Technical efficiency in competing panel data models: A study of Norwegian grain farming

Subal C. Kumbhakar (corresponding author)
Department of Economics, State University of New York, Binghamton, NY 13902, USA,
University of Stavanger Business School, Stavanger, Norway, and Norwegian Agricultural Economics Research Institute, Oslo, Norway. E-mail: kkar@binghamton.edu, Tel: (+1 607) 777 4762, Fax: (+1 607) 777 2681

Gudbrand Lien
Norwegian Agricultural Economics Research Institute, Oslo, Norway and
Faculty of Economics and Organisation Science, Lillehammer University College, Lillehammer, Norway

J. Brian Hardaker
School of Business, Economics and Public Policy, University of New England, Armidale, Australia

Paper prepared for presentation at the EAAE 2011 Congress
Change and Uncertainty
Challenges for Agriculture,
Food and Natural Resources

August 30 to September 2, 2011
ETH Zurich, Zurich, Switzerland

1 Introduction

Since its introduction by Aigner et al. (1977), stochastic frontier (SF) estimation has been extensively used to estimate technical efficiency in applied economic research.[1] Both cross-sectional and panel data are used for this purpose. Estimates of technical efficiency measures in these models often depend on model specification, distributional assumptions, temporal behavior of inefficiency, etc. Given the interest of these efficiency measures in policy discussions, there is a need to examine the robustness of such results in both cross-sectional and panel data models.

In cross-sectional modeling specific distributions on inefficiency and noise terms are assumed in order to estimate the frontier function. The distributional assumptions are necessary in separating inefficiency from noise (Jondrow et al. 1982). The Jondrow et al. estimator of inefficiency is not consistent in cross-sectional models. The advantage of panel data is that, if inefficiency is time invariant, one can estimate inefficiency consistently without distributional assumptions (Schmidt and Sickles 1984). However, the assumption that inefficiency is time-invariant is quite strong, although the model is relatively simple to estimate if inefficiency is specified as fixed parameters instead of as a random variable (Pitt and Lee 1981; Kumbhakar 1987; Battese and Coelli 1988). The other extreme is to assume that both inefficiency and noise terms are independently and identically distributed (iid). This assumption makes the panel nature of the data irrelevant. There are also models that fall between these extremes.

Among panel data models, that are our main focus in this study, the inefficiency specification used by Battese and Coelli (1995) is most frequently used in empirical studies. Their model allows inefficiency to depend on some exogenous variables so that one can investigate how exogenous factors influence inefficiency. Although this model is designed for cross-sectional data, it can readily be used for panel models. The panel data model due to Battese and Coelli (1992) is somewhat restrictive because it only allows inefficiency to change over time exponentially.[2] Furthermore, these models mix firm effects with inefficiency. Two other models, viz., the 'true-fixed' and 'true-random' effects frontier models for panel data (Greene 2005a, 2005b) have become popular in recent years. These models separate firm effects (fixed or random) from inefficiency, where inefficiency can either be iid or can be a function of exogenous variables. Although there are many other specifications, empirical researchers mostly seem to use either the Battese and Coelli or the Greene models, apparently often without fully considering the assumptions behind these models. So the questions are: (i) Why are these particular models preferred? (ii) How do they compare with others that are seldom applied or even discussed?

The goal in this study is neither to give an exhaustive review of SF models for panel data, nor to recommend a particular model. Rather we have selected some alternative panel models that address inefficiency with or without heteroskedasticity and have applied these to the same dataset to illustrate the extent to which results from such studies are model dependent. Some of these models can also be used to analyze cross-sectional data.

In a standard panel data model, the focus is mostly on controlling firm effects (heterogeneity due to unobserved time-invariant factors). This notion is adapted from the earlier panel data models (Pitt and Lee 1981; Schmidt and Sickles 1984; Kumbhakar 1987) in which inefficiency is treated as time-invariant. The only innovation in the efficiency models was to make these firm effects one-sided so as to give them an inefficiency interpretation. Models were developed to treat these firm effects as fixed as well as random. Several models have been developed based on the assumption that all the time-invariant (fixed or random) effect is (persistent) inefficiency (e.g. Schmidt and Sickles 1984; Pitt and Lee 1981). This is in contrast to the 'true' random or fixed effect models by Greene (2005a, 2005b) in which firm-specific effects are not parts of inefficiency. The models proposed by Kumbhakar (1991) and Kumbhakar and Heshmati (1995) are in between. These models treat firm effects as persistent inefficiency and include another component to capture time-varying technical inefficiency. Since none of these assumptions outlined above may be wholly satisfactory, we introduce a new model that may overcome some of the limitations of earlier approaches. In this model we decompose the time-invariant firm effect as a firm effect and a persistent technical inefficiency effect.

It is clear from the above discussion that to get meaningful results from SF panel data models one

[1] Reviews of models used and recent applications are given in, e.g., Kumbhakar and Lovell (2000), Coelli et al. (2005), Kumbhakar (2006) and Greene (2008).

[2] Wang and Ho (2011) generalized the Battese-Coelli formulation in which the temporal pattern of inefficiency is made firm-specific by specifying it as a function of covariates that can change both temporally and cross-sectionally.

should consider several aspects carefully. The results obtained are likely to depend on the modeling approach taken and on the way inefficiency is interpreted. Applying several different models to the same data set to expose differences in results, as we do herein, is likely to provide deeper insights into the implications of choosing different models.

The rest of the article is organized as follows. We first outline the panel data models applied in the empirical applications. Then, we discuss the Norwegian grain farm data that are used in the models, followed by a presentation and discussion of the different model results. Finally, some concluding comments are provided.

2 Survey of the panel data models: A partial view

Our goal here is not to investigate all existing panel data models, since we know, a priori, that different models give different results. So we have selected six panel data models, and investigated the results from these when applied to the same data set. The first three of the selected models include heteroskedasticity in the inefficiency/noise term. The last three models include heterogeneity in the intercept, which may or may not be part of inefficiency. These last three models also account for time-varying inefficiency. These six models are briefly summarized in Table 1.

Table 1 Some main characteristics of the six panel data models investigated

	Model 1	***Model 2***	***Model 3***	***Model 4***	***Model 5***	***Model 6***
General firm effect	No	No	Fixed	Random	No	Random
Technical inefficiency						
Persistent	No	No	No	No	Yes	Yes
Residual	No	No	No	No	Yes	Yes
Overall technical inefficiency						
Mean	Time -inv.[1]	Time -inv.	Time -inv.	Zero trunc.[2]	Zero trunc.	Zero trunc.
Variance	Homo.	Hetero.	Hetero.	Hetero.	Homo.	Homo.
Symmetric error term						
Variance	Homo.	Hetero.[3]	Hetero.	Homo.	Homo.	Homo.

Notes: [1] Time-inv. mean inefficiency models include determinants of inefficiency in the mean function.
[2] Zero truncation models assume inefficiency distribution to be half-normal.
[3] Hetero. (Homo.) refers to models in which variances are functions of covariates that are both firm-specific and time-varying (constant).

Model 1

Here we consider generalization of the first generation panel data models (Pitt and Lee 1981; Schmidt and Sickles 1984; Kumbhakar 1987; Battese and Coelli 1988), which are of the form:

$$y_{it} = \alpha + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + v_{it} - u_i \tag{1}$$

where y_{it} is the log of output (revenue) for firm i at time t; α is a common intercept; $f(\boldsymbol{x}_{it}; \boldsymbol{\beta})$ is the production technology; $\boldsymbol{x}_{it}$ is the vector of inputs (in logs); $\boldsymbol{\beta}$ is the associated vector of technology parameters to be estimated; v_{it} is a random two-sided noise term (exogenous production shocks) that can increase or decrease output (*ceteris paribus*); and $u_i \geq 0$ is the non-negative one-sided inefficiency term. The parameters of the model are estimated by the maximum likelihood (ML) method using the following distributional assumptions:

$$u_i \sim N^+(0, \sigma^2) \text{ or } N^+(\mu, \sigma^2), \tag{2}$$

$$v_{it} \sim N(0, \sigma_v^2) \tag{3}$$

The estimated parameters are then used to obtain firm-specific estimates of technical inefficiency in (1) using the Jondrow et al. (1982) technique.

If the u_i are fixed parameters (as in Schmidt and Sickles 1984), then the u_i term can be combined with the common intercept, i.e., $\alpha_i = \alpha + u_i$ so that all the α_i parameters can be identified, for example, from the coefficients of the firm dummies. Inefficiency u_i can then be estimated from $\hat{u}_i = \max_i\{\hat{\alpha}_i\} - \hat{\alpha}_i \geq 0$ where $\hat{\alpha}_i$ is the fixed firm effect in the standard panel data model. This makes the best firm (highest intercept) fully efficient and thus inefficiency for other firms is relative to the best firm. The advantage of this approach is that it is not necessary to make any distributional assumptions about the inefficiency term. The disadvantage is that we cannot use any time-invariant covariates to explain inefficiency.

If u_i is assumed to be a random variable (Pitt and Lee 1981; Kumbhakar 1987; Battese and Coelli 1988) that is distributed as either half- or truncated normal, as in (2), the parameters of the model can be estimated by the ML method. It can be shown (Kumbhakar 1987) that the conditional distribution of $u_i \mid \boldsymbol{\varepsilon}_i$ is truncated normal where $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, ..., \varepsilon_{iT})$ and $\varepsilon_{it} = v_{it} - u_i$. The mean and/or mode of $u_i \mid \boldsymbol{\varepsilon}_i$ can then be used to obtain firm-specific estimates of inefficiency. These estimates are consistent when $T \to \infty$ (Kumbhakar 1987).

To make this model comparable with several other models that are used in this paper, we consider a generalization, viz.

$$u_i \sim N^+(\mu_i, \sigma^2) = N^+(\delta_0 + \boldsymbol{z}_i'\boldsymbol{\delta}, \sigma^2), \tag{4}$$

$$v_{it} \sim N(0, \sigma_v^2) \tag{5}$$

In this specification inefficiency is explained by a vector of time-invariant covariates $\boldsymbol{z}_i$ (or the means of time varying covariates for each firm), and $\boldsymbol{\delta}$ is the vector of parameters associated with these covariates. Using these covariates lets one examine the marginal effect of these variables on inefficiency. If some of these are policy variables, implications can be drawn about the effect of changing the policy on the measure of inefficiency. If all the $\boldsymbol{\delta}$-parameters are zero, then the model reduces to the one considered by Pitt and Lee (1981), Schmidt and Sickles (1984), Kumbhakar (1987), and Battese and Coelli (1988). Model 1 in this study refers to the specification in (1), (4) and (5).

Model 2

Model 1 (based on the specification in either (1), (2) and (3) or (1), (4) and (5)) is based on the assumptions that the two-sided error term v_{it} and the one-sided error term u_i are homoscedastic, i.e., that both σ^2 and σ_v^2 are constants. However, there may be no reason to assume that this is so in reality. Ignoring heteroskedasticity could lead to inconsistent parameter estimates. After a detailed discussion of the issues Kumbhakar and Lovell (2000, chapter 3.4) concluded that:

- Ignoring heteroskedasticity of the symmetric error term v_{it} gives consistent estimates of the frontier function parameters (β). Heteroskedasticity refers to models in which variances are functions of covariates that are both firm-specific and time-varying, except that the intercept (α) is downward biased. Estimates of technical efficiency will also be biased.
- Ignoring heteroskedasticity of the one-sided technical inefficiency error component u_i causes biased estimates of both the parameters of the frontier function and the estimates of technical efficiency.

The other problem in Model 1 is that inefficiency is time-invariant, which is quite restrictive. Model 2 is an extension of Model 1 that allows for heteroskedasticity in both the one-sided technical inefficiency error component and in the symmetric noise term. This model is frequently termed the doubly heteroskedastic model in the literature. It is specified as:

$$y_{it} = \alpha + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + v_{it} - u_{it} \tag{6}$$

$$u_{it} \sim N^+(\mu, \sigma_{it}^2) = N^+\left(\mu, exp(\omega_{u0} + \boldsymbol{z}_{u,it}'\boldsymbol{\omega}_u)\right) \tag{7}$$

$$v_{it} \sim N(0, \sigma_{v,it}^2) = N\left(0, exp(\omega_{v0} + \boldsymbol{z}_{v,it}'\boldsymbol{\omega}_v)\right) \tag{8}$$

Although both the Kumbhakar and the Battese-Coelli models are based on assumptions that *u* and *v* are homoskedastic (cf. (4) and (5) above), such assumptions are not necessary. The model in (6)-(8) generalizes the models proposed by Kumbhakar and by Battese-Coelli by making both *u* and *v* heteroskedastic. In the variance function ω_{u0} is a constant term, the $\boldsymbol{z}_{u,it}$ vector includes exogenous variables associated with variability in the technical inefficiency function, and $\boldsymbol{\omega}_u$ is the corresponding coefficient vector. Similarly, ω_{v0} is the constant term and the vector $\boldsymbol{z}_{v,it}$ includes exogenous variables (that can be time-varying) associated with variability in the noise term, and $\boldsymbol{\omega}_v$ is the corresponding coefficient vector.

Parameterizing $\sigma_{v,it}^2$ as done here to model production variability within a stochastic production function framework, is an alternative to well-known 'production risk' specification of Just and Pope (1978).

It is also possible to use (6)-(8) and change (7) to $u_{it} \sim N^+(0, \sigma_{it}^2) = N^+\left(0, \exp(\omega_{u0} + \boldsymbol{z}'_{u,it}\boldsymbol{\omega}_u)\right)$ (see Caudill and Ford 1993; Caudill et al. 1995; Hadri 1999). Another option is to consider a further generalization in which both the mean and variance of u are functions of $\boldsymbol{z}$ variables (Wang 2002), i.e.,

$$u_{it} \sim N^+(\mu_{it}, \sigma_{it}^2) = N^+\left(\delta_0 + \boldsymbol{z}'_{it}\boldsymbol{\delta}, \exp(\omega_{u0} + \boldsymbol{z}'_{u,it}\boldsymbol{\omega}_u)\right) \tag{7a}$$

Wang demonstrated that parameterizing both the mean and variance of the one-sided technical inefficiency error component allows non-monotonic efficiency effects, which can be useful for understanding the relationships between the inefficiency and its exogenous determinants. The models of Huang and Liu (1994) and Battese and Coelli (1995), in which variances are assumed to be constant, are special cases of the Wang (2002) model. Given all these generalizations, there is no reason for using the Battese-Coelli model without scrutinizing it carefully, viz., testing it against more general specifications. In other words, many alternative specifications are possible. In the analysis reported below we used the specification in (6), (7a) and (8) as Model 2.

Model 3

Both Model 1 and various versions of Model 2 account for the panel data structure by including time as an exogenous variable in the model components. In Model 3, which is an extension of the model by Kumbhakar and Wang (2005), we accommodate the panel nature of the data by introducing firm-specific intercepts, i.e.,

$$y_{it} = \alpha_i + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + v_{it} - u_{it} \tag{9}$$

$$u_{it} = G_t u_i \tag{10}$$

$$G_t = exp(\gamma\ t) \tag{11}$$

$$u_i \sim N^+(\mu_i, \sigma_i^2) = N^+\left(\delta_0 + \boldsymbol{z}'_i\boldsymbol{\delta}, exp(\omega_{u0} + \boldsymbol{z}'_{u,i}\boldsymbol{\omega}_u)\right) \tag{12}$$

$$v_{it} \sim N(0, \sigma_{v,it}^2) = N\left(0, exp(\omega_{v0} + \boldsymbol{z}'_{v,it}\boldsymbol{\omega}_v)\right) \tag{13}$$

The above equations describe Model 3 used in the analysis below. Compared to Models 1 and 2, Model 3 exploits the panel structure of the data better, since the intercept term α_i in equation (9) controls for unobserved heterogeneity or firm-specific fixed effects. Note that, in this specification, firm effects (α_i whether fixed or random) are not regarded as part of inefficiency. In other words, this model can separate technical inefficiency (time-varying) from time-invariant firm effects simply by assuming (without any explanation) that firm effects do not include inefficiency.

Note that in specifying inefficiency as $u_{it} = G_t u_i$ in (10) we are making the assumption that it can be represented as a product of G_t − a deterministic function of time, and u_i − a non-negative random variable. This is one way of exploiting the panel feature of the data without introducing additive firm effects. Kumbhakar (1991) formulated $G_t = (1 + \exp(b_1 t + b_2 t^2))^{-1}$ so that G_t can be monotonically increasing (decreasing) or concave (convex) depending on the signs and magnitudes of b_1 and b_2. Battese and Coelli (1992) simplified the formulation by assuming $G_t = exp(-\gamma(t-T))$. Their specification allows inefficiency to increase or decrease exponentially depending on the sign of γ. Thus the Kumbhakar (1991) model is slightly more general because it allows more flexibility in the temporal behavior of inefficiency. Feng and Serletis (2009) extended the Battese-Coelli formulation by specifying $G_t = exp\left(-\gamma_1(t-T) - \gamma_2(t-T)^2\right)$. Wang and Ho (2010) further generalized the model by introducing covariates in the G function that are both firm- and time-specific. Our Model 3 constitutes (9)-(13). Note that the parameters and firm effects in this model are identified through distributional assumptions.

Model 4

Greene (2005a, 2005b) proposed two models, which he called 'true' fixed-effects frontier model and 'true' random-effects frontier model. The purpose of these models is to disentangle firm-heterogeneity or firm effects from technical efficiency. His 'true' random effect frontier model, which we label Model 4, is specified as:

$$y_{it} = (\alpha + \omega_i) + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + v_{it} - u_{it} \tag{14}$$

$$u_{it} \sim N^+(0, \sigma_{it}^2) = N^+\left(0, exp(\omega_{u0} + \boldsymbol{z}'_{u,it}\boldsymbol{\omega}_u)\right) \tag{15}$$

$$v_{it} \sim N(0, \sigma_v^2) \tag{16}$$

$$\omega_i \sim \text{with mean 0 and constant variance} \tag{17}$$

The main difference between Models 3 and 4 is the way inefficiency is modelled. In Kumbhakar and Wang (2005) inefficiency is first specified as the product of G_t, which is usually a function of time, and u_i. The latter is a truncated normal variable the mean and variance of which depend on the vector of firm-specific variables. These variables cannot be time-varying because u_i is time-invariant. In contrast, inefficiency in Model 4 is not a product of G_t and u_i and therefore the mean and variance of u_{it} can depend on variables that are not necessarily time-invariant. Naturally the likelihood functions of these two models are different. Kumbhakar and Wang (2005) treated α_i as fixed while Greene (2005a, 2005b) treated it as both random and fixed.[3] Because of its complexity the Greene model is estimated by the Maximum Simulated Likelihood method.

Model 5

In Model 4 the firm effects are not regarded as part of inefficiency. This is in contrast to Model 1 in which firm effects are regarded as inefficiency. Whether firm effects (fixed or random) are parts of inefficiency or not depends on how these effects are interpreted. However, hardly any economic rationale is provided either in favor of against treating firm effects as inefficiency. To avoid the criticism that inefficiency is time-invariant, Kumbhakar and Heshmati (1995) proposed a model in which technical inefficiency is assumed to have a persistent firm-specific (time-invariant) component and a time-varying residual component. Thus, in their model firm effects are treated as persistent inefficiency. Kumbhakar and Heshmati argued that identifying the magnitude of persistent inefficiency may be important, at least in panel data with a short time span, because it reflects the effects of inputs such as management that vary between firms but not over time. Thus, unless there are changes in something that affects the management style of individual firms (for example, a change in firm ownership or a change in the operating environment, such a change in government regulations, taxes or subsidies), it is very unlikely that the persistent inefficiency component will change. On the other hand, the residual component of inefficiency might change over time. It is possible to explain the persistent component by making it a function of covariates that are time-invariant (for example a manager's innate ability and skills). Similarly, the residual component can be explained by factors such as experience that might vary over time and across firms. It is likely that a part of inefficiency is firm effects (effects of omitted/unobserved time-invariant factors). However, as argued by Kumbhakar and Heshmati (1995), the distinction between the persistent and residual components of inefficiency is important because they have different policy implications. Thus our Model 5 is the Kumbhakar-Heshmati model that is specified as:

$$y_{it} = \alpha_0 + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + v_{it} - \eta_i - u_{it} \tag{18}$$

where v_{it} is noise; $\eta_i \geq 0$ represents persistent technical inefficiency; $u_{it} \geq 0$ represent time-varying inefficiency; and $\eta_i + u_{it}$ is overall technical inefficiency. The error components are assumed to be independent of each other and also independent of $\boldsymbol{x}_{it}$. For estimation purposes we rewrite (18) as

$$y_{it} = \alpha_0^* + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + v_{it} - u_{it}^* - \eta_i^* \tag{19}$$

where $\alpha_0^* = \alpha_0 - E(\eta_i) - E(u_{it})$; $u_{it}^* = u_{it} - E(u_{it})$; and $\eta_i^* = \eta_i - E(\eta_i)$.

The model can be estimated in three steps. In step 1 we estimate equation (19) via a standard random effect regression model for panel data. In step 2, the persistent technical efficiency is estimated. In particular, from step 1 we obtain the firm-specific estimates of the η_i^* component. Persistent technical efficiency can then

[3] The 'true' fixed-effect frontier models by Greene (2005a, 2005b) include a potential incidental parameters problem. Wang and Ho (2010) proposed an alternative estimation model that is immune to the incidental parameters problem. A fixed-effect panel stochastic frontier model is estimated by applying first-difference and/or within transformation methods.

be estimated from

$$\hat{\eta}_i = \max(\eta_i^*) - \eta_i^* \quad (20)$$

Finally, the persistent technical efficiency measure (PTE) is obtained from $\exp(-\hat{\eta}_i)$. In step 3 the residual technical efficiency is estimated. For this we go back to step 1 and obtain the residuals (i.e., $y_{it} - f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + \eta_i = \alpha_0 + v_{it} - u_{it}$). By assuming that v_{it} is iid $N(0, \sigma_v^2)$, and u_{it} is iid $N^+(0, \sigma^2)$, we can simply maximize the log-likelihood function for the following standard normal-half normal SF model for pooled data

$$r_{it} = \alpha_0 + v_{it} - u_{it} \quad (21)$$

where $r_{it} = y_{it} - f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + \eta_i$. In practice we use the estimated values of $\boldsymbol{\beta}$ and η_i to define r_{it}. That is, sampling variability associated with $\boldsymbol{\beta}$ and η_i is ignored. Using the standard frontier model on (21) we get estimates of α_0, σ_v^2 and σ^2. The Jondrow et al. (1982) result can then be used to estimate residual technical inefficiency, $\hat{u}_{it}$, conditional on the estimated residuals, $(v_{it} - u_{it})$. We can use these $\hat{u}_{it}$ to calculate time-varying residual technical inefficiency defined as $RTE = exp(-\hat{u}_{it})$, and then find overall technical efficiency defined as $OTE = PTE * RTE$.

Model 6

Unlike Model 4, Model 5 does not take into account any fixed or random effects associated with unobserved factors that are not related to inefficiency. Model 6 is a version of Model 5, modified and extended to include random firm effects. The presence of such effects can be justified, for example, by making an argument that there are unobserved time-invariants inputs that are not inefficiency. In agriculture one such might be land quality. Our Model 6 is specified as:

$$y_{it} = \alpha_0 + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + \mu_i + v_{it} - \eta_i - u_{it} \quad (22)$$

where μ_i are random firm effects that capture unobserved time-invariant inputs. This model can be rewritten as

$$y_{it} = \alpha_0^* + f(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + \alpha_i + \varepsilon_{it} \quad (23)$$

where $\alpha_0^* = \alpha_0 - E(\eta_i) - E(u_{it})$; $\alpha_i = \mu_i - \eta_i + E(\eta_i)$; and $\varepsilon_{it} = v_{it} - u_{it} + E(u_{it})$. With this specification α_i and ε_{it} have zero mean and constant variance. This model can be estimated in three steps. Since equation (23) is the familiar panel data model, in step 1 the standard random effect panel regression is used to estimate $\widehat{\boldsymbol{\beta}}$. This procedure also gives predicted values of α_i and ε_{it}, denoted by $\hat{\alpha}_i$ and $\hat{\varepsilon}_{it}$. In step 2, the time-varying technical inefficiency, u_{it}, is estimated. Let

$$\begin{aligned}\hat{\varepsilon}_{it} &= \varepsilon_{it} + (\hat{\varepsilon}_{it} - \varepsilon_{it}) = v_{it} - u_{it} + E(u_{it}) + (\hat{\varepsilon}_{it} - \varepsilon_{it}) \\ &= E(u_{it}) + [v_{it} + (\hat{\varepsilon}_{it} - \varepsilon_{it})] - u_{it} \quad (24)\end{aligned}$$

By assuming v_{it} is iid $N(0, \sigma_v^2)$, and u_{it} is iid $N^+(0, \sigma^2)$ and ignoring the $(\hat{\varepsilon}_{it} - \varepsilon_{it})$ term that has zero mean, we can estimate equation (24) using the standard SF technique. This procedure gives estimates of the time-varying residual technical inefficiency components, u_{it}, and time-varying residual technical inefficiency $\hat{u}_{it}$ which can be used to estimate residual technical efficiency, $RTE = \exp(-\hat{u}_{it})$. In step 3 we estimate η_i, following a similar procedure as in step 2. For this we write:

$$\begin{aligned}\hat{\alpha}_i &= \alpha_i + (\hat{\alpha}_i - \alpha_i) = \mu_i - \eta_i + E(\eta_i) + (\hat{\alpha}_i - \alpha_i) \\ &= E(\eta_i) + [\mu_i + (\hat{\alpha}_i - \alpha_i)] - \eta_i \quad (25)\end{aligned}$$

By ignoring the $(\hat{\alpha}_i - \alpha_i)$ term that has a zero mean, and assuming μ_i is iid $N(0, \sigma_\mu^2)$, η_i is iid $N^+(0, \sigma_\eta^2)$; we can estimate equation (25) using the standard pooled normal-half normal SF model to obtain estimates of the persistent technical inefficiency components, η_i, and persistent technical inefficiency measure, $PTE = \exp(-\hat{\eta}_i)$. Overall technical efficiency (OTE) is then obtained from $OTE = PTE * RTE$.

Compared to Model 5, this model includes two additional features. First, it includes random firm effects. Second, it includes and separates persistent technical inefficiency from random firm effects. Identification of these components comes from distributional assumptions on various error components.

It is also possible to extend Models 6 (in step 2 and 3) to include non-zero mean of persistent and time-varying inefficiency and also to account for heteroskedasticity in either or both. These extensions are left for the future. Also, the finite sample behaviour of estimators of persistent and residual inefficiency is left for the future.

3 Data

The data source is the Norwegian Farm Accountancy Survey. This is an unbalanced set of farm-level panel data, collected by the Norwegian Agricultural Economics Research Institute (NILF). It includes farm production and economic data collected annually from about 1000 farms, divided between different regions, farm size classes, and types of farms.

The data set used in the analysis is an unbalanced panel with 687 observations on 533 grain farms from 2004 to 2008. We included farms in the lowlands of Eastern Norway, Jæren, and Mid-Norway. Within each of these three regions the growing conditions are reasonably similar, and these are the main grain producing regions of Norway.

Grain farms usually produce several types of grains (wheat, barley, oats etc.), and, by the classification system applied, have few (if any) farm activities besides grains. The total output, y_1, is aggregated and measured as the farm revenue (exclusive of coupled and environmental subsidies) in Norwegian kroner (2008 NOK) per year, obtained by deflating the annual farm revenues to 2008 revenues using the consumer price index (CPI).

The production function $f(\boldsymbol{x}_{it};\boldsymbol{\beta})$ in Models 1 and 6 is specified with the following input variables: x_1 is labor hours used on the farm, measured as total number of hours worked, including management, family and hired workers; x_2 is productive farmland in hectares; x_3 is variable farm inputs, measured by variable costs, deflated by the CPI to 2008 NOK prices; x_4 is farm fixed and capital costs, also deflated by the CPI to 2008 NOK prices; and t $(1,\ldots,5)$ is a time-trend. The fixed and capital costs include incurred expenditure on fixed costs items plus depreciation and required return on farm capital tied up in machinery, buildings and livestock.

The $\boldsymbol{z}$-variables in this study consist of the following: z_1 is income share from off-farm work, measured as the farmer's net income off the farm as a proportion of the farmer's total net income on and off the farm within a year; z_2 is share of income from coupled (output related) subsidies, measured as coupled subsidies received as a proportion of the total farm net income within a year; z_3 is environmental payments share, measured as the farm environmental payments received as a proportion of the total farm net income within a year; z_4 is entrepreneurial orientation index; z_5 is experience, measured as number of years as a farmer; z_6 is a dummy variable of one if the farmer has only primary education (i.e. no secondary or higher education); and z_7 is a dummy variable of one if the farmer has secondary education (i.e. high school, but no higher education). Descriptive statistics of the variables used in the study are reported in Table 2.

Table 2 Descriptive statistic (N = 687)

Variable	Label	Mean	Std.dev	Min	Max
Production function variables					
y_1	Farm revenue (2008 NOK)	1085411	948950	80610	9386916
x_1	Labor (hours)	2579	1480	74	15200
x_2	Farmland (hectare)	35.3	19.5	0.5	129.4
x_3	Variable farm inputs (2008 NOK)	480793	509090	21397	3383073
x_4	Fixed farm input and capital costs (2008 NOK)	415583	262028	40238	2231058
t	Year (1=2004, 5=2008)	3.1	1.4	2	5
Inefficiency determinant variables and heteroskedasticity variables in the inefficiency and error comp. function					
z_1, z_{u1}, z_{v1}	Farm-specific off-farm income share	0.57	0.23	0.08	0.94
z_2, z_{u2}, z_{v2}	Farm-specific coupled subsidy income share	0.20	0.09	0.01	0.41
z_3, z_{u3}, z_{v3}	Farm-specific environmental subsidy income share	0.04	0.03	0	0.16
z_4, z_{u4}, z_{v4}	Entrepreneurial orientation index	2.85	1.19	1	5.58
z_5, z_{u5}, z_{v5}	Farmer experience, years	18.98	9.41	2	39
z_6, z_{u6}, z_{v6}	Primary education, dummy	0.29	0.42	0	1
z_7, z_{u7}, z_{v7}	Secondary education, dummy	0.45	0.50	0	1

We choose a translog specification of the $f(\boldsymbol{x}_{it};\boldsymbol{\beta})$ function in our empirical analysis in Models 1 to 6 because of its flexibility (Christensen et al. 1973).

We used log values for the input variables in the translog production function. Prior to taking log the x-variables were scaled (divided by their geometric means). Consequently, the first-order coefficients in the model can be interpreted as elasticities of output evaluated at the means of the data.

4 Results and interpretations

4.1. Technical efficiency

Fig. 1 presents the kernel density distribution of the technical efficiency estimates for Models 1 to 6 (overall technical efficiency for Models 5 and 6), and the mean, first quartiles and third quartiles scores per year for the same models. The various models clearly produce different empirical distributions, in some instances markedly so.

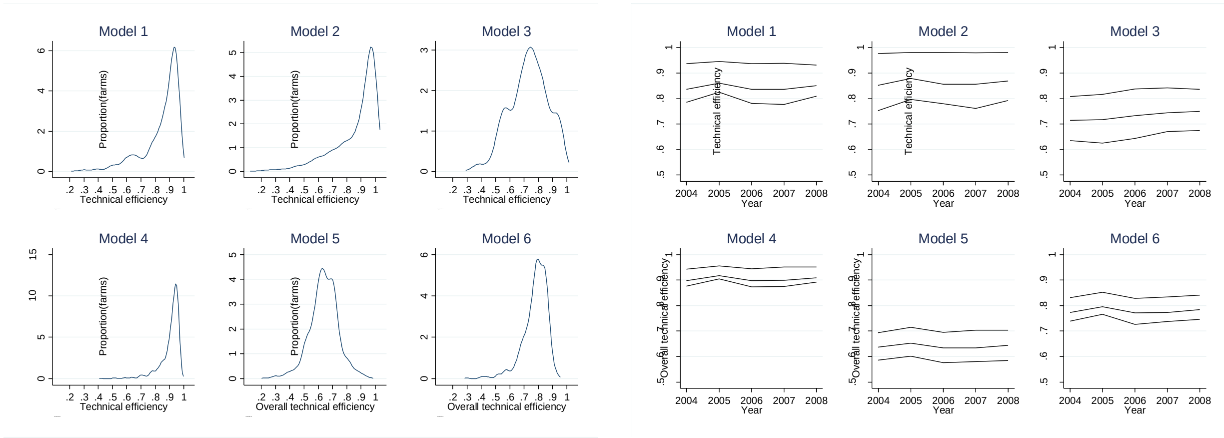

Fig. 1 Technical efficiency distributions of sample farms (left part of the figure) and **t**he mean, first and third quartile values (middle, bottom and top lines) of technical efficiency of sample farms (right part of the figure) for Models 1 to 6

The mean technical efficiency of Model 4 (0.91) is the highest, while it is smallest (0.64) for Model 5. The spread of the efficiency scores can be seen from Fig. 1, where spread is illustrated by the inter-quartile ranges. Model 2 has the widest spread of efficiency scores, slightly wider than Model 3, while Model 4 has an appreciably narrower spread than the other models. The figures show that, despite the difference in spreads, the time-series patterns for the different models are quite similar.

The mean efficiency in Model 1 and Model 2 (which allow both *u* and *v* to be heteroskedastic) are almost similar. However, the maximum and especially the minimum efficiency scores are more extreme when allowance is made for heteroskedasticity.

In Model 3, the combined effect of accounting for heteroskedasticity and firm-specific random effect resulted in a reduction in mean technical efficiency (to 0.73), with a spread about the same as for Model 2. These results are largely consistent with earlier empirical findings. Hadri et al. (2003a, 2003b), in their studies of cereal farms in UK, found that correcting for heteroskedasticity had a significant effect on the spread of the measure of technical efficiency. Caudill et al. (1995) found a dramatic decrease in the technical efficiency measures from inclusion of heteroskedasticity in their study of bank cost data. It appears that inclusion of a random effect (in Model 3, compared to Model 2) has also contributed to reduced estimates of technical efficiency. Yet such an effect seems counter-intuitive, since the random effect component should pick up some of the technical efficiency effect found in models without a random effect. Evidently, it is hard to form firm conclusions about the interaction between the firm-specific random effect, the mean efficiency function, the heteroskedasticity in the efficiency function and the heteroskedasticity in the symmetric error component.

Instead of focusing on heteroskedasticity, Models 4, 5 and 6 provide ways to account for heterogeneity between firms and to specify and eventually decompose the technical efficiency component. The results of decomposition of technical efficiency into persistent technical efficiency and time-varying technical efficiency for Model 5 and Model 6 are plotted in Fig. 2.

One problem with Model 5 is that all time-invariant noise is measured as persistent technical inefficiency. It seems plausible that the persistent technical inefficiency part in Model 5 is the main reason for the low overall technical efficiency estimates. In other words, some potential firm effects may have been captured in the inefficiency measures, as the distributions in the upper part of Fig. 2 appear to confirm. The mean persistent technical efficiency score in Model 5 is 0.71 while the mean residual technical efficiency score is 0.89. As shown in the lower part of Fig. 2 for Model 6, the spread of efficiency is significantly higher for the persistent component than for the residual component. These results suggest that the persistent technical inefficiency is a larger problem than residual technical inefficiency in Norwegian grain farming. Kumbhakar and Heshmati (1995) also found a larger persistent technical efficiency than residual technical efficiency in their analysis of dairy farms in Sweden for the period 1976 to 1988. A high degree of persistent technical efficiency could be a problem in the long-run. For example, in Norwegian agriculture there has been a high level of support in many decades. The high level of support may have caused some persistent inefficiency. If there is a wish to reduce the governmental support level, and still keep the living standard in the long run, focus should be on measures to reduce persistent technical inefficiency. For example, it may help to consider measures that encourage long-term structural adjustment toward fewer larger farms or switches to other production activities.

Results from the model by Schmidt and Sickles (1984) using exactly the same data gave the same technical efficiency estimates as the persistent technical efficiency estimates from Model 5.

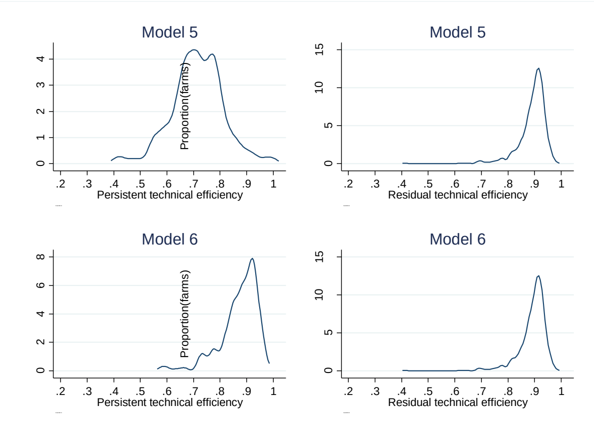

Fig. 2 Distributions of persistent technical efficiency (to left) and residual technical efficiency (to right) distributions for Model 5 and Model 6

In Model 4, the ‘true’ random effects frontier model, the firm effects are not considered to be inefficiency, leading to high efficiency scores and low dispersion (Fig. 1). Greene (2005a, 2005b) also found in a study of the U.S. banking industry that the ‘true’ random effects results had higher and less dispersed technical efficiency scores than results for other models considered.

Neither the assumption that all time-persistent noise is inefficiency (Model 5) nor that no firm-specific effects are inefficiency (Model 4) might be true. Model 6 overcomes these problems by decomposing the time-persistent noise into a firm effect and a persistent technical inefficiency effect. The results are efficiency scores between the scores for Model 4 and Model 5 (Fig. 1). The mean overall efficiency score for Model 6 is 0.78, and the score spread is between that of Model 4 (low spread) and Model 5 (high spread) (Fig. 1). Compared to Model 5, Model 6 has a higher persistent technical efficiency score (0.87) with a less dispersed distribution (Fig. 2).

As the above results illustrate, the efficiency scores are, as expected, sensitive to model specification. How then are the technical efficiency ranking of farms affected by different model specifications used? In Table 3 pairwise rank-order correlations for Models 1 to 6 (overall technical efficiency for Model 5 and 6) illustrate the differences between the models in technical efficiency ranking of the sample farms.

Table 3 Kendall's rank-order correlation between of technical efficiency estimates for Model 1 to Model 6. For Model 5 and Model 6 overall technical efficiency (OTE) is calculated

	Model 1	Model 2	Model 3	Model 4	Model 5 OTE
Model 2	0.79				
Model 3	0.73	0.69			
Model 4	0.73	0.59	0.58		
Model 5 OTE	0.52	0.38	0.49	0.68	
Model 6 OTE	0.54	0.38	0.45	0.73	0.88

The patterns of results between Model 1 and Model 2 and between Model 5 and Model 6 seem to be the most consistent, i.e., these model comparisons have the most consistent rankings. The correlations in Table 3 confirm this observation. Both Models 1 and 6 have quite similar technical efficiency assessments to those for Model 4. On the other hand, the results for Models 2 and 3 are quite different and these two models also show quite inconsistent patterns and rankings of results relative to the other models investigated.

Further, the efficiency assessments of residual technical efficiency for Model 5 and Model 6 are perfectly positively correlated, while the results based on persistent and residual technical efficiency are to a large extent independent or random, with a rank-order correlation of 0.14 (not reported in a table in this version of the paper).

4.2. Elasticities, returns to scale, technical change and efficiency determinants

For all models the estimated output elasticities with respect to labor, land, variable farm inputs and fixed farm input/capital all differed from zero at the 1% significance level (5% significance level for fixed farm input/capital in Model 3). The elasticities for labor, land and variable farm inputs were larger than 0.23 for all models. Estimates of technological change were statistically significant and positive for all models at 1.7% to 2.7% per year. The scale elasticity was highest for Models 5 and 6 (at 1.21 on average) and lowest for Model 2 (1.04 on average) and Model 4 (1.08 on average). The estimated scale elasticity was not significantly (at 10% level) different from 1 only for Model 2. All models showed a weak decreasing time trend in scale elasticity, and the spread of elasticity estimate was highest for Models 3, 5 and 6. All these results in this paragraph are not reported in any table or figure in this version of the paper. To further save space, the effect of the different determinants of efficiency scores are dropped in this version of the paper (but available from the authors upon request).

5 Concluding comments

Panel data frontier model estimation has been widely used to estimate technical efficiency. Yet the technical efficiency measures may be distorted by specification error. This study has demonstrated that allowing for heteroskedasticity in the error terms can lead to appreciably different technical efficiency estimates and can also change the ranking of farms based on efficiency scores. Further, it may not be valid to assume either that all time-persistent noise is inefficiency, or that no firm-specific effects are inefficiency. Our results suggest that models structured to capture inefficiency that is time-invariant (and mixes with firm effects) may lead to very low efficiency estimates, while models in which firm effects are not considered to be part of inefficiency may give high efficiency scores. It seems that the ‘true’ measure of efficiency may be somewhere between these extremes and we have presented a new model which might be said to come closer to capturing this ‘true’ efficiency. This new model also decomposes the overall technical inefficiency into a persistent component and a residual component. For the Norwegian grain farms included in the study, the persistent component of inefficiency was much larger than the residual inefficiency component, implying that policy measures that could reduce persistent inefficiency should be prioritized. Unless persistent inefficiency is reduced, farmers might not be able to survive in the long run, especially if competitors are more efficient.

The variability of the results from the different models clearly demonstrates the difficulty in ‘correctly’ measuring efficiencies. No model can be held to be ‘correct’, and the efficiencies will always be a kind of unobserved or modeled effect. For the future, model choice in empirical research should not be based on ‘standard practice’, but on a reasoned choice. A good understanding of the institutional and production environments of the industry under study, and of the data applied, are crucial in deciding which estimator should be utilized.

References

Aigner, DJ, CAK Lovell and P Schmidt (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics* 6, 21-37.

Battese, GE and TJ Coelli (1988). Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data. *Journal of Econometrics* 38, 387-399.

Battese, GE and TJ Coelli (1992). Frontier production functions, technical efficiency and panel data: with application to paddy farmers in India. *Journal of Productivity Analysis* 3, 153-169.

Battese, GE and TJ Coelli (1995). A model for technical inefficiency effects in a stochastic frontier production function for panel data. *Empirical Economics* 20, 325–332.

Caudill, SB and JM Ford (1993). Biases in frontier estimation due to heteroskedasticity. *Economics Letters* 41, 17–20.

Caudill, SB, JM Ford and DM Gropper (1995). Frontier estimation and firm-specific inefficiency measures in the presence of heteroskedasticity. *Journal of Business and Economic Statistics* 13, 105–111.

Christensen, L, D Jorgenson and L Lau (1973). Transcendental logarithmic production frontiers. *Review of Economics and Statistics* 55, 28-45.

Coelli, TJ, DSP Rao, CJ O'Donnell and GE Battese (2005). *An Introduction to Efficiency and Productivity Analysis,* Second Edition. Springer, New York.

Feng, G and A Serletis (2009). Efficiency and productivity of the US banking industry, 1998-2005: Evidence from fourier cost functions satisfying global regularity conditions. *Journal of Applied Econometrics* 24, 105-138.

Giannakas, K, R Schoney and V Tzouvelekas (2001). Technical efficiency, technological change and output growth of wheat farms in Saskatchewan. *Canadian Journal of Agricultural Economics* 49, 135-152.

Greene, W (2005a). Fixed and random effects in stochastic frontier models. *Journal of Productivity Analysis* 23, 7-32.

Greene, W (2005b). Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics* 126, 269-303.

Greene, W (2008). The econometric approach to efficiency analysis. In: HO Fried, CAK Lovell and SS Shelton (eds.): The Measurement of Productivity Efficiency and Productivity Growth, pp. 92-250. Oxford University Press, New York.

Hadri, K (1999). Estimation of a doubly heteroskedastic stochastic frontier cost function. *Journal of Business and Economic Statistics* 17, 359–363.

Hadri, K, C Guermat and J Whittaker (2003a). Estimating farm efficiency in the presence of double heteroskedasticity using panel data. *Journal of Applied Economics* 6, 255-268.

Hadri, K, C Guermat and J Whittaker (2003b). Estimating of technical inefficiency effects using panel data and doubly heteroskedastic stochastic production frontiers. *Empirical Economics* 28, 203-222.

Huang, CJ and JT Liu (1994). Estimation of a non-neutral stochastic frontier production function. *Journal of Productivity Analysis* 5, 171–180.

Jondrow, J, CAK Lovell, IS Materov and P Schmidt (1982). On the estimation of technical inefficiency in stochastic frontier production function model. *Journal of Econometrics* 19, 233-238.

Just, RE and RD Pope (1978). Stochastic specification of production functions and economic implications. *Journal of Econometrics* 7, 67-86.

Karagiannis, G and A Sarris (2005). Measuring and explaining scale efficiency with the parametric approach: The case of Greek tobacco growers. *Agricultural Economics* 33, 441-451.

Kumbhakar, SC (1987). The specification of technical and allocative inefficiency in stochastic production and profit frontiers. *Journal of Econometrics* 34, 335-348.

Kumbhakar, SC (1991). Estimation of technical inefficiency in panel data models with firm- and time-specific effects. *Economics Letters* 36, 43-48.

Kumbhakar, SC (2006). Productivity and efficiency measurement using parametric econometric methods. In: M Bagella, L Becchetti and I Hasan (eds.), *Transparency, Governance, and Markets*, pp. 21-61. Elsevier, Oxford, UK.

Kumbhakar, SC and CAK Lovell (2000). *Stochastic Frontier Analysis*. Cambridge: Cambridge University Press.

Kumbhakar, SC and A Heshmati (1995). Efficiency measurement in Swedish dairy farms: An application of rotating panel data, 1976-88. *American Journal of Agricultural Economics* 77, 660-674.

Kumbhakar, SC and G Lien (2010). Impacts of subsidies on farm productivity and efficiency. In: E Ball, R Fanfani and L Gutierrez. (eds), *The Economic Impact of Public Support to Agriculture, An International Perspective*, pp. 109-124. Springer, New York.

Kumbhakar, SC and H-J Wang (2005). Estimation of growth convergence using a stochastic production function approach. *Economics Letter* 88, 300-305.

Pitt, M and LF Lee (1981). The measurement and sources of technical inefficiency in the Indonesian weaving industry. *Journal of Development Economics* 9, 43-64.

Schmidt, P and R Sickles (1984). Production frontiers and panel data. *Journal of Business and Economic Statistics* 2, 367-374.

Wang, H-J (2002). Heteroskedasticity and non-monotonic efficiency effects of a stochastic frontier model. *Journal of Productivity Analysis* 18, 241-253.

Wang, H-J and C-W Ho (2010). Estimating fixed-effect panel data stochastic frontier models by model transformation. *Journal of Econometrics* 157, 286-296.

Wilson, P, D Hadley and C Asby (2001). The influence of management characteristics on the technical efficiency of wheat farmers in eastern England. *Agricultural Economics* 24, 329–338.