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**Economics of Controlling Invasive Species: A  
Stochastic Optimisation Model for a Spatial-  
Dynamic Process**

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**Economics of controlling invasive species: a stochastic optimisation model for a spatial-dynamic process**

**by**

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## **Abstract**

Invasive species are significant threats to biodiversity, natural ecosystems and agriculture leading to large worldwide economic and environmental damage. Spread and control of invasive species are stochastic processes with important spatial dimensions. Most economic studies of invasive species control ignore spatial and stochastic aspects. This paper covers this gap in the previous studies by analysing a spatially explicit dynamic process of controlling invasive species in a stochastic setting. We show how stochasticity, spatial location of infestation and control can influence the spread, control efficiency and optimal control strategies. The main aim of this paper is to analyse the relationship between economic parameters and stochastic spatial characteristics of infestation and control. In the model used, there are two ways to control infestation: *border control*, under which the spread of invasive species from any of its infested neighbouring cell is prevented, and *cell control*, which removes the infestation from the existing cell. An integer optimisation model is applied to find the optimal strategies to deal with invasive species. Results show that it is optimal to eradicate or contain for a larger range of border control and cell control costs when the invasion is in the corner or on the edge as compared to the case where the initial infestation is in the middle of the landscape. Decrease in the probability of successful border control makes containment an unfavourable control option even for low border control costs. We show that decrease in the rate of spread can result in switching optimal strategies from containment to abandonment of control, or from eradication to containment. We also showed when the probability of successful cell control decreases, a lower eradication cost is required for eradication to remain the optimal strategy. In summary, this paper shows that in order to avoid providing misleading recommendations to environmental managers, it is important to include uncertainty in the spatial dynamic analysis of invasive species control.

## **1. Introduction:**

Invasive species are significant threats to biodiversity, natural ecosystems and agriculture, leading to major economic and environmental damage worldwide (Costello *et al.* 2007; Olson and Roy 2010).

The spread and control of invasive species is a stochastic spatial process. Even though there have been a number of spatially-explicit studies of invasive species problems by ecologists (e.g. Brow *et al.* 2002; Latimer *et al.* 2009; Espanchin-Niell and Wilen 2010), most economic studies ignore spatial aspects and focus on the performance of particular management strategies (e.g. Olson and Roy 2002; Odom *et al.* 2002; Burnett *et al.*, 2007).

This paper covers this gap in the previous studies by analysing a spatially explicit dynamic process of controlling invasive species in a stochastic setting. We show how stochasticity can influence spread, control efficiency and optimal control strategies in a spatially explicit model. The main aim of this paper is to analyse the relationship between economic parameters and stochastic spatial characteristics of infestation and control. We tackle the challenging task of developing a spatial dynamic model of invasive species control and dealing with uncertainty in a numerical model that builds on the work of Espanchin-Niell and Wilen (2010).

## **2. Method**

### **2.1. Modelling biological spread and Economics**

We develop a stochastic and spatially explicit dynamic optimisation model. A series of square cells represent the landscape where invasion in each cell can spread to the neighbouring cell and eventually can cover the entire landscape.

In a landscape with  $i \times j$  cells, cells are presented as  $a_{i,j}$ . If the cell is invaded with a pest species,  $a_{i,j}=1$  and if the cell is not invaded  $a_{i,j}=0$ . Without any control, the neighbouring cells of an invaded cell will be infested with some probability in the next time period. For example, if, in year  $t$ , cell  $a_{i,j}$  is infested, in year  $t+1$ , cells  $a_{i,j+1}$ ,  $a_{i,j-1}$ ,  $a_{i+1,j}$  and  $a_{i-1,j}$  may be invaded with a certain probability.

Total economic damage caused by the invasive species depends on the number of infested cells. Damage to each cell is represented by  $d$ . Thus the economic damage in the landscape are equal to  $d * \text{number of infested cells}$ . There are two control costs: cell control cost ( $cc$ ) and border control cost ( $bc$ ). Cell control cost refers to the cost of removing infestation from an infested cell. When a cell is infested, it will remain infested unless it is removed by cell control. Border control cost ( $bc$ ) refers to the cost of preventing a cell being infested by its neighbours through their shared boundary. Each cell has 4 boundaries with neighbouring cells and the border control cost for each cell is  $bc * \text{number of boundaries with uninvaded cells}$ .

We solve the model considering three stochastic variables each with three assigned probabilities. The three stochastic processes are: spread, cell control and border control. We assigned separate probabilities to each of these processes and for each cell and time ( $t$ ).

Each cell is either invaded or clear at the end of time  $t$ . Decision (abandon, cell control or border control) is taken in year  $t+1$ . In year  $t+1$ , an uninvaded cell will be invaded with a probability if in year  $t$  it had an invaded neighbour and an effective border control has not been applied to the relevant border.

## 2.2. Optimisation model

We minimise net present value of the damages caused by invasion and control costs in an optimisation framework.

The optimisation model is:

$$\text{Min } \sum_{t \in T, t > 0} \delta_t \left( \sum_{(i,j) \in C_t} a_{i,j,t} d + \sum_{(i,j) \in C} c_{i,j,t} cc + \sum_{(i,j,k,l) \in N} b_{i,j,k,l,t} bc \right) \quad (1)$$

subject to

$$a_{i,j,0} = a_{i,j} \quad \forall (i,j) \in C \quad (2)$$

$$c_{i,j,0} = 0 \quad \forall (i,j) \in C \quad (3)$$

$$b_{i,j,k,l,0} = 0 \quad \forall (i,j,k,l) \in N \quad (4)$$

$$0 \leq \text{rand}(a)_{i,j,t} \leq 1 \quad \forall (i,j) \in C \quad (5)$$

$$0 \leq \text{rand}(c)_{i,j,t} \leq 1 \quad \forall (i,j) \in C \quad (6)$$

$$0 \leq \text{rand}(b)_{i,j,k,l,t} \leq 1 \quad \forall (i,j,k,l) \in N \quad (7)$$

$$p(a)_{i,j,t} \in \{0.2, 0.8, 1\} \quad \forall (i,j) \in C \quad (8)$$

$$p(c)_{i,j,t} \in \{0.2, 0.8, 1\} \quad \forall (i,j) \in C \quad (9)$$

$$p(b)_{i,j,k,l,t} \in \{0.2, 0.8, 1\} \quad \forall (i,j,k,l) \in N \quad (10)$$

$$s(a)_{i,j,t} = 1 \quad \forall (i,j) \in C, \quad \text{rand}(a)_{i,j,t} \leq p(a)_{i,j,t} \quad (11)$$

$$s(a)_{i,j,t} = 0 \quad \forall (i,j) \in C, \quad \text{rand}(a)_{i,j,t} \geq p(a)_{i,j,t} \quad (12)$$

$$s(c)_{i,j,t} = 1 \quad \forall (i,j) \in C, \quad \text{rand}(c)_{i,j,t} \leq p(c)_{i,j,t} \quad (13)$$

$$s(c)_{i,j,t} = 0 \quad \forall (i,j) \in C, \quad \text{rand}(c)_{i,j,t} \geq p(c)_{i,j,t} \quad (14)$$

$$s(b)_{i,j,k,l,t}=1 \quad \forall (i, j, k, l) \in N, \text{rand}(b)_{i,j,k,l,t} \leq p(c)_{i,j,k,l,t} \quad (15)$$

$$s(b)_{i,j,k,l,t}=0 \quad \forall (i, j, k, l) \in N, \text{rand}(b)_{i,j,k,l,t} \geq p(c)_{i,j,k,l,t} \quad (16)$$

$$a_{i,j,t} \geq a_{i,j,t-1} - c_{i,j,t} s(c)_{i,j,t} \quad \forall (i, j) \in C, t \in T, t \geq 1 \quad (17)$$

$$a_{i,j,t} \geq a_{k,l,t-1} s(a)_{i,j,t} - b_{i,j,k,l,t} s(b)_{i,j,k,l,t} - c_{i,j,t} s(c)_{i,j,t} \quad \forall (i, j, k, l) \in N, t \in T, t \geq 1 \quad (18)$$

$$a_{i,j,t} \in \{0,1\} \quad \forall (i, j) \in C, t \in T \quad (19)$$

where

$i$  indexes row and  $j$  indexes the column in a rectangular set of cells ( $C$ ).

$k$  and  $l$  index pairs of the neighbors of cells  $a_{i,j}$ .

$t$  represents time (year) and  $T$  is the number of years considered.

$a_{i,j,t}$  represents the cells in row  $i$ , column  $j$  at time  $t$ . When a cell is invaded  $a_{i,j,t} = 1$ , otherwise  $a_{i,j,t} = 0$ .

$c_{i,j,t}$  is a binary decision variable to remove the pest from cell  $a_{i,j}$  in time  $t$ .  $c_{i,j,t} = 1$  if the decision is to remove invasion and  $c_{i,j,t} = 0$  otherwise.

$b_{i,j,k,l,t}$  is a binary decision variable to control the spread of invasion along the border between  $a_{i,j,t}$  and  $a_{k,l,t}$ .  $b_{i,j,k,l,t} = 1$  if the border control is applied and  $b_{i,j,k,l,t} = 0$  otherwise.

$a_{i,j,b}$ ,  $c_{i,j,t}$  and  $b_{k,l,b}$  are random variables. For the purpose of this illustrative analysis, it is assumed that they have uniform distributions.

$p(a)_{i,j,b}$ ,  $p(b)_{k,l,j,t}$  and  $p(c)_{i,j,t}$  are assigned probabilities of successful spread, border control and cell control respectively.

$\delta$  is the real discount factor at time  $t$  ( $t > 0$ ).  $\delta = \frac{1}{(1+r)^{t-1}}$  where  $r$  is real discount rate.

$d$  is the economic damages caused by invasive species for each cell in year  $t$ .

$cc$  is cell control cost, representing the cost of removing the pest from a cell.

$bc$  is border control cost, representing the cost of avoiding pest spread between neighbouring cells.

Equation (2) indicates initial infestation in year  $t_0$ . Equations (2) and (3) show that cell control and border control start in the next time period. Equations (11-16) represent stochastic binary multipliers for spread, border control and cell control at time  $t$  and cells  $a(i,j)$ . The value of these stochastic multipliers depends on the assigned probabilities of successful spread, cell control and border control (equations 8-10). If these assigned probabilities are bigger than the random variables (equations 5-7) at time  $t$ , the stochastic variable equals 1 otherwise 0. Equation (17) shows that a cell that has been invaded in year  $t-1$  will be invaded in year  $t$  unless a cell control measure is applied and the assigned probability of cell control is bigger than the random variable for cell control. Equation (18) shows that cell  $a(i,j)$  is invaded at time  $t$ , if it had an invaded neighbour at time  $t-1$  and the assigned probability of spread is larger than the random variable for spread. However, the spread will not occur if cell control is applied and the assigned probability of successful control is larger than the border control random variable. Spread also would not occur if border control measure is applied along the relevant border and the assigned probability for successful border control is larger than border control random variable.

The problem is solved for a finite time horizon using Bellman's principal of optimality. When solving for an infinite time horizon, the system can reach a state that will remain the same. However, this is not the case in a finite time horizon where the system can reach the steady state and depart from it and reach it again. To solve this problem, we lock in the steady state equilibrium using constraints after the steady state has been reached. To do this, a terminal value function has been added that

accounts for economic values (damages and control costs) after the fixed time horizon. The following equation has been added to lock in the steady state solution:

$$a_{i,j,t} = a_{i,j,t_m} \quad \forall (i, j) \in C, t \in T, t > t_m \quad (20)$$

where  $t_m$  is smaller than  $T$ . We allow enough time for the steady state to be reached before  $t_m$  and  $T$ .

The terminal value is calculated from the followings that will be added to the objective function:

$$\sum_{t=T+1}^{\infty} \delta_t \left( \sum_{(i,j) \in C_t} a_{i,j,T} d + \sum_{(i,j) \in C} c_{i,j,T} cc + \sum_{(i,j,k,l) \in N} b_{i,j,k,l,T} bc \right) \quad (21)$$

### 3. Results

The binary integer programming model was solved using GAMS (General Algebraic Modelling System). When  $p(a)_{i,j,t} = 0.8$  or 1,  $T_m$  and  $T$  are set at 50 and 100 years respectively. However, when  $p(a)_{i,j,t} = 0.2$ , it takes longer for the system to reach the steady state and  $t_m$  and  $T$  are set at 100 and 150, respectively.

We solved the model for all possible combinations of the cases where the probability of spread, successful cell control and border control are 0.2, 0.8 and 1. Here we present a selection of the results obtained.

#### 3.1. Optimal decisions for the deterministic case

Here we illustrate how optimal control strategies change depending on the eradication and cell control costs when the probability of spread, border control and cell control are deterministic. Three cases are considered: (1) when the initial infestation is in the middle of the landscape (Figure 1-A); (2) when the initial infestation is in the corner of the landscape (Figure 2-B); and (3) when the initial infestation is on the edge

(Figure 2-C). For all these cases, when the border control costs are low and eradication cost is relatively high, the optimal strategy is to contain (Figure 1).

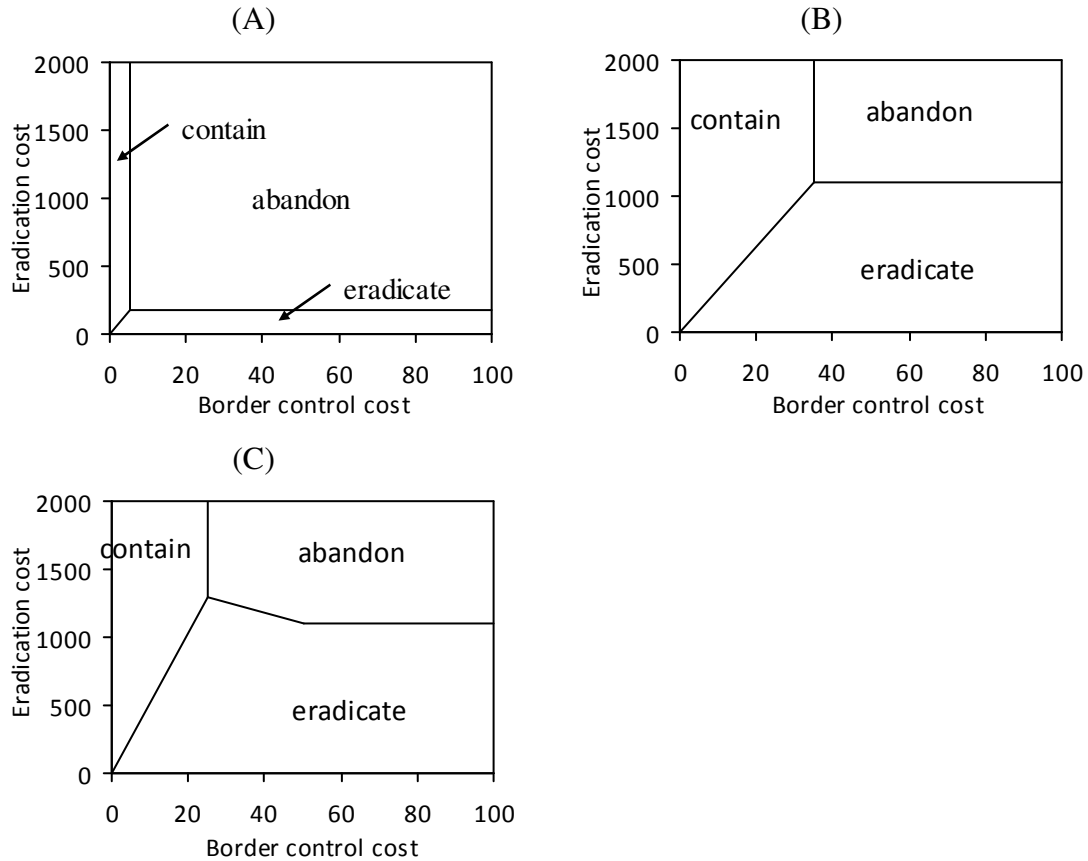


Figure 1. Optimal strategies for the deterministic model when A) the initial infestation is in the middle, B) initial infestation is in the corner and C) initial infestation is on the edge.

When the eradication and border control costs are both high, the optimal strategy is to abandon control and when the eradication costs are low and border control costs sufficiently high, the optimal strategy is to eradicate. When the initial infestation is in the middle of landscape, infestation can potentially cover the entire landscape more quickly. This means that controlling invasion when the initial infestation is in the middle of the landscape can be harder relative to the cases where the invasion is on the edge or in the corner. Therefore when the invasion is in the corner or on the edge, it is optimal to eradicate or contain for a larger range of border control and cell control

costs as compared to the case where the initial infestation was in the middle of the landscape.

### 3.2. Optimal decisions when the probability of spread, successful cell and border control is 80%

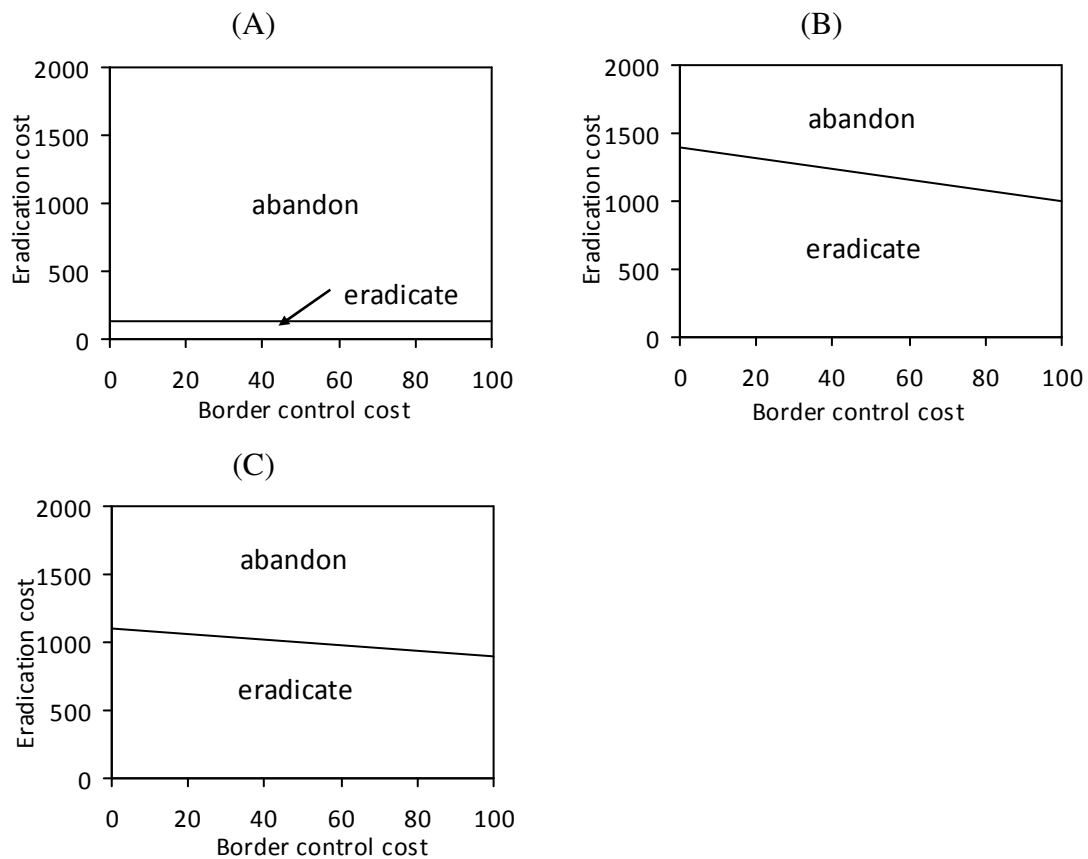


Figure 2. Optimal strategies when the probability of spread, successful cell and border control is 80%. The initial invasion is either in the middle (A), in the corner (B) or on the edge (C).

Optimal control strategies when the probabilities of spread, cell control and border control are 80% are different to the deterministic case. With stochastic border control, containment is not strictly possible, so containment does not appear as an optimal strategy in any of the panes of Figure 2. When the initial infestation is located in the middle of landscape (Figure 2A) it is optimal to eradicate only when eradication cost

is low. However, when the initial infestation is in the corner (Figure 2B) or on the edge of landscape (Figure 2C), it is relatively less costly to contain the infestation, as there are fewer boundaries over which spread threatens to occur. Therefore it is optimal to eradicate for a larger range of eradication costs.

Border control together with cell control helps eradication of the invasive species.

When the initial infestation is in the corner or edge of the landscape, an increase in border control cost makes border control less cost-effective. Thus the increase in border control costs make eradication a less favourable strategy and the optimal strategy may become to abandon control (Figure 2A and 2B).

### **3.3. Optimal decisions when the probability of spread, successful cell and border control is 20%**

Optimal control strategies when the probability of spread, cell control and border control are 20% are presented in Figure 3A-C. Similar to the case where probability of spread, successful cell and border control is 80%, border control is not effective so containment is not optimal in any scenario. When the initial infestation is in the middle or on the edge, due to the relative difficulty of controlling and containing the pest, invasion spreads quickly and eradication is not optimal (Figure 3A and 3C). However, when the initial infestation is in the corner, there are fewer boundaries over which spread threatens to occur and it is easier to eradicate the invasion. In this case for a range of low eradication cost it is optimal to eradicate (Figure 3B). Note that the optimality of eradication mainly depends on eradication costs, but also depends to some extent on border control costs. For example in the cases that eradication is

optimal, increase in border control cost alone can result in replacement of eradication by abandonment of control.

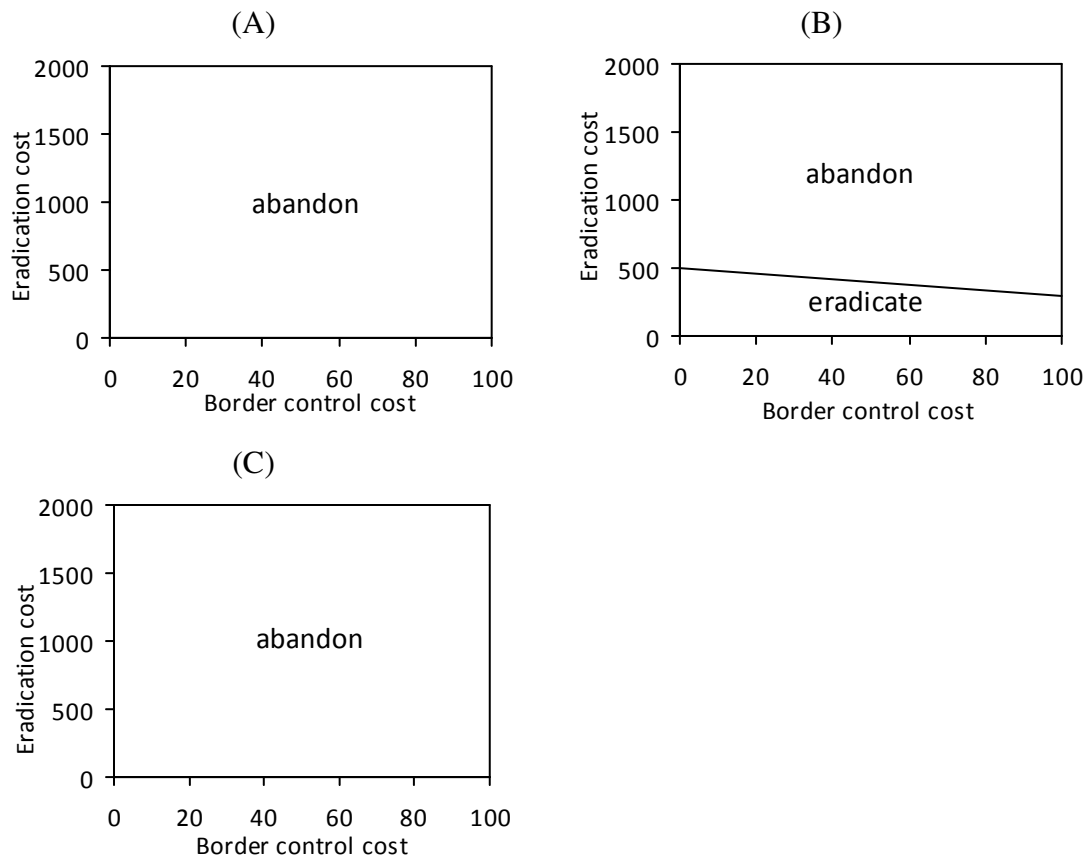


Figure 3. Optimal strategies when the probability of spread, successful cell and border control is 20%. The initial invasion is either in the middle (A), in the corner (B) or on the edge (C).

### 3.4. Optimal strategies for different probabilities of spread

Here we analyse how change in the probability of spread affects the optimal strategies when the probability of successful cell and border control are deterministic. This analysis focuses on the case where initial infestation is on the edge of the landscape. The optimal decision does not change significantly when the probability of spread decreases from 100% (Figure 3A) to 80% (Figure 3B). Only for a small range of eradication costs at higher border control costs, decrease in the probability spread to

80% results in replacement of abandonment by eradication. With the reduced probability of spread, eradication becomes slightly more feasible, and so becomes optimal in a few more cases.

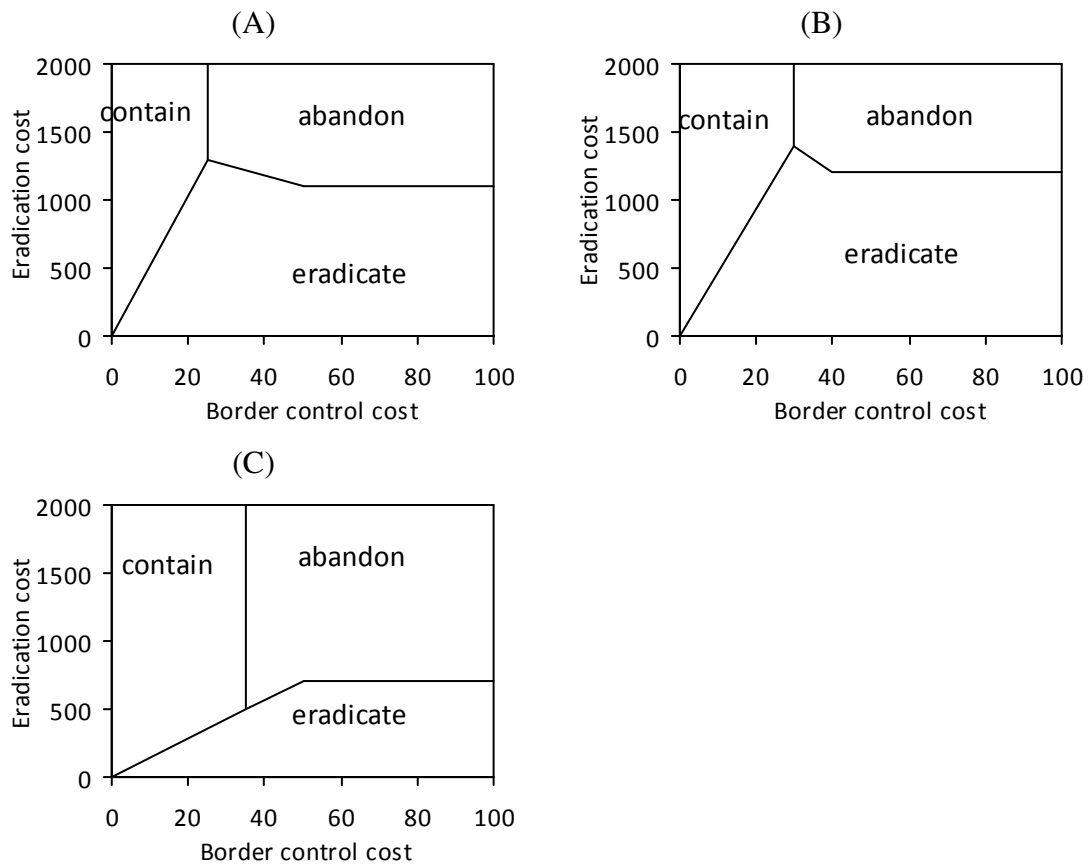


Figure 4. Optimal strategies when the probability of spread is either deterministic (A), 80%(B) or 20% (C). Cell control and border control are deterministic and the initial infestation is on the edge.

However, when the probability of spread reduces to 20% (Figure 4C), the benefits of eradication are reduced – failure to eradicate causes less damage because of the lower probability of spread. In this case, eradication becomes a less favourable strategy and is optimal in fewer scenarios.

On the other hand, when the probability of spread decreases, containment becomes a more cost-effective control option. It becomes the preferred option for a larger range of border control costs.

### 3.5. Optimal strategies for different probabilities of successful cell control

Figure 5A-C show how change in the probability of successful cell control affects optimal strategies when the probability of spread and successful border control are deterministic and initial infestation is in the middle. Decrease in the probability of successful cell control to 80% does not have a significant impact on the optimal decision.

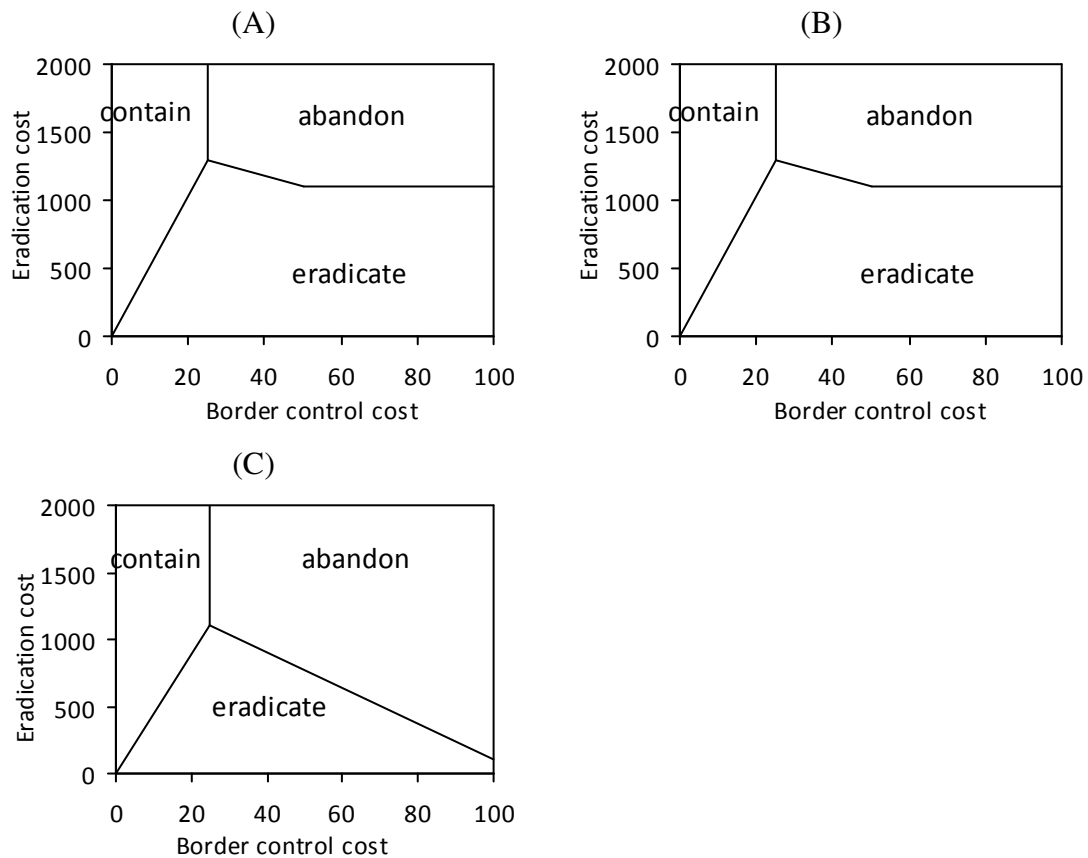


Figure 5. Optimal strategies when the probability of cell control is either deterministic (A), 80% (B) or 20% (C). Spread and successful border control are deterministic and initial infestation is on the edge.

However, when the probability successful cell control decreases to 20%, eradication is optimal only at a lower eradication cost. Border control plays an important role when the probability of cell control decreases to 20%. For the deterministic case when

border control cost is larger than 48, an increase in border control cost does not affect optimal decision. However, when the probability of successful cell control decreases to 20%, increases in border control cost rapidly lead to replacement of eradication with abandonment.

### 3.6. Optimal strategies for different probabilities of successful border control

The effect of change in the probability of successful border control on the optimal decision options are analysed here.

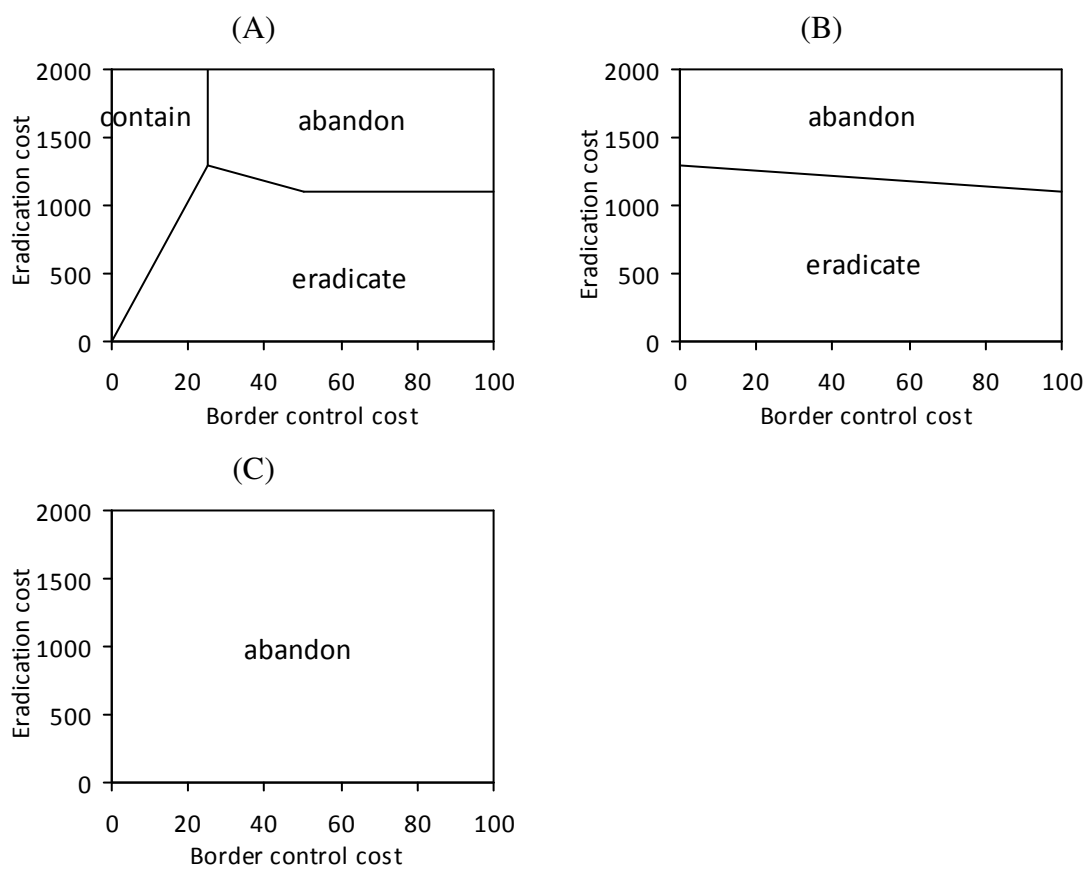


Figure 6. Optimal strategies when the probability of border control is either deterministic (A), 80%(B) or 20% (C). Spread and successful cell control are deterministic and initial infestation is on the edge.

The probability of spread and successful cell control are deterministic and initial invasion is on the edge of the landscape. Results show that a decrease in the probability of successful border control has an important affect on the optimal

strategies. As the probability of successful control decreases invasion can spread through borders and containment is no longer an optimal option even for low levels of border control costs (Figures 6B and 6C). When the probability of successful border control decrease to 80%, eradication remains optimal if the eradication cost is low enough. However, when the probability of border control decreases to 20%, it is no longer optimal to eradicate at any eradication cost.

#### **4. Conclusions**

In this study we have analysed optimal strategies to deal with invasive species in a stochastic, spatial, dynamic, setting. We have extended the work of Espanchin-Niell and Wilen (2010) by introducing stochasticity to a spatial dynamic process in an optimisation formwork. Results showed that stochasticity and spatial location play important roles in determining the optimal strategy adopted.

We confirm the finding of Espanchin-Niell and Wilen (2010) that it is optimal to eradicate or contain for a larger range of border control and cell control costs when the invasion is in the corner or on the edge as compared to the case where invasion is in the middle of the landscape. This remains generally true in the stochastic framework. For low probabilities of successful border control and cell control, it is not optimal to eradicate unless the invasion is in the corner of landscape.

Decreases in the probability of successful border control make containment an unfavourable control option even for low border control costs. As the probability of spread decreases, it takes longer for the invasion to cover the land and economic damages become smaller. Thus economic benefits of control become smaller and for some ranges of parameter values abandonment can replace containment and containment can replace eradication. We also showed that when the probability of

successful cell control decreases to 20% it is optimal to eradicate only at a low eradication costs.

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