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A note on the concordance correlation coefficient

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Abstract.

Program `concord` implements L. I. Lin's concordance correlation coefficient (Lin, 1989), as well as the limits-of-agreement procedure (Bland and Altman, 1986). Recently, Lin (2000) issued an erratum reporting a number of typographical errors in his seminal 1989 paper. Further, changes in Stata Version 7 required modification of `concord`. This note reports the effect of the errors and provides a corrected and updated program.

Keywords: st0015, concordance correlation, graphics, measurement comparison, limits-of-agreement

1 Description

`concord` computes the concordance correlation coefficient for agreement on a continuous measure obtained by two persons or methods, and provides optional graphical displays. It also provides the Bland and Altman limits-of-agreement assessment. A full description of the method and of the operation of the command was given by Steichen and Cox (1998a), with revisions and updates in Steichen and Cox (1998b, 2000, 2001). The operation of the program remains as previously documented.

Before publication of this note, changes required for proper operation under the new sort-stability functionality of Stata 7 and for the handling of long variable names were incorporated in a new version made available only at the SSC archive site at *http://ideas.uqam.ca/ideas/data/bocbocode.html*. Those changes are also incorporated into the current version.

This note implements the corrections required by an erratum issued by Lin in 2000, analyzes the effect of the errors, and provides the corrected and updated program.

2 Explanation

Lin's erratum reports a number of typographical errors, only one of which affects the computations in `concord`. In particular, the variance of $\widehat{\rho}_c$, the concordance correlation coefficient, was reported in the 1989 paper as

$$\sigma^2_{\widehat{\rho}_c} = \frac{1}{n-2}\left\{(1-\rho^2)\rho_c^2(1-\rho_c^2)/\rho^2 \;+\; 4\rho_c^3(1-\rho_c)u^2/\rho \;-\; 2\rho_c^4u^4/\rho^2\right\}$$

and was implemented as such in `concord`. The corrected formula is

$$\sigma^2_{\widehat{\rho}_c} = \frac{1}{n-2}\left\{(1-\rho^2)\rho_c^2(1-\rho_c^2)/\rho^2 \;+\; 2\rho_c^3(1-\rho_c)u^2/\rho \;-\; \rho_c^4u^4/2\rho^2\right\}$$

The formulas differ only in that the constant 4 in the second term of the section in square brackets should be 2 and the constant 2 in the third term should be in the denominator.

Lin claims that these errors have "negligible effect since the second and third terms are usually small in the opposite directions". We evaluate the correctness of this assertion in the next section.

3 Assessment of the error

The variance of $\widehat{\rho}_c$ is a function of four values, n, ρ, $\widehat{\rho}_c$, and u (and an assumption of bivariate normality in the underlying data). Of the four values, only ρ, the usual (Pearson) correlation coefficient, and n, the sample size, can be used directly in defining bivariate datasets that might be used to assess the impact of the error in the variance formula. Note, however, that $\widehat{\rho}_c$ is defined as $2\rho/(v+1/v+u^2)$, where $v=\sigma_1/\sigma_2$ (the scale shift), and $u=(\mu_1-\mu_2)/\sqrt{\sigma_1\sigma_2}$ (the location shift relative to the scale). These formulas state u in terms of the means and variances from the underlying bivariate data. Thus, given the scale shift, the location shift, and the correlation between the two bivariate normal variables, we can control all four values in the variance formulas by manipulating simple parameters of the underlying data.

3.1 Simulation data

For our assessment, we generated (using a modified version of Jenkins' `mkbilogn` program) 5000 bivariate normal random datasets of size $n=50$, where the correlation (ρ) ranged from -1 to 1, the scale shift ($v=\sigma_1/\sigma_2$) ranged from .25 to 4, and the offset ($\mu_1-\mu_2$) ranged from -2.5 to 2.5. We held n, the sample size, constant at 50 as it only affects the magnitude of the error, not the form or proportionality of the error.

The difference between the old (incorrect) standard error, $\sigma_{\widehat{\rho}_c}$, and the new (corrected) value was computed for each of these datasets and then analyzed.

3.2 Results

Figure 1 shows the resulting distributions of the old (incorrect) and the new (correct) standard errors, $\sigma_{\hat{\rho}_c}$. It is evident that the correct formula results in a smoother, more compact distribution.

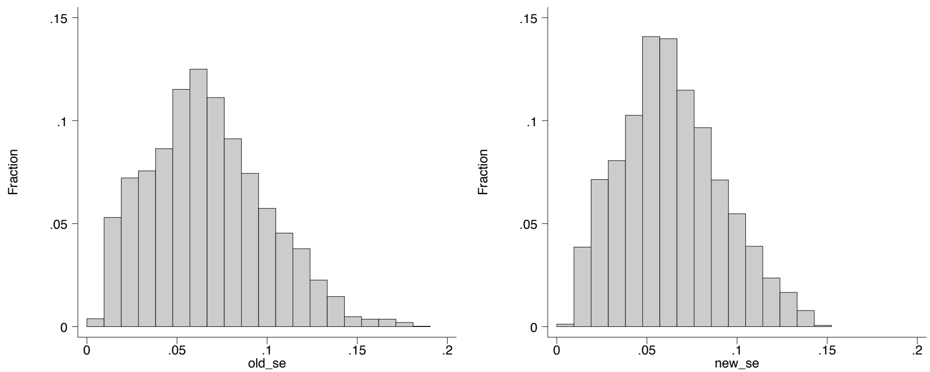

Figure 1: Distributions of the incorrect and correct standard errors

Figure 2 shows the error, i.e., the difference computed as the correct standard error minus the incorrect standard error, plotted against the correct standard error and against the computed value of ρ_c.

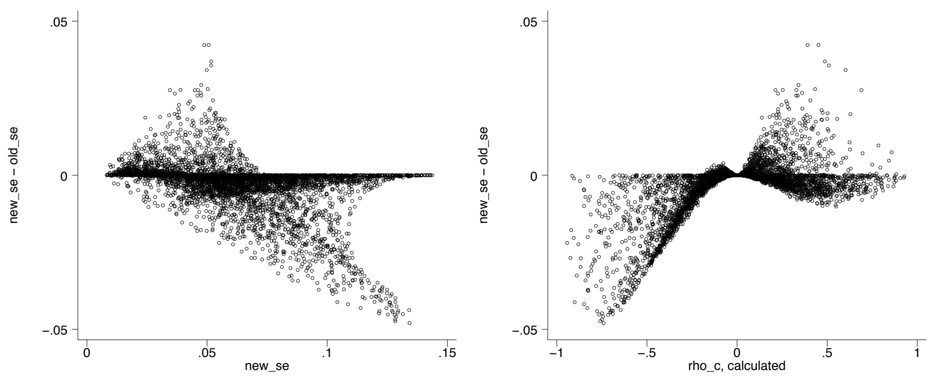

Figure 2: Error in $\sigma_{\hat{\rho}_c}$ versus the correct $\sigma_{\hat{\rho}_c}$ and ρ_c

The errors were strongly patterned and in the range of −.05 to .05, with most differences near 0. As the values of the correct standard errors range from near 0 to about .15, a difference of .05 can be considered to be quite large.

We extend this analysis, noting that one would likely most require an accurate variance calculation when the two measures under investigation are nearing perfect concordance. This occurs when the location shift is small and the sd ratio and underlying correlation are near 1. Figure 3 shows that small offsets had the least effect on the error and that the greatest differences occurred for underlying correlations with absolute value near 1 and for scale shifts (sd ratios) near 1.

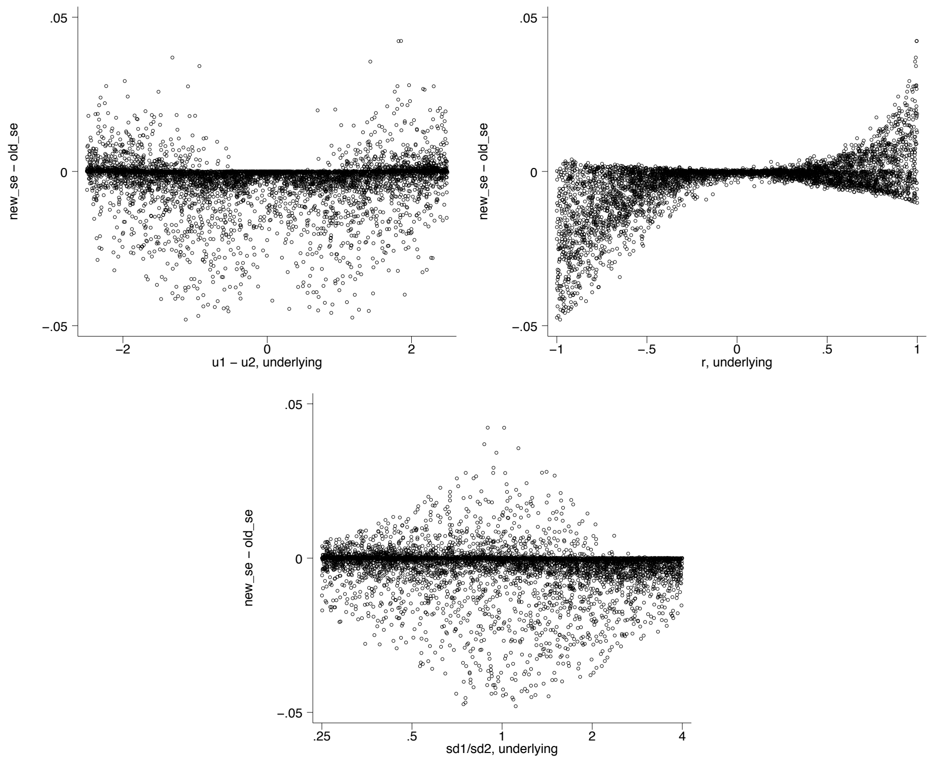

Figure 3: Error in $\sigma_{\hat{\rho}_c}$ versus the underlying location shift, correlation, and sd ratio.

The most egregious errors occurred for datasets with underlying strong positive correlation and sd ratio 1 (they are the upper-left edge of points rising from 0 up toward .05 in the left-side graph in Figure 2). These points are of particular concern because the old (incorrect) formula generated near-zero variances when the true variance was substantially higher. Such an error leads to claims of highly reliable concordance when such a claim may be unwarranted by the data.

Situations where the incorrect formula overestimates the true variance (i.e., the negative points in the right-side graph of Figure 2) are not of as great a concern, as such errors generally occurred when there was a strong negative concordance, a situation seldom of interest.

3.3 Concordance

It seems only reasonable that the relationship of the old (incorrect) variance formula to the new (correct) variance formula be assessed by a program designed for this purpose: `concord`. Here is the analysis of the simulated data using the corrected program:

```
. concord new_se old_se
Concordance correlation coefficient (Lin: 1989, 2000)
 rho_c   SE(rho_c)   Obs    [   95% CI   ]      P        CI type
-----------------------------------------------------------------
 0.954     0.001    5000     0.952  0.956    0.000   asymptotic
                             0.952  0.956    0.000  z-transform
Pearson's r =  0.971  Pr(r = 0) = 0.000  C_b = rho_c/r =  0.983
Reduced major axis:   Slope =     0.859   Intercept =     0.006
   Difference (new_se - old_se)    95% Limits Of Agreement
   Average    Std. Dev.             (Bland & Altman, 1986)
-----------------------------------------------------------------
    -0.003       0.009                  -0.020      0.014
```

We first comment on the concordance correlation results. The printout reveals a substantial concordance (`rho_c`) of .954, but one that does not approach 1 (95% CI: .952, .956). This results both from a lack of perfect correlation (`Pearson's r = .971`) and from bias (`C_b = .983`). The reduced major axis reveals a slope less than one (thus, the true variance does not rise as rapidly as the incorrect values) and a positive intercept (thus, the true variance is higher than indicated by low values of the incorrect variance). The concordance plot in Figure 4 supports this assessment, as it shows systematic deviation from the (dashed) line of perfect concordance.

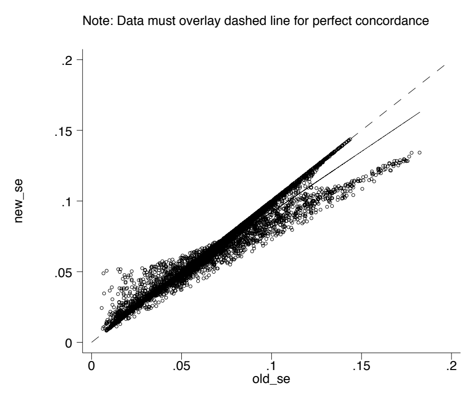

Figure 4: Concordance plot.

We now consider Bland and Altman's limits-of-agreement results. The limits-of-agreement measure, at −.003 (95% CI: −.020, .014), does not indicate significant *average* departure from agreement, which provides some support for Lin's assertion of negligible effect. However, this measure is only interpretable when there are not significant departures from normality. Figure 5 provides the limits-of-agreement plots. Both the limit plot (on the left) and the Normal plot (on the right) reveal departures from normality. The limit plot shows that the points outside of the 95% confidence interval are clearly not randomly distributed. The Normal plot likewise reveals departures throughout the data range.

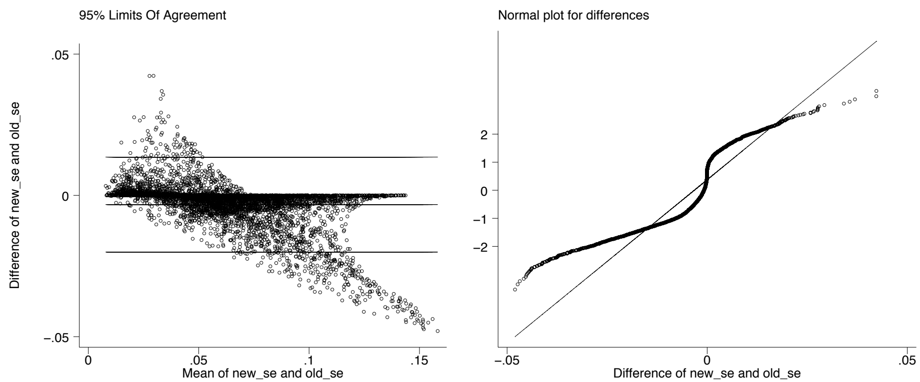

Figure 5: Limits-of-agreement plots.

3.4 Conclusions

Lin's erratum suggests that his error should have "negligible effect". Our examination suggests that, while there are situations where he is reasonably correct, there are important situations where the calculation error leads to important interpretational error. In particular, the error is most egregious when the assessed relationship approaches a strong concordance.

We strongly recommend that any analyses performed using the incorrect formula be repeated with the corrected program.

4 A presentational change

After some consideration, we have also decided to make, for internal consistency reasons, an additional minor change in the output of `concord`. For the limits-of-agreement calculations, prior versions of `concord` computed the difference measure as $x - y$ (x being the second variable on the command line and y being the first). For this and any subsequent versions, we are reversing the direction of the difference and will now compute $y - x$. This will result only in a reversal of the sign of the difference and a flipping of the related confidence interval and graphs. Nonetheless, the change will yield more consistent output from the `concord` command.

5 Acknowledgment

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6 References

Bland, J. M. and D. G. Altman. 1986. Statistical methods for assessing agreement between two methods of clinical measurement. *Lancet* I: 307–310.

Jenkins, S. P. 1999. sg105: Creation of bivariate random lognormal variables. *Stata Technical Bulletin* 48: 18-19. In *Stata Technical Bulletin Reprints*, vol. 8, 260–261. College Station, TX: Stata Press.

Lin, L. I. 1989. A concordance correlation coefficient to evaluate reproducibility. *Biometrics* 45: 255–268.

Lin, L. I.-K. 2000. A note on the concordance correlation coefficient. *Biometrics* 56: 324–325.

Steichen, T. J. and N. J. Cox. 1998a. sg84: Concordance correlation coefficient. *Stata Technical Bulletin* 43: 35-39. In *Stata Technical Bulletin Reprints*, vol. 8, 137–143. College Station, TX: Stata Press.

—. 1998b. sg84.1: Concordance correlation coefficient, revisited. *Stata Technical Bulletin* 45: 21-23. In *Stata Technical Bulletin Reprints*, vol. 8, 143–145. College Station, TX: Stata Press.

—. 2000. sg84.2: Concordance correlation coefficient, update for Stata 6. *Stata Technical Bulletin* 54: 25-26. In *Stata Technical Bulletin Reprints*, vol. 9, 169–170. College Station, TX: Stata Press.

—. 2001. sg84.3: Concordance correlation coefficient, minor corrections. *Stata Technical Bulletin* 58: 9. In *Stata Technical Bulletin Reprints*, vol. 10, 137. College Station, TX: Stata Press.

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