamma)$$

$$- (\Delta^2 X_{t-i}' \Delta^2 X_{t-i})^{-1}(\Delta^2 X_{t-i}' X_{t-2}\Pi)].$$

Substituting $\alpha B'$ in for Π makes (B.6) the same as regressing $\Delta^2 X_t$, $\Delta X_{t-1}\Gamma$, and $\alpha B'X_{t-2}$ on $\Delta^2 X_{t-1},...,\Delta^2 X_{t-k+2}$. This yields the residuals R_{0t}, R_{1t}, and R_{2t}, and the residual product moment matrices $S_{ij} = T^{-1} \sum R_{it}R_{jt}'$. Johansen (1992b) suggests manipulating the following regression equation

$$(B.7) \quad R_{ot} = \Gamma R_{1t} + \alpha B'R_{2t} + e_t.$$

The next step is to determine the values for r, α, and B from the I(1) model (Γ unrestricted in (B.7)). Solve the I(1) portion of the model with the additional parameter Γ (Johansen, 1992b). Once the parameters and value of r are found, assume that they are fixed and multiply (B.7) by $\alpha^{1'}$ and $\alpha_{\perp}^{1}$ to get

$$(B.8) \quad \alpha^1 {}'R_{ot} = \alpha^1 {}'\Gamma R_{1t} + B'R_{2t} + \alpha^1 {}'e_t, \text{ and}$$

$$(B.9) \quad \alpha_{\perp}^1 {}'R_{ot} = \alpha_{\perp}^1 {}'\Gamma R_{1t} + \alpha_{\perp}^1 {}'e_t.$$

The I(2) model eigenvalue problem is based on solving (B.9). Define $I = B^1 B' + B_{\perp}^1 B_{\perp}'$ and substitute this definition into (B.9) to get

$$(B.10) \quad \alpha_{\perp}^1 {}'R_{ot} = \alpha_{\perp}^1 {}'\Gamma(B^1 B' + B_{\perp}^1 B_{\perp}')R_{1t} + \alpha_{\perp}^1 {}'e_t.$$

This equation can be rewritten as

$$(B.11) \quad \alpha_{\perp}^1 {}'R_{ot} = \alpha_{\perp}^1 {}'\Gamma B^1(B'R_{1t}) + \phi\eta{}'(B_{\perp}{}'R_{1t}) + \alpha_{\perp}^1 {}'e_t.$$

The parameter $\alpha_{\perp}^1 \Gamma B^1$ can be eliminated via maximizing with respect to that parameter. The actual eigenvalue problem is determined by solving for Λ. This is accomplished by maximizing with respect to Λ and substituting in the result from the maximization of $\alpha_{\perp}^1 \Gamma B^1$ where needed. The resulting eigenvalue problem is

$$(B.12) \quad \Lambda = |\rho B_{\perp}'(S_{11} - S_{11}B(B'S_{11}B)^{-1}B'S_{11})B_{\perp}$$

$$- [B_{\perp}'(S_{10} - S_{11}B(B'S_{11}B)^{-1}B'S_{10})\alpha_{\perp}^1]$$

$$[\alpha_{\perp}^1 {}'(S_{00} - S_{10}B(B'S_{11}B)^{-1}B'S_{10})\alpha_{\perp}^1]$$

$$[B_{\perp}'(S_{10} - S_{11}B(B'S_{11}B)^{-1}B'S_{10})\alpha_{\perp}^1]| = 0.$$

The solution to (B.12) gives eigenvalues $\rho_1 > ... > \rho_{p-r} > 0$ and eigenvectors $W = (w_1,...,w_{p-r})$ normalized by $W'[B_{\perp}'(S_{11} - S_{11}B(B'S_{11}B)^{-1}B'S_{11})B_{\perp}]W = I$. Note, B and $B_{\perp}$ transform the differences which are I(1)

variables. The $\alpha_\perp^1$ coefficient transforms the second differences which are stationary by assumption. The likelihood ratio test to determine the number of eigenvalues that are significantly different than zero is

$$(\text{B}.13) \qquad -2\ln(Q_{r,s}) = -T \sum_{i=s+1}^{p-r} \ln(1 - \hat{\rho}_i)$$

where s = (0,1,...,p-r-1). The maximum likelihood estimators (for fixed values r, α, and B) are $\eta = (w_1,...,w_s)$, and $\phi = [B_\perp^{'}(S_{10} - S_{11}B(B'S_{11}B)^{-1}B'S_{10})\alpha_\perp^1]\eta$. The variance matrix Λ is equal to (B.12) without the absolute value symbols and ρ.

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Table 1.   Income, Government, Investment and industry Inequality

	Income Inequality				Government Inequality				Investment Inequality				Industry Inequality			
Year (1)	J (2)	J_R (3)	N. Am. J_g (4)	Oth. 5 J_g (5)	J (6)	J_R (7)	N. Am. J_g (8)	Oth. 5 J_g (9)	J (10)	J_R (11)	N. Am. J_g (12)	Oth. 5 J_g (13)	J (14)	J_R (15)	N. Am. J_g (16)	Oth. 5 J_g (17)
1950	0.2182	0.1432	0.0018	0.1195	0.1374	0.0633	0.0100	0.1130	0.2579	0.1602	0.0001	0.1571	0.0246	0.0002	0.0000	0.0392
1951	0.2023	0.1366	0.0023	0.1048	0.2152	0.1261	0.0142	0.1351	0.1675	0.1140	0.0000	0.0865	0.0297	0.0009	0.0002	0.0464
1952	0.1888	0.1310	0.0018	0.0925	0.2254	0.1412	0.0149	0.1272	0.1690	0.0981	0.0008	0.1144	0.0270	0.0012	0.0004	0.0415
1953	0.1811	0.1246	0.0020	0.0906	0.2333	0.1463	0.0166	0.1311	0.1736	0.0941	0.0011	0.1287	0.0281	0.0011	0.0005	0.0436
1954	0.1660	0.1092	0.0024	0.0911	0.2009	0.1200	0.0151	0.1225	0.1452	0.0640	0.0000	0.1324	0.0318	0.0042	0.0004	0.0447
1955	0.1628	0.1076	0.0025	0.0887	0.1893	0.1121	0.0135	0.1177	0.1561	0.0678	0.0000	0.1446	0.0329	0.0045	0.0004	0.0463
1956	0.1520	0.1007	0.0017	0.0831	0.1849	0.1062	0.0134	0.1207	0.1298	0.0545	0.0022	0.1223	0.0285	0.0053	0.0002	0.0380
1957	0.1396	0.0919	0.0018	0.0775	0.1900	0.1105	0.0163	0.1205	0.0918	0.0334	0.0027	0.0946	0.0276	0.0080	0.0001	0.0322
1958	0.1260	0.0819	0.0016	0.0719	0.1799	0.1081	0.0159	0.1084	0.0919	0.0246	0.0016	0.1104	0.0305	0.0132	0.0000	0.0286
1959	0.1209	0.0797	0.0022	0.0670	0.1685	0.0981	0.0173	0.1054	0.0875	0.0274	0.0004	0.0994	0.0278	0.0116	0.0002	0.0268
1960	0.1036	0.0679	0.0024	0.0577	0.1568	0.0888	0.0169	0.1021	0.0553	0.0096	0.0002	0.0760	0.0301	0.0145	0.0004	0.0257
1961	0.0867	0.0586	0.0026	0.0452	0.1527	0.0890	0.0142	0.0969	0.0291	0.0029	0.0000	0.0438	0.0332	0.0198	0.0003	0.0222
1962	0.0840	0.0590	0.0024	0.0403	0.1474	0.0873	0.0147	0.0908	0.0387	0.0062	0.0000	0.0544	0.0308	0.0197	0.0002	0.0186
1963	0.0767	0.0559	0.0022	0.0335	0.1352	0.0810	0.0140	0.0816	0.0302	0.0062	0.0000	0.0403	0.0302	0.0202	0.0002	0.0166
1964	0.0705	0.0530	0.0021	0.0281	0.1297	0.0790	0.0143	0.0755	0.0200	0.0036	0.0001	0.0276	0.0215	0.0127	0.0004	0.0145
1965	0.0735	0.0560	0.0021	0.0279	0.1240	0.0761	0.0137	0.0713	0.0301	0.0101	0.0001	0.0336	0.0204	0.0109	0.0004	0.0158
1966	0.0695	0.0561	0.0021	0.0212	0.1328	0.0873	0.0144	0.0668	0.0247	0.0105	0.0003	0.0237	0.0177	0.0087	0.0004	0.0150
1967	0.0608	0.0515	0.0021	0.0143	0.1386	0.0929	0.0150	0.0668	0.0093	0.0045	0.0000	0.0080	0.0145	0.0082	0.0007	0.0102
1968	0.0534	0.0467	0.0021	0.0100	0.1342	0.0929	0.0136	0.0605	0.0060	0.0015	0.0000	0.0077	0.0147	0.0087	0.0008	0.0095
1969	0.0461	0.0408	0.0017	0.0078	0.1274	0.0882	0.0127	0.0576	0.0075	0.0002	0.0002	0.0123	0.0144	0.0084	0.0008	0.0095
1970	0.0367	0.0324	0.0015	0.0062	0.1149	0.0791	0.0085	0.0548	0.0115	0.0026	0.0001	0.0150	0.0164	0.0109	0.0008	0.0087
1971	0.0364	0.0323	0.0011	0.0061	0.1026	0.0682	0.0066	0.0538	0.0096	0.0003	0.0001	0.0158	0.0184	0.0136	0.0004	0.0079
1972	0.0348	0.0309	0.0019	0.0052	0.0952	0.0610	0.0060	0.0540	0.0116	0.0000	0.0000	0.0198	0.0159	0.0113	0.0004	0.0076
1973	0.0315	0.0284	0.0017	0.0039	0.0854	0.0520	0.0042	0.0538	0.0077	0.0000	0.0000	0.0131	0.0142	0.0093	0.0003	0.0081
1974	0.0296	0.0264	0.0016	0.0044	0.0833	0.0493	0.0034	0.0554	0.0092	0.0000	0.0010	0.0149	0.0132	0.0093	0.0001	0.0066
1975	0.0276	0.0250	0.0006	0.0040	0.0783	0.0438	0.0024	0.0571	0.0131	0.0007	0.0063	0.0168	0.0150	0.0121	0.0000	0.0051
1976	0.0275	0.0249	0.0007	0.0039	0.0697	0.0373	0.0019	0.0540	0.0080	0.0000	0.0029	0.0117	0.0110	0.0082	0.0000	0.0048
1977	0.0278	0.0257	0.0003	0.0034	0.0643	0.0350	0.0013	0.0493	0.0087	0.0012	0.0009	0.0123	0.0095	0.0068	0.0002	0.0045
1978	0.0280	0.0259	0.0007	0.0031	0.0597	0.0312	0.0012	0.0481	0.0107	0.0030	0.0001	0.0132	0.0075	0.0044	0.0004	0.0049
1979	0.0249	0.0231	0.0007	0.0025	0.0540	0.0267	0.0012	0.0462	0.0102	0.0012	0.0013	0.0146	0.0066	0.0033	0.0003	0.0054
1980	0.0211	0.0197	0.0005	0.0020	0.0507	0.0243	0.0010	0.0451	0.0159	0.0000	0.0035	0.0248	0.0074	0.0044	0.0001	0.0051
1981	0.0214	0.0202	0.0003	0.0018	0.0463	0.0222	0.0008	0.0412	0.0215	0.0017	0.0045	0.0311	0.0073	0.0042	0.0000	0.0055
1982	0.0169	0.0159	0.0004	0.0014	0.0456	0.0223	0.0006	0.0400	0.0152	0.0009	0.0030	0.0228	0.0103	0.0070	0.0002	0.0056
1983	0.0183	0.0176	0.0002	0.0010	0.0432	0.0211	0.0005	0.0381	0.0091	0.0002	0.0021	0.0139	0.0107	0.0066	0.0003	0.0070
1984	0.0216	0.0209	0.0005	0.0008	0.0441	0.0230	0.0008	0.0364	0.0141	0.0076	0.0001	0.0113	0.0091	0.0037	0.0005	0.0091
1985	0.0219	0.0213	0.0005	0.0006	0.0500	0.0292	0.0013	0.0356	0.0122	0.0051	0.0010	0.0118	0.0100	0.0036	0.0004	0.0120
1986	0.0215	0.0211	0.0003	0.0005	0.0483	0.0297	0.0016	0.0316	0.0117	0.0037	0.0019	0.0126	0.0101	0.0031	0.0003	0.0122
1987	0.0212	0.0209	0.0003	0.0002	0.0508	0.0305	0.0019	0.0344	0.0113	0.0032	0.0025	0.0122	0.0099	0.0027	0.0002	0.0126
1988	0.0203	0.0199	0.0006	0.0002	0.0480	0.0282	0.0015	0.0338	0.0105	0.0016	0.0037	0.0128	0.0114	0.0031	0.0001	0.0146

N. Am. (Canada, USA); Other 5 (Japan, W. Germany, U.K., France, Italy); J=Total Inequality, J_R =Regional Inequality, and J_g = Within Inequality

Table 2. Initial, Final, and the Growth of Investment Per Capita

Year (1)	Canada (2)	U.S. (3)	Japan (4)	U.K. (5)	W. Germany (6)	France (7)	Italy (8)
1950	1562	1640	185	516	752	845	569
1988	4661	3513	3878	2465	3050	3094	2921
Rate of Increase[*]	3	2	21	4.8	4	3.7	5

[*] Growth is calculated by dividing the final Investment per capita value by the initial value.

Table 3. Unit Root Tests[a]

Tests	Income	Government	Investment	Industry
ADF[b,c]	9.40	4.65	10.01	10.23
Phillips[d]	9.67	10.16	10.30	10.54

[a] Only the second differenced results are reported.
[b] The reported values are for 0 lagged difference terms.
[c] The critical values for the Augmented Dickey-Fuller test and Phillips test are
   3.56, and 2.94 for the 0.01, and 0.05 confidence levels respectively.
[d] The reported values are for 1 autocorrelation term.

Table 4. Pairwise Tests for Cointegration

Tests	Government	Investment	Industry
Durbin-Watson Regression[a]	0.374	0.524	0.263
Augmented Dickey-Fuller Regression[b,c]	7.25	3.14	3.76

[a]The critical values for the Durbin-Watson regression are .511, .386, .322 for the 0.01, 0.05, and 0.10 confidence levels, respectively (Engle and Granger, 1987).
[b]The critical values for the Augmented Dickey-Fuller regression are 4.02, 3.4, and 3.09 for the 0.01, 0.05, and 0.10 confidence levels, respectively.
[c]The reported values are for 0 lags.

Table 5. Estimated Eigenvalues and Eigenvectors from Johansen's Multiple Cointegration Test[a]

Eigenvalue ($\hat{\rho}$):

0.638	0.536	0.280	0.123

Eigenvectors ($\hat{W}$):

55.047	-251.057	107.747	224.610
-6.439	32.678	-12.882	-133.018
-32.331	177.140	-40.991	-106.753
125.960	283.937	-323.738	-17.699

[a]These results were calculated using a RATS program written and provided by Dr. Johansen.

Table 6. Test Statistics from Johansen's Multiple Cointegration Tests[b]

p-r	r	$Q_{r,s}$				Q_r	$C_{p-r}(95\%)$
4	0	103.411 s=0	48.233 s=1	16.545 s=2	3.056 s=3	80.870	49.10
3	1		71.819 s=0	27.182 s=1	1.756 s=2	44.241	31.62
2	2			36.999 s=0	2.303 s=1	16.567	17.65
1	3				29.536 s=0	4.742	8.11

p-r-s	4	3	2	1
C_{p-r-s} (95%)	49.10	31.62	17.65	8.11

[a]These results were calculated using a RATS program written and provided by Dr. Johansen. P = 4 represents the number of variables, r the number of significant eigenvalues, and s the number of common I(1) trends.

Table 7. Cointegrating Adjustment Coefficients from the G-7

$B_\perp^2$	$\alpha_\perp^2$
10.533	-0.011
5.693	0.006
14.729	-0.001
-0.531	0.034