PRIVATE LABELS: A MECHANISM FOR FULFILLING CONSUMER DEMAND FOR HEALTHY FOOD?

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ABSTRACT
As consumer preferences rapidly evolve with respect to health-related attributes of food, many have questioned the performance of the conventional food industry to fulfill new demand. As the structure of the industry has shifted toward conditions where retailers have access to and incentive to respond to rapidly changing information concerning consumer preferences, it is relevant to consider the conditions under which retailers will offer private labeled healthy food products not available from national brand manufacturers. This paper presents a theory of the dynamic, stochastic decision of retailers to offer private label products that are close substitutes to national brands. Within the context of salient features of healthy foods, we examine the role of uncertainty with respect to consumer demand, the relative unit cost of private label vs. national brand products, and the optimal pricing of the private label products.

Keywords: Private labels, store brands, health claims, functional food, real options, product cannibalization, food choice.
1. INTRODUCTION

A widely held view is that a dramatic shift is well underway in the structure of the food system. A key element of this change has been a shift from what Weaver (2008) called push innovation to pull innovation. That is, from a system that emphasizes commodity or standardized food to one that is highly responsive to consumer preferences and offers highly differentiated products. First noticed in the livestock and vegetable sectors, processor-farm contracts signaled a shift in market structure to one where processors could actively specify and procure differentiated products with quality attributes that were perceived to have strategic advantage. Similar contracting between retailers and fruit and vegetable growers for specific quality attributes further signaled a shift from commodity style market procurement to relational transactions managed by retail grocers operating on massive spatial scales. By contracting directly with suppliers, retailers found they could more efficiently procure and manage characteristics of products and transactions that affect their performance. As a result of these shifts in strategy and market structure, retailers have broadly recognized new opportunity to define product specifications, contract production, and label and market these new products. This new private label (PL) strategy constitutes a shift from a manufacturer dominated push system offering national brands (NBs) coordinated by markets operating at grower, assembler, processor, wholesaler, and retailer levels, to a pull system coordinated by bilateral contracts and agreements. This paper considers the implications this new system may have for fulfilling consumer demands for healthy food through private label products or store brands. We use the general term “healthy food” to encompass conventional food with health claims including organic with and without government sanction and functional foods.

Our paper is motivated by the following logic. Two observations appear supportable as premises of this logic. First, the food system changes noted have resulted in a new feasibility for retailers to design and offer private label products (both in and outside of their shopping venues). Second, well known changes in information technology now provide retailers with an information advantage rendering greater and more rapid access to consumer preferences relative to that possible for national brand
manufacturers. Finally, we proceed with the premise that the combination of information advantage, globalization, and evolution of capital markets has shifted bargaining power to retailers that has led to a new feasibility for them to actively manage their product lines with a high level of control that allows them respond to observed changes in consumer demand. Accepting these premises, the question is raised as to under what conditions food retailers may now have incentives to serve as key innovators by identifying emerging consumer preference shifts and developing new products that fulfill related demands. The answer to this question is, of course, conditioned on the existence of competitive imperatives at the retail level. This paper contributes to this question by considering the existence and nature of conditions under which food retailers might strategically lead national brand manufacturers in offering healthy foods. Of specific interest in this paper is the timing of introduction of private labels that offer health claims relative to existing national brands. In this case, we specify PLs as near equivalent products and consider the optimality of retailer leadership in introduction of new product attributes. Within this context, we provide a basis for considering the role PLs might play in offering a substantial opportunity for retailers to innovate to fulfill evolving consumer demand. The paper presents a theory of the introduction of healthy foods under private labels by retailers, examines the conditions under which such strategies are optimal for retailers, and provides a numerical illustration that supports consideration of how changes in underlying conditions might impact the timing of launches of healthy, PL food products.

2. Background

Current trends in consumer demand for healthy food, in the use of private labels by retailers, and of evidence with respect to innovation in products offered as private labels are all well known and have been reviewed extensively. Only a few points deserve reiteration. According to the Private Label Manufacturer’s Association, the market for PL products encompasses at least $88 billion in sales revenue with more than half of U.S. customers purchasing baskets of products composed or 25% or more of private label products (PLMA (2010)). Private label products play an increasingly important role in retailing business, especially in the food industry, where they hold market shares of 16%
in the US and 30% in Europe, and where these brands are gaining ground on once dominant national brands (Groznik and Heese (2009)). Most recent actions by Walmart in dropping Biglow tea for a private label have received limited press within this context, however, serves to further evidence consumer acceptance of PLs as near equivalent substitutes to NBs rather than cheap, lower quality alternatives, see e.g. McKinsey (2007). Within the context of healthy food, causal observation confirms the rapid emergence of PL brands in this realm.

We build our work on several threads of literature. Literature regarding private labels has largely focused on how their introduction affects retailer profits and customer welfare under different conditions. Narasimhan and Wilcox (1998) showed the role of a private label in the channel relationship by comparing strategy with and without a private label. Morton and Zettelmeyer (2004) investigated why retailers value control over private labels. Many papers have assumed NB manufacturers game with retailers and often play dominant roles as leaders. However, we assert that evidence in the retail food sector refutes the relevance of such specifications. With respect to approach, we rely on a microeconomic theory of choice of the timing and price of PLs. Our approach recognizes that the opportunity to control timing is interpretable as a real option, see Dixit and Pindyck (1994), Schwartz and Zozaya-Gorostiza (2003). To our knowledge, past work has not exploited this theory for considering the optimal timing to introduce a private label. Although Moorthy and Png (1992) discussed timing of a product introduction, they did not consider its option value. Weaver and Wesseler (2004) considered the option value of adoption of GM crops for the case where such technology is not universally attractive. Within this context, they argued the option value of the technology’s introduction may be reduced. To the extent that PLs for healthy foods offer attributes that are not universally valued, their theory has implications for the present problem. Our approach offers two contributions. First, we recognize the real option aspect of the problem of PL introduction. Second, we explicitly recognize the retailer’s interest in preserving a market for the NB, i.e. avoiding cannibalization.

3. MICROECONOMICS OF PRODUCT INTRODUCTION

The logic of our enquiry is simple. We suppose a retailer has an option to introduce a PL that is responsive to a change in consumer demand that expresses a relative preference for
a new product attribute such as a health claim. We suppose this relative preference is not fulfilled by the NB manufacturer and look for conditions under which the retailer will introduce the PL. Underlying our specification is the assumption that retailers have an option to respond. This might follow from consumer demand information that is asymmetrically distributed across retailers and NB manufacturers, stickiness in response by manufacturers due to inflexible technologies relative to PL manufacturers, or market power perceived to be held by NB manufacturers.

We suppose the consumer population is segmented into at least two segments, one that we label as the “conventional” customer segment and the other as the “health conscious” and “experimental” customer segment. We suppose the retailer buys national brand (NBs) products from national brand manufacturers (NBMs). We suppose the retailer is able to procure private label (PL) products either from an independent manufacturer or the national brand manufacturer.

3.1 Demand function with customer’s behavior
We suppose the consumer population defines market potential demand for the food product category over time. While human population may be predictable, consumer population relevant for a product category is uncertain. Thus, we specify potential consumer demand function to be uncertain. We sharply focus on the problem faced within a retail firm, abstracting from a strategic response by competitors. We suppose the retail firm faces an uncertain market of size, or potential demand, \( N(t) \) that evolves over time following geometric Brownian motion (GBM):

\[
dN(t) = \mu N(t) dt + \sigma N(t) dz,
\]

where \( dz \) is a standard Wiener process for demand, \( \mu \) is the mean drift in demand, and \( \sigma \) is the uncertainty rate (volatility) of such a process. Here, the mean-drift and the volatility characterize the risk intrinsic to potential demand fluctuate.

We suppose the retailer faces the problem of deciding whether and when to introduce a private label that is a close substitute for an existing national brand that the retailer offers. The fact that the products are not perfect substitutes may allow the firm to profit from selling both products. We suppose the NB product is priced exogenously to
the retailer. We suppose the PL product introduces a set of health claims not available in the NB and this implies the PL product is perceived as having higher or equal quality relative to the NB. We note this is the inverse assumed in past literature. For example, Choi and Coughlan (2006) supposed national brand product has a higher quality or loyalty than a private label product. Also, on average, store brands are priced 25%–30% below national brands (e.g., Kumar and Steenkamp (2007). However, as is well-known, health claims typically result in a price premium at least in the short-run that implies customers prefer the health claim product to the NB.

To model this customer’s preference, we consider the case that each customer has a reservation price $V_i$ for product $i$, $i = nb, pl$, where $V_i \geq 0$ and $V_i$ has a known density function $f_i(\cdot)$ and a cumulative distribution function $F_i(\cdot)$. We suppose that consumers are either indifferent to the health claim or uncertain of the health claim relative to the conventional attribute bundle embodied in the NB. Thus, we suppose that consumers hold relative preferences for the product $pl$ versus the NB product $nb$ and represent this by defining $\theta$ such that $V_{nb} = \theta V_{pl}$. This specification is intuitive and has been widely used in related literature, see e.g. Amrouche et al. (2008), and Yan and Ghose (2009)). Where the NB has established quality reputation or the PL quality is uncertain leading consumers to demand a discount, it may be that $\theta \geq 1$. In this paper, we consider this case as resulting from a balance across consumer preference for health attributes, uncertainty with respect to their value, and established reputation of NBs. In related work, we consider the case where the PL may be preferred implying that $0 \leq \theta \leq 1$. We define the price for a private label product and a national brand product as $p_{pl}(t) \in \mathbb{R}^+$ and $p_{nb}(t) \in \mathbb{R}^+$ and suppose that $p_{nb}(t)$ is determined exogenously in a global market while $p_{pl}(t)$ is a control available to the retailer. Thus, we suppose the retailer has at least local market power and rule out the presence of competing store offerings of a similar PL product.

It follows we can define the customer surplus associated with the national brand product ($U_{nb}$) and a private label product ($U_{pl}$) as:
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(2) \[ U_{nb} = V_{nb} - p_{nb} = \theta V_{pl} - p_{nb} \text{ and } U_{pl} = V_{pl} - p_{pl}. \]

Based on this, consumer choice can be specified and demand functions for each product derived. Quite simply, we suppose a customer will purchase the NB if \( U_{nb} \geq 0 \) and \( U_{nb} \geq 0 \), or the PL if \( U_{pl} \geq 0 \) and \( U_{pl} \geq 0 \). If the surpluses for each product are negative, customers will purchase neither. Based on this utility theory and the probability density function defined above, we derive demand functions for a private label and a national brand \((Q_{pl}(t))\text{ and } Q_{nb}(t))\), respectively: (an Appendix on derivation is available from the authors):

(3) \[ Q_{pl}(t) = \begin{cases} 
0 & \text{if } p_{nb}(t) < \theta p_{pl}(t) \\
F_1 \left( \frac{p_{nb}(t) - p_{pl}(t)}{\theta - 1} \right) - F_1 \left( p_{pl}(t) \right) N(t) & \text{if } p_{nb}(t) \geq \theta p_{pl}(t) 
\end{cases} \]

(4) \[ Q_{nb}(t) = \begin{cases} 
1 - F_1 \left( p_{nb}(t) \right) N(t) & \text{if } p_{nb}(t) < \theta p_{pl}(t) \\
1 - F_1 \left( \frac{p_{nb}(t) - p_{pl}(t)}{\theta - 1} \right) N(t) & \text{if } p_{nb}(t) \geq \theta p_{pl}(t) 
\end{cases} \]

For demand to exist for both products, consider the case where \( p_{nb} \geq \theta p_{pl} \). In this case, the PL has been strategically discounted. Without loss of generality, suppose that \( V_{pl} \) is uniformly distributed between 0 and 1; i.e., \( V_{pl} = [0,1] \). In this case, since \( f_1(x) = 1 \) and \( F_1(x) = x \), we have demand functions for each brand

(5) \[ Q_{pl}(t) = \left( \frac{p_{nb}(t) - p_{pl}(t)}{\theta - 1} - p_{pl}(t) \right) N(t), \; Q_{nb}(t) = \left( 1 - \frac{p_{nb}(t) - p_{pl}(t)}{\theta - 1} \right) N(t). \]

To consider introduction of the PL, we will consider its implications for profits of the retailer who already offers the NB and will require demand in the absence of the PL.
Based on the customer surplus condition $V_{nb} - p_{nb} = \theta V_{pl} - p_{nb} \geq 0$, we denote the demand function when only the national brand is offered as $Q_{nbo}(t)$ and note its form as:

(6) $Q_{nbo}(t) = \left(1 - \frac{p_{nb}(t)}{\theta}\right)N(t)$.

### 3.2 Timing introduction of PL functional food

We define a procurement cost for the national brand and private label as $c_{nb}$ and $c_{pl}$, respectively. Based on the demand functions above, we exploit a real option model to derive the optimal investment timing decisions. The retailer currently selling NB products faces instantaneous profits at time $t$ defined as:

(7) $(p_{nb} - c_{nb})Q_{nbo} = (p_{nb} - c_{nb})\left(1 - \frac{p_{nb}}{\theta}\right)N$

We define the time of PL introduction as $T$. After introducing a private brand, retailer profit is defined as $\pi(N(T))$ and we assume is conditional on an investment of product introduction expense represented as irreversible sunk cost $K$. As in an American option, we suppose the retailer is free to choose the optimal time of product launch. The value function ($\Psi(N)$) for based on such an optimal timing is defined as:

(8) $\Psi(N) = \max_T E\left[\int_0^T e^{-\rho t} (p_{nb} - c_{nb})(1 - \frac{p_{nb}}{\theta})N(t)dt + e^{-\rho T} \pi(N(T))\right]$,

where

(9) $\pi(N(T)) = E\left[\int_T^T e^{-\rho(t-T)}\left((p_{nb} - c_{nb})\left(1 - \frac{p_{nb} - p_{pl}^*}{\theta - 1}\right)N + (p_{pl}^* - c_{pl})\left(p_{nb} - \theta p_{pl}^*\right)\left(1 - \frac{p_{nb} - p_{pl}^*}{\theta - 1}\right)N\right]dt - K\right]$,

and we note as $p_{pl}^*$ the optimal price of a private label product. While the final aim is to determine an optimal time to introduce private label, we determine the optimal private
label price as the first step. Consistent with observation, in this paper we suppose that the PL product will be priced such that both demand for each of the products persists. As Proposition 1 indicates this implies the optimal PL price is constrained by a necessary condition for both labels to coexist.

**Proposition 1.** The optimal price of a private label after its introduction \((p_{pl}^*)\) is

\[
p_{pl}^* = \frac{2p_{nb} + \theta c_{pl} - c_{nb}}{2\theta} \text{ and if } \theta c_{pl} \leq c_{nb}, \text{ both national brand and private label products will co-exist. (See Appendix for proof).}
\]

We can interpret the optimal private label price as a function of national brand price and wholesale price \(c_{nb}\) as well as procurement cost of private label product \(c_{pl}\). Several interesting comparative-statics are apparent. If the private label cost increases or the national brand firm increases price, the optimal PL price increases. If the wholesale price \(c_{nb}\) increases, the optimal PL price increases as well. Finally, the second condition suggests articulates the condition under which introduction a private label product will cannibalize the NB, i.e. when the procurement cost for a private label is greater than \(c_{nb} / \theta\).

Next, we consider the optimal timing problem conditioned on the optimal price strategy. By substitution, we have the expression that defines retail profits after PL launch:

\[
\pi(N(T)) = E \left[ \int_T^\infty e^{-p(t-T)} \left( p_{nb} - c_{nb} \right) \left( \frac{2(\theta - 1)p_{nb} - \theta c_{pl} + c_{nb}}{2\theta} \right) \left( \frac{2p_{nb} + \theta c_{pl} - c_{nb}}{2} \right) dt - K \right]
\]
Optimal profits follow from determining the optimal timing of PL launch to maximize by considering the evolution of (10) as well as profits before launch. The resulting problem is a dynamic, stochastic optimal stopping problem where the optimal timing represents a threshold that maximizes profits before and after launch as indicated in Proposition 2.

Proposition 2. The firm’s value for the private label conditional on managerial flexibility (option) and optimal market scale threshold \((N^*)\) are as follows: (See the proof in the Appendix)

\[
\Psi(N) = \begin{cases} 
    a_1 (N)^{\beta_1} + \left(-p_{nb}^2 + (c_{nb} + \theta)p_{nb} - \partial c_{nb}\right)N \frac{\theta(r-\mu)}{r-\mu} & \text{if } N < N^* \\
    -p_{nb}^2 + \left(\theta + \frac{c_{nb}}{\theta}\right)p_{nb} + \frac{(\partial c_{pl} - c_{nb})^2}{4\theta(\theta-1)} - c_{nb} \frac{N}{r-\mu} - K & \text{if } N \geq N^*
\end{cases}
\]

where \(N^* = \frac{\beta_1 (r-\mu)K}{(\beta_1 - 1)} \left(\frac{1-\theta}{\theta}\right)p_{nb}^2 + (\theta - 1)p_{nb} + \left(\frac{(\partial c_{pl} - c_{nb})^2}{4\theta(\theta-1)}\right)^{-1}
\)

\[
\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1
\]

\[
a_1 = \frac{(N^*)^{1-\beta_1}}{\beta_1} \left(\frac{1-\theta}{\theta}\right)p_{nb}^2 + (\theta - 1)p_{nb} + \left(\frac{(\partial c_{pl} - c_{nb})^2}{4\theta(\theta-1)}\right)^{-1}
\]

The value function \(\Psi(N)\) can be interpreted as follows: when a firm’s current market potential \(N\) is lower than a certain threshold \(N^*\), the firm will offer only the conventional national brand to earn a net present value \(\frac{-p_{nb}^2 + (c_{nb} + \theta)p_{nb} - \partial c_{nb})N}{\theta(r-\mu)}\) plus the value of the option \(a_1 (N)^{\beta_1}\). The second case for \(\Psi(N)\) follows from profits after
the launch of the private label. That is, when the potential demand market exceeds the threshold, the firm’s option to launch the PL becomes profitable to exercise.

3.3. Analysis of Launch Timing

We now exploit the theory presented to investigate the impacts of key determinants on the product launch timing decision. As launch timing is driven by the value of the threshold $N^*$, we focus analysis on the impacts of exogenous factors on that threshold in Proposition 3.

**Proposition 3.** The optimal threshold is strictly increasing in the market volatility ($\sigma$) and product introduction, sunk cost, and increasing (decreasing) in the procurement cost for private label (national brand), respectively. (See Appendix for the proof)

$$\frac{\partial N^*}{\partial \sigma} > 0, \frac{\partial N^*}{\partial c_{pl}} \geq 0, \frac{\partial N^*}{\partial c_{nb}} \leq 0 \text{ and } \frac{\partial N^*}{\partial K} > 0.$$  

These results are of particular interest in the food industry. The first result clarifies that as the extent of uncertainty increases with respect to the level of market scale ($N(t)$) the optimal market scale for launch of a private label increases. This suggests that as consumers are more highly segmented and less loyal to particular product attributes, the optimal scale of market needed for profitable launch of a private label increases. Within the context of healthy food, the observation that retailers delayed PL launches of food products with organic and other health-related claims is consistent with this theory. Similarly, the second result above appears consistent with observation. As the unit procurement cost of the PL product increases relative to that of the NB, the optimal market scale for launch increases.

4. Numerical Illustration

To provide further illustration, we implemented the optimal stochastic, dynamic control problem using numerical methods. Figure 1(a), (b), (c) and (d) illustrates these results based on a suggestive parameterization summarized in Table 1. The numerical examples were developed for different uncertainty rates ($\sigma$), private label cost ($c_{pl}$), national brand
cost \((c_{pl})\), and an investment sunk cost \((K)\). Here, we consider only the case where although the healthy food may be preferred, the credibility of its health claims are questioned implying that \(\theta \geq 1\). Alternative specifications are available from the authors.

Table 1. Parameters for numerical examples

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Figure 1(a)</th>
<th>Figure 1(b)</th>
<th>Figure 1(c)</th>
<th>Figure 1(d)</th>
<th>Figure 2(a)</th>
<th>Figure 2(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>(\mu)</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.35/0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>(p_{nb})</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.5~0.9</td>
</tr>
<tr>
<td>(c_{ab})</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5/0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>(c_{pl})</td>
<td>0.3</td>
<td>0.3/0.08</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>(K)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1/1.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(\theta)</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1~1.3</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Figure 1(a) illustrates how a firm’s optimal launching timing strategy changes when market potential demand changes. The red (blue) line represents a firm’s value when the market is characterized by high (low) levels of uncertainty. For each case, the retailer finds it optimal to launch at dates in the feasible region defined by dotted lines. The corresponding optimal thresholds \((T^*)\) for low and high uncertainty are 1.2 and 2.0, respectively. As derived in Proposition 3, the optimal threshold increases by 0.8 (66% increment) when market volatility increases from 0.15 to 0.35 (133% increment). This translates into a response rate of about 0.50% per 1% change in uncertainty. Note, this is not a elasticity as changes are not small. As shown in the figure, the firm’s value when a market is uncertain is larger than the value under stable market conditions. Further, the increased threshold implies that the firm needs to wait longer until increased profit flows are guaranteed.
When private label cost increases, Proposition 3 and corresponding Figure 1(b) indicate that the product launch threshold increases. From the retailer’s perspective, it would like to optimally set a private label price relative to the national brand price.

As shown in Figure 1(c), as the unit cost of the NB increases, the relationship between the retailer’s profits (indicated as Firm’s value) associated with PL introduction and market potential demand rotates counter-clockwise. Intuitively, this implies that at any level of market potential demand, greater profits are associated with a PL launch. However, Figure 1(d) implies that as sunk cost of a launch increases, that relationship rotates clockwise implying that as sunk cost increases, the firm value of a launch decreases for any given market potential demand. This is particularly interesting with respect to the question of whether the retailer or NB manufacturer is better suited to launch a new healthy food product. If the NB manufacturer has substantially higher sunk costs for a launch, as is often claimed, it is clear that the retailer may have a substantial advantage.

**Figure 1. Dynamics of the Value of PL Launch**

![Diagram](image)

a. Effect of market uncertainty

b. Effect of PL unit cost
Next, we illustrate the role of consumer preferences and NB price within the context of this particular parameterization. Recall, that $\frac{\partial N^*}{\partial \theta}$ and $\frac{\partial N^*}{\partial p_{nb}}$ are nonlinear, in general. Figure 2(a) illustrates that consumer preferences derive the optimal market scale necessary for profitable introduction of the PL. Where consumer preferences are exogenous to the retailer, an optimal market scale threshold is indicated. For alternative parameterizations, the curve could be shifted leftward as in the case where the PL is more highly valued than the NB. In other cases, the curvature of the relationship might be dramatically less than depicted in the figure. Finally, where consumer preferences are endogenous, perhaps dependent on advertising, such a control would be an important determinant of the shape of the relationship. The curvature is further of interest as an illustration of brand conflict where PL introduction can reduce profits and imply a much larger market scale is necessary given consumer preferences to ensure profitable launch. Clearly, the NB price, $P_{nb}$, is a crucial determinant of the optimal market scale threshold. Intuitively, we might expect that the retail firm can launch earlier when the national brand price goes up. However, Figure 2(b) shows that at low NB prices, as that price increases the market scale threshold increases.
Figure 2. Market scale relationships

<table>
<thead>
<tr>
<th>a. Relative consumer valuation</th>
<th>b. National brand price</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph of Customer behavior vs. Investment Threshold" /></td>
<td><img src="image2.png" alt="Graph of Price of national brand vs. Investment Threshold" /></td>
</tr>
</tbody>
</table>

**CONCLUSIONS AND FURTHER STUDY**

Private labels logically present a strategic tool for retailers to respond to consumer demand, perhaps allowing them to innovate faster than national brand manufacturers that face high sunk costs generated by advertising and marketing. While such a strategy may offer increased profits, the timing of product introduction depends on consumer preferences, characteristics of stochastic and dynamic demand, and market power held by the retailer that allows timing to be controlled, rather than specified by competitors. In this paper, we presented the problem of timing and pricing healthy food products as private labels when close substitute national brand products may exist. For successful PL introduction, our theory notes such a retailer must recognize that i) introduction of the private label is *optional*, ii) that the PL introduction may negatively impact profits as the retailer offers close substitute NBs, iii) the investment has irreversible sunk cost, and iv) customer preferences across the PL and NB products. Within this context, we offer analysis the investment strategy under various scenarios. We show at when the market volatility ($\sigma$) increases, when a cost for private label (national brand) increases (decreases), and when higher sunk cost is expected, it is optimal for the retail firm to delay launching the private label product. We also show a counter-intuitive result that i) inappropriately positioned private label products may reduce total profits, i.e. profits from both brands and render a delay in introduction optimal, and ii) an increase in the national brand price may increase profits and support postponed rollout of a PL.
These results motivate several extensions including 1) the case where PLs are preferred to NBs, 2) the case where the consumer population is heterogeneous, and 3) the presence of competition with other firms. When many firms compete, the timing strategy of adding the private label will be changed by a national brand firm’s strategy. Even though a national brand firm is not reacting immediately to each retailer’s policy, it also is obvious that the firm would set up their price strategies against a retailer in the long run as discussed in Sloot and Verhoef (2008).
APPENDIX

PROOF OF PROPOSITION 1

We assume the retailing firm hopes to maximize its profit by setting the national brand price. Therefore, the firm’s optimization problem at each time is written as:

$$\max_{p_{pl}} \left\{ (p_{nb} - c_{nb}) \left[ 1 - \frac{p_{nb} - p_{pl}}{\theta - 1} \right] N + (p_{pl} - c_{pl}) \left[ \frac{p_{nb} - \theta p_{pl}}{\theta - 1} \right] N \right\}$$

By the first order condition, we have an optimal price for the private label $p_{pl}^*$ as follows:

$$\max_{p_{pl}} \left\{ (p_{nb} - c_{nb}) \left[ 1 - \frac{1}{\theta - 1} \right] + \left( \frac{p_{nb} - \theta p_{pl}}{\theta - 1} \right) + (p_{pl} - c_{pl}) \left[ -\frac{\theta}{\theta - 1} \right] \right\}$$

$$(p_{nb} - c_{nb}) + p_{nb} - \theta p_{pl} + (-\theta p_{pl} + \theta c_{pl}) = 0$$

$$2\theta p_{pl} = (2p_{nb} + \theta c_{pl} - c_{nb})$$

$$p_{pl}^* = \frac{2p_{nb} + \theta c_{pl} - c_{nb}}{2\theta}.$$

Since $p_{nb} \geq \theta p_{pl}$, $p_{pl}^* = \frac{2p_{nb} + \theta c_{pl} - c_{nb}}{2\theta} \geq \frac{2\theta p_{pl}^* + \theta c_{pl} - c_{nb}}{2\theta} = p_{pl}^* + \theta c_{pl} - c_{nb}.

In order not to violate the solution, we need the condition $\theta c_{pl} \leq c_{nb}$. Q.E.D.

PROOF OF PROPOSITION 2.

The derivation of firm’s value is very similar to that of valuing a financial American option as shown in Pindyck (1991). The optimal investment rule for a firm is determined by solving a stochastic dynamic optimal stopping problem. As aforementioned in the text, the firm is currently selling products of a national brand and incurs cash flows

$$(p_{nb} - c_{nb})q_{nho} = (p_{nb} - c_{nb}) \left[ 1 - \frac{p_{nb}}{\theta} \right] N \text{ for each instant time } dt \text{ until an optimal time } T \text{ to develop a private brand. After introducing a private brand at time } T, \text{ the firm would make profit } \pi(N(T)) \text{ and incur an irreversible sunk cost } K \text{ associated with the product’s introduction. Hence, as shown in Pindyck (1991) and Dixit and Pindyck (1994) (Chapter 4), in the continuation region, the Bellman equation is}$$

$$[\rho \Psi(N) - (p_{nb} - c_{nb}) \left[ 1 - \frac{p_{nb}}{\theta} \right] N] dt = E(d\Psi)$$
Using Itô’s Lemma to manipulate $d\Psi$ (for the details, refer to Wilmott et al. (1995) and Oksendal (2003)) we have

\[ d\Psi(N) = \Psi(N)dt + (1/2)\sigma^2 N^2 \Psi_{NN}(N)dt + \Psi(N)dN. \]

By substitution and manipulation (see Pindyck (1991)), the Bellman equation becomes the following non-homogeneous differential equation.

\[
\frac{1}{2} \sigma^2 N^2 \Psi_{NN}(N) + N \Psi_{N}(N) - \rho \Psi + (p_{nb} - c_{nb}) \left( 1 - \frac{P_{nb}}{\theta} \right) N = 0
\]

where $\Psi_{N}(N) = \frac{\partial \Psi(N)}{\partial N}$ and $\Psi_{NN}(N) = \frac{\partial^2 \Psi(N)}{\partial N^2}$.

In addition, $\Psi(N)$ should satisfy the following boundary conditions:

\[ \Psi(0) = 0 \]

\[ \Psi(N) = \left( -p_{nb}^2 + \left( \theta + \frac{c_{nb}}{\theta} \right) p_{nb} + \frac{(\theta c_{pl} - c_{nh})^2}{4\theta(\theta - 1)} - c_{nb} \right) \frac{N}{r - \mu} - K \]

\[ \frac{\partial \Psi(N)}{\partial N} = \left( -p_{nb}^2 + \left( \theta + \frac{c_{nb}}{\theta} \right) p_{nb} + \frac{(\theta c_{pl} - c_{nh})^2}{4\theta(\theta - 1)} - c_{nb} \right) \frac{1}{r - \mu} \]

The first condition indicates profits that the a firm makes when $N = 0$. The second and third conditions are smooth pasting and value matching conditions coming from optimality. The general solution for the non-homogeneous differential equation must take the form $\Psi(N) = a^1N + a^2N$, where $a^1, a^2$ are constants to be determined, and

\[ \beta_1 = 1 - \frac{\mu}{\sigma^2} + \sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}} > 1 \quad \beta_2 = 1 - \frac{\mu}{\sigma^2} - \sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2}} < 0. \]

However, because of the first condition, we can take the value of $\beta_1$. The solution for non-homogeneous part is $\frac{(-p_{nb}^2 + (c_{nb} + \theta)p_{nb} - \theta c_{nb})N}{\theta(r - \mu)}$. Using the value-matching and smooth-pasting conditions, we can find the value function $\Psi(N)$ as in Proposition 2.

Q.E.D.

PROOF OF PROPOSITION 3

From the optimal threshold, we take derivatives with respect to volatility.
\[
\frac{\partial N^*}{\partial \sigma} = (r - \mu)K \left[ \left( \frac{1-\theta}{\theta} \right) p_{nb}^2 + \frac{(\theta^2 - \theta)}{\theta} p_{nb} + \left( \frac{(\theta \cdot c_{pl} - c_{nh})^2}{4\theta(\theta - 1)} \right) \right]^{-1} \frac{\partial \beta_1}{\partial \sigma} \frac{\partial}{\partial \beta_1} \left( \beta_1 \right) \left( \beta_1 - 1 \right)
\]

From differential equation (1) and (3), \( \beta_1 > 1 \) and \( \beta_2 < 0 \) should satisfy the following equation:

\[
\frac{1}{2} \sigma^2 \beta(\beta - 1) + \mu \beta - r = 0
\]

Let \( Q(\beta_i) = \frac{1}{2} \sigma^2 \beta(\beta - 1) + \mu \beta - r \) and take a derivative:

\[
\frac{\partial Q}{\partial \beta_i} \frac{\partial \beta_i}{\partial \sigma_i} + \frac{\partial Q}{\partial \sigma_i} = 0, \quad \text{for } i = 1, 2
\]

where derivatives are evaluated at \( \beta_1 \) and \( \beta_2 \).

And we know \( \frac{\partial Q}{\partial \beta} < 0 \) at \( \beta_2 \) and \( \frac{\partial Q}{\partial \beta} > 0 \) at \( \beta_1 \) since \( Q \) is quadratic, \( \beta_1 > 1 \) and \( \beta_2 < 0 \).

Since \( \frac{\partial \beta_1}{\partial \beta} < 0 \) and \( \frac{\partial}{\partial \beta_1} \left( \frac{\beta_1}{\beta_1 - 1} \right) = -\frac{1}{(\beta_1 - 1)^2} < 0 \), then \( \frac{\partial N^*}{\partial \sigma} > 0 \)

\[
\frac{\partial N^*}{\partial c_{pl}} = \frac{\beta_1 (r - \mu)K}{(\beta_1 - 1)} \left[ \left( \frac{1-\theta}{\theta} \right) p_{nb}^2 + \frac{(\theta^2 - \theta)}{\theta} p_{nb} + \left( \frac{(\theta \cdot c_{pl} - c_{nh})^2}{4\theta(\theta - 1)} \right) \right]^{-2} \left( \frac{2\theta(\theta \cdot c_{pl} - c_{nh})}{4\theta(\theta - 1)} \right)
\]

Since \( \theta \cdot c_{pl} \leq c_{nh} \) by Proposition 1, \( \frac{\partial N^*}{\partial c_{pl}} \geq 0 \).

Similarly, we can have

\[
\frac{\partial N^*}{\partial c_{nh}} = \frac{\beta_1 (r - \mu)K}{(\beta_1 - 1)} \left[ \left( \frac{1-\theta}{\theta} \right) p_{nb}^2 + \frac{(\theta^2 - \theta)}{\theta} p_{nb} + \left( \frac{(\theta \cdot c_{pl} - c_{nh})^2}{4\theta(\theta - 1)} \right) \right]^{-2} \left( \frac{2(\theta \cdot c_{pl} - c_{nh})}{4\theta(\theta - 1)} \right) \leq 0
\]

\[
\frac{\partial N^*}{\partial K} = \frac{\beta_1 (r - \mu)K}{(\beta_1 - 1)} \left[ \left( \frac{1-\theta}{\theta} \right) p_{nb}^2 + \frac{(\theta^2 - \theta)}{\theta} p_{nb} + \left( \frac{(\theta \cdot c_{pl} - c_{nh})^2}{4\theta(\theta - 1)} \right) \right]^{-1} > 0
\]
REFERENCES


