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NE-165

# PRIVATE STRATEGIES, PUBLIC POLICIES & FOOD SYSTEM PERFORMANCE

## Alternative Measures of Benefit for Nonmarket Goods Which are Substitutes or Complements for Market Goods

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## WORKING PAPER SERIES



A Joint USDA Land Grant University Research Project

April 1991

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ABSTRACT

Nonmarket goods include quality aspects of market goods and public goods which may be substitutes or complements for private goods. Traditional methods of measuring benefits of exogenous changes in nonmarket goods are based on Marshallian demand: change in spending on market goods or change in consumer surplus. More recently, willingness to pay and accept have been used as welfare measures. This paper defines the relationships among alternative measures of welfare for perfect substitutes, imperfect substitutes, and complements. Examples are given to demonstrate how to obtain exact measures from systems of market good demand equations.

Thanks to Professor Deb Brown, Purdue University, for her encouragement and help over the period in which this paper was written.

Thanks also to the very helpful anonymous reviewers for Social Choice and Welfare.

# Alternative Measures of Benefit for Nonmarket Goods Which are Substitutes or Complements for Market Goods

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## Introduction

This paper concerns the measurement of benefits for nonmarket goods. Nonmarket goods are not priced directly in a market. They include public goods and quality aspects of market goods. The need for benefit measurement arises from the need to evaluate government programs or policies when nonmarket goods are provided or are regulated by a government.

Nonmarket goods may be perfect substitutes for market goods: for example, irrigation water can be either publically or privately provided. Market and nonmarket goods may also be complements: for example water quality may enhance fishing which may be privately provided. The case of imperfect substitutes has been given less attention but is also very relevant for policy purposes: for example, public health programs may substitute for private health care; also, improvements in air quality may substitute for health care and air filters.

Traditional methods of valuing nonmarket goods may be classified as being of two main types - based on either a consumer surplus measure or on change in spending for related private goods (Prest and Turvey; Mishan). The justification for use of consumer surplus is that it measures the excess of willingness to pay over actual payment for a market commodity (Currie, Murphy, and Schmitz). Consumer surplus measures are obtained as areas under Marshallian demand curves for which consumer income is held constant.

Two different consumer surplus areas have been applied for nonmarket goods. The area under the inverse demand curve, integrated over the quantity change, has been used when public and private goods are perfect substitutes (eg., irrigation water) (p. 34, Principles and Standards for Planning, Water Resources Council). The area between two demand curves has been used to measure benefits when a nonmarket good is complementary to a private good and the demand curve for the related private good shifts with change in the nonmarket good. This method has been used to evaluate recreation benefits when demand for visits to a location shifts with environmental quality (Freeman).

The change in spending for transportation with and without a transportation project has been used to measure the benefits of an investment in transportation (Prest and Turvey). For health benefits related to air quality, spending on health care which is avoided by improvements in air quality has been used to measure benefits (Ridker). Change in property values (spending on housing) is a method used to estimate air quality benefits (Freeman). Change in spending is also based on Marshallian demand, but it is more simply computed than consumer surplus since it is price times quantity for goods related to a nonmarket good.

Another method of obtaining benefit measures used particularly in environmental economics literature is "willingness to pay" and "willingness to accept" (Freeman) measured through surveys. Willingness to pay and accept measures are related to equivalent and compensating variation which are based on Hicksian rather than Marshallian demand (Maler; Currie, Murphy, and Schmitz). Compensating and equivalent variation have measures that also have been called "exact" measures because they can be expressed in terms of change in utility from a given reference point (Hause, McKenzie). Although no measure can be truly "exact" (Ng, p. 99), exact measures exhibit preferred mathematical properties such as "integrability" and "acceptability" reviewed below.

Consumer surplus has been shown to approximate exact measures under certain conditions for the case of price changes in market goods (Willig). Willig developed some rules of thumb for determining when the size of the error (the difference between exact and surplus measures) is small based on the income elasticity of demand. More recently the comparison of exact measures and consumer surplus has been made for quotas on market goods (Randall and Stoll) and for household production with public goods (Bockstael and McConnell). Consumer surplus may not be very close to exact measures in some cases (Lankford).

This paper considers the relationship among alternative types of welfare measures (change in consumer surplus areas, change in spending, willingness to pay, willingness to accept) for a general relationship (demand interdependence) between market and nonmarket goods. Although the literature has primarily focussed on complementarity between market and nonmarket goods, "demand interdependence" means more generally that a nonmarket good will observably affect the demand for related market goods, either as complements or as substitutes (perfect or imperfect).

A unified framework is given here for defining exact measures and for comparing alternative types of welfare measures for nonmarket goods. Size relationships are compared for willingness to pay, willingness to accept, consumer surplus, and change in spending, for both substitutes and complements. Knowing such size relationships is useful for policy analysis purposes - e.g. if one type of measure is more convenient to estimate than another.

Some well-known demand systems are used to demonstrate how exact welfare measures for nonmarket goods can be obtained from related market demand information. Similar to the method used by Hausman and Varian for price changes in market goods, the method of obtaining exact measures requires solving a system of differential equations based on market demand to derive the expenditure function and then using the expenditure function directly to define welfare measures.

Issues in Welfare Measurement

Past work in welfare measures concerned evaluation of price changes, while more recent literature has included evaluation of public goods and quality aspects of market goods. Below we briefly review recent welfare measurement issues regarding price changes in market goods which are applied to nonmarket goods in succeeding sections. Two mathematical criteria for welfare measures are "acceptability" and "integrability". "Observability" has been another major concern for welfare measures.

Acceptability has to do with whether a ranking of situations obtained from applying a measure would be consistent with a utility function representing a preference ordering (Hause, Chipman and Moore, McKenzie and Pearce). For a price change in a market good, the equivalent variation is considered to be an acceptable measure whereas compensating variation and consumer surplus measures are not. Because it is a monetary measure which orders choices the same as a utility function would, McKenzie and Pearce apply the term "money metric" to the equivalent variation measure. However, McKenzie and Pearce still support the application of compensating variation for compensation purposes rather than for making welfare comparisons.

"Integrability" (or path independence) is required so that market demand relations are well-defined in the case of multiple price changes (Silberberg, Takayama). As discussed by Takayama, observed demands cannot correspond to a utility maximization solution if integrability conditions do not hold. Exact measures automatically satisfy integrability because they are derived from the expenditure function whereas Marshallian demand may not satisfy these conditions.

Hausman showed that approximation of exact measures by consumer surplus or other means is not necessary because exact measures may be obtained by solving for the expenditure function from observed demand relations. In the case of demand systems the method was incompletely applied by Hausman to obtain "quasi-expenditure" functions. Varian demonstrated how to obtain the expenditure function with multiple goods by solving a system of differential equations based on market demand. Vartia and McKenzie developed related numerical methods for obtaining exact measures for demand systems without explicitly solving the system of demand equations. Earlier, Hurwicz and Uzawa gave the same representation of this solution as Vartia. Bergland and Randall have recently developed computation techniques based on control theory to implement the Vartia method.

Evaluation of nonmarket good changes is considered to present a greater problem than price changes for welfare measurement because of observability. Bradford and Hildebrandt proposed "demand interdependence" as a general condition necessary to obtain welfare measures from observable demands for private goods. However, only special cases have been studied in the literature. The case when there is a quota on a good otherwise available in a market was studied by Randall and Stoll. The quota case is a special case of perfect substitutes. Consumer benefits are obtained when restrictions are eased so that more of this good can be provided. In this case, the observable inverse demand relation can be used to measure the benefit of a change in the quota. Randall and Stoll's analysis comparing surplus and exact measures for this special case paralleled Willig's analysis for price changes, and they similarly demonstrated that the area under an inverse Marshallian demand curve, integrated over quantity, may be close in value to exact measures. Lankford disagreed that such error is small because of income effects, using a numerical example to demonstrate the potential size of the error. He showed that two income effects must be considered -- the direct effect due to change in consumption of the good with the quota and the indirect effect due to subsequent utility adjustments;

Maler introduced the concept of "weak complementarity" -- when a nonmarket good produces no benefit in the absence of the market good -- and showed that this relationship can be used to derive exact measures for a nonmarket good from observable market demand. Bradford and Hildebrandt also studied only the weak complementarity case.

Willig studied another market relationship case -- when market goods have a quality aspect which affects demand. Consumer surplus was shown to be a valid measure of quality benefit for the special case when the demand function is of the "repackaging" form which gives rise to an expenditure function which is weakly separable in price and quality. To ensure that Marshallian surplus is finite, the qualitative good is also required to be "non-essential" -- with zero consumption being admissable.

Generally speaking, demand interdependence includes the quota case, Willig's repackaging case, and Maler's weak complementarity case. It also includes cases considered here in which a public good may substitute imperfectly or perfectly for a private good.

Definition of the Expenditure Function with Demand Interdependence

The consumer purchases private goods $x_i$ at a price $p_i$ and Marshallian demand for some goods are observably affected by nonmarket goods. A market good ($x_i$) exhibits "demand interdependence" with a nonmarket good Y if $\frac{\partial x_i}{\partial Y}$ is not zero. The problem is to measure the benefit to consumers of an exogenous nonmarket good change based on market observations. In contrast to consumer surplus, so-called exact measures of welfare defined here do not require distinctions to be made as to whether goods are complements or substitutes. The analysis below assumes that market good prices are constant. Appendix 2 extends the definition of exact welfare measures to include both nonmarket good changes and exogenous price changes.

The preference model for market good choice as related to a nonmarket good Y is given as follows where $\bar{U}(M,Y,p)$ denotes the indirect utility function:

$$\bar{U}(M,Y,p) = \underset{x}{\text{Max}}\ U(x,Y) \qquad (1)$$
$$\text{s.t. } p \cdot x \leq M;$$

x denotes private good consumption (a vector with components $x_i$), p is a vector of market good price, Y denotes an exogenous quantity of a nonmarket good, and M is income of the consumer. We assume Y is a nonsatiated "good", i.e., $\bar{U}_Y > 0$ and goods related to Y are noninferior.

As discussed by Hanemann, this model is more general than Lancaster's model of characteristics in which x and Y are linearly related. The preference model is also similar mathematically to the household production model in which utility is a function of household output $Z = Z(x,Y)$ where Y denotes nonpurchased inputs and x denotes purchased inputs. In the case of household production, Y may be a vector with some components endogenously determined and some (eg. public goods) exogenously determined.

The expenditure function, obtained from the dual of (1), is used to define welfare measures (similar to the method for price changes). $\mu(\bar{U},Y,p)$ denotes the expenditure function for the dual problem:

$$\mu(\bar{U},Y,p) = \underset{x}{\text{Min}}\ p \cdot x \qquad (2)$$
$$\text{s.t. } U(x,Y) \geq \bar{U}$$

In the following discussion $x_i$ will denote Marshallian demand (the solution to (1)) whereas $x_i^*$ will denote compensated demand (the solution to (2)).

The following theorem defines the system of differential equations derived from the duality of problems (1) and (2) which are the basis for obtaining welfare measures from demands for market goods observably related to nonmarket goods. Property for private goods are well-known and are found in many texts but are repeated here as part of the demand system. Property (v) has also been included elsewhere (Maler). Property (v) is used repeatedly to obtain the results given below.

<u>Theorem 1</u>: Provided a solution exists, $\mu(\overline{U},Y,p)$ in (2) and $\overline{U}(M,Y,p)$ in (1) satisfy a system of differentiable equations:

i) $\frac{\partial \mu}{\partial p_i} = x_i^*(\overline{U},Y,p)$ (Shephard's Lemma)

with boundary conditions

$$\mu(\overline{U},Y,p) = M$$

$$x_i^*(\overline{U},Y,p) = x_i(M,Y,p)$$

$$\overline{U} = \overline{U}(M,Y,p).$$

Properties satisfied by this system are:

ii) $\frac{\partial^2 \mu}{\partial p_i \partial p_j} = \frac{\partial x_j^*}{\partial p_i} = \frac{\partial x_j}{\partial M} \frac{\partial \mu}{\partial p_i} + \frac{\partial x_j}{\partial p_i}$ (Slutsky Equation)

iii) $\frac{\partial^2 \mu}{\partial Y \partial p_i} = \frac{\partial x_i^*}{\partial Y} = \frac{\partial x_i}{\partial M} \frac{\partial \mu}{\partial Y} + \frac{\partial x_i}{\partial Y}$

iv) $\frac{\partial \mu}{\partial p_i} + \frac{\partial \mu}{\partial \overline{U}} \frac{\partial \overline{U}}{\partial p_i} = 0$ (Roy's Identity)

v) $\frac{\partial \mu}{\partial Y} + \frac{\partial \mu}{\partial \overline{U}} \frac{\partial \overline{U}}{\partial Y} = 0$

vi) $\frac{\partial \mu}{\partial \overline{U}} \frac{\partial \overline{U}}{\partial M} = 1$ for $\partial \overline{U}/\partial M > 0$

vii) $\frac{\partial \overline{U}}{\partial Y} = \frac{\partial U(x,Y)}{\partial Y}$ .

<u>Proof</u>: Boundary conditions are the conditions for equivalence of the solutions to (1) and (2). (i), (ii), (iv), and (vi) may be found in any advanced microeconomics textbook (e.g. Deaton and Muellbauer); the proof that these conditions derive from (1) and (2) is the same with Y included as a parameter as without.

Property (iii) follows from (i) and differentiation of the boundary condition $x_i(M,Y,p) = x_i^*(\overline{U},Y,p)$ with respect to Y for $M = \mu(\bar{U},Y,p)$ holding $\bar{U}$ constant.

The proof of properties, (v) and (vii) follow immediately from the "envelope theorem" (Varian). For example, from the constraint in (2):

$$\sum_i \frac{\partial U}{\partial x_i} \frac{\partial x_i^*}{\partial Y} + U_Y = 0;$$

from the first order condition for (2) and (vi),

$$p_i = U_{x_i} / \bar{U}_M;$$

thus

$$\frac{\partial \mu}{\partial Y} = \sum_i p_i \frac{\partial x_i^*}{\partial Y} = - \bar{U}_Y / \bar{U}_M.$$

QED.

Similar to the method for market goods (e.g., Hausman; Varian), observable market good demand functions can be used to solve for the expenditure function with nonmarket goods included as a parameter. If $\partial x_i/\partial Y \neq 0$, then also $\partial^2\mu/\partial Y \partial p_i \neq 0$ so that a system of differential equations (i) for $\partial\mu/\partial p_i$ with Y as a parameter can be observed and then solved for the expenditure function. Procedures are demonstrated in examples below.

Note that recoverability of $\mu$, $\bar{U}$ as functions of Y from observable data on demand is not always possible. For example, for

$$U(x,Y) = \prod_i x_i^{\alpha_i} + \beta Y,$$

$\partial x_i^*/\partial Y \neq 0$ but $\partial x_i/\partial Y = 0$; the relation of the expenditure function to Y cannot be recovered in this case from observations of $x_i$.

Integrability conditions for demand systems are necessary to obtain a well-defined solution of the differential equation system (i). The integrability condition for private goods is that

$$\frac{\partial^2 \mu}{\partial p_i \partial p_j} = \frac{\partial^2 \mu}{\partial p_j \partial p_i} .$$

For nonmarket goods, parallel to the Slutsky condition, the integrability requirement

$$\partial^2\mu/\partial Y\partial p_i = \partial^2\mu/\partial p_i\partial Y,$$

is not directly testable from observables. However, differentiating both sides above with respect to $p_j$,

$$\frac{\partial^3\mu}{\partial Y\partial p_i\partial p_j} = \frac{\partial^3\mu}{\partial Y\partial p_j\partial p_i}$$

is also required. Equivalently,

$$\partial^2 x_i^*/\partial p_j\partial Y = \partial^2 x_j^*/\partial p_i\partial Y.$$

Or, from (ii), the integrability requirement is

$$\frac{\partial}{\partial Y}\left(\frac{\partial x_j}{\partial M} x_i + \frac{\partial x_j}{\partial p_i}\right) = \frac{\partial}{\partial Y}\left(\frac{\partial x_i}{\partial M} x_j + \frac{\partial x_i}{\partial p_j}\right).$$

Given a potential demand system which includes both market and nonmarket goods, the above integrability requirement can be used to test it for consistency with utility maximization.

The following result defines second order properties of the expenditure function. The term $-\partial\mu/\partial Y$ has been termed the marginal bid for a nonmarket good Y (Maler). The following corollary shows under what conditions the marginal bid value declines with y and increases with utility and hence has properties similar to a demand relation.

Lemma 1: With the requirement $\bar{U}_M > 0$, $\frac{-\partial\mu}{\partial Y}$ is positive if $\bar{U}_Y > 0$. $\bar{U}$ quasiconcave in M and Y implies that $-\partial\mu/\partial Y$ is nonincreasing in Y. If also $\bar{U}_{MM} < 0$, $\bar{U}_{YM} > 0$, then $-\partial\mu/\partial Y$ is increasing in $\bar{U}$. Also, diminishing marginal utility of income implies that $\partial^2\mu/\partial\bar{U}^2$ is positive.

Proof: From property (v) of Theorem 1,

$$-\frac{\partial\mu}{\partial Y} = -\frac{\partial\bar{U}}{\partial Y}\Big/\frac{\partial\bar{U}}{\partial M}.$$

The second order properties of the expenditure function follow from properties (v) and (vi) of Theorem 1:

$$\frac{\partial}{\partial Y}\left(-\frac{\partial\mu}{\partial Y}\right) = \frac{\bar{U}_M^2\bar{U}_{YY} - 2\bar{U}_Y\bar{U}_{YM}\bar{U}_M + \bar{U}_Y^2\bar{U}_{MM}}{\bar{U}_M{}^3}$$

$$\frac{\partial}{\partial \bar{U}}\left(-\frac{\partial\mu}{\partial Y}\right) = \frac{\bar{U}_M\bar{U}_{YM} - \bar{U}_Y\bar{U}_{MM}}{\bar{U}_M{}^3}$$

$$\frac{\partial}{\partial \bar{U}}\left(\frac{\partial\mu}{\partial \bar{U}}\right) = -\bar{U}_{MM}/\bar{U}_M{}^3.$$

QED.

## Definition of Exact Welfare Measures for Nonmarket Goods

Below, two types of exact welfare measures (compensating and equivalent variation) are defined for a change in nonmarket good. To emphasize the acceptability criterion, definitions are given directly in terms of the expenditure function, as in McKenzie and Pearce, rather than in terms of indirect utility as in Randall and Stoll, and the definitions relate changes in expenditure to utility change from a base reference point. This definition can be extended to include cases in which multiple changes in prices, incomes, and nonmarket goods occur (see Appendix).

The equivalent measure is defined to be the money metric equivalent of the change in utility from $U^o$ to $U'$ from the reference point $(M_o, Y_o, p^o)$:

$$E = \mu(U', Y_o, p^o) - \mu(U^o, Y_o, p^o) \tag{3}$$

where

$$U' = \bar{U}(M_o, Y_o + y, p^o) \tag{4}$$

and

$$U^o = \bar{U}(M_o, Y_o, p^o). \tag{5}$$

To obtain an interpretation of the equivalent measure in terms of utility, from the boundary conditions of theorem 1,

$$\bar{U}(\mu(U', Y_o, p^o),\ Y_o,\ p^o) = U'. \tag{6}$$

$$M_o = \mu(U^o,\ Y_o,\ P^o). \tag{7}$$

Thus, from (3), (6) and (7),

$$\bar{U}( M_o+E, Y_o, p^o) = U' = \bar{U}(M_o, Y_o+y, p^o) \tag{8}$$

so the interpretation obtained for the equivalent measure is that it gives the same utility effect as a change in the nonmarket good.

The <u>compensating measure</u> is similarly defined from the expenditure function as the change in utility from $U^o$ to $U'$ from the new reference $(M_o,Y_o+y,p^o)$:

$$C = \mu(U',Y_o+y,p^o) - \mu(U^o,Y_o+y,p^o). \tag{9}$$

To obtain the usual interpretation of the compensating measure, use the boundary conditions

$$M_o = \mu(U',Y_o+y,p^o) = \mu(U^o,Y_o,p^o) \tag{10}$$

$$\bar{U}(\mu(U^o,Y_o+y,p^o), Y_o+y,p^o) = U^o. \tag{11}$$

From (9), (10), and (11)

$$\bar{U}(M^o-C, Y_o+y, p^o) = U^o = \bar{U}(M^o, Y_o, p^o). \tag{12}$$

Thus, the interpretation obtained for the compensating measure is that it is an income change which results in holding utility constant at the original level when the nonmarket good changes.

Acceptability properties of the two types of measures can readily be seen from (3) and (9). Since the reference point varies with the change y, the C measure may not provide a consistent ranking of changes in utility when there are more than two alternatives. Thus, as in the price change case (Chipman and Moore), the "C" measure will not be an acceptable measure. However, the equivalent measure will be acceptable welfare measure (Chipman and Moore) since it ranks changes in a nonmarket good consistent with the resulting utility change.

From the duality condition (10), the E and C measures can also equivalently be defined as expenditure change holding utility constant:

$$E = \mu(U',Y_o,p^o) - \mu(U',Y_o+y,p^o); \tag{13}$$

$$C = \mu(U^o,Y_o,p^o) - \mu(U^o,Y_o+y,p^o). \tag{14}$$

These two definitions will be used in the results below.

The following result is not new (see for example Randall and Stoll or Bockstael and McConnell) but is presented here for completeness.

Lemma 2: Compensating and equivalent measures can be defined as an integral of the marginal bid $-\dfrac{\partial\mu(\bar{U}, Y, p^o)}{\partial Y}$ over the range of Y, with utility held constant.

Proof: From the fundamental theorem of calculus applied to (13), (14):

$$E = \int_{Y_o}^{Y_o+y} -\frac{\partial\, \mu(U', Y, p^o)}{\partial Y}\, dY; \tag{15}$$

$$C = \int_{Y_o}^{Y_o+y} -\frac{\partial\, \mu(U^o, Y, p^o)}{\partial Y}\, dY. \tag{16}$$

QED.

Relative Sizes of Willingness to Pay and Willingness to Accept Measures

The question of the relative sizes of compensating and equivalent variation measures has been examined by Willig in the case of price changes and by Maler in the case of public goods. Knowledge of the relative sizes of these measures is important for policy analysis purposes, particularly when some measures are easier to estimate then others. Rather than questioning whether compensating or equivalent measures are larger, the size relationship question has recently been raised in terms of willingness to pay and willingness to accept. Randall and Stoll studied the size relationship of willingness to pay and accept for the case of a quota. Below, for the general demand interdependence case, we show that willingness to accept is generally greater than willingness to pay based on results in Theorem 1 and Lemma 1.

Randall and Stoll defined four separate welfare measures in terms of willingness to pay and accept depending on whether changes in a nonmarket good are increases or decreases. Similarly, four Hicksian measures can be derived from the compensating and equivalent measures defined above by considering a positive or negative change (y) from the reference point $(M_o,Y_o,p^o)$. Utility is held constant at either the original $(U^o)$ or new values as in (13) and (14). Figure 1 illustrates these four values, both in terms of the indifference curves and as areas. The representation of the measures as areas under a bid curve follows from Lemma 1 and Lemma 2.

Expressed as positive values, these four Hicksian measures are:

$$WTP^c_{Y_o,y} = \mu(U^o,Y_o,p^o) - \mu(U^o,Y_o+y,p^o) \tag{17}$$

$$WTA^c_{Y_o,y} = \mu(U^o,Y_o-y,p^o) - \mu(U^o,Y_o,p^o) \tag{18}$$

$$WTP^e_{Y_o,y} = \mu(U^1,Y_o-y,p^o) - \mu(U^1,Y_o,p^o) \tag{19}$$

$$WTA^e_{Y_o,y} = \mu(U^2,Y_o,p^o) - \mu(U^2,Y_o+y,p^o) \tag{20}$$

where the "c" denotes compensating measures and the "e" denotes equivalent measures and WTP indicates payment whereas WTA indicates income gain. The subscripts $Y_o$ and y denote the starting level of nonmarket good and change, and resulting utility levels are

$$U^o = \bar{U}(M_o,Y_o,p^o)$$

$$U^1 = \bar{U}(M_o,Y_o-y,p^o)$$

$$U^2 = \bar{U}(M_o,Y_o+y,p^o)$$

with

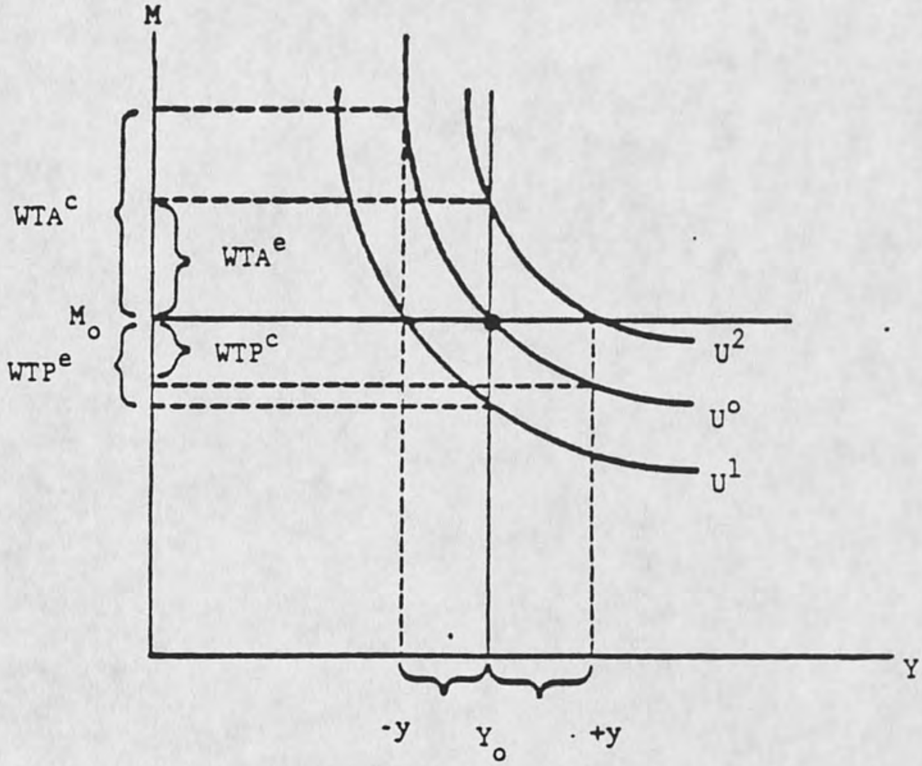
$$U^2 > U^o > U^1.$$

Note that these four willingness to pay and willingness to accept definitions are defined in terms of the reference point $(M_o,Y_o,p^o)$.

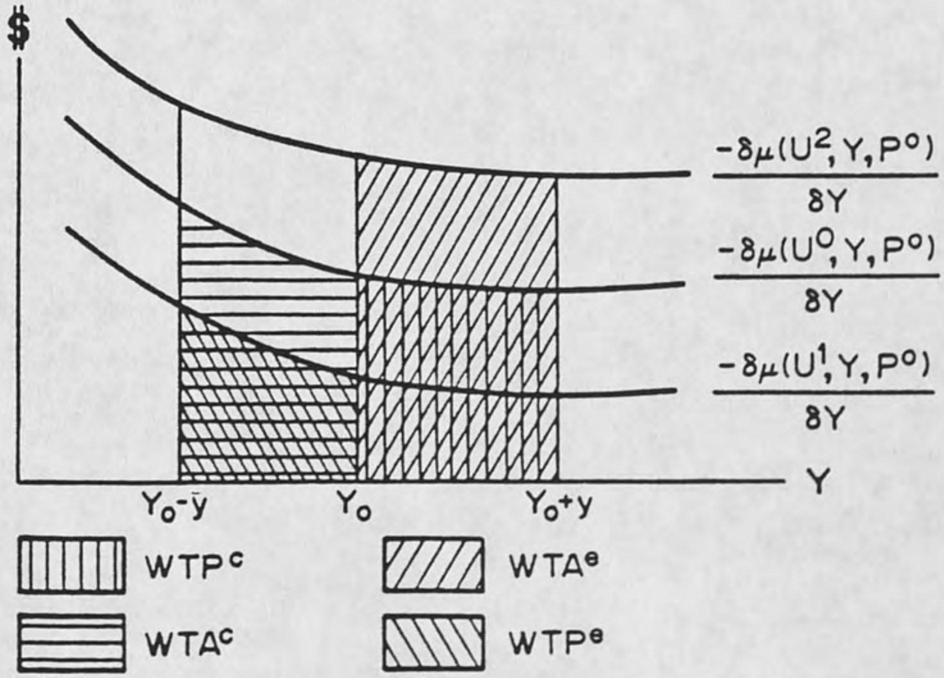
Although mathematically there are only two types of exact measures, the four Hicksian measures have different interpretations. Similar to the analysis of (8) and (12), these four values may be interpreted respectively as:

Figure 1. Alternative welfare measures for changes in nonmarket good

1a. Indifference Curves




1b. Areas



$WTP^c$ - the maximum a consumer would pay as a lump sum to obtain an increase in a nonmarket good (holding utility constant at the base level);

$WTA^c$ - the minimum lump sum income increase required to compensate for a given decrease in a nonmarket good (holding utility constant at the base level);

$WTP^e$ - the maximum income deduction that is equivalent to a given decrease in a nonmarket good;

$WTA^e$ - the minimum lump sum income increase that is equivalent to an increase in the non-market good.

The lemma below shows that willingness to accept measures are bounded from below by willingness to pay measures. However, there is no required general relationship between compensating and equivalent measures.

Theorem 2: $\bar{U}_Y > 0$, $\bar{U}_M > 0$, $\bar{U}_{MM} \leq 0$, $\bar{U}_{YY} \leq 0$, $\bar{U}_{MY} \geq 0$ and $\bar{U}$ quasiconcave imply that

$$WTA^c_{Y_o,y} \geq WTP^c_{Y_o,y}$$

$$WTA^e_{Y_o,y} \geq WTP^c_{Y_o,y}$$

$$WTA^c_{Y_o,y} \geq WTP^e_{Y_o,y}.$$

Equality holds in the last two comparisons if $\bar{U}_{YM}$ and 0 and $\bar{U}_{MM} = 0$. For relatively small changes y, if utility elasticities also satisfy

$$\left(\frac{-\bar{U}_{MM}}{\bar{U}_M}\right) \geq \left(\frac{-\bar{U}_{YY}}{\bar{U}_Y}\right)\left(\frac{\bar{U}_M}{\bar{U}_Y}\right)$$

then also

$$WTA^c_{Y_o,y} \geq WTA^e_{Y_o,y} \geq WTP^e_{Y_o,y} \geq WTP^c_{Y_o,y}.$$

Proof: From corollary 1, quasiconcavity implies that the marginal bid is nonincreasing in y $(-\partial^2\mu/\partial y^2 \leq 0)$ so that

$$-\frac{\partial\mu(U^o, Y_o - y, p^o)}{\partial y} \geq -\frac{\partial\mu(U^o, Y_o + y, p^o)}{\partial y}.$$

Also from corollary 1, $\bar{U}_{MY} \geq 0$ and $\bar{U}_{YY} \geq 0$ imply $-\frac{\partial^2 \mu}{\partial U \partial y} \geq 0$ so that

$$-\frac{\partial \mu(U^2, Y_o+y, p^o)}{\partial y} \geq -\frac{\partial \mu(U^o, Y_o+y, p^o)}{\partial y}.$$

and

$$-\frac{\partial \mu(U^o, Y_o-y, p^o)}{\partial y} \geq -\frac{\partial \mu(U^1, Y_o-y, p^o)}{\partial y}.$$

Thus, from Lemma 1, the first three indicated inequalities are obtained for any change y. $\bar{U}_{MY} = 0$ and $\bar{U}_{YY} = 0$ would imply by Lemma 1 that $\frac{-\partial \mu}{\partial y}$ does not shift with utility.

Now we show the following size relationships for relatively small changes in y satisfying the elasticity condition:

$$WTA^e_{Y_o,y} \geq WTP^e_{Y_o,y}$$
$$WTP^e_{Y_o,y} \geq WTP^c_{Y_o,y}$$

and

$$WTA^c_{Y_o,y} \geq WTA^e_{Y_o,y}.$$

From (17)-(20), all bid measures are equal to zero at $y = 0$. Thus the Taylor series representations for relatively small changes y are:

(21) $$WTP^c(y;\ Y^o) = \left.\frac{\partial WTP^c}{\partial y}\right|_{y=0} y + \frac{1}{2} \left.\frac{\partial^2 WTP^c}{\partial y^2}\right|_{y=0} y^2 + \text{Remainder}$$

(22) $$WTP^e(y;\ Y^o) = \left.\frac{\partial WTP^e}{\partial y}\right|_{y=0} y + \frac{1}{2} \left.\frac{\partial^2 WTP^e}{\partial y^2}\right|_{y=0} y^2 + \text{Remainder}.$$

(23) $$WTA^c(y;\ Y^o) = \left.\frac{\partial WTA^c}{\partial y}\right|_{y=0} y + \frac{1}{2} \left.\frac{\partial^2 WTA^c}{\partial y^2}\right|_{y=0} y^2 + \text{Remainder}$$

(24) $$WTA^e(y;\ Y^o) = \left.\frac{\partial WTA^e}{\partial y}\right|_{y=0} y + \frac{1}{2} \left.\frac{\partial^2 WTA^e}{\partial y^2}\right|_{y=0} y^2 + \text{Remainder}.$$

Differentiating (17)-(20), because of income equivalences (10) and property (v),

$$(25)\qquad \frac{\partial\ WTP^c}{\partial y} = -\ \frac{\partial\mu(U^o,\ Y_o+y,\ p^o)}{\partial y}$$

$$= \bar{U}_y(M\ -WTP^c,\ Y^o+y,\ p^o)/\ \bar{U}_M(M_o\text{-}WTP^c,\ Y^o+y,\ p^o);$$

$$(26)\qquad \frac{\partial\ WTP^e}{\partial y} = -\ \frac{\partial\mu(U^1,\ Y_o-y,\ p^o)}{\partial y}$$

$$= \bar{U}_y(M_o,\ Y^o-y,\ p_o)/\ \bar{U}_M(M_o\text{-}WTP^e,\ Y^o,\ p^o);$$

$$(27)\qquad \frac{\partial\ WTA^c}{\partial y} = -\ \frac{\partial\mu(U^o,\ Y_o-y,\ p^o)}{\partial y}$$

$$= \bar{U}_y(M_o+WTA^c,\ Y^o-y,\ p^o)/\ \bar{U}_M(M_o+WTA^c,\ Y^o-y,\ p^o);$$

$$(28)\qquad \frac{\partial\ WTA^e}{\partial y} = -\ \frac{\partial\mu(U^2,\ Y_o+y,\ P^o)}{\partial y}$$

$$= \bar{U}_y(M_o,\ Y^o+y,\ p^o)/\ \bar{U}_M(M_o+WTA^e,\ Y^o,\ p^o).$$

The marginal bids (25)-(28) all have the same value at $y = 0$.

Second derivatives of willingness to pay are found by differentiating (25)-(28) again with respect to y:

$$(29)\qquad \frac{\partial^2\ WTP^c}{\partial y^2} = [\bar{U}_M^2\ \bar{U}_{yy} + \bar{U}_{MM}\ \bar{U}_y^2 - 2\ \bar{U}_{My}\ \bar{U}_y\ \bar{U}_M)]/\ \bar{U}_M.$$

$$(30)\qquad \frac{\partial^2\ WTP^e}{\partial y^2} = \frac{-\ \bar{U}_{yy}\ \bar{U}_M^2 + \bar{U}_y^2\ \bar{U}_{MM}}{\bar{U}_M^3}.$$

$$(31)\qquad \frac{\partial^2\ WTA^c}{\partial y^2} = -\ [\bar{U}_M^2\ \bar{U}_{yy} + \bar{U}_{MM}\ \bar{U}_y^2 - 2\ \bar{U}_{My}\ \bar{U}_y\ \bar{U}_M)]/\ \bar{U}_M.$$

$$(32)\qquad \frac{\partial^2\ WTA^e}{\partial y^2} = \frac{\bar{U}_M^2\ \bar{U}_{yy} - \bar{U}_y^2\ \bar{U}_{MM}}{\bar{U}_M^3}.$$

Quasiconcavity of $\bar{U}$ implies that $\frac{\partial^2 \ WTP^c}{\partial y^2}$ is negative and $\frac{\partial^2 \ WTA^c}{\partial y^2}$ is positive, implying that $WTA^c$ is convex and $WTP^c$ is concave. So, the Taylor series implies, $WTA^c \geq WTP^c$ for small y values. The signs of (30) and (32) are also opposite; for the assumed elasticity size condition, $WTA^e$ is convex and $WTP^e$ is concave. So, the Taylor series implies $WTA^e \geq WTP^e$ for small y values.

Subtracting (30) from (29),

$$\frac{\partial^2 \ WTP^c}{\partial y^2} - \frac{\partial^2 \ WTP^e}{\partial y^2} = \frac{2(\bar{U}_M^2\bar{U}_{yy} - \bar{U}_{My}\bar{U}_y\bar{U}_M)}{\bar{U}_M^3}$$

Therefore, $\bar{U}_{yy} \leq 0$ and $\bar{U}_{My} \geq 0$ imply, from the Taylor series, that $WTP^e \geq WTP^c$. Similarly subtracting (32) from (31), under the same assumptions, $WTA^c \geq WTA^e$. QED.

## Comparison of Consumer Surplus and Exact Measures

Here, we compare consumer surplus for complements and substitutes with exact welfare measures. As for price changes, consumer surplus may be viewed as an approximation of exact welfare measures. Since the approximation may not be very good, rather than making numerical comparisons for a special case (eg, constant elasticity as in Willig and Randall and Stoll), we compare the integral forms of Marshallian and exact measures to provide a justification for use of consumer surplus type diagrams. Relationships are based on the properties in Theorem 1.

Below, market goods are separated into two sets - those related and those unrelated to a nonmarket good Y in terms of Hicksian rather than Marshallian demand. A market good ($z_i$) <u>related</u> to Y is defined to have the property $\partial z_i^*/\partial y \neq 0$ and a good which is <u>independent</u> of Y is defined to have the property $\partial z_i^*/\partial y = 0$. The separability of market goods into these two sets relates to the possibility of two stage budgeting discussed by Phlips.

It is not always possible to separate a system of goods in this way as a later example will illustrate.

We further classify related goods as either <u>Hicksian</u> <u>substitutes</u> ($\partial z_i^*/\partial y < 0$) or <u>Hicksian</u> <u>complements</u> ($\partial z_i^*/\partial y > 0$). From Theorem 1, (iii), the relation between Marshallian and Hicksian demand effects is given by

$$\partial z_i^*/\partial y = \partial z_i/\partial M \; \partial \mu/\partial y + \partial z_i/\partial y. \tag{33}$$

Therefore, if it is observed that $\partial z_i/\partial y < 0$, then also $\partial z_i^*/\partial y < 0$ since $\partial \mu/\partial y < 0$. For Hicksian complementary goods ($\partial z_i^*/\partial y > 0$), observed demands must also satisfy $\partial z_i/\partial y > 0$. For goods <u>independent</u> of Y, since $\partial z_i^*/\partial y = 0$,

$$\partial z_i/\partial y = (\bar{U}_y/\bar{U}_M) \; \partial z_i/\partial M,$$

so a nonmarket good affects market good demand only through the income effect.

In the following results, relationships between Hicksian demand curves and the Marshallian demand curve are based on properties defined in Theorem 1. We will assume that goods related to Y are noninferior ($\frac{\partial z_i}{\partial M} \geq 0$). A zero income effect ($\partial z_i/\partial M = 0$) is sufficient for the Hicksian and Marshallian demand curves to coincide by (33).

Also, the set of goods related to Y will be denoted by $\{z_i\}$ whereas $\{q_i\}$ will denote the set of goods independent of Y. Since expenditure can be decomposed as:

$$\mu(\bar{U}, Y, p) = p_z \cdot z^*(\bar{U}, Y, p) + p_q \cdot q^*(\bar{U}, Y, p),$$

by definitions of goods related and unrelated to Y,

$$\frac{\partial \mu}{\partial y} = - p_z \cdot \frac{\partial z^*}{\partial y}. \tag{34}$$

The boundary conditions imply that the Marshallian and Hicksian demand curves intersect at certain specified points. For normal goods, the Slutsky condition

$$\frac{\partial z_i^*}{\partial p_i} = \frac{\partial z_i}{\partial p_i} + \frac{\partial \mu}{\partial p_i} \frac{\partial z_i}{\partial M} \tag{35}$$

says that the slope of the compensated demand curve is less negative than the slope of Marshallian demand curve at points where they intersect. For the inverse demand functions diagrammed in Figures 2-4, the slopes are the inverses of $\partial z_i^*/\partial p_i$ and $\partial z_i/\partial p_i$, and so Hicksian demand curves are more steeply sloped than inverse Marshallian demand curves.

The following result specifies allowable relationships between market and nonmarket goods.

Lemma 3: For goods related to Y, either all goods are Hicksian substitutes; or, if there is a Hicksian complementary good, there must also be at least one substitute good.

Proof: By property (v), $-\frac{\partial \mu}{\partial y} = \bar{U}_y/\bar{U}_M > 0$. By (34),

$$-\frac{\partial \mu}{\partial y} = -p_z \cdot \frac{\partial z^*}{\partial y} > 0$$

is implied. The case of all substitute goods satisfies this sign condition. If the only good related to Y were a Hicksian complement, then the above inequality would be contradicted. Thus for any complementary good, there must also be at least one substitute good to obtain the proper sign for $-\frac{\partial \mu}{\partial y}$. QED.

## Imperfect Substitutes and Complements

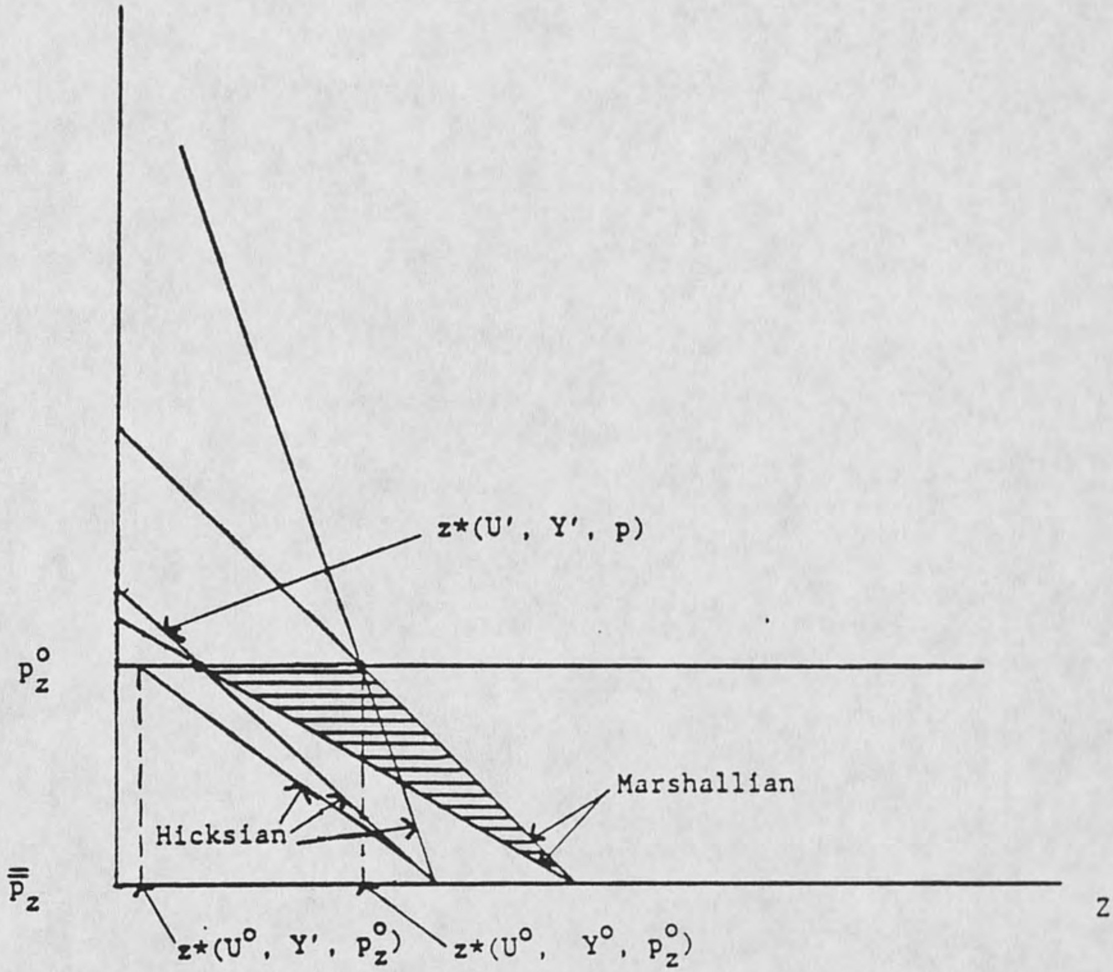
The change in consumer surplus (the area between Marshallian demand curves) is illustrated in Figure 2a,b for both substitutes and complements. The following theorem gives a sufficient condition for a Hicksian measure to be equivalently expressed as an area between compensated demand curves. Therefore, by analogy, consumer surplus provides an appropriate geometrical representation, regardless whether or not it is a good numerical approximation of a Hicksian measure.

The "weak complementarity" property (Maler; Bradford) is a restriction for when the consumer surplus area may be used to measure benefits. Maler defined "weak complementarity" to hold if $U_y(0, q, Y) = 0$. Willig used a similar property to ensure finiteness of consumer surplus for quality changes in the "repackaging" model. Problems with this condition are that it is only defined for complements, and tests of it may be difficult to observe. Later examples show it may also be difficult to fulfill.

First, we define a price restriction which applies to substitutes as well as complements. Define the "CCD" (constant compensated demand) price vector $\bar{\bar{p}}$ to be a finite price vector for market goods related to Y such that compensated private demand is constant (not necessarily zero) with respect to the nonmarket good. For a complement, $\bar{\bar{p}}_{z_i}$ is a price <u>sufficiently high</u> such that compensated demand for market good $z_i$ becomes constant (at a minimum consumption level or zero); thus $\partial z_i^*/\partial y = 0$ for $p_{z_i} \geq \bar{\bar{p}}_{z_i}$. For a substitute, the CCD price $\bar{\bar{p}}_{z_i}$ is a price <u>sufficiently low</u> such that compensated demand is constant for $p_{z_i} \leq \bar{\bar{p}}_{z_i}$ (i.e., the consumer becomes satiated with the market good). Figure 2a,b illustrates substitute and complement cases. Note that such a price may not always exist.

"<u>Weak substitution</u>" is defined to be the property $\mu_y(\bar{U}, Y, \bar{\bar{p}}) = 0$ with $\partial z_i^*/\partial y \leq 0$, whereas for <u>weak complementarity</u>, $\mu_y(\bar{U}, Y, \bar{\bar{p}}) = 0$ is satisfied with $\partial z_i^*/\partial y \geq 0$.

<u>Lemma 4</u>: Existence of a CCD price vector implies the property of "weak complementarity" for goods related to Y as complements; similarly existence of the CCD price vector implies "weak substitutability" for the substitutes case.




$Y' > Y^o$ —> z decreases

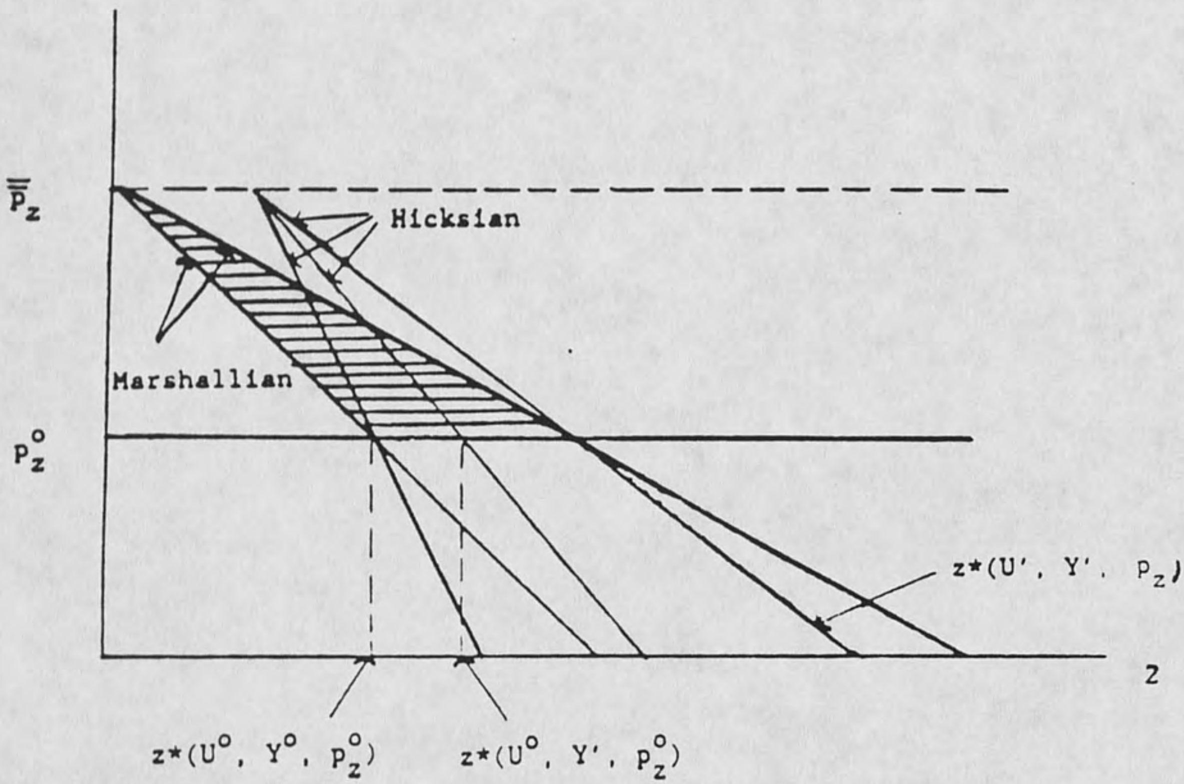
z* decreases

$\bar{\bar{p}}_z$ : z constant

z* constant

**Marshallian surplus**

Figure 2a. Consumer surplus and exact areas for substitutes




$Y' > Y^o \longrightarrow$ z increases

z* increases

$\bar{\bar{p}}_z$ : z constant ( = 0)

z* constant ( $\neq$ 0)

Marshallian surplus

Figure 2b. Consumer surplus and exact areas for complements

Proof: Suppose the CCD price exists. Then by definition of the CCD price, compensated demand is constant for $p_{z_i}$ larger than the CCD price for complements, or smaller than the CCD price for substitutes. By (34), the expenditure function satisfies

$$\partial\mu/\partial y = p_z \cdot \partial z^*/\partial y.$$

Evaluating this derivative at prices in the relevant price range (above $\bar{\bar{p}}_{z_i}$ for complements and below $\bar{\bar{p}}_{z_i}$ for substitutes), the right hand side is zero. Thus, $\partial\mu/\partial y$ is zero for prices in the relevant range. QED.

Marshallian demand for related market goods will also be constant with respect to y at the CCD price vector since

$$\partial z/\partial y = \partial z/\partial M \;\; \partial\mu/\partial y \;\; + \partial z^*/\partial y = 0.$$

However, as Figure 2 illustrates, Marshallian and Hicksian demand are constant at different values, not both at zero in the case of complements.

By property (v) of Theorem 1, the Maler definition that $\bar{U}_y(0, q, Y) = 0$ is obtained for complements and $\bar{U}_y(z^{max}, q, Y) = 0$ is obtained for substitutes where $z^{max}$ is the satiation level for the related market good.

The following theorem shows that existence of the CCD price vector is a sufficient condition so that an exact measure can be equivalently expressed as an area between compensated demand curves. The proof may be extended to apply to sets of goods related to Y, particularly for complementary goods.

Theorem 3: Suppose that a CCD price exists for a single substitute good (z) related to Y. Then an exact measure of benefit for a change in a nonmarket good from $Y_o$ to Y′ for the relevant $\bar{U}$ is equal to the change in area between the compensated demand curves $z^*(\bar{U},Y_o,p)$ and $z^*(\bar{U},Y',p)$ for prices between $p_z^o$ and the CCD price $\bar{\bar{p}}_z$.

For a set of goods related to Y, if the CCD price vector $\bar{\bar{p}}_z$ exists, a Hicksian measure is equal to the difference in the integrals over the <u>sum</u> of related compensated demands, integrated between price vectors $p_z^o$ and $\bar{\bar{p}}_z$ and holding utility constant at the appropriate level as Y changes from $Y_o$ to Y'.

<u>Proof</u>: First, consider the case when only one good (z) is related to Y as a substitute and let Y' denote a level of nonmarket good greater than $Y_o$; Figure 2a illustrates this case. (The proof is similar for a quantity decrease.) Suppose existence of the CCD price $\bar{\bar{p}}_z$ such that $z^*(\bar{U},Y,p_q,p_z)$ is constant for $p_z < \bar{\bar{p}}_z$. Then by the lemma above,

$$\frac{\partial\mu(\bar{U},\ Y,\ p_q^o,\ \bar{\bar{p}}_z)}{\partial y} = 0 \quad \text{for all } Y \geq Y_o.$$

Thus, adding a zero term to the integral form of an exact measure:

$$\Delta\mu = \int_{Y_o}^{Y'} - \frac{\partial\mu(\bar{U},y,p_q^o,p_z^o)}{\partial y}\, dy = \int_{Y_o}^{Y'} - \frac{\partial\mu(\bar{U},y,p_q^o,p_z^o)}{\partial y} + \frac{\partial\mu(\bar{U},y,p_q^o,\bar{\bar{p}}_z)}{\partial y}\, dy.$$

By the Fundamental Theorem of Calculus, the right hand side is

$$= \int_{Y_o}^{Y'} \int_{\bar{\bar{p}}_z}^{p_z^o} - \frac{\partial^2\mu(\bar{U},y,p_q^o,p_z)}{\partial p_z\, \partial y}\, dp_z\, dy.$$

By Theorem 1, property (i), and interchange of order of integration,

$$= - \int_{Y_o}^{Y'} \frac{\partial}{\partial y} \int_{\bar{\bar{p}}_z}^{p_z^o} z^*(\bar{U},y,p_q^o,p_z)\, dp_z\, dy;$$

again by the Fundamental Theorem,

$$= \int_{\bar{\bar{p}}_z}^{p_z^o} z^*(\bar{U},Y_o,p_q^o,p_z)dp_z - \int_{\bar{\bar{p}}_z}^{p_z^o} z^*(\bar{U},Y',p_q^o,p_z)dp_z.$$

For multiple goods related to Y, $\bar{p}_z$ is a price vector such that the marginal bid is zero. The same proof applies for sets of substitutes or complements and substitutes where the inner integral is defined over a sum of demands for related goods. Then, the exact measure is equal to a difference in integrals defined over the sum of related compensated demands. The Slutsky symmetry conditions are also required for path independence but are satisfied because of the use of compensated demand. QED.

Existence of a price such that compensated and Marshallian demands are constant is a major difficulty in application of the result. (See the examples below.) Another issue is that even when both exact and surplus measures can be expressed as areas between demand curves, the relative sizes of the Hicksian and the Marshallian surplus areas could be very different (as in Lankford).

Perfect Substitutes

The vertical area under the inverse Marshallian demand curve is illustrated in Figure 3; it has been used to measure benefits when the nonmarket good (Y) is a "perfect substitute" for a market good (z). The theorem below justifies this measurement of benefits.

We define z and Y to be perfect substitutes when $\partial z^*/\partial y = -1$. Here, although the nonmarket good Y is essentially the same as the private good z in terms of producing utility, market price is only charged for z. The public good is either free or is financed through income tax. (This case is different from the quota case which has an upper bound restriction on a private good regardless of price.)

Lemma 5: Perfect substitutes are obtained when the consumer problem (1) is of the form

$$\max_{z,\,q} U(\, z + Y,\, q) \qquad (36)$$
$$\text{s.t.} \quad p_z z + p_q q \le M.$$

Proof: Taking the derivative of the indirect utility in (36),

$$U_y\ (z_y^* + 1) + U_q\ (q_y^*) = 0.$$

By definition of q, $q_y^* = 0$, thus, $z_y^* = -1$ is obtained. QED.

For perfect substitutes, the following theorem compares exact measures, consumer surplus (CS), and the market value of a change in the nonmarket good from an initial point $(Y_o, p^o)$.

Theorem 4: For perfect substitutes, assuming $\bar{U}_{My} > 0$, $\bar{U}_{MM} < 0$, $\bar{U}_M > 0$, $\bar{U}_y > 0$ and $\bar{U}$ is quasiconcave, the following relationships hold for a gain (g) or a loss (l) of y:

$$p_z^o y > CS_g > WTP^c$$
$$WTA^c > CS_l > p_z^o y.$$

When $\bar{U}_{MM} = \bar{U}_{MY} = \bar{U}_{yy} = 0$, then

$$WTA^c = WTP^c = p_z^o y.$$

Proof: The proof is illustrated in Figure 3. In Figure 3, the marginal bid curve with utility constant at $U^o$ is graphed against Y on the horizontal axis whereas the Marshallian demand (z) with public good at $Y_o$ is graphed against $p_z$ on the vertical axis.

To show the relation between the Marshallian demand curve and the bid curve with utility held constant at $U^o$, the following relationship for perfect substitutes is used:

$$\bar{U}(M, Y, p) = U(z(M, Y, p) + Y, q(M, Y, p)).$$

Taking the derivative of both sides with respect to y and applying the envelope theorem,

$$\begin{aligned}\bar{U}_y &= U_z(z_y + 1) + U_q q_y \\ &= \bar{U}_M(p_z z_y + p_q q_y + p_z) \\ &= \bar{U}_M\ p_z\end{aligned}$$

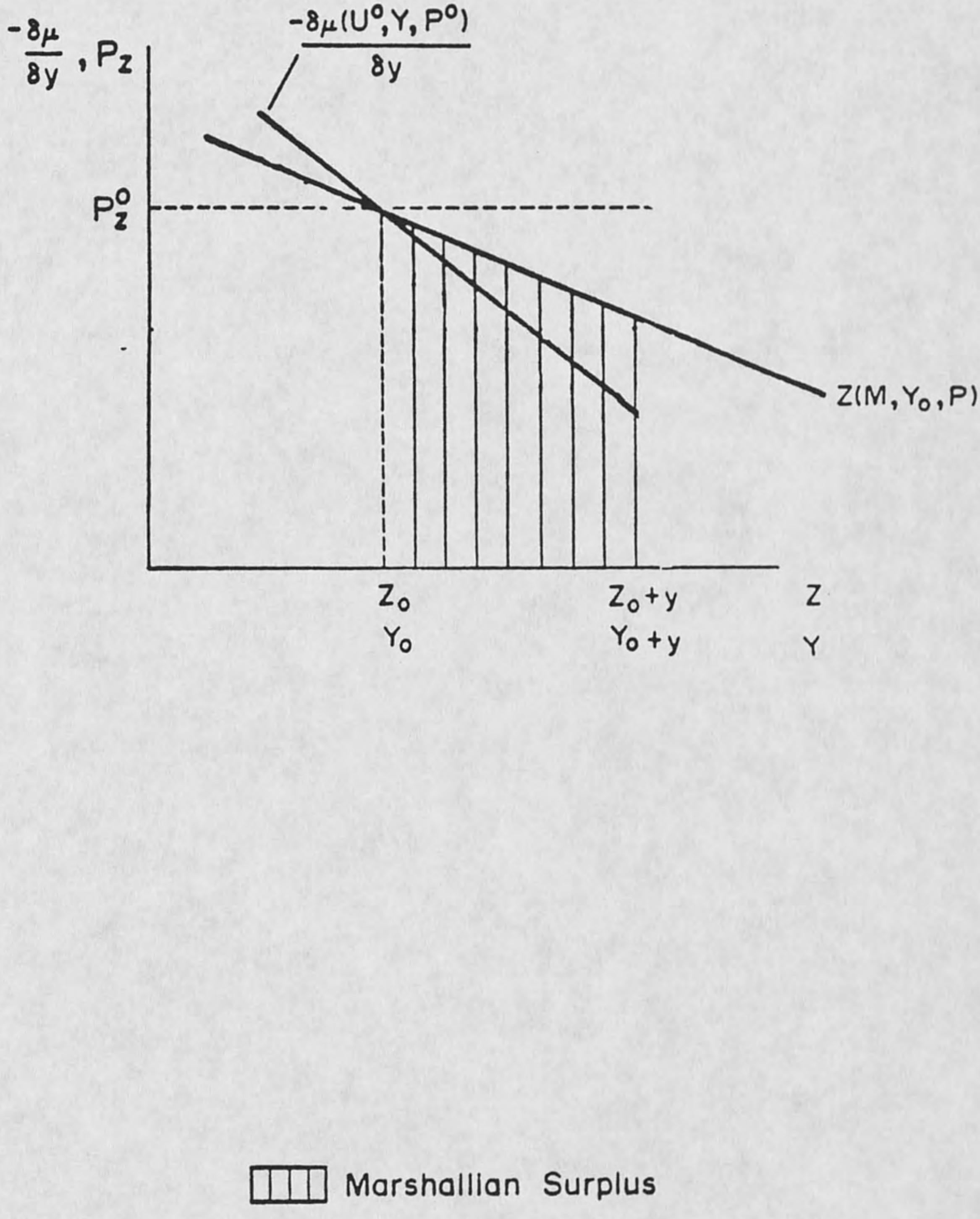



Figure 3. Comparison of Marshallian and Hicksian Measures for a Perfect Substitute

so

$$p_z = \bar{U}_y(M, Y, p)/\bar{U}_M(M, Y, p).$$

Therefore, by property (v) of Theorem 1, at the initial point $(M_o, Y_o, p^o)$ the Marshallian inverse demand and marginal bid take on the same value of $p_z^o$.

To compare the slopes of the marginal bid curve and inverse demand curve with respect to y, differentiating the above with income held constant at $M_o$,

$$\frac{\partial p_z}{\partial y} = \frac{\bar{U}_M^{\ 2}\ \bar{U}_{yy} - \bar{U}_y\ \bar{U}_M\ \bar{U}_{My}}{\bar{U}_M^{\ 3}}.$$

By Lemma 1, the bid curve slope satisfies

$$\frac{-\partial^2\mu}{\partial y^2} = \frac{\bar{U}_M^{\ 2}\bar{U}_{yy} - 2\ \bar{U}_y\ \bar{U}_M\ \bar{U}_{My} + \bar{U}_y^{\ 2}\bar{U}_{MM}}{\bar{U}_M^{\ 3}}.$$

Therefore, the assumptions $\bar{U}_{MY} > 0$ and $\bar{U}_{YY} > 0$ imply

$$\frac{\partial p_z}{\partial y} - \left(\frac{-\partial^2\mu}{\partial y^2}\right) = \frac{\bar{U}_y\ \bar{U}_M\ \bar{U}_{My} - \bar{U}_y^{\ 2}\bar{U}_{MM}}{} > 0.$$

That is, the area under the marginal bid curve, integrated with respect to y, will be less than the area under the inverse demand curve, integrated with respect to y.

By Theorem 1, property (i) $\partial z^*/\partial y = -1$ implies

$$\partial^2\mu/\partial y\partial p_z = -1;$$

integration with respect to $p_z$ implies

$$-\frac{\partial\mu(U^o, y, p^o)}{\partial y} = p_z^o + C(y, U^o, p^o).$$

Thus $C(y,U^o,p^o)$ represents the difference between the marginal bid curve and the constant price line at $p_z = p_z^o$. Differentiating the above with respect to y,

$$-\frac{\partial^2\mu}{\partial y^2}(U^o, y, p^o) = \frac{\partial C(y, U^o, p^o)}{\partial y}.$$

By Lemma 1, $C(y,U^o,p^o)$ is decreasing as y increases when $\overline{U}$ is quasiconcave. Since the Marshallian and compensated demand coincide at the initial point $(M_o,Y_o,p^o)$, $C(0,U^o,p^o) = 0$. Therefore $C(y,U^o,p^o)$ must be positive for $Y_o - y < Y_o$ and negative for $Y_o + y > Y_o$. If second derivatives of $\overline{U}$ with respect to M and y are all zero, then C is identically zero. QED.

## Comparison of Change in Spending and Exact Measures

Since exact measures are changes in expenditure, the change in spending for related goods is immediately suggested as an alternative to consumer surplus as an approximation method. (Obviously, since income is held constant, the change in total spending must be zero, but here only related goods are to be considered.) For the perfect substitutes case, a change (y) in a public good will not reduce spending for the market good by the full market value of y (by (33)). The results below extend this observation to compare change in spending and exact measures.

Define the change in spending for related goods as the absolute value of the change in price times quantities demanded, summed over all market goods related to Y, for old ($Y_o$) and new (Y') levels for nonmarket goods:

$$\Delta E = \left| p_z^o \cdot [z(M,Y_o,p^o) - z(M,Y',p^o)] \right| .$$

Consider an increase in nonmarket good from $Y_o$ to $Y_o+y$. From (34) and the fundamental theorem of calculus, the Hicksian benefit measures for an increase in Y are equivalently defined by

$$\Delta\mu = \int_{Y_o}^{Y_o+y} - \frac{\partial\mu(\overline{U},y,p^o)}{\partial y}\, dy = \int_{Y_o}^{Y_o+y} - p_z \cdot \frac{\partial z^*(\overline{U},y,p^o)}{\partial y}\, dy$$

$$= p_z \cdot [z^*(\overline{U},Y_o,p^o) - z^*(\overline{U},Y_o+y,p^o)]$$

for alternative $\overline{U}$ values. Similarly, for a decrease in nonmarket good,

$$\Delta\mu = \int_{Y_o-y}^{Y_o} - \frac{\partial\mu(\overline{U},y'p^o)}{\partial y}\, dy = p_z \cdot [z^*(\overline{U},Y_o-y,p^o) - z^*(\overline{U},Y_o,p^o)].$$

The similarity of $\Delta E$ and $\Delta\mu$ is clear: exact measures are obtained from the market value of compensated demand differences for related goods whereas the change in spending is obtained from the market value of Marshallian demand differences for related goods.

The following theorem compares the relative sizes of $\Delta E$ and $\Delta\mu$. Figure 4 illustrates the theorem for a substitute good.

Theorem 5: Let z be a vector of normal market goods related to a nonmarket good Y, either as substitutes or with substitutes and complements. Under the conditions of Lemma 1, a change in spending for related goods z is less than all exact measures. A zero income effect ($\partial z/\partial M=0$) implies that the measures are equal.

Proof: The results obtain from the property

$$\frac{\partial z^*}{\partial \bar{U}} = \frac{\partial z}{\partial M}\frac{\partial \mu}{\partial \bar{U}} > 0.$$

First, consider one substitute good and the effect of an increase in Y. Define $U^2 = U(M_o,Y_o+y,p^o)$ and $U^o = U(M_o,Y_o,p^o)$ so $U^2 > U^o$. Duality implies

$$z(M_o,Y_o,p^o) = z^*(U^o,Y_o,p^o)$$

and

$$z(M_o,Y_o+y,p^o) = z^*(U^2,Y_o+y,p^o).$$

Then,

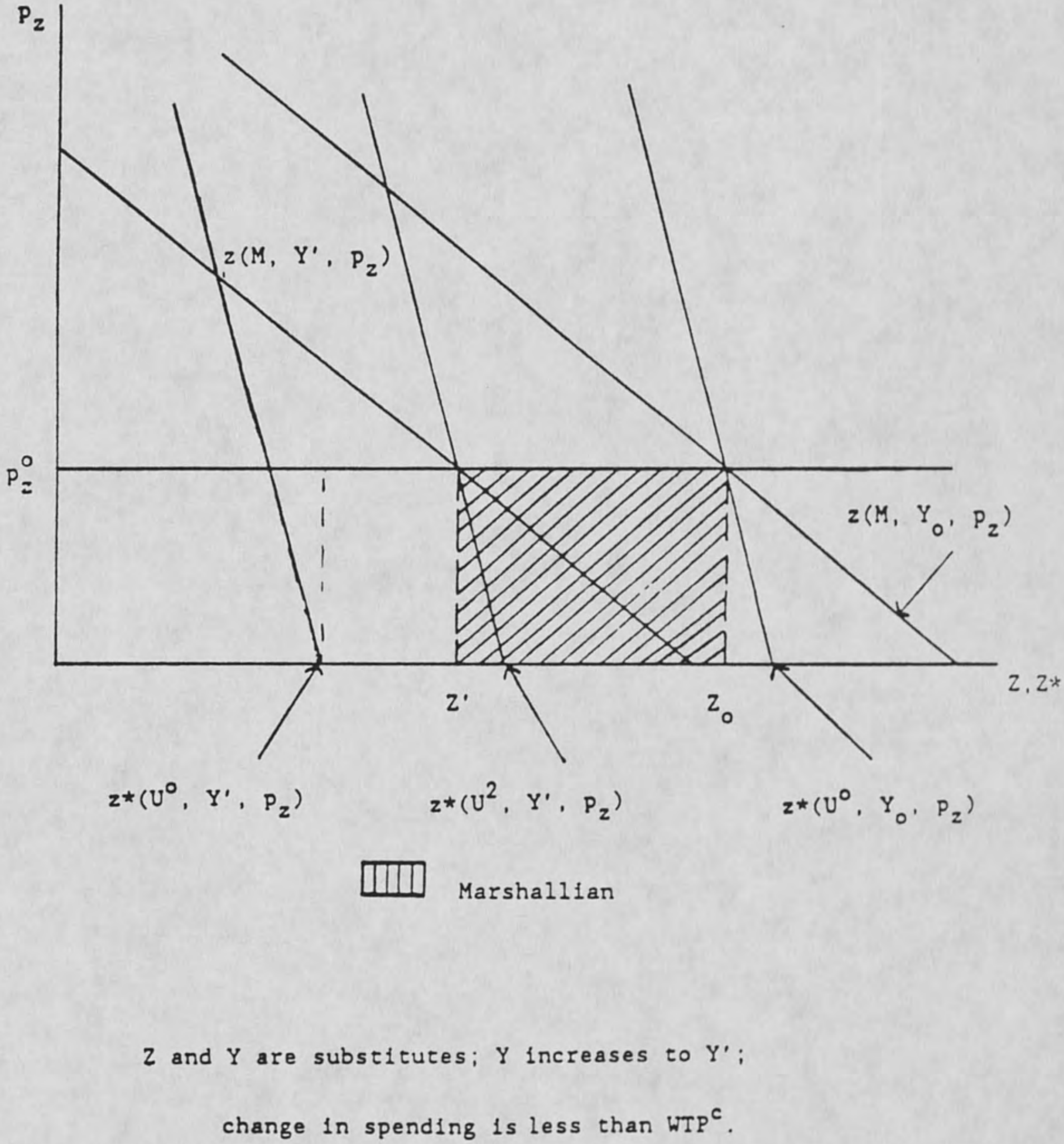
$$\begin{aligned}[z(M_o,Y_o,p^o) - z(M_o,Y_o+y,p^o)] &= [z^*(U^o,Y_o,p^o) - z^*(U^2,Y_o+y,p^o)]\\ &= [z^*(U^o,Y_o,p^o) - z^*(U^o,Y_o+y,p^o)]\\ &+ [z^*(U^o,Y_o+y,p^o) - z^*(U^2,Y_o+y,p^o)].\end{aligned}$$

Thus,

$$\Delta E = WTP^c + p_z \cdot [z^*(U^o,Y_o+y,p^o) - z^*(U^2,Y_o+y,p^o)].$$

Since compensated demand $z^*(\bar{U},Y,p)$ increases with $\bar{U}$, the bracketed term above is negative so we obtain $\Delta E \leq WTP^c$. Similarly,

$$\begin{aligned}z(M_o,Y_o,p^o) - z(M_o,Y_o+y,p^o) &= z^*(U^o,Y_o,p^o) - z^*(U^2,Y_o+y,p^o)\\ &= [z^*(U^2,Y_o,p^o) - z^*(U^2,Y_o+y,p^o)] + [z^*(U^o,Y_o,p^o) - z^*(U^2,Y_o,p^o)]\end{aligned}$$




Z and Y are substitutes; Y increases to Y';

change in spending is less than $WTP^c$.

Figure 4. Comparison of spending and expenditure measures

so that $\Delta E \leq WTA^e$.

For a decrease in Y,

$$\Delta E = WTA^c + p_z \cdot [z^*(U^1, Y_o - y, p^o) - z^*(U^o, Y_o^o - y, p^o)].$$

Since the bracketed term is negative, the change in spending is less than $WTA^c$. Likewise $\Delta E \leq WTP^e$ for a decrease since

$$\Delta E = WTP^e + p_z \cdot [z^*(U^1, Y_o, p^o) - z*(U^o, Y_o, p^o)].$$

If there is more than one substitute good related to Y, then the proof is easily extended by considering z to be a vector with vector multiplication indicated by $p_z \cdot z$.

If some goods are complements and some are substitutes, the vector z represents the set of related complements and substitutes. Eg., for an increase in Y,

$$\Delta E = |p_z \cdot [z^*(U^o, Y_o, p) - z^*(U^o, Y_o + y, p^o)] + p_z \cdot [z^*(U^o, Y_o + y, p^o) - z^*(U^2, Y_o + y, p^o)]|.$$

The first term inside the absolute value is $WTP^c$ and the second term is negative; thus $\Delta E$ is less then $WTP^c$. Similarly for other cases, the terms inside the absolute value operator will have opposite signs. QED.

Note that Lemma 3 and Theorem 5 imply that it is necessary to consider all goods related to Y in order to obtain the greatest lower bound estimate for exact measures. In particular, for complements, changes in spending for related substitute goods should also be considered. If it is not feasible to consider all goods related to a nonmarket good, then at least those with large effects on $\Delta\mu$ should be considered. Considering the relation of $\Delta\mu$ to $\partial z_i^*/\partial y$ and (33), such goods are those which have higher market prices, larger income elasticities, and are more impacted by the nonmarket good.

## Derivation of Exact Measures from Market Data

The purpose of the following examples is to demonstrate how to use the results of Theorem 1 to obtain exact measures of welfare from market observations. As in the case of price changes in private goods, there is no need to approximate exact welfare measures if the expenditure function can conveniently be obtained since exact measures can be expressed in closed form as differences in expenditure.

The method for solving the system of differential equations given in Theorem 1 is similar to the method used by Hausman for price changes in private goods. However, Hausman's paper did not derive exact measures in the case of demand systems. Here we demonstrate how to obtain exact measures for demand systems including nonmarket goods. The examples demonstrate that exact measures can be obtained even when consumer surplus cannot be defined. The following examples include two commonly used demand systems (linear expenditure and AIDS) extended to incorporate nonmarket goods.

### Example 1: Maler's Example

First, we use Maler's example (p. 187) as a simple demonstration. The method of solution is different from Maler's since weak complementarity is not assumed as he does. Actually, the nonmarket and market goods are substitutes, not complements!

The demand system for two goods $z$ and $q$ related to a public good Y is:

$$z = \frac{M}{2p_z} - \frac{\theta Y}{2}$$

$$q = \frac{M}{2p_q} + p_z \frac{\theta Y}{2p_q}$$

and it is easily shown that the system satisfies integrability conditions. Thus, the differential equations to be solved are

$$\frac{\partial \mu}{\partial p_z} - \frac{\mu}{2p_z} = -\frac{\theta Y}{2}$$

$$\frac{\partial \mu}{\partial p_q} - \frac{\mu}{2p_q} = p_z \frac{\theta Y}{2p_q}$$

The system is solved by finding the homogeneous solution and a particular solution for the system. A solution is given by

$$\mu = \bar{U}\, p_q^{1/2}\, p_z^{1/2} - \theta Y p_z$$

and

$$\bar{U} = \frac{M + \theta Y p_z}{p_q^{1/2} p_z^{1/2}}$$

defined to satisfy the boundary condition $\mu(\bar{U},Y,p) = M$. Note in this case that $U_{MY} = 0$ and $U_{YY} = 0$. The compensated demand equations are

$$z^* = \frac{\bar{U}\, p_q^{1/2}}{2\, p_z^{1/2}} - \theta Y$$

$$q^* = \frac{\bar{U}\, p_z^{1/2}}{2\, p_g^{1/2}}$$

Note that $\partial z^*/\partial Y < 0$ (a substitute) while $\partial q^*/\partial Y = 0$ ($q^*$ is independent of Y). In this case

$$WTA^e = WTP^c = (\theta p_z^o)\ [(Y_o+y) - Y_o] = \theta p_z^o\, y = WTP^e = WTA_o^c.$$

The change in spending on z (the good related to Y) is half this amount: $\Delta E = p_z^o\, \theta y/2$.

There is no price such that demand goes to zero. Therefore, the appropriate consumer surplus area cannot be computed when z and Y are imperfect substitutes. Perfect substitution holds for $\theta$=1; in this case, the Marshallian area for gains is

$$MM_g = \frac{M}{3} \ln(1 + \frac{3p_z^o\, y}{M}) \leq p_z^o\, y.$$

Example 2: Linear Expenditure System

The following example, a modified linear expenditure system, is a simple example of a larger demand system which includes a nonmarket good. Suppose a nonmarket good affects the minimum consumption requirement for good one to reduce it to $\gamma_1/\theta Y$; other minimum requirements, not affected by Y, are given by $\gamma_i$. The demand system is then given by :

$$x_1 = \gamma_1/\Theta Y + \beta_1/p_1[M - \sum_{i\neq 1} p_i\gamma_i - p_1\gamma_1/\Theta Y]$$

$$x_i = \gamma_i + \beta_i/p_i[M - \sum_{i\neq 1} p_i\gamma_i - p_1\gamma_1/\Theta Y], \quad \text{for } i = 2, n$$

with $\sum\beta_i = 1$. The corresponding differential equation system to be solved is

$$\partial\mu/\partial p_1 = \gamma_1/\Theta Y + \beta_1/p_1[\mu - \sum_{i\neq 1} p_i\gamma_i - p_1\gamma_1/\Theta Y]$$

$$\partial\mu/\partial p_i = \gamma_i + \beta_i/p_i[\mu - \sum_{i\neq 1} p_i\gamma_i - p_1\gamma_1/\Theta Y].$$

Solving for homogeneous and particular solutions, the general solution is

$$\mu = \bar{U}\,\Pi p_i^{\beta_i} + \sum_{i\neq 1} p_i\gamma_i + p_1\gamma_1/\Theta Y$$

where

$$\bar{U} = [M - \sum_{i\neq 1} p_i\gamma_i - p_1\gamma_1/\Theta Y]/\Pi p_i^{\beta_i}.$$

Compensated demands are

$$x_1^* = \gamma_1/\Theta Y + \beta_1\bar{U}\,\Pi p_j^{\beta_j}/p_1$$

$$x_i^* = \gamma_i + \beta_1\bar{U}\,\Pi p_j^{\beta_j}/p_1$$

so that all goods except good one are independent of Y. Note that Y is a substitute for $x_1$ since $\partial x_1^*/\partial y < 0$ for $\Theta > 0$.

Exact measures are

$$WTP^c = p_1\gamma_1\left[\frac{1}{Y_o} - \frac{1}{Y_o+y}\right]/\Theta$$

and

$$WTP^e = p_1\gamma_1\left[\frac{1}{Y_o-y} - \frac{1}{Y_o}\right]/\Theta$$

so that $WTP^e > WTP^c$.

Change in spending on good one for an increase in Y is

$$\Delta E = \gamma_1 p_1 (1-\beta_1)[\frac{1}{Y_o} - \frac{1}{Y_o+y}]/\theta$$

which is a smaller than $WTP^c$. $\Delta E$ will be a good approximation of $WTP^c$ if $\beta_1$ is small (consumption of good one is close to the minimum requirement). There is no finite price such that demand for $x_1$ is constant and so the consumer surplus area cannot be computed.

Example 3: Almost Ideal Demand System

This example demonstrates a more complicated demand system (the "almost ideal demand system" or AIDS) modified to include nonmarket goods related to market goods. The AIDS system allows both substitutes and complements.

The AIDS system (Deaton and Muellbauer) has the following properties. Marshallian budget share equations are:

$$w_i = \alpha_i + \sum_k \gamma_{ik} \log p_k + \beta_i \log M/P$$

where P is a price index defined by

$$\log P = \sum \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_i \gamma_{ki} \log p_k \log p_i$$

with parameter restrictions

$$\sum_1^n \beta_k = 0, \ \sum_k \gamma_{ki} = 0, \ \sum_i \gamma_{ki} = 0, \ \gamma_{ki} = \gamma_{ik}, \ \sum \alpha_k = 1$$

so that expenditure shares of income "add up" to total income. The term $\gamma_{ki}$ indicates the interaction effect on expenditure between good i and good k. The resulting expenditure function is of the form

$$\log \mu = \log P + \overline{U} b(p)$$

where

$$b(p) = \beta_o \Pi p_k^{\beta_k}$$

and the indirect utility function is

$$\overline{U} = \frac{1}{b(p)} \log M/P.$$

Suppose a nonmarket good reduces the budget share of a market good (use good 1 for example). This effect may be modeled by replacing $p_1$ by $p_1/\theta Y$ in the AIDS system where $\theta$ is a scaling parameter. Then, the budget shares in the AIDS system are modified as follows:

$$w_i' = \alpha_i + \sum\gamma_{ki} \log p_k + \beta_i \log M/P' - \gamma_{i1} \log \theta Y$$

and the overall price index is also modified:

$$\log P' = \log P - \alpha_1 \log\theta Y - \sum\gamma_{k1} \log p_k \log \theta Y.$$

Here, the expenditure share for each good is affected by a change in nonmarket good both due to direct interaction effect with good one and the effect of the nonmarket good on the overall price index.

The solution of the modified system is

$$\log \mu' = \log P - (\alpha_1+\sum\gamma_{k1} \log p_k) \log \theta Y + \bar{U}' b(p)(\theta Y)^{-\beta_1}$$

where

$$\bar{U}' = \frac{1}{b(p)(\theta Y)^{-\beta_1}} \log M/P'.$$

That is, increasing the nonmarket good reduces expenditure for market goods at any fixed level of indirect utility. (To verify that the above is the appropriate solution, it must be shown that the conditions of Theorem one are satisfied.) The resulting compensated expenditure shares are

$$w_i^* = \frac{\partial\mu'}{\partial p_i} \frac{p_i}{\mu'} = \frac{\partial \log \mu'}{\partial p_i} p_i$$

$$= \alpha_i + \sum\gamma_{ki} \log p_k + \bar{U}'\beta_i b(p)(\theta Y)^{-\beta_1} - \gamma_{i1} \log \theta Y.$$

Compensated expenditure shares for each good are affected by Y both through the $\gamma_{i1}$ term and through the indirect utility term.

A market good and the nonmarket good are Hicksian substitutes if $\gamma_{i1} \geq 0$. Hicksian complements require $\gamma_{i1} \leq 0$. A good would be independent of Y for any level of utility if and only if $\gamma_{i1} = \beta_i = 0$ . Thus, all goods with expenditure shares related to income ($\beta_i > 0$) must also be related to Y. (Fortunately, this system is amenable to aggregation in terms of goods!)

Consumer surplus measures may not be used because there is no CCD price unless demand is identically constant ($w_i \equiv \alpha_i$). However, Hicksian welfare measures for an exogenous change in Y are readily computed as a change in expenditure, holding utility constant. For example, we obtain

$$WTP^c = M_o\left[1 - \left[\frac{Y_o}{Y_o+y}\right]^{(\alpha_1+\sum\gamma_{k1}\log p_k^o)} e^{[\theta(Y_o+y)]^{-\beta_1} - [\theta Y_o)^{-\beta_1}}\right]$$

$$WTP^e = M_o\left[1 - \left[\frac{Y_o-y}{Y_o}\right]^{(\alpha_1+\sum\gamma_{k1}\log p_k^o)} e^{[\theta Y_o]^{-\beta_1} - [\theta Y_o-y)]^{-\beta_1}}\right]$$

so $WTP^e \geq WTP^c$. Note that willingness to pay is a fraction of income depending on the percent change in public good weighted by the price effect of the public good and the income share effect.

## Conclusions

Traditional methods to measure benefits of nonmarket good changes have used procedures based on Marshallian demand to value benefits, either change in spending for related market goods or change in consumer surplus. Different consumer surplus areas apply for complements, imperfect substitutes, and perfect substitutes. Use of surplus measures also requires a condition of weak substitutability or complementarity. In contrast, no such requirements are needed for computing exact measures, and distinctions with regard to the type of demand relationship need not be made to compute exact welfare measures. Exact measures are obtained from differences in the expenditure function and the same methods apply regardless of the type of demand relationship. Furthermore, exact measures can be applied for simultaneous changes in prices, income, and public goods.

This paper demonstrated that exact welfare calculations for public goods can be based on observable market relationships in many cases. Demand systems with known expenditure functions (e.g. linear expenditure or AIDS) can be used. Therefore, procedures based on the expenditure function can be applied even when the necessary conditions to use consumer surplus (finiteness of the relevant area and path independence) do not hold. When expenditure functions are not given in closed form, or demand systems are incomplete, numerical methods such as those suggested by Vartia or McKenzie can be used to approximate welfare measures consistent with expenditure theory.

For policy purposes, lower bound approximations of benefits may be adequate to make decisions regarding nonmarket goods. Considering the complexity of evaluating welfare measures, it may be more convenient to approximate welfare effects by the change in spending for goods related to a nonmarket good. However, to obtain a good lower bound welfare estimate, all goods with a strong relationship to a nonmarket good should be identified. When the relationship between a market and nonmarket good is one of complementarity, this paper showed that there must also be some substitute goods in the set of goods related to a nonmarket good.

Besides welfare evaluation, a traditional use of consumer surplus has been to illustrate "net" welfare effects of policy changes in diagrams including both supply and demand. Because exact measures can also be expressed as area differences in terms of compensated demands, consumer surplus type diagrams still apply to illustrate the nature of net welfare effects. However, it is not necessary to perform computations in the same way that illustrative diagrams are constructed!

Since alternative exact welfare measures depend on specified initial conditions and differ in size, the appropriateness of which measure to use for policy analysis remains as an ethical question.

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## Appendix 1
## Definition of Compensating and Equivalent Measures for Combinations of Price, Quantity, and Income Changes

Policy evaluation may require comparisons of situations in which combined changes in prices, incomes, and nonmarket goods occur. Generalizing from the work here and corresponding work for price changes in McKenzie and Pearce, the definition of exact measures is easily extended for cases with combined changes. In each case, a money metric measure is defined as the change in the expenditure function due to a change in utility from a given starting point.

Considering an initial point $(M_o, Y_o, p^o)$ and alterred income, public goods, and prices, M′, Y′, and p′, six alternative utility levels apply:

$$U^0 = \bar{U}(M_o, Y_o, p^o)$$

$$U^1 = \bar{U}(M', Y_o, p^o)$$

$$U^2 = \bar{U}(M_o, Y_o, p')$$

$$U^3 = \bar{U}(M', Y_o, p')$$

$$U^4 = \bar{U}(M_o, Y', p')$$

$$U^5 = \bar{U}(M', Y', p')$$

First consider a combined price and public good quantity change. Suppose $p' < p^o$ and $Y' > Y_o$. Then $U^4 > U^0$ and there exists $E^4 > 0$ such that

$$E^4 = \mu(U^4, Y_o, p^o) - \mu(U^0, Y_o, p^o).$$

By duality

$$\overline{U}(\mu(U^4,Y_o,p^o),Y_o,p^o) = U^4,$$

$$\overline{U}(M_o + E^4,Y_o,p^o) = \overline{U}(M_o,Y',p').$$

Thus, the equivalent measure gives the amount of income which is <u>equivalent</u> in utility to <u>both</u> the price and income change. Such a number may also be defined in other cases as well, e.g., $p' > p^o$ and $Y' > Y_o$ but then we may not know the sign of E.

For the compensating measure, define

$$C^4 = \mu(U^4,Y',p') - \mu(U^0,Y'p')$$

by duality,

$$\overline{U}(\mu(U^0,Y',p'),Y',p') = U^0$$

and substituting,

$$\overline{U}(M_o-C^4,Y',p') = \overline{U}(M_o,Y_o,p^o).$$

Thus $C^4$ is the number which, when subtracted from income after both price and nonmarket good changes, gives the same utility as initially.

For combined income and price changes, similarly defining

$$E^3 = \mu(U^3,Y_o,p^o) - \mu(U^o,Y_o,p^o),$$

by duality

$$\overline{U}(M_o+E^3,Y_o,p^o) - \overline{U}(M',Y_o,p').$$

Also, defining

$$C^3 = \mu(U^3,Y_o,p') - \mu(U^o,Y_o,p')$$

by duality

$$\overline{U}(M'-C^3,Y_o,p') = \overline{U}(M_o,Y_o,p^o).$$

For combined changes in prices, income, and nonmarket goods, defining

$$E^5 = \mu(U^5,Y_o,p^o) - \mu(U^o,Y_o,p^o),$$

by duality

$$U(\mu(U^5,Y_o,p^o),Y_o,p^o) = \overline{U}(M',Y',p')$$

or

$$U(M_o+E^5,Y_o,p^o) = \overline{U}(M',Y',p').$$

Similarly, defining

$$C^5 = \mu(U^5,Y',p') - \mu(U^0,Y',p')$$

gives

$$\overline{U}(M'-C^5,Y',p') = \overline{U}(M_o,Y_o,p^o).$$

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