Measuring farmers' risk aversion:
the unknown properties of the value function

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Abstract:

We argue in this paper that available econometric estimates of farmers' risk aversion do not measure true farmers' preferences towards risky outcomes. Available analyses are mostly of static nature and indeed measure the parameters of the synthetic optimal value function rather than the deep parameters of the utility functions. We derive analytical and empirical results in a simple dynamic and stochastic framework showing that that there is not a simple relationship between utility functions and value functions when agents have many decision variables. In particular we find that the value function does not necessarily exhibit DARA when the instantaneous utility function satisfies DARA and conversely. We recommend performing dynamic econometric estimation with at least farm production and consumption data.

1. Introduction

The agricultural sector is facing many new risky and uncertain challenges such as those induced by the climate change. Accordingly measuring the risk aversion of farmers turns out to be nowadays a major topic in agricultural economics so as to understand economic behaviours and design relevant policy responses. This was already crucial in order to, for example, assess the impacts of current policy instruments, such as the decoupled nature of direct payments granted to farmers in developed countries. It has been shown (for instance, Hennessy, 1998) that these payments have positive production effects if farmers' preferences exhibit Decreasing Absolute Risk Aversion (DARA), i.e. that their risk aversion decreases with their wealth.

Many researches have already been conducted on the identification of farmers risk aversion, leading without surprise to a great heterogeneity of available estimates. From the recent OECD review of these estimates, it appears that studies generally conclude that farmers preferences exhibit risk aversion (OECD, 2009). Much more disputed is the exact structure of farmers' preferences with respect to risky outcomes. For instance, focusing on analyses imposing Constant Relative Risk Aversion (CRRA) preferences, average estimates of the Arrow Pratt coefficient of relative risk aversion vary from 0 to 10. This rightfully raises some scepticism on our ability to quantify several risk issues. Several factors can indeed explain such discrepancy, even when we remain with the expected utility paradigm. First Meyers and Meyers (2005) attribute part of the difference to the fact that different arguments are specified in the utility function: wealth, income, profit, consumption. When converted to the same argument, they show much less variation across estimates. Second Lence (2009) argue that farm production data do not exhibit sufficient variability in order to reveal the preferences towards risks. More precisely, the information contained in farm production data is generally too poor for the identification of both preferences and technologies, leading to potentially biased estimates when flexible forms are specified. Just and Just (2009) make the same point arguing in addition that perceived risks by farmers also need to be identified. Third Pope *et alii* (2010) recall that farmers have different solutions to cope with risk, such as consumption or investment decisions. These decisions should be estimated simultaneously with production (input) decisions to correctly assess farmers' preferences.

Our main objective in this paper is to emphasize that available estimates do not measure true farmer preferences. While risk is inherently a dynamic issue, most studies develop static theoretical and empirical frameworks. By such they implicitly measure the parameters of the optimal value function rather the deep parameters of farmers' preferences. In their dynamic analysis, Pope *et alii* more clearly recognize that they estimate the parameters of the value function rather than the parameters of the utility function. We argue that in non degenerate cases we cannot identify the parameters of the utility function from the estimation of the parameters of the value function, hence available estimates do not reflect farmers preferences towards risky outcomes. The intuition is that the value function captures the behaviour of farmers, for instance the maximization of the discounted expected utility, subject to technological and budget constraints. Accordingly the parameters of the value function are not only determined by the parameters of the farmers' preferences but also by, at least, the parameters of production technologies.

From a technical point, value functions have not in general cases closed form solutions. In order to overcome this technical difficulty, we adopt two complementary strategies. First we theoretically demonstrate that the value function does not inherit from all properties of the utility function. In particular, we prove that the value function does not necessarily exhibit decreasing absolute risk aversion (DARA) if the underlying utility function is DARA. Our analytical demonstration shows that the properties of the value function depend on the properties of the response functions, in particular the consumption function. It appears that these properties cannot be determined unambiguously when economic agents have many decision variables. It should be noted here that Carroll and Kimball (1996) for instance were able to demonstrate the concavity of the consumption function and the DARA property of the value function because consumers maximise their value function only over consumption. This result cannot be extended to the case of multiple decision variables as in our setting where producers determine, at least, both consumption and production levels. Second we conduct numerical experiments where we first impose parameters of the utility and production functions using flexible forms and then compute the optimal levels of production, consumption and ultimately the value function. We then estimate the parameters of the value function using again flexible forms and a classical adjustment method. We test the equality of these estimates to the original preference parameters. We conduct several sensitivity analyses of our results to the original values of preferences and technological parameters. Our empirical results reveal that estimates of value function parameters strongly differ from the values of our deep parameters. We also find that the value function does not necessarily exhibit DARA when the instantaneous utility function satisfies DARA. We also find that the Arrow Pratt coefficient of absolute risk aversion is systematically lower for the value function. The bias depends, as expected, on the extent of supply responses to economic incentives.

This paper is organized in two parts. In the first part, we conduct our analytical demonstration using a voluntary simple setting such as to already reveal that the properties of the value function are different from the properties of the utility function. More precisely, we first show that the value function may not exhibit DARA even if the utility function is DARA. On the other hand, we show that the value function is concave with respect to initial wealth if the utility function is concave with respect to consumption. The second part is devoted to the empirical analysis. We first detail our calibration assumptions and then comment our quantitative results. We conclude the paper with methodological recommendations.

2. Theoretical analysis

2.a. Assumptions

Farmers around the world are confronted with many sources of risks (such as price, production, environmental, quality,…) and can manage their exposure to and the consequences of risks through many technical and financial decisions (such as the choice of activities, cropping patterns, investment, insurance contracts, participation to future and options markets, savings and borrowing decisions, …). Pope *et alii* convincingly argue that all these dimensions should be integrated in the analysis in order to identify the deep parameters.

In this paper, we consider a very simple framework in order to analyze the relationship between the instantaneous utility function and the dynamic value function. We consider a farmer with two independent decisions only, his periodic final consumption and the periodic production of an agricultural good. Periodic savings are directly given by the difference between the initial periodic wealth and periodic final expenditures. We also assume that only the price of this agricultural good is stochastic while wealth is accumulated in a riskless asset. Regarding the dynamic behavior of our farmer, we again simplify the framework by assuming that he maximizes a time additive present discounted value of utility from consumption. As usual (see for instance, Carroll and Kimball), we focus on the last two periods for analytical computations. We work backwards from the last period where we assume that the farmer consumes all remaining wealth.

Formally, we consider a two-period consumption-production model with an economic agent (farmer) choosing his consumption level c_t and his production level x_t (or input level) at time t. His wealth at period t is given as $w_t = c_t + s_t$ where s_t is the agent's saving level at time t. We assume that the agent has enough wealth to live at period t, thus $w_t - c_t - s_t > 0$. For the last period, the agent's random wealth is given by

$$\tilde{w}_{t+1} = r(w_t - c_t) + R_t(x_t; \tilde{\theta}_{t+1}) \qquad (1)$$

where R_t is his net random return of production x_t in state $\tilde{\theta}_{t+1}$. We assume that this return function R_t is concave in x. r is the fixed total rate of return of the safe asset. Note that (x_t, c_t) are functions of w_t and we do not a priori know the curvature of $x_t(w_t)$ and $c_t(w_t)$. The agent's utility function for consumption at time t is $u(c_t)$ which is increasing and strictly concave in c and we assume first that this function exhibits DARA, thus

$$\frac{dA_u(c)}{dc} = -\frac{u'''(c)}{u'(c)} + \left(\frac{u''(c)}{u'(c)}\right)^2 < 0 \leftrightarrow \frac{u'''u'}{(u'')^2} > 1$$

or equivalently, $u'''(c)u'(c) - (u''(c))^2 > 0$

It is clear that $u''' > 0$ since the other two terms are always positive. It means that the marginal utility function for consumption is convex in the agent's consumption level c.

Now, the agent's value function can be written as follows:

$$V_t(w_t) = u(c_t(w_t)) + \beta E_t[V_{t+1}\left(r(w_t - c_t(w_t)) + R_t\left(x_t(w_t); \tilde{\theta}_{t+1}\right)\right)] \qquad (2)$$

In this Bellman equation, E_t is a conditional expectation on the information set available at t, β is the agent's discount factor. We assume that the additive utility function at the last period $V_{t+1}(\tilde{w}_{t+1})$ is monotonic increasing, strictly concave and it also exhibits DARA, thus we have $V'_{t+1} > 0$, $V''_{t+1} < 0$ and $V'''_{t+1} > 0$. In Carroll and Kimball, they have succeed to show that $E_t[V_{t+1}]$ is DARA if V_{t+1} is DARA, but this is just a particular case and this is no longer true in our model which will be shown later. For the agent, his objective is to find optimal (c, x) in order to maximize his value function V_t. Thus the above equation can be given as

$$\max_{c_t, x_t}\{u\left(c_t(w_t)\right) + \beta E_t[V_{t+1}(\tilde{w}_{t+1}(w_t))]\} \qquad (3)$$

Subject to $\tilde{w}_{t+1} = r(w_t - c_t) + R_t(x_t; \tilde{\theta}_{t+1})$

The first order condition (FOC) give us the optimal choices (c^*, x^*) which are

$$\partial c_t: \quad u'(c_t^*) - \beta r E_t[V'_{t+1}(\tilde{w}_{t+1})] = 0$$
$$\leftrightarrow \quad u'(c_t^*) = \beta r E_t[V'_{t+1}(\tilde{w}_{t+1})] \tag{4}$$

$$\partial x_t: \quad \beta E_t\left[R'_t(x_t^*; \tilde{\theta}_{t+1}) V'_{t+1}(\tilde{w}_{t+1})\right] = 0 \tag{5}$$

Using the implicit theorem, we can also obtain the first derivative of the value function as:

$$V_t'(w_t) = \beta r E_t[V'_{t+1}(\tilde{w}_{t+1})] \tag{6}$$

Equation (4) and (6) give us the follows equality

$$V'_t(w_t) = u'(c_t^*) \tag{7}$$

It should be clear here that, even if we know the properties of u, we still do not know the properties of the agent's consumption function, so we can't already determine the properties of the first derivate of the value function.

2.b. On the DARA property of the value function

We now show that the DARA property of V_t cannot be proved even when we know that the utility function for consumption $u(c_t)$ and the additive utility function $V_{t+1}(\tilde{w}_{t+1})$ (for the last period) exhibit DARA. To prove this point, we need to determine if the following inequality is satisfied $V'_t * V'''_t - (V''_t)^2 > 0$. So we derive equation (2) in w_t to get the three terms V'_t, V''_t, V'''_t.

We now differentiate equation (6) to get:

$$\frac{dV_t'}{dw_t} = V_t''(w_t) = \frac{\partial \beta r E_t\left[V'_{t+1}\left(r\left(w_t - c_t(w_t)\right) + R_{t+1}\left(x_t(w_t)\right)\right)\right]}{\partial w_t}$$
$$= \beta r E_t[(r - rc_t' + R'_{t+1}x_t') V''_{t+1}(\tilde{w}_{t+1})] \tag{8}$$

A third derivation gives us

$$\frac{dV_t''}{dw_t} = V_t'''(w_t) = \frac{\partial \beta r E_t\left[(r - rc_t' + R'_{t+1}x_t') V''_{t+1}\left(r\left(w_t - c_t(w_t)\right) + R_{t+1}\left(x_t(w_t)\right)\right)\right]}{\partial w_t}$$
$$= \beta r E_t[(r - rc_t' + R'_{t+1}x_t')^2 V'''_{t+1} + (R'_{t+1}x_t'' - rc_t'' + R''_{t+1}(x_t')^2) V''_{t+1}] \tag{9}$$

Now, we want to show that $V'_t * V'''_t - (V''_t)^2 > 0$ is impossible to be proved. Using equations (7) to (9), this expression is given by:

$$\frac{V'_t V'''_t - (V_t''(w_t))^2}{\beta^2 r^2}$$
$$= E_t[V'_{t+1}] E_t[(r - rc_t' + R'_{t+1}x_t')^2 V'''_{t+1} + (R''_{t+1}(x_t')^2 + R'_{t+1}x_t'' - rc_t'') V''_{t+1}]$$
$$- (E_t[(r - rc_t' + R'_{t+1}x_t') V''_{t+1}])^2$$

$$
\begin{aligned}
&= \underbrace{E_t[V'_{t+1}]E_t[(r-rc'_t)^2 V'''_{t+1}]}_{1} + \underbrace{E_t[V'_{t+1}]E_t[2(r-rc'_t)R'_{t+1}x'_tV'''_{t+1}]}_{2} \\
&\quad + \underbrace{E_t[V'_{t+1}]E_t[(R'_{t+1}x'_t)^2 V'''_{t+1}]}_{3} \\
&\quad + \underbrace{E_t[V'_{t+1}]E_t[(R''_{t+1}(x'_t)^2 + R'_{t+1}x''_t - rc''_t)V''_{t+1}]}_{4} - \underbrace{(E_t[(r-rc'_t)V''_{t+1}]^2}_{5} \\
&\quad + \underbrace{2(E_t[(r-rc'_t)V''_{t+1}]E[R'_{t+1}x'_tV''_{t+1}]}_{6} + \underbrace{E[R'_{t+1}x'_tV''_{t+1}]^2}_{7})
\end{aligned}
$$

$$
\begin{aligned}
&= \overbrace{\underbrace{\underbrace{(r-rc'_t)^2}_{>0}\underbrace{(E_t[V'_{t+1}]E_t[V'''_{t+1}] - E_t[V''_{t+1}]^2)}_{>0 \text{ since } V_{t+1} \text{ is DARA}}}_{1-5}}^{>0} \\
&\quad + \overbrace{\underbrace{2\underbrace{(r-rc'_t)}_{>0}\underbrace{x'_t}_{?}\underbrace{(E_t[V'_{t+1}]E_t[R'_{t+1}V'''_{t+1}] - E_t[V''_{t+1}]E_t[R'_{t+1}V''_{t+1}])}_{?}}_{2-6}}^{unknown} \\
&\quad + \overbrace{\underbrace{\underbrace{(x'_t)^2}_{>0}\underbrace{(E_t[V'_{t+1}]E_t[(R'_{t+1})^2V'''_{t+1}] - E_t[R'_{t+1}V''_{t+1}]^2)}_{?}}_{3-7}}^{unknown} \\
&\quad + \overbrace{\underbrace{E_t[V'_{t+1}]}_{>0}E_t[\underbrace{(R''_{t+1}(x'_t)^2 + R'_{t+1}x''_t - rc''_t)}_{?}\underbrace{V''_{t+1}}_{<0}]}^{unknown}
\end{aligned}
\tag{10}
$$

While we know the derivatives of the value function evaluated at the last period, we are unable to unambiguously determine the sign of the expression (10) for the three following reasons. First the sign of this equation depends on the curvature of x and c which are unknown (see the last term of the right hand side). Second both the second and third terms in the right hand side includes a weighted formula of the DARA condition of the utility function. The weight is given by the derivative of the stochastic return function in the second term, by the square of this derivate in the third term. Third, the second term involves the derivatives of the consumption and production function with respect to initial wealth. This dependency on first order derivatives of response functions is not new. In their portfolio choice problems, Roy and Wagenwoort (1996) also find that the value function exhibit DARA if the underlying utility function is DARA and if the derivative of their investment function with respect to wealth is lower than one. Our formula is more complex because we have two independent decision variables: the derivative of the production function with respect to the initial wealth is also crucial. We can unambiguously determine the sign of the consumption derivative but not the derivate of production with respect to production (see below equations 17 and 18).

So we are left with three sources of indeterminacy. Now we show that this expression can be unambiguously determined only in very specific cases. Let first assume that the return function is linear which means that $R_t(x_t) = \alpha x_t + \gamma$ where only the last parameter is stochastic. In this case $R'_{t+1} = \alpha$ and $R''_{t+1} = 0$ so that we can simplify the equation (10) and resolve one source of ambiguity:

$$\frac{V'_t V'''_t - (V''_t(w_t))^2}{\beta^2 r^2}$$

$$= \underbrace{(r - rc'_t)^2}_{>0} \overbrace{\underbrace{(E_t[V'_{t+1}]E_t[V'''_{t+1}] - E_t[V''_{t+1}]^2)}_{>0 \text{ since } V_{t+1} \text{ is DARA}}}^{>0}$$

$$+ \overbrace{\underbrace{(x'_t)^2}_{>0} \underbrace{(E_t[V'_{t+1}]E_t[\alpha^2 V'''_{t+1}] - E_t[\alpha V''_{t+1}]^2)}_{>0 \text{ since } V_{t+1} \text{ is DARA}}}^{>0} + \overbrace{\underbrace{E_t[V'_{t+1}]}_{>0} E_t[\underbrace{(\alpha x''_t - rc''_t)}_{?} \underbrace{V''_{t+1}}_{<0}]}^{unknown}$$

$$+ 2 \underbrace{(r - rc'_t)}_{>0} \underbrace{x'_t}_{?} \underbrace{(E_t[V'_{t+1}]E_t[\alpha V'''_{t+1}] - E_t[V''_{t+1}]E_t[\alpha V''_{t+1}])}_{>0} \qquad (11)$$

From the above, we can see that even the return function is linear, we still cannot distinguish the properties of the value function. In particular we still don't know the curvature of x and c which ensures our finding. In the second case, let's assume that the agent's return function is exogenous but stochastic, thus $R'_{t+1} = R''_{t+1} = 0$. In this case, the equation (10) simplifies to:

$$\frac{V'_t V'''_t - (V''_t(w_t))^2}{\beta^2 r^2} = \overbrace{\underbrace{(r - rc'_t)^2}_{>0} \underbrace{(E_t[V'_{t+1}]E_t[V'''_{t+1}] - E_t[V''_{t+1}]^2)}_{>0 \text{ since } V_{t+1} \text{ is DARA}}}^{>0} \qquad (12)$$

This is in fact the case specified by Carroll and Kimball(1996). These authors were able to determine that the value function is DARA because there is only one choice variable which is not "directly" to the stochastic variable. From this result, these authors were then able to show that the optimal consumption rule $c(w^*)$ is concave.

2.c. On the concavity of the value function

To show the concavity of the agent's value function V_t, Fama(1970) has shown that $V_t(w_t, c_{t-1})$ is monotone increasing and strictly concave in (w_t, c_{t-1}) if the agent's additive utility function for the next period $V_t(w_{t+1}, c_t)$ is monotone increasing and strictly concave in (w_{t+1}, c_t). One big different between our model and Fama's model is that Fama's model is based on multi-period while our model has only 2 period. Nonetheless we can follow the logic of his demonstration.

We first need to show first that V_{t+1} is monotonic increasing and strictly concave in (x, s). We change the consumption function c_t by $w_t - s_t$ in order to simplify notations. To demonstrate this point, we take the FOC of equation (2)

$$g'_{z,t}(x_t, s_t; w_t) = \begin{bmatrix} \beta E_t[R'_t(x^*_t; \tilde{\theta}_{t+1}) V'_{t+1}(\tilde{w}_{t+1})] \\ -u'(w_t - s^*_t) + \beta r E_t[V'_{t+1}(\tilde{w}_{t+1})] \end{bmatrix} \qquad (13)$$

where $z = (x, s)$. Differentiating these FOCs leads us to:

$$G''_{z,t}(x_t, s_t; w_t) = \begin{bmatrix} g''_{xx,t}(x_t, s_t; w_t) & g''_{xs,t}(x_t, s_t; w_t) \\ g''_{xs,t}(x_t, s_t; w_t) & g''_{ss,t}(x_t, s_t; w_t) \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{\beta\left(E_t\left[\overbrace{R''_t V'_{t+1}}^{<0}\right] + E_t[\overbrace{(R'_t)^2 V''_{t+1}}^{<0}]\right)}_{<0} & r\beta E_t[R'_t V''_{t+1}] \\ r\beta E_t[R'_t V''_{t+1}] & \underbrace{u'' + \beta r^2 E_t[V''_{t+1}]}_{<0} \end{bmatrix} \qquad (14)$$

In this matrix, $g''_{xx,t}(x_t, s_t; w_t)$ and $g''_{ss,t}(x_t, s_t; w_t)$ are both negative since V_{t+1} is monotone increasing and strictly concave in w_t and $R(x)$ is concave in x, so we can say that V_{t+1} is concave in (x, s). Nevertheless, we can show that the determinant of matrix $G''_{z,t}(x_t, s_t; w_t)$ is positive by using Cauchy-Schwartz inequality:

$$detG''_{z,t} = g''_{xx,t}(x_t, s_t; w_t) g''_{ss,t}(x_t, s_t; w_t) - (g''_{xs,t}(x_t, s_t; w_t))^2$$

$$= \beta (E_t[R''_t V'_{t+1}] + E_t[(R'_t)^2 V''_{t+1}])(u'' + \beta r^2 E_t[V''_{t+1}]) - \beta^2 r^2 E_t[R'_t V''_{t+1}]^2$$

$$= \beta E_t[R''_t V'_{t+1}](u'' + \beta r^2 E_t[V''_{t+1}]) + \beta E_t[(R'_t)^2 V''_{t+1}] u'' + \beta^2 r^2 \{E_t[(R'_t)^2 V''_{t+1}] E_t[V''_{t+1}] - E_t[R'_t V''_{t+1}]^2\}$$

$$\geq \beta \overbrace{\underbrace{E_t[R''_t V'_{t+1}]}_{<0} \underbrace{(u'' + \beta r^2 E_t[V''_{t+1}])}_{<0}}^{>0} + \beta \overbrace{\underbrace{E_t[(R'_t)^2 V''_{t+1}]}_{<0} \underbrace{u''}_{<0}}^{>0} + \beta^2 r^2 \underbrace{\{E_t[(R'_t)^2 (V''_{t+1})^2] - E_t[R'_t V''_{t+1}]^2\}}_{variance(R'_t V''_{t+1})>0} > 0 \qquad (15)$$

Then, we want to show that $V''_t < 0$. To do this, we define first $g''_{zw,t}$ which is the derivative of equation (7) in w:

$$g''_{zw,t}(x_t, s_t; w_t) = \begin{bmatrix} 0 \\ -u''(w_t - s^*_t) \end{bmatrix} \qquad (16)$$

with $G''_{z,t} = \begin{bmatrix} g''_{xx,t} & g''_{xs,t} \\ g''_{xs,t} & g''_{ss,t} \end{bmatrix}$ and $[\frac{d\bar{z}_t}{dw_t}]^T = -[g''_{zw,t}(x_t, s_t; w_t)]^T G''^{-1}_{z,t}$, then we have

$$\frac{d\bar{z}_t}{dw_t} = -(detG''_{z,t})^{-1} [g''_{zw,t}]^T \begin{bmatrix} g''_{ss,t} & -g''_{xs,t} \\ -g''_{xs,t} & g''_{xx,t} \end{bmatrix}$$

$$= \frac{1}{detG''_{z,t}} [0 \quad u''_t(w_t - s_t)] \begin{bmatrix} g''_{ss,t} & -g''_{xs,t} \\ -g''_{xs,t} & g''_{xx,t} \end{bmatrix}$$

$$= \frac{u''_t(w_t - s_t)}{det\, G''_{z,t}} \begin{bmatrix} -g''_{xs,t} \\ g''_{xx,t} \end{bmatrix}$$

Returning to the production and saving functions, we get the expression:

$$\frac{d\bar{x}_t}{dw_t} = \frac{\overbrace{-u''_t g''_{xs,t}}^{>0}}{\underbrace{g''_{xx,t} g''_{ss,t} - (g''_{xs,t})^2}_{>0}} \qquad (17)$$

And

$$\frac{d\bar{s}_t}{dw_t} = \frac{\overbrace{u''_t g''_{xx,t}}^{>0}}{\underbrace{g''_{xx,t} g''_{ss,t} - (g''_{xs,t})^2}_{>0}} > 0$$

$$\frac{d\bar{s}_t}{dw_t} = \frac{u''_t g''_{xx,t}}{u''_t g''_{xx,t} + g''_{xx,t}(g''_{ss,t} - u''_t) - (g''_{xs,t})^2}$$

$$= \frac{u_t'' g_{xx,t}''}{\underbrace{u_t'' g_{xx,t}''}_{>0} + \underbrace{\beta r^2 g_{xx,t}'' E_t[V''_{t+1}] - (g_{xs,t}'')^2}_{>0}} < 1 \tag{18}$$

The sign of $\frac{d\bar{x}_t}{dw_t}$ is the same of that of $g_{xs,t}'' = r\beta E_t[R'_t V''_{t+1}]$. At this stage, it is interesting to note that if the derivative of the return function to the production is always positive, then an increase in initial wealth induces a production decrease. By extension, a lump sum payment has a negative production effect if the marginal profit evaluated at the initial point is positive. This result is at the opposite of Hennessy's conclusion (1998) where the distinction between value and utility functions is not made. Conversely, this lump-sum payment is "coupled" to the production if the marginal profit evaluated without this lump sum payment is negative. More generally our result may partly explain the debates of the decoupled nature of farm subsidy. When consumption is included in the analysis, their coupling effect may be lower as empirically suggested by Whitaker (2009) in the US case.

Returning to our demonstration, we can now determine the sign of V_t'' which is given by

$$V_t''(w_t) = \frac{\partial u_t'(w_t - s_t)}{\partial w_t} + \frac{\partial u_t'(w_t - s_t)}{\partial z_t'} \frac{d\bar{z}_t}{dw_t}$$

$$= \frac{\partial u_t'(w_t - s_t)}{\partial w_t} - [g_{zw,t}''(x_t, s_t; w_t)]^T G_{z,t}''^{-1} g_{zw,t}''$$

$$= u_t'' - [0 \quad -u_t''(w_t - s_t)] \begin{bmatrix} g_{ss,t}'' & -g_{xs,t}'' \\ -g_{xs,t}'' & g_{xx,t}'' \end{bmatrix} \begin{bmatrix} 0 \\ -u_t''(w_t - s_t) \end{bmatrix} (detG_{z,t}'')^{-1}$$

$$= u_t'' - (u_t'')^2 g_{xx,t}'' (detG_{z,t}'')^{-1}$$

$$= \frac{u_t''(g_{xx,t}'' g_{ss,t}'' - (g_{xs,t}'')^2 - u_t'' g_{xx,t}'')}{g_{xx,t}'' g_{ss,t}'' - (g_{xs,t}'')^2} \tag{19}$$

Form the matrix $G_{z,t}''(x_t, s_t; w_t)$, we know that $g_{ss,t}'' = u'' + \beta r^2 E_t[V''_{t+1}]$, thus

$$g_{xx,t}'' g_{ss,t}'' - (g_{xs,t}'')^2 - u_t'' g_{xx,t}'' = g_{xx,t}''(u'' + \beta r^2 E_t[V''_{t+1}]) - (g_{xs,t}'')^2 - u_t'' g_{xx,t}''$$
$$= g_{xx,t}'' \beta r^2 E_t[V''_{t+1}] - (r\beta E_t[R'_t V''_{t+1}])^2 \tag{20}$$

Thus,

$$V_t''(w_t) = u_t'' \frac{g_{xx,t}'' r^2 E_t[V''_{t+1}] - (r\beta E_t[R'_t V''_{t+1}])^2}{g_{xx,t}'' u_t'' - g_{xx,t}'' u_t'' + g_{xx,t}'' g_{ss,t}'' - (g_{xs,t}'')^2}$$

$$= u_t'' \underbrace{\frac{\overbrace{g_{xx,t}'' \beta r^2 E_t[V''_{t+1}] - (r\beta E_t[R'_t V''_{t+1}])^2}^{>0}}{\underbrace{g_{xx,t}'' u_t''}_{>0} + \underbrace{g_{xx,t}'' r^2 E_t[V''_{t+1}] - (r\beta E_t[R'_t V''_{t+1}])^2}_{>0}}}_{<1} \in (u_t'', 0) \tag{21}$$

We thus find that the value function is concave even when the agents have two independent decision variables.

3. Empirical analysis

The theoretical analysis shows that the value function may not be of the DARA type when the instantaneous utility is DARA. We are nevertheless unable to identify explicit cases where we are certain that the value function is not DARA. Furthermore we are not able to link parameters of the utility function to the parameters of the value function. We now proceed with simulations to eventually find cases where inheritance properties fail. We first justify our calibration procedures and then comments on our results.

3.a. Empirical assumptions

Any empirical analysis inevitably depends on the specifications of functional forms or data used. In order to be quite general, we specify rather flexible forms and test our results to different calibration points. More precisely, we use the Expo-Power utility function initially proposed by Saha (1993). This function is given by:

$u(c) = u_0 - \exp\left(-u_1 c^{u_2}\right)$ with $u_0 > 1, u_1 \neq 0, u_2 \neq 0, u_1 u_2 > 0$

The Arrow Pratt coefficient of absolute risk aversion is given by:

$$A_u(c) = \frac{1 - u_2 + u_2 u_1 c^{u_2}}{c}$$

Thus the utility function exhibits DARA if $u_2 < 1$, CARA when $u_2 = 1$ and IARA when $u_2 > 1$. In all simulations, we fix the constant u_0 to 1.1 and will consider different values for the two other parameters. In the central case, we assume that these two parameters equal 0.5.
Regarding the production side of our model, we assume that the technology is resumed by a quadratic cost function. The marginal cost function is thus linear with respect to the production level. Parameters of the marginal cost function are calibrated such as to target a price supply elasticity when both production and output price equal one. There are obviously also great uncertainties to the true value of this elasticity because they depend on the risk aversion of farmers. We thus consider two successive calibrations, one when the supply elasticity equals 0.5 and one when it equals 0.1. We make clear here that these are the values for the calibration at one point (when both price and production equals one); the true ex post elasticity obviously varies with the production (and price) levels.
The price of the agricultural good is the sole stochastic variable. We assume that it follows a normal law with mean one and standard error 0.2, so a coefficient of variation of 20 per cent. Finally we assume that the discount factor (β) equals one and the return of the safe asset (r) equals one as well.

3.b. Empirical results

We solve the optimization program for different (one hundred) initial values of the initial wealth of our farmer (from 0.5 to 5, so that initial wealth range from 50 per cent to 500 per cent of annual average revenue). We then obtain the optimal values of production, consumption and ultimately the optimal level of the value function. We then perform a simple adjustment of these optimal values to the initial level of wealth assuming an Expo Power value function:
$v(w) = v_0 - \exp\left(-v_1 c^{v_2}\right)$
Because this function is highly nonlinear, we provide initial points to facilitate convergence. The initial points equal the values of the deep parameters. Results of our estimation for different values of utility parameters are reported in the following tables.

In the table 1, we only change the value of the u_2 deep parameter which mostly governs the Arrow Pratt coefficient of absolute risk aversion. The parameters of the cost function ensure 0.5 supply elasticity at the mean point and without risk. When both deep parameters equal 0.5 and thus the utility function is DARA, we also find that the value function parameters ensure DARA. The estimation value of the v_2 parameter is significantly lower than one. On the other hand, the estimated parameters are significantly different from the deep parameters. One is greater and the other is lower. We then compute the Arrow Pratt coefficient of absolute risk aversion at different points. For instance, at the mean point of consumption, this coefficient equals 0.45 while it equals 0.23 at the mean wealth level (figures not shown in table 1). In fact we observe at all points that the Arrow Pratt coefficient computed with the value function is lower (roughly equal one half) than the coefficient computed with the utility function at the optimal consumption level. Both coefficients are always positive and thus the concavity is effectively satisfied.

We then perform the same computations now assuming that the u_2 parameter equals 0.8. While the utility function still exhibit DARA preferences, it appears that the estimated parameters of the value function no longer ensure DARA: the estimate of the v_2 parameter is statistically greater than one, so the value function exhibit IARA. At the mean point, we again find that the Arrow Pratt coefficient of absolute risk aversion evaluated with the value function is lower (0.28) than the coefficient with the utility function (0.46). So the concavity is still satisfied at the mean point. Table 1 also provides results with other values for the u_2 parameters. These results basically suggest a positive relationship between the deep and estimated parameters.

Table 2 provides the same results when we now assume a supply elasticity of 0.1 at the mean point without risk. As expected results are much positive in the sense that the value function always exhibit DARA when the utility function is also DARA. This makes sense because, as we show in the analytical section, the properties of the value function depend on the curvature of the supply function. By reducing the response of production to economic incentives, we expect less production effects due to changes in initial wealth levels. Again there are some differences between the deep parameters and the estimated ones. The Arrow Pratt coefficients of absolute risk aversion are again positive with the value one being roughly half the utility one. On the other hand, we find that the value function exhibit DARA while the utility function is IARA (when u_2 equals 1.3). So a DARA value function may be also generated by non DARA utility functions. While this is not contradicted by the theory, this raises another difficulty with present available estimates of risk aversion.

Finally we modify the range of initial wealth levels. We now assume a narrow range (from 50 per cent to 150 per cent of average revenue) because additional production effects are very limited when we start from high initial wealth. Results reported in table 3 show the robustness of our central results.

4. Concluding comments

While knowing the risk aversion of farmers is of crucial interest in many agricultural economic issues, we are far from a consensus on the magnitude of farmers' risk aversion. We argue that available econometric estimates do not measure farmers' preferences towards risky outcomes. Analyses are mostly of static nature and indeed measure the parameters of the optimal value function rather than the deep parameters of the utility functions.

This paper shows theoretically and empirically that there is not a simple relationship between utility functions and value functions when agents have many decision variables. More precisely we analytically prove in a simple setting with two independent decision variables (production and consumption) and no market failures that the value function used in many studies may not verify DARA when the utility satisfies this condition. On the other hand, we find that the value function is concave with respect to wealth when utility is concave with respect to consumption. When we restrict our framework to one decision variable, we end up with well established results. Furthermore our empirical results reveal that the estimates of value function parameters strongly differ from the values of our deep parameters. We also find that the value function does not necessarily exhibit DARA when the instantaneous utility function satisfies DARA and conversely. The estimated Arrow Pratt coefficient of absolute risk aversion is systematically lower for the value function. The bias with the deep parameter depends, as expected, on the extent of supply responses to economic incentives.

Accordingly our recommendation is to perform new econometric estimation of farmers' attitude toward risks with at least two new dimensions. First these estimations should be dynamic using Euler type equations. Second these estimations should include final consumption expenditures of farmers so as to identify the true deep parameters. More generally, we believe like Pope *et alii* that due account should be made on the different decision variables available to farmers.

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Table 1. Econometric estimates of the value function parameters given deep parameters (supply elasticity calibrated to 0.5)

$u_0 = 1.1, u_1 = 0.5$					
u_2	0.2	0.5	0.8	1	1.3
v_2	0.49 (0.0026)	0.81 (0.0007)	1.24 (0.0058)	1.61 (0.0133)	2.25 (0.049)
v_1	0.15 (0.0014)	0.24 (0.0005)	0.23 (0.0035)	0.19 (0.0049)	0.11 (0.0069)

Source: our computations (standard errors in parentheses)

Table 2. Econometric estimates of the value function parameters given deep parameters (supply elasticity calibrated to 0.1)

$u_0 = 1.1, u_1 = 0.5$					
u_2	0.2	0.5	0.8	1	1.3
v_2	0.76 (0.0029)	0.83 (0.002)	0.82 (0.0029)	0.71 (0.0054)	0.39 (0.0054)
v_1	0.03 (0.0002)	0.08 (0.0004)	0.14 (0.001)	0.21 (0.0033)	0.55 (0.0288)

Source: our computations (standard errors in parentheses)

Table 3. Econometric estimates of the value function parameters given deep parameters (supply elasticity calibrated to 0.5, restricted range for the initial wealth)

$u_0 = 1.1, u_1 = 0.5$					
u_2	0.2	0.6	0.8	1	1.3
v_2	0.61 (0.0017)	0.89 (0.0001)	1.03 (0.0009)	1.19 (0.002)	1.43 (0.004)
v_1	0.11 (0.0003)	0.28 (0.00004)	0.33 (0.0004)	0.38 (0.001)	0.43 (0.002)

Source: our computations (standard errors in parentheses)