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From the help desk: Swamy's random-coefficients model

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Abstract. This article discusses the Swamy (1970) random-coefficients model and presents a command that extends Stata's `xtrchh` command by also providing estimates of the panel-specific coefficients.

Keywords: st0046, panel data, random-coefficients models

1 Introduction

Fixed- and random-effects models incorporate panel-specific heterogeneity by including a set of nuisance parameters that essentially provide each panel with its own constant term. However, all panels share common slope parameters. Random-coefficients models are more general in that they allow each panel to have its own vector of slopes randomly drawn from a distribution common to all panels. Stata's `xtrchh` command provides estimates of the parameters characterizing the distribution from which the panel-specific parameters are drawn. The command included with this article extends Stata's implementation by also providing best linear unbiased predictors of the panel-specific draws from that distribution.

Section 2 develops the Swamy (1970) random-coefficients model. Section 3 then presents the syntax and usage of a command called `xtrchh2` that implements the estimator in Stata, and section 4 presents an example. Section 5 lists the results stored by `xtrchh2`.

2 Swamy's random-coefficients model

Following Swamy (1970), consider a random-coefficients model of the form

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i \quad (1)$$

where $i = 1 \dots P$ denotes panels, \mathbf{y}_i is a $T_i \times 1$ vector of observations for the i th panel, \mathbf{X}_i is a $T_i \times k$ matrix of nonstochastic covariates, and $\boldsymbol{\beta}_i$ is a $k \times 1$ vector of parameters specific to panel i . The error term vector $\boldsymbol{\epsilon}_i$ is distributed with mean zero and variance $\sigma_{ii}\mathbf{I}$. The panels do not need to be balanced.

Each panel-specific $\boldsymbol{\beta}_i$ is related to an underlying common parameter vector $\boldsymbol{\beta}$:

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \mathbf{v}_i \quad (2)$$

where $E\{\mathbf{v}_i\} = \mathbf{0}$, $E\{\mathbf{v}_i\mathbf{v}_i'\} = \Sigma$, $E\{\mathbf{v}_i\mathbf{v}_j'\} = \mathbf{0}$ for $j \neq i$, and $E\{\mathbf{v}_i\epsilon_j'\} = \mathbf{0}$ for all i and j . Combining (1) and (2),

$$\begin{aligned}\mathbf{y}_i &= \mathbf{X}_i(\boldsymbol{\beta} + \mathbf{v}_i) + \epsilon_i \\ &= \mathbf{X}_i\boldsymbol{\beta} + \mathbf{u}_i\end{aligned}$$

with $\mathbf{u}_i \equiv \mathbf{X}_i\mathbf{v}_i + \epsilon_i$. Moreover,

$$\begin{aligned}E\{\mathbf{u}_i\mathbf{u}_i'\} &= E\{(\mathbf{X}_i\mathbf{v}_i + \epsilon_i)(\mathbf{X}_i\mathbf{v}_i + \epsilon_i)'\} \\ &= \mathbf{X}_i\Sigma\mathbf{X}_i' + \sigma_{ii}\mathbf{I} \\ &\equiv \boldsymbol{\Pi}_i\end{aligned}$$

Stacking the equations for the P panels,

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \quad (3)$$

where

$$\boldsymbol{\Pi} \equiv E\{\mathbf{u}\mathbf{u}'\} = \begin{bmatrix} \boldsymbol{\Pi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Pi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Pi}_P \end{bmatrix}$$

Estimating the parameters of (3) is a standard problem in generalized least squares (GLS), so

$$\begin{aligned}\hat{\boldsymbol{\beta}} &= (\mathbf{X}'\boldsymbol{\Pi}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Pi}^{-1}\mathbf{y} \\ &= \left(\sum_i \mathbf{X}_i'\boldsymbol{\Pi}_i^{-1}\mathbf{X}_i\right)^{-1} \sum_i \mathbf{X}_i'\boldsymbol{\Pi}_i^{-1}\mathbf{y}_i \\ &= \sum_i \mathbf{W}_i\mathbf{b}_i\end{aligned} \quad (4)$$

where

$$\mathbf{W}_i = \left[\sum_j \left\{ \Sigma + \sigma_{jj}(\mathbf{X}_j'\mathbf{X}_j)^{-1} \right\}^{-1} \right]^{-1} \left\{ \Sigma + \sigma_{ii}(\mathbf{X}_i'\mathbf{X}_i)^{-1} \right\}^{-1}$$

and $\mathbf{b}_i \equiv (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{y}_i$, showing that $\hat{\boldsymbol{\beta}}$ is a weighted average of the panel-specific OLS estimates. The final equality in (4) makes use of the fact that

$$(\mathbf{A} + \mathbf{BDB}')^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{BEB}'\mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{BE}(\mathbf{E} + \mathbf{D})^{-1}\mathbf{EBA}'^{-1}$$

where $\mathbf{E} \equiv (\mathbf{B}'\mathbf{A}^{-1}\mathbf{B})^{-1}$. See Rao (1973, 33).

The variance of $\hat{\boldsymbol{\beta}}$ is

$$\text{Var}(\hat{\boldsymbol{\beta}}) = (\mathbf{X}'\boldsymbol{\Pi}^{-1}\mathbf{X})^{-1} = \sum_i \left\{ \Sigma + \sigma_{ii}(\mathbf{X}_i'\mathbf{X}_i)^{-1} \right\}^{-1}$$

In addition to estimating $\boldsymbol{\beta}$, one often wishes to obtain estimates of the panel-specific $\boldsymbol{\beta}_i$ vectors as well. As discussed by Judge et al. (1985, 541), if attention is restricted

to the class of estimators $\{\beta_i^*\}$ for which $E\{\beta_i^* | \beta_i\} = \beta_i$, then the panel-specific OLS estimator \mathbf{b}_i is appropriate. However, if one does not condition on β_i , then the best linear unbiased predictor is

$$\begin{aligned}\widehat{\beta}_i &= \widehat{\beta} + \Sigma \mathbf{X}_i' (\mathbf{X}_i \Sigma \mathbf{X}_i' + \sigma_{ii} \mathbf{I})^{-1} (\mathbf{y}_i - \mathbf{X}_i \widehat{\beta}) \\ &= (\Sigma^{-1} + \sigma_{ii}^{-1} \mathbf{X}_i' \mathbf{X}_i)^{-1} (\sigma_{ii}^{-1} \mathbf{X}_i' \mathbf{X}_i \mathbf{b}_i + \Sigma^{-1} \widehat{\beta})\end{aligned}$$

Greene (1997, 672) suggests using the following method to obtain the variance of $\widehat{\beta}_i$. Define $\mathbf{A}_i \equiv (\Sigma^{-1} + \sigma_{ii}^{-1} \mathbf{X}_i' \mathbf{X}_i)^{-1} \Sigma^{-1}$. Then

$$\widehat{\beta}_i = [\mathbf{A}_i \quad (\mathbf{I} - \mathbf{A}_i)] \begin{bmatrix} \widehat{\beta} \\ \mathbf{b}_i \end{bmatrix}$$

and

$$\text{Var}(\widehat{\beta}_i) = [\mathbf{A}_i \quad (\mathbf{I} - \mathbf{A}_i)] \text{Var} \begin{pmatrix} \widehat{\beta} \\ \mathbf{b}_i \end{pmatrix} \begin{bmatrix} \mathbf{A}_i' \\ (\mathbf{I} - \mathbf{A}_i) \end{bmatrix}$$

Note that

$$\text{Var} \begin{pmatrix} \widehat{\beta} \\ \mathbf{b}_i \end{pmatrix} = \begin{bmatrix} \text{Var}(\widehat{\beta}) & \text{Cov}(\widehat{\beta}, \mathbf{b}_i) \\ \text{Cov}(\widehat{\beta}, \mathbf{b}_i) & \text{Var}(\mathbf{b}_i) \end{bmatrix}$$

The GLS estimator $\widehat{\beta}$ is both consistent and efficient; and, although inefficient, \mathbf{b}_i is nevertheless also a consistent estimator of β . Thus, making use of Lemma 2.1 of Hausman (1978), $\text{Asy.Cov}(\widehat{\beta}, \mathbf{b}_i) = \text{Asy.Var}(\widehat{\beta}) - \text{Asy.Cov}(\widehat{\beta}, \widehat{\beta} - \mathbf{b}_i) = \text{Asy.Var}(\widehat{\beta})$. After some algebraic manipulation,

$$\text{Asy.Var}(\widehat{\beta}_i) = \text{Var}(\widehat{\beta}) + (\mathbf{I} - \mathbf{A}_i) \left\{ \text{Var}(\mathbf{b}_i) - \text{Var}(\widehat{\beta}) \right\} (\mathbf{I} - \mathbf{A}_i)'$$

To make the above formulas feasible, each σ_{ii} may be replaced with the consistent OLS estimate

$$\widehat{\sigma}_{ii} = \frac{(\mathbf{y}_i - \mathbf{X}_i \mathbf{b}_i)' (\mathbf{y}_i - \mathbf{X}_i \mathbf{b}_i)}{T_i - k}$$

Swamy (1970) showed that a consistent estimator of Σ is

$$\widehat{\Sigma} = \frac{1}{P-1} \left(\sum_{i=1}^P \mathbf{b}_i \mathbf{b}_i' - P \bar{\mathbf{b}} \bar{\mathbf{b}}' \right) - \frac{1}{P} \sum_{i=1}^P \widehat{\sigma}_{ii} (\mathbf{X}_i' \mathbf{X}_i)^{-1}$$

where $\bar{\mathbf{b}} \equiv \frac{1}{P} \sum_i \mathbf{b}_i$. However, that estimator may not always be positive definite in finite samples. A practical solution is to ignore the final term, and both Stata's `xtrchh` command and the `xtrchh2` command accompanying this article do that.

A natural question to ask is whether the panel-specific β_i s differ significantly from one another. Under the null hypothesis

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_P \quad (5)$$

the test statistic

$$T \equiv \sum_{i=1}^P (\mathbf{b}_i - \boldsymbol{\beta}^\dagger)' \{ \hat{\sigma}_{ii}^{-1}(\mathbf{X}_i \mathbf{X}_i) \} (\mathbf{b}_i - \boldsymbol{\beta}^\dagger)$$

where

$$\boldsymbol{\beta}^\dagger \equiv \left\{ \sum_{i=1}^P \hat{\sigma}_{ii}^{-1}(\mathbf{X}_i \mathbf{X}_i) \right\}^{-1} \sum_{i=1}^P \hat{\sigma}_{ii}^{-1}(\mathbf{X}_i \mathbf{X}_i) \mathbf{b}_i$$

is distributed χ^2 with $k(P - 1)$ degrees of freedom.

3 Stata implementation

3.1 Syntax

```
xtrchh2 depvar varlist [if exp] [in range] [, i(varname) t(varname)
    level(#) offset(varname) noconstant nobetas]
```

Syntax for predict

```
predict [type] newvarname [if exp] [in range] [, [xb|stdp|xbi] group(#)
    nooffset]
```

3.2 Options

`i(varname)` specifies the variable that contains the unit to which the observation belongs. You can specify the `i()` option the first time you estimate, or you can use the `iis` command to set `i()` beforehand. Note that it is not necessary to specify `i()` if the data have been previously `tsset`, or if `iis` has been previously specified—in these cases, the group variable is taken from the previous setting. See [XT] `xt`.

`t(varname)` specifies the variable that contains the time at which the observation was made. You can specify the `t()` option the first time you estimate, or you can use the `tis` command to set `t()` beforehand. Note that it is not necessary to specify `t()` if the data have been previously `tsset`, or if `tis` has been previously specified—in these cases, the time variable is taken from the previous setting. See [XT] `xt`.

`level(#)` specifies the confidence level, in percent, for confidence intervals. The default is `level(95)` or as set by `set level`; see [U] **23.6 Specifying the width of confidence intervals**.

`offset(varname)` specifies that `varname` is to be included in the model with its coefficient constrained to be 1.

`noconstant` suppresses the constant term (intercept) in the regression.

`nobetas` requests that the panel-specific $\widehat{\boldsymbol{\beta}}_i$ s not be displayed.

Options for predict

xb, the default, calculates the linear prediction based on $\hat{\beta}$.

stdp calculates the standard error of the linear prediction based on $\hat{\beta}$.

xbi calculates the linear prediction based on the group-specific $\hat{\beta}_i$, where i is specified with the **group(#)** option. The predictions are calculated for all available observations in the dataset, not just those in group i ; you can use **if** or **in** to restrict that behavior.

group(#) specifies which group-specific $\hat{\beta}_i$ to use with the **xbi** option. The default is **group(1)**. **group(#)** has no effect if **xbi** is not specified.

nooffset is relevant only if you specified **offset(varname)** for **xtrchh2**. It modifies the calculations made by **predict** so that they ignore the offset variable; the linear prediction is treated as $\mathbf{x}_{it}\mathbf{b}$ instead of $\mathbf{x}_{it}\mathbf{b} + \text{offset}_{it}$.

3.3 Remarks

The **xtrchh2** command fits Swamy's random-coefficients model as described in the previous section. The estimates of β are identical to those produced by **xtrchh**. Additionally, **xtrchh2** displays the best linear unbiased estimates of the panel-specific coefficients; an option allows that output to be suppressed. Note that one can simply use the **statsby** command to obtain the panel-specific OLS estimates if they are desired. Saved results are stored in **e()** macros; see *Saved results* below.

4 Example

To illustrate the usage of **xtrchh2**, the following example uses the same dataset as [XT] **xtrchh**.

```
. webuse invest2, clear
. xtrchh2 invest market stock, i(company) t(time)
```

The output is shown on the next page. The header displays the number of observations and summarizes the structure of the panel data. It also contains a Wald test of the joint significance of the slope parameters in $\hat{\beta}$. Below the estimate of $\hat{\beta}$ is the test statistic for the null hypothesis shown in (5). The remainder of the output consists of the estimated panel-specific $\hat{\beta}_i$ s.

(Continued on next page)

Swamy random-coefficients regression
 Group variable (i): company

Number of obs = 100
 Number of groups = 5

Obs per group: min = 20
 avg = 20.0
 max = 20

Wald chi2(2) = 17.55
 Prob > chi2 = 0.0002

invest	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
market	.0807646	.0250829	3.22	0.001	.0316031	.1299261
stock	.2839885	.0677899	4.19	0.000	.1511229	.4168542
_cons	-23.58361	34.55547	-0.68	0.495	-91.31108	44.14386

Test of parameter constancy: chi2(12) = 603.99 Prob > chi2 = 0.0000

Group-specific coefficients

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Group 1						
market	.1027848	.0108566	9.47	0.000	.0815062	.1240634
stock	.3678493	.0331352	11.10	0.000	.3029055	.4327931
_cons	-71.62927	37.46663	-1.91	0.056	-145.0625	1.803978
Group 2						
market	.084236	.0155761	5.41	0.000	.0537074	.1147647
stock	.3092167	.0301806	10.25	0.000	.2500638	.3683695
_cons	-9.819343	14.07496	-0.70	0.485	-37.40575	17.76707
Group 3						
market	.0279384	.013477	2.07	0.038	.0015241	.0543528
stock	.1508282	.0286904	5.26	0.000	.0945961	.2070603
_cons	-12.03268	29.58083	-0.41	0.684	-70.01004	45.94467
Group 4						
market	.0411089	.0118179	3.48	0.001	.0179461	.0642717
stock	.1407172	.0340279	4.14	0.000	.0740237	.2074108
_cons	3.269523	9.510794	0.34	0.731	-15.37129	21.91034
Group 5						
market	.147755	.0181902	8.12	0.000	.1121028	.1834072
stock	.4513312	.0569299	7.93	0.000	.3397506	.5629118
_cons	-27.70628	42.12524	-0.66	0.511	-110.2702	54.85766

5 Saved results

`xtrchh2` saves in `e()`:

Scalars

<code>e(N)</code>	number of observations	<code>e(g_avg)</code>	average group size
<code>e(chi2)</code>	χ^2	<code>e(N_g)</code>	number of groups
<code>e(df_m)</code>	model degrees of freedom	<code>e(chi2_c)</code>	χ^2 for comparison test
<code>e(g_max)</code>	largest group size	<code>e(df_chi2c)</code>	degrees of freedom for comparison test
<code>e(g_min)</code>	smallest group size		

Macros

<code>e(cmd)</code>	<code>xtrchh2</code>	<code>e(depvar)</code>	name of dependent variable
<code>e(predict)</code>	program used to implement <code>predict</code>	<code>e(chi2type)</code>	Wald; type of model χ^2 test
<code>e(ivar)</code>	variable denoting groups	<code>e(title)</code>	title in estimation output

Matrices

<code>e(b)</code>	$\hat{\beta}$ vector	<code>e(V_i)</code>	estimated $\text{Var}(\hat{\beta}_i)$, $i=1\dots P$
<code>e(V)</code>	estimated $\text{Var}(\hat{\beta})$	<code>e(Sigma)</code>	$\hat{\Sigma}$
<code>e(beta_i)</code>	$\hat{\beta}_i$ vector, $i=1\dots P$		

Functions

<code>e(sample)</code>	marks estimation sample
------------------------	-------------------------

6 References

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Brian P. Poi received his Ph.D. in economics from the University of Michigan and joined Stata Corporation as a staff statistician.