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# **Modelling farm production risk with copulae instead of correlations**

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# Modelling farm production risk with copulae instead of correlations

## Abstract

The optimisation of production plans is an important topic in agriculture, often related to diversification and specialisation as the classical instruments of coping with production risk. Although the measurement of embedded risk is often inaccurate, it is nevertheless necessary for decision making to describe the common behaviour of different variables in a model. Imprecisely defined relationships influence the “right” choice, why it is important to find a good approximation of the real circumstances. In financial science, copula functions are frequently used instead of correlation coefficients to model joint price behaviour, because of the possibility to link the marginal distributions on multifarious ways. By now, agricultural science makes less use of this method. This research uses the concept of “partly nested Archimedean copula” to model the relationship between different crop yields and compares it with a correlation based approach. The analysis focuses the differences of the approaches in the context of production planning and the use of weather derivatives.

**Keywords:** Copula, risk, weather derivatives

## 1 Introduction

The way variables are connected significantly influences the results of a model. Many authors use correlations to link variables (cf. Berg 2003, Hirschauer, Mußhoff 2009), assuming an elliptical structure of the scatter plot. However, in real world often extreme events occur simultaneously, e.g. the collective loss of stocks in crises. In such situations there is a strong dependence in one tail, which cannot be modelled by correlation. This illustrates, among others (e.g. linearity, exact only for symmetric distributions), one problem of correlation concept, related to the joint marginal distribution (cf. Schulz 2008, p. 241). SAVU and TREDE consider the copula-concept as a “[...] powerful tool to create more flexible and more realistic multivariate distributions [...]” (Savu, Trede 2006, p. 1) e.g. compared to multivariate normal distributions or t-distributions. The concept allows to separate the dependency structure from the marginal distributions of the random variables, which overcomes some of the disadvantages related to the use of correlations (Härdle 2009, p. 3). Many families of copulae are available, to describe the dependency structure, differentiated in simple, elliptical and Archimedean copulas (Härdle 2009, p. 6).

Several studies using the copula concept have been conducted in the finance sector. For instance, SAVU and TREDE applied this method to a portfolio consisting of a subset of Euro-Stoxx-50 stocks (Savu, Trede 2006). KOZIOL analyses the ability of different copulae to model basket credit derivatives (Koziol 2006). In agricultural science copulae are used by XU ET AL. to describe the relationship between weather data related to different weather stations (Xu et al. 2010) and LIU ET AL. to diversify the yield risk on a global level (Liu et al. 2010). BOKUSHEVA did some research about the aptitude of copulae to link a weather index and an index related to an underlying used for weather derivatives (Bokusheva 2010).

For the purpose of this research two farm models are constructed. The only difference is the way the relationship between variables is represented. One approach uses correlation matrices, the other one copula functions. To reduce the complexity, only yields and precipitation are assumed to be random. To compare the models, one part of the analysis deals with the gen-

eral difference in the cropping plans. The second part investigates the influence of a weather derivative for risk reduction. The construction of a correlation model is assumed to be widely known, so that the paper focuses on the less common copula approach. The next section of this paper describes the basics of copula functions. This includes the construction, the estimation of the parameters and the simulation of realisations for a given function. In the third section copula functions are used to link crop yields and precipitation and an introduction to the concept of partly nested Archimedean copulae is given, which links variables at different levels. This enables to construct a “more realistic” dependency between variables compared to “simple” copula functions. In the fourth section, the optimisation model is set up and the optimal production plan for given risk aversion is shown. Adding weather derivatives to the model and showing the potential of the instrument in the context of the different models is done in section five. The last section gives a short conclusion and discusses further research questions.

## 2 Functionality of copulae

Copula functions generate the dependence between variables, so they are an alternative to correlation. The basis of the copula concept is an  $n$ -dimensional (joint) distribution function ( $F^n(x)$ ). If  $\mathbf{X}' = (X_1, \dots, X_n)$  is a random vector, the joint distribution function is given by:

$$F^n(x) = F(x) = \mathbb{P}(X_1 \leq x_1, \dots, X_n \leq x_n) = \mathbb{P}(\mathbf{X} \leq \mathbf{x}) \quad (1)$$

where  $\mathbb{P}$  is the probability that  $\mathbf{X}'$  is smaller or equal to  $\mathbf{x}'$ . Given by SKLAR’S theorem (Sklar 1973, p. 449) a (copula) function  $C^n$  exists, representing the distribution function ( $F^n(x)$ ) with the margins  $F_1, F_2, \dots, F_n$  – defined as  $F_i(x_i) = \mathbb{P}(X_i \leq x_i) = \mathbb{P}(X_1 \leq \infty, \dots, X_i \leq x_i, \dots, X_n \leq \infty)$  – such that:

$$F^n(x) = C^n(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) = C^n(u_1, u_2, \dots, u_n) \quad (2)$$

with  $u_i = F_i(x_i)$

Equation (2) says that each multivariate distribution function could be represented by the cumulative marginal distributions – which are uniform on  $[0, 1]$  – and a structural relationship given by the functional form (Härdle 2009, p. 4f.)

In this article the family of Archimedean copulae is used. This copula function describes the dependency by generators ( $\rho$ ), such that:

$$C(u_1, u_2, \dots, u_n) = \rho^{[-1]}(\rho(u_1) + \rho(u_2) + \dots + \rho(u_n)) \quad (3)$$

$\rho$  is a continuous, strictly decreasing function with  $\rho(1) = 0$  and  $\rho(0) \leq \infty$ . The range is given by  $[0, 1]$  and the domain by  $[0, \infty]$ . The pseudo-inverse<sup>1</sup> of  $\rho$  is the function  $\rho^{[-1]}$ , given by (Nelsen 1999, p. 90):

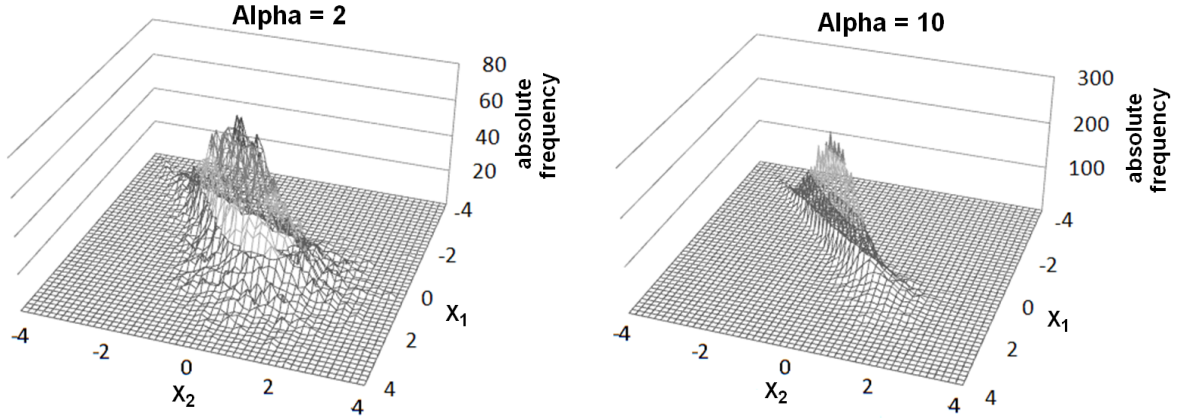
$$\rho^{[-1]}(x) = \begin{cases} \rho^{-1} & , 0 \leq x \leq \rho(0) \\ 0 & , \rho(0) \leq x \leq \infty \end{cases} \quad (4)$$

Reasons for applying Archimedean copulae are e.g. the easy way of construction, the great variety of families of copulae which belong to this class, and the useful properties possessed by the members of this class (Nelsen 1999, p. 89). One of the properties of a Clayton copula (one special family) is the lower tail dependence and a weight distribution of the values at the upper part (if  $\alpha$  is chosen greater than 0). In principle, this seems to fit the behaviour of crop

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<sup>1</sup>In case of  $\rho(0)=\infty$  the inverse, because  $\rho^{[-1]} = \rho^{-1}$

Figure 1: Density functions for different Clayton copulas



The density functions are generated by 10,000 simulated values

yields depending on a precipitations index. Without any water, there would be no yield at all, with enough water the yield would increase, but the influence of other factors leads to more variation<sup>2</sup>. The generator function in the case of a Clayton copula is:

$$\rho_{\alpha}(u) = \frac{1}{\alpha}(u^{-\alpha} - 1) \quad , \text{with } \alpha \in [-1; \infty) \setminus \{0\} \quad (5)$$

$\alpha$  is a parameter which determines the strength of the relationship between the two variables. In case of  $n$  dimensions the Clayton copula is given by (Schulz 2008, p.522):

$$C^n(u_1, \dots, u_n) = \max\left([u_1^{-\alpha} + \dots + u_n^{-\alpha} - n + 1]^{-1/\alpha}; 0\right) \quad (6)$$

To get a feeling for the behaviour of the Clayton copula and its properties, figure 1 shows two functions, with different values of  $\alpha$  and standard normal marginal distributions  $F_i(x_i)$ . Using asymmetric marginal distributions could lead to a kink in the scatter plot. This would be a usefull property to model a behaviour related to non-linear production functions, but for better comparison to the correlation model, normal distributions are chosen for this example.

## 2.1 Estimation of copula parameters

A common approach to estimate parameters of a copula function ( $\alpha$ ) is the canonical maximum likelihood method (cf. Savu, Trede 2006, Xu 2010). This method, presented in equation (7) for two dimensions, maximizes the sum of the logarithm of the copula density functions (c) by adjusting  $\alpha$  (Savu, Trede 2006, p. 12)

$$\hat{\alpha} = \operatorname{argmax} \sum_{t=1}^T \ln c_{\alpha}^2(u_1^t, u_2^t | \alpha) \quad , \text{with } c_{\alpha}^2(u_1^t, u_2^t | \alpha) = \frac{\delta^2 C_{\alpha}^2(u_1^t, u_2^t)}{\delta u_1^t \delta u_2^t} \quad (7)$$

For small datasets this method is fragile against outliers. Time series for agricultural yields are in most cases either short or the influence of technical progress is hard to measure, why the non-parametric method by GENEST und RIVEST is used. This method is robust against outliers, better adapted to small sample sizes and does not require exact knowledge about the marginal distributions (Schulz 2008, p. 294).

<sup>2</sup>E.g. Mitcherlich (1922, S. 53ff.) uses the logarithm for two factor production functions. This leads in principle to this behaviour.

To estimate  $\alpha$  by this method, the value of Kendall's tau( $\tau$ ) calculated from the dataset (8) has to be equal to the  $\tau$  resulting by the functional form of the copula (9) (Schulz 2008, p. 285f.). The solution for (9) using a Clayton copula is given in (10). Kendall's  $\tau$  is a method for the measuring dependency between random variables, in the case of two random vectors  $(\mathbf{X}', \mathbf{Y}')$  calculated as the difference between the probability of concordant pairs and discordant pairs<sup>3</sup> (Nelsen 1999, p. 126ff.).

$$\tau = \mathbb{P}[(X_i - X_j) * (Y_i - Y_j) > 0] - \mathbb{P}[(X_i - X_j) * (Y_i - Y_j) < 0] \quad (8)$$

$$\tau = 1 + 4 \int_0^1 \frac{\rho_\alpha(u)}{\rho'_\alpha(u)} \delta u \quad (9)$$

$$\tau = \frac{\alpha}{\alpha + 2} \quad (10)$$

## 2.2 Simulation of realizations

An approach to obtain realizations for the  $n$  random variables of a given (Archimedean) copula  $(C = C^n(u_1, \dots, u_n))$  is the conditional inversion method. It is  $C^k(u_1, \dots, u_k) = C^k(u_1, \dots, u_k, 1, \dots, 1)$ , with  $k = 1, \dots, n$ , and for  $k = 1$  the resulting copula is  $C^1(u_1) = u_1$ ,  $C^n(u_1, \dots, u_n)$  for  $k = n$ .<sup>4</sup> Then the conditional distribution function for  $U_k$  with known  $U_1$  to  $U_{k-1}$  is given by:

$$C_{\alpha,k}^n(u_k | u_1, \dots, u_{k-1}) = \mathbb{P}(U_k \leq u_k | U_1 = u_1, \dots, U_{k-1} = u_{k-1}) = \frac{\frac{\delta^{k-1} C_{\alpha,k}^n(u_1, \dots, u_k)}{\delta u_1 \dots \delta u_{k-1}}}{\frac{\delta^{k-1} C_{\alpha,k-1}^n(u_1, \dots, u_{k-1})}{\delta u_1 \dots \delta u_{k-1}}} \quad (11)$$

with  $k = 2, \dots, n$

The simulation algorithm involves the following steps:

- Generating  $n$  independent, uniform distributed random variables ( $v$ )
- Setting  $u_1$  equal to  $v_1$
- building the conditional distribution function (11) with  $k = 2, \dots, n$  and solving step by step the equation  $u_k = C_k^{-1}(v_k | u_1, \dots, u_{k-1})$

The inverse function can either be obtained analytically or numerically (Savu, Trede 2006, p. 10f.).

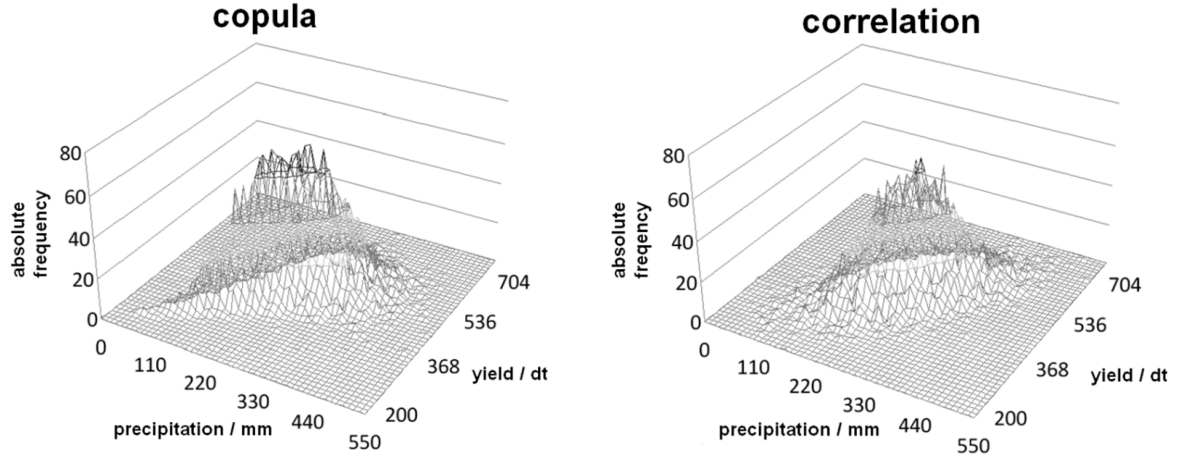
## 3 Farm structure modelled by copulae

The dependence between variables is often simulated by correlation matrices (cf. Berg 2003, Hirschauer, Mußhoff 2009). This approach has several disadvantages, e.g. tail dependence cannot be modelled appropriately, because linear correlation leads to elliptically spread realizations (Schulz 2008, p. 243). But it seems to be realistic that missing precipitation certainly leads to low yields, whereas higher yields, however with increasing uncertainty occur if precipitation is sufficient, because of other yield generating factors. Since the amount of precipitation is the

<sup>3</sup>Concordant: pairs of two vectors  $(\mathbf{X}', \mathbf{Y}')$  for which either  $x_i < x_j$  and  $y_i < y_j$  or  $x_i > x_j$  and  $y_i > y_j$ . discordant: pairs of two vectors  $(\mathbf{X}', \mathbf{Y}')$  for which either  $x_i < x_j$  and  $y_i > y_j$  or  $x_i > x_j$  and  $y_i < y_j$

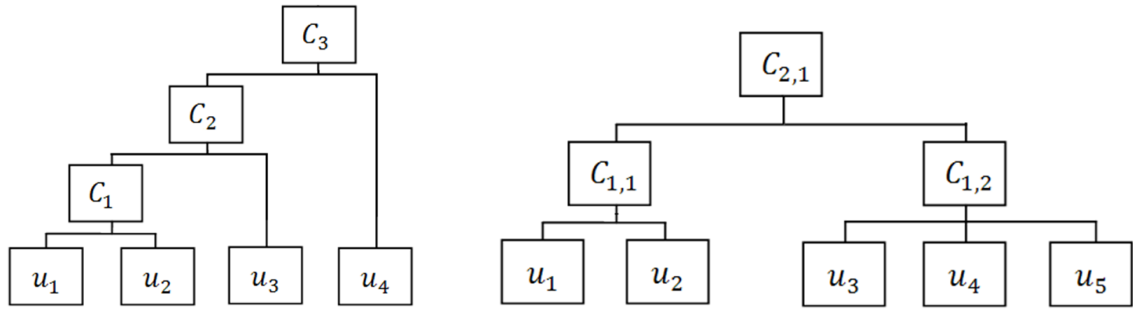
<sup>4</sup>Because  $C^k(1, \dots, 1, u_m, 1, \dots, 1) = u_m$  (Sklar 1973, p. 451);  $u_i = 1 = F(\infty) = \mathbb{P}(x_i \leq \infty)$

Figure 2: Comparison of dependency structures



Absolute frequency given 10,000 simulated values

Figure 3: Structure of (left) FNA and (right) PNA Copula



similar to: Köck 2008, p. 68–69

same for all plants in a given small area, the relationship between different crops is similar, so that the Clayton copula<sup>5</sup> is also suitable to connect the different crops in this model. The impact of the chosen model is shown exemplarily in figure 2, connecting precipitation and potato yield. The parameters are calculated from the same dataset assuming a normal distribution to describe the margins. The distribution parameters are placed in table 1, the correlation coefficient is 0.65, the copula parameter 2.9.

A disadvantage of Archimedean copula is, that the position ( $i$ ) of a marginal distribution ( $u_i$ ) in the copula function is unimportant, because the dependence is simply given by the chosen generator and the copula parameter. To overcome this problem, the concept of hierarchical Archimedean copulae is used (Köck 2008, p. 67).

The literature distinguishes between the concept of “fully nested Archimedean” (FNA) and “partially nested Archimedean” (PNA) Copula. The principle structure is given in figure 3. With both concepts the marginal distributions ( $u_i$ ) are linked at different levels, and for each copula a new family and parameter could be chosen (Härdle 2009, p. 14).

The dependence between the cultures and the weather index (precipitation) shall be modelled by PNA-copula.  $C_{l,j}$  describes a copula, with  $l = 1, \dots, L$  the number of levels and  $j = 1, \dots, n_l$  the number of copulae on each level. The variables ( $u_i$ ) of the lowest level ( $l = 1$ )

<sup>5</sup>Also other copula families have similar properties, clayton copula is simply chosen to show general effects of the applied method.

are connected with  $n_l$  copulae, each copula (equation 12) connects  $d_{l,j}$  variables with the copula parameters ( $\alpha_{l,j}$ ) and the generators ( $\rho_{l,j}$ ).

$$C_{1,j}(u_{1,j,1}, \dots, u_{1,j,d_{1,j}}) = \rho_{1,j}^{-1} \left( \sum_{m=1}^{d_{1,j}} \rho_{1,j}(u_{1,m}) \right) \quad (12)$$

This procedure continues, connecting copulae of the different levels ( $l$ ) with copulae of the level ( $l-1$ ) below (see equation 13), until the highest level ( $L$ ) is achieved.

$$C_{L,1}(C_{L-1,1}, \dots, C_{L-1,d_{L,j}}) = \rho_{L,j}^{-1} \left( \sum_{m=1}^{d_{L,j}} \rho_{L,j}(C_{L-1,m}) \right) \quad (13)$$

It is a condition for the hierarchical copula that the parameters for each level satisfy  $\alpha_{l+1,i} < \alpha_{l,j}$  for all  $l = 1, \dots, L$ ,  $j = 1, \dots, n_l$  and  $i = 1, \dots, n_{l+1}$  (Savu, Trede 2006, p. 5ff.). Following this rules, the model with the variables precipitation (cumulated over the period May to August) ( $u_1$ ), potatoes ( $u_2$ ), silage maize ( $u_3$ ), grain maize ( $u_4$ ), winter barley ( $u_5$ ) and winter rye ( $u_6$ ) is build up with data of a farm located nearby Bremervörde (Germany). The copula parameters are calculated by equation (8) and (10), connecting the highest realisations before calculating the parameters of the next level. It is a useful property, if the structure of the copulae results in a naturally interpretation (cf. Savu, Trede 2006, S. 2). In this dataset potatoes and precipitation are paired because the time horizon for precipitation is chosen to fit best for potatoes, according to the correlation coefficient. Maize ( $C_{1,2}$ ) and cereals ( $C_{1,3}$ ) are also paired, so that there is a relation between the variables. This leads to the hierarchical structure shown in equation (14).

$$C_{2,1} = \left( \left[ (u_1^{-\alpha_{1,1}} + u_2^{-\alpha_{1,1}} - 1)^{\frac{-1}{\alpha_{1,1}}} \right]^{-\alpha_{2,1}} + \left[ (u_3^{-\alpha_{1,2}} + u_4^{-\alpha_{1,2}} - 1)^{\frac{-1}{\alpha_{1,2}}} \right]^{-\alpha_{2,1}} + \left[ (u_5^{-\alpha_{1,3}} + u_6^{-\alpha_{1,3}} - 1)^{\frac{-1}{\alpha_{1,3}}} \right]^{-\alpha_{2,1}} - 2 \right)^{\frac{-1}{\alpha_{2,1}}} \quad (14)$$

with  $\alpha_{1,1} = 2,2$ ,  $\alpha_{1,2} = 2,9$ ,  $\alpha_{1,3} = 1,2$  and  $\alpha_{2,1} = 1,1$

## 4 Model optimisation

The model represents a hypothetical 100 hectare arable farm with the crops named before. Rotational restrictions are set at a 25% share of potatoes ( $u_2$ ) and 60% share of cereals ( $u_5 + u_6$ ). The workload for fieldwork is limited at 1,500 hours, as well as by the monthly available days for fieldwork ( $lm$ ) (cf. Betriebsplanung Landwirtschaft, 2006). The yields are assumed to be normally distributed (table 1) and the parameters are calculated from historical data. Normal marginal distributions are not necessary in principle, but they are used in the example because of better comparison to the correlation based model. Recognize that the assumption of normally distributed margins does not mean that the whole distribution is described by a multivariate normal distribution. The prices ( $p_i$ ) in table 1 are arbitrary chosen, not listed working hours per crop, month and hectare ( $l_{m,n}$ ) are based on the "KTBL Betriebsplanung Landwirtschaft" (Betriebsplanung Landwirtschaft, 2006) database. The area payment ( $PL$ ) in 2010 for Lower Saxony was 266 €/ha arable land.

The optimisation is based on 1,000 ( $k$ ) realisations for each crop yield ( $y_{i,n}$ ) and the respective precipitation. The common behaviour of the variables is generated by copulae on one hand (see equation 14; section 2.2), and by correlation matrices (table 2) on the other hand. For each realised yield the gross margin per hectare is calculated using the figures given in table 1. This leads for a given production plan to a sample of 1,000 whole farm gross margins ( $GM_i$ ). The optimisation of the cropping plan is based on the lower partial moment 1 ( $LPM_1$ ),



Table 1: Basis data for the optimised farm

		Precipitation (may-aug)	Potatoes ( $u_2$ )	Silage maize ( $u_3$ )	Grain maize ( $u_4$ )	W. barley ( $u_5$ )	W. rye ( $u_6$ )
Distribution- parameters	Mean Standard derivation	279 mm 66 mm	517 dt/ha 73 dt/ha	453 dt/ha 80 dt/ha	98 dt/ha 17 dt/ha	74 dt/ha 15 dt/ha	84 dt/ha 9 dt/ha
Production	Price / dt		7.00 €	2.80 €	13.50 €	11.00 €	9.00 €
	Variable costs / ha		1,176 €	715 €	725 €	457 €	457 €
Farm labour	year( $ly$ ) / ha		24.7 h	16.1 h	7.6 h	6.4 h	6.4 h

Table 2: Correlation matrix

	Precipitation (may-aug)	Potatoes	Silage maize	Grain maize	W. barley	W.rye
Precipitation (may-aug)	1	0.65	0.50	-0.22	0.16	-0.09
Potatoes		1	0.57	-0.01	0.31	0.09
Silage maize			1	0.64	0.12	0.08
Grain maize				1	0.08	0.22
W. barley					1	0.68
W.rye						1

which represents the shortfall expectation (cf. Starp 2006, p. 62ff.). The results are presented in an expected value- shortfall expectation diagram. In both models the optimisation is done by maximising the expected gross margin ( $E(GM_{opt})$ ) subject to an upper limit on the shortfall expectation ( $SE$ ) by means of varying the crop mix ( $u_n$ ):

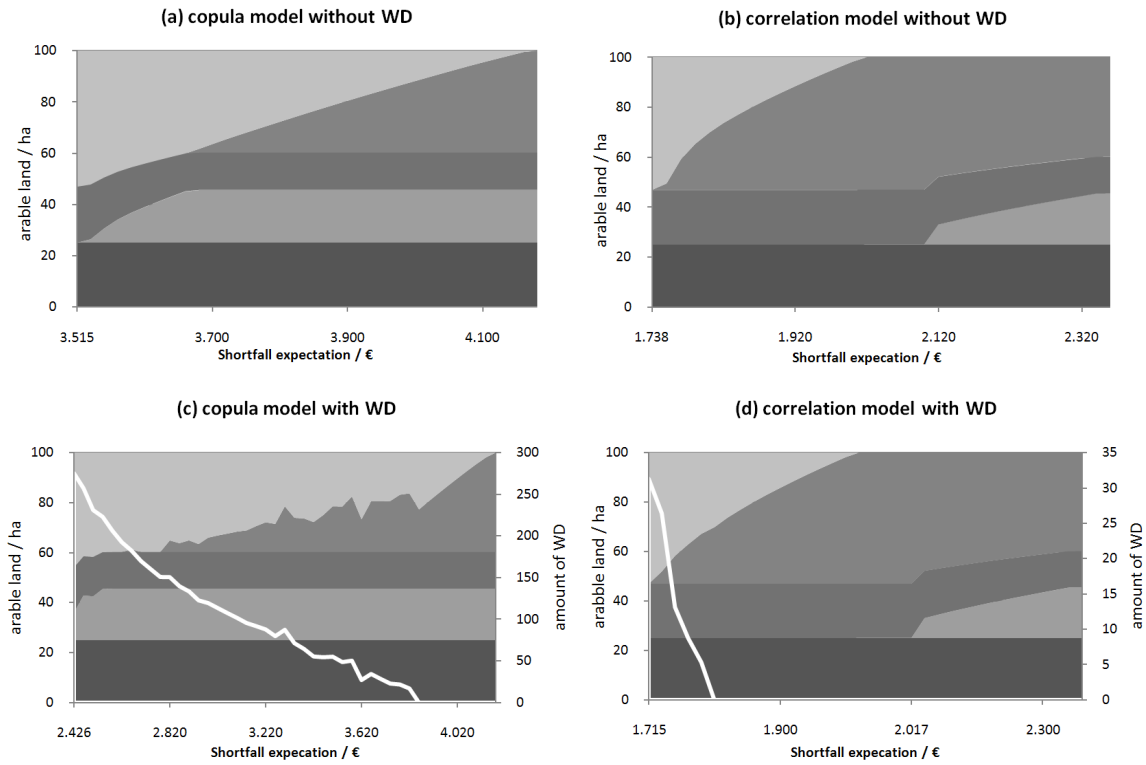
$$\begin{aligned}
E(GM_{opt}) &= \frac{1}{k} \cdot \operatorname{argmax} \sum_{i=1}^k GM_i \\
&= \frac{1}{k} \cdot \operatorname{argmax} \sum_{i=1}^k \left[ \sum_{n=2}^6 (y_{i,n} \cdot p_n - c_n^{var} + PL) u_n \right. \\
&\quad \left. + (\max(SL - pr_i, 0) - P) \cdot u_{WD} \right]
\end{aligned} \tag{15}$$

s.t.

$$\begin{aligned}
LPM_1(T_{GM}) &= \sum_{i=1}^k \varphi_i (T_{GM} - GM_i) \cdot a & , \text{ with } a & \begin{cases} 1, \text{ if } GM_i \leq T_{GM} \\ 0, \text{ if } GM_i > T_{GM} \end{cases} \\
u_n &\geq 0 & , \text{ for each } n &\in \{2, \dots, 6\}; & u_{WD} &= 0 \\
\sum_{n=2}^6 u_n &\leq 100; & & & \sum_{n=2}^6 ly_n \cdot u_n &\leq 1,500 \\
\sum_{n=2}^6 l_{m,n} \cdot u_n &\leq lm_n & , \text{ for each } n &\in \{2, \dots, 6\} \\
u_2 &\leq \frac{u_2}{\sum_{n=2}^6 u_n}; & & & u_5 + u_6 &\leq \frac{u_5 + u_6}{\sum_{n=2}^6 u_n}
\end{aligned}$$

The second term of the objective function in (15) is zero as long as  $u_{WD} = 0$ , which excludes the use of derivatives. To obtain the shortfall expectation, the difference between a target gross margin ( $T_{GM}$ ) and all realisations (of the gross margin;  $GM_i$ ) below this target are calculated, multiplied with their associated probabilities ( $\varphi_i$ ) and summed up. In principle, the target value could be chosen free. In the example it seems to be realistic to choose a value of 100,000 €, which is needed to cover the fixed cost as well as the personal living expenses of the farm family. In this model the shortfall expectation is used to describe the risk component in the model,

Figure 4: a-d: Optimal production programme by fixed shortfall expectation



Crops from bottom to top: potatoes, silage maize, grain maize, winter barley, winter rye; white line: weather derivatives (WD)

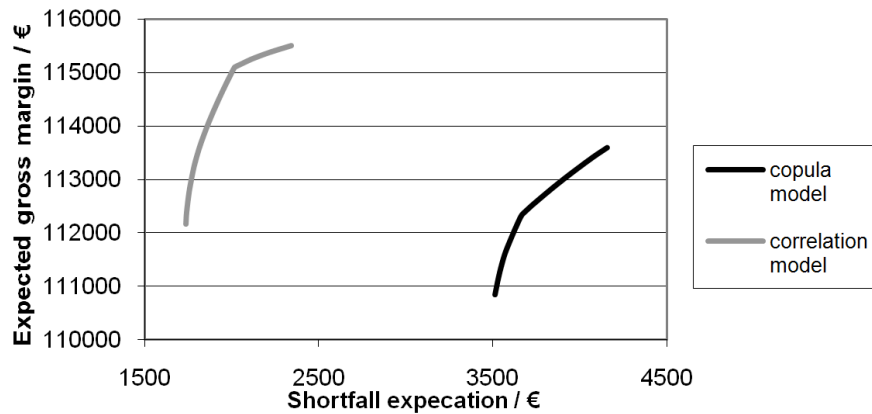
Table 3: Comparison of the models

		for both max	without WD min	dif. to max	with WD min	dif. to max
Correlation	SE / €	2,341	1,738	603	1,715	626
	$E(GM_{opt})$ / €	115,507	112,167	3,339	111,637	3,870
Copula	SE / €	4,168	3,515	654	2,426	1,742
	$E(GM_{opt})$ / €	113,624	110,837	2,786	107,365	6,259

so only downside risk is taken into account (cf. Starp 2006, p. 63). This instrument seems to fit quite well because of the later introduced weather derivatives, which aim at compensating shortfalls in crop yield. Variance based instruments would be less appropriate, because increasing skewness of the outcome distribution would bias the risk measure (Starp 2006, p. 60). An optimally constructed weather derivative should cut the lower part of the distribution, thus resulting in asymmetry (cf. Berg et al. 2005, p. 164).

A risk neutral farmer will maximise the expected profit and would not care of the shortfall expectation. That leads in both models to ca. 25 ha potatoes, 20 ha silage maize, 15 ha grain maize and 40 ha winter barley as the optimal production program. The shortfall expectation in the copula model case amounts to 4,168 €, in the correlation model to 2,341 €. The possible reduction of the shortfall expectation (i.e. the potential risk reduction) is approximately equal in both models, the amount in the correlation model is 603 € and in the copula model 653 €. This reduction comes at a price of a 3,339 € (2,786 €) reduction of the expected gross margin in the correlation (copula) model (cf. table 3). Interpreting the  $E(GM_{opt})$  in table 3, an amount of 100,000 € must be subtracted to obtain the surplus after living costs. The impact of risk reduction on the optimal crop mix of both models is shown in figure 4 a-b. In both models the production programme changes similarly. With increasing accepted shortfall expectation the

Figure 5: Expected gross margin- shortfall expectation diagrams for the copula and correlation model



using 100,000 € as target value

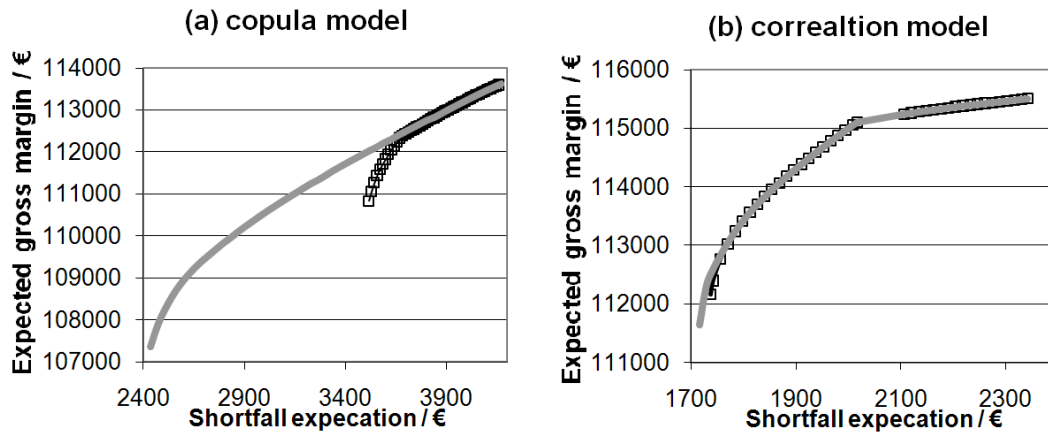
crops with lower variance are reduced (grain maize and rye) and substituted by the crops with higher variance (silage maize and barley). Figure 5 shows the expected gross margin -shortfall expectation diagram. It can be seen, that the correlation model always exhibits higher expected values for given shortfall expectations.

## 5 Implementation of a weather derivate

In this section a risk management instrument is added to the model. A weather derivative (WD) is implemented, paying a compensation if an index value ( $pr$ ), measured as the cumulated precipitation, falls below a fixed level (strike level). The derivative provides a compensation of one € per index point below the strike level, the expected indemnity (fair premium plus 10%) are charged as premium ( $P$ ) for each weather derivative. The strike level ( $SL$ ) is set at 279 mm cumulative precipitation (the mean of the distribution). This leads to a premium of ca. 29 € per weather derivative. The optimisation is done as described in section 4, the only difference is the opportunity to buy weather derivatives (now  $u_{WD} \geq 0$ ) to stabilise the gross margin. Not least, this leads to an asymmetric - and thus not normally - distributed gross margins in both models, making variance based risk measures inappropriate. The model results are presented in figure 4 c-d. In the correlation model a maximum of 31 weather derivative contracts are purchased. This small number of derivatives is caused by diversification effects of the cropping program caused by the partly negative correlation between crops and precipitation (cf. table 2). The purchased weather derivatives causes only minor changes in the risk behaviour, presented in figure 6 b, and hardly any modification of the production programme. This leads only to (little) higher expected values of gross margin in the area below 1,800 € shortfall expectation. A boundary value is given in this point, where no weather derivative is bought any more, because higher expected shortfall expectations lead to the same results as in the model without weather derivatives. The possible reduction of the shortfall expectation is 626 €. Compared to 603 € in the model without weather derivatives, there is only little risk reduction. This causes an extra decrease of expected gross margin by 531 € compared to the model without derivatives.

The copula model shows a different behaviour. Using weather derivatives, the shortfall expectation could be reduced to 2,426 € (from 3,515 €). This is an absolute reduction of 1,742€, which is a high amount compared to 653€ in the model without weather derivatives. At the point of lowest shortfall expectation 275 weather derivatives are bought. This leads to

Figure 6: a-b: Expected gross margin - shortfall expectation diagrams with and without weather derivatives



using 100,000 € as target value; gray lines (black lines with squares) model with (without) weather derivatives

an expected gross margin of 107,365 €. Figure 6a shows, that the shortfall expectation below 3,860 € (boundary value with no weather derivatives) are clearly below the values of the model without weather derivatives.

## 6 Conclusions

The comparison of the two models illustrates the differences between the applied methods of connecting random variables. Without weather derivatives, the general pattern of crop mix changes in the course of risk reduction (i.e. reduction of accepted shortfall expectation) is similar in both modes: crops with higher variability of gross margins are reduced and substituted by those with lower variability. The main difference occurs with respect to the estimated level of risk exposure. If the underlying assumptions used in the copula approach approximate the reality reasonably well, a model based on correlations significantly underestimates the risk embedded in the production program. Consequently, the potential risk reducing effect of an instrument like weather derivatives is also underestimated by a correlation based approach.

In modeling weather related crop yield uncertainty, copula based models appear advantageous because they allow incorporating prior knowledge about nonlinearities and relationships between crops by selecting the appropriate type of copula function. Particularly the tails of yield distributions can be represented more accurately, which is highly relevant if it comes to assessing the potential benefits of index based risk management instruments like weather derivatives or index based insurance policies.

In our example correlation model showed hardly any risk reduction through the use of weather derivatives, due to the significant basis risk implied by the correlation matrix. Contrary to this, the copula model indicated a high risk mitigation potential of the same instrument, due the presumably more appropriate modeling of existing tail dependency. Since the impacts of weather derivatives so far have mainly been assessed through correlation based models, the results of our paper indicate that the potential benefits of these risk management instruments might often be underestimated.

In summary, copula based modeling approaches appear quite promising. Because of the vast variety of possibilities provided by the copula approach in general, further research is needed to identify ten most suitable types. Further studies could use modified copula models, e.g. considering skewed marginal distributions to capture nonlinear structures or adding more random

variables (e.g. prices) to the model. Incorporating weather forecasts along with conditional distributions could be another useful direction to expand the models.

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