The CDET Profit Function: Could it generate a Parsimonious Agricultural Sector Model?

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Abstract: While the single- output Constant Difference of Elasticities (CDE) cost function has been applied several times, its profit counterpart called "the Constant Difference of Elasticities of Transformation" (CDET) profit frontier has not yet been applied econometrically. It is an indirect, implicit, non-homothetic and non-separable frontier that may be viewed as more flexible than the commonly used CES and Cobb-Douglas specifications, while demanding less parameters to be estimated than fully flexible functional forms commonly do. We therefore introduce the CDET profit function and illustrate its potential usefulness as a parsimonious econometric model of agricultural production in Switzerland. Results indicate plausible elasticities and a satisfactory fit to the data; however, successful estimation requires that certain theoretical characteristics of the CDET are exactly obeyed.

Keywords: CDE, Profit function, Agricultural Sector, Functional Form, Switzerland

1. Introduction

Choosing appropriate functional forms is one of the most important methodological tasks for empirical analyses of economic systems. For instance, *ex ante* analyses of sectoral changes due to changing prices and/or the corresponding policies in agriculture often make predictions based on the underlying assumption that a chosen functional form is an appropriate representation of reality (Griffin *et al.,* 1987). Even though a large number of functional forms exist and have been studied extensively by applied and agricultural economists as well as within general production economics (e.g. Färe and Mitchell 1989), in practice two distinct approaches seem to dominate within applied economics: On the one hand fully flexible functional forms such as the transcendental logarithmic (Translog) or the quadratic (Chambers 1988) seem to dominate farm level analyses. On the other hand, empirical models such as standard CGEs (Hertel 1997) which are used for the analysis of more aggregated data (e.g. trade and sector models) heavily rely on relatively restrictive functional forms. Perhaps most frequently used in this respect is the Constant Elasticitiy of Substitution (CES) or the Constant Elasticity of Transformation (CET). While the application of such restrictive functional forms reduces the amount of exogenous parameters to be specified or to be estimated econometrically, their use has with respect to agricultural sector models repeatedly been criticized as only weakly justified from an empirical point of view.

However, functional forms that are less restrictive than the CES but at the same time globally regular very well exist. Such a functional form is given by the Constant Difference of Elasticities (CDE) (Hanoch 1975). This functional form has been applied several times in the past (e.g. Dar and Dasgupta, 1985), and also with respect to analyses of the agricultural sector (Surry 1993), or to trade of food products (Surry, Herrard, and Le Roux, 2002). Furthermore, this function is an important component of the representation of regional household consumption within the widely used CGE model developed by the Global Trade Analysis Project (GTAP) (Hertel 1997). All these applications have in common that they refer to the single- output (or utility) version of the Constant Difference of Elasticities (CDE) cost or expenditure function.

Hanoch (1978a, p. 307), however, also concisely derives the Constant Difference of Elasticities of Transformation (CDET) profit frontier, which according to our knowledge has not yet been applied econometrically. It is an indirect, implicitly additive, non-homothetic and non-separable profit (and hence production) frontier that may be viewed as more flexible than the commonly used CES or Cobb-Douglas specifications. On the cost side, however, in general terms the CDE cost function nests the widely used CES as a special (homogeneous) case and therefore should potentially allow for empirical applications. "Implicit additivity" in this context means "implicit strong separability" and is

characterized by isoquant surfaces that are strongly separable with respect to input prices, which clearly poses some rigidity on the underlying technology.
Given these considerations, the objective of this paper is to provide a brief description of the CDE profit frontier (section 2) and then show an econometric application to the Swiss agricultural sector (sections 3 and 4). Section 5 discusses directions for future research.

2. The CDE Profit Function: A Brief Overview

Hanoch (1978a, 1978b) outlines the derivation of dual (polar) cost-, production- and profit functions based on commonly used functional forms that exhibit a constant ratio of elasticities of substitution (CRES) or constant difference of elasticities of Substitution (CDE). According to Hanoch, a CRES frontier (with m outputs and n inputs; $m,n > 2$) is given by the following equations:

$$\sum_{i=1}^{m} D_i f^{-e_i d_i}(x_i)^{d_i} \equiv 1 \qquad , d_i < 1 \qquad [1]$$

$$\sum_{j=1}^{m} B_j g^{h_j b_j}(y_j)^{b_j} \equiv 1 \qquad , b_j > 1 \quad \text{and } f(x) = g(y) \qquad [2]$$

Dual to this CRES frontier, a CDE frontier exists that can be derived from [1] and [2] by transforming inputs x_i to cost (C^*) and output (y_j) to revenue (R^*):

$$\sum_{i=1}^{m} D_i f^{e_i d_i}\left(\frac{w_i}{C^*}\right)^{d_i} \equiv 1 \quad ; \quad \sum_{j=1}^{m} B_j g^{h_j b_j}\left(\frac{p_j}{R^*}\right)^{b_j} \equiv 1 \quad ; \quad f\left(\tfrac{w}{C^*}\right) = g\left(\tfrac{p}{R^*}\right) \qquad [3]$$

Hanoch (1978b, p. 304) then shows that the condition for existence of a (unit) profit function $\Pi^*(p,w)$ dual to [1] and [2] is that the set $T\{(y,x):f(x) \geq g(y)\}$ is convex. If such a profit function exists, [3] can, under the assumption of positive profits, be transformed into

$$\sum_{i=1}^{m} D_i f^{-e_i d_i}\left(\frac{w_i}{\Pi^*}\right)^{d_i} \equiv 1 \quad ; \quad \sum_{j=1}^{m} B_j g^{-h_j b_j}\left(\frac{p_j}{\Pi^*}\right)^{b_j} \equiv 1 \qquad [4]$$

By assuming further that $h_j b_j = e_i d_i = c > 0$ $(j = 1,...,m;\ i=1,...,n)$, sufficient parameter restrictions are B_j and $D_i > 0$, $b_j > 1$, $0 \leq d_i < 1$ or $d_i \leq 0$ for all i,j and $\log(x_i)$ if $d_i = 0$. Furthermore, if x_i and y_i are optimal quantities, positive maximum profit is given by:

$$\Pi = \sum_{j=1}^{m} p_j y_j^* - \sum_{i=1}^{n} w_i x_i^* > 0$$

and the two frontiers (given in [5] and [6]) look as follows, with the CRE of transformation (CRET) being non-homothetic and separable,

$$\sum_{j=1}^{m} B_j (y^*)^{bj} - \sum_{i=1}^{n} D_i (x^*)^{d_i} = 0 \qquad [5]$$

and in analogy the CDE of Transformation (CDET) profit function is given in [6] and represents a non-homothetic and non-separable frontier under the assumption of positive profits and under the same parameter restrictions as stated for [4]. Furthermore, Hanoch defines the CDE profit function in an implicit way through the following expression:

$$\sum_{i=1}^{m} B_i \left(\frac{p_i}{\Pi}\right)^{b_i} - \sum_{k=1}^{n} D_k \left(\frac{w_k}{\Pi}\right)^{d_k} = 0 \quad , \quad \text{where } \Pi = \sum_{i=1}^{m} p_k y_k - \sum_{k=1}^{n} w_k x_k > 0 \qquad [6]$$

The CDE profit function in [6] is thus defined for m outputs and n inputs. Subscript i is defined for output while k is defined for the n inputs. The parameters b_i are greater than one while the parameters

d_k are smaller than one. However, these parameters do not reflect any elasticity directly, nor should it be immediately obvious how [6] can be operationalised for empirical studies. The following sub-section therefore defines the output supply and input demand equations corresponding to [6] for the supply of the commodity *i* and the demand for input *k*. Then we proceed with the derivation of the output and input price elasticities. An econometric estimation is presented in section 3.

2.1 **Supply function for commodity *i* and demand for input *k***

Applying Hotelling's lemma to [6] yields the output supply functions corresponding to the profit frontier. The supply function for commodity *i* is then given by the following expression:

$$y_i = \frac{\partial \Pi}{\partial p_i} = \frac{b_i B_i \left(\frac{p_i}{\Pi}\right)^{b_i - 1}}{K} \quad \text{where} \quad K = \sum_{i=1}^{m} b_i B_i \left(\frac{p_i}{\Pi}\right)^{b_i} - \sum_{k=1}^{n} d_k D_k \left(\frac{w_k}{\Pi}\right)^{d_k} \qquad [7]$$

The supply function can also be expressed in a profit share form:

$$\delta_i = \frac{p_i y_i}{\Pi} = \frac{b_i B_i \left(\frac{p_i}{\Pi}\right)^{b_i}}{K} = \frac{p_i \frac{\partial \Pi}{\partial p_i}}{\Pi} = \left(\frac{\partial \Pi}{\partial p_i}\right)\left(\frac{p_i}{\Pi}\right) = \frac{\frac{\partial \Pi}{\Pi}}{\frac{\partial p_i}{p_i}} = \frac{\partial \ln(\Pi)}{\partial \ln(p_i)} \qquad [8]$$

The input demand function for input *k* is also obtained in the usual way through implicit differentiation of profit Π in [6] with respect to input prices w_k, which corresponds to the following equation:

$$\sum_{i=1}^{m} (-1) b_i B_i (p_i)^{b_i} (\Pi)^{-b_i - 1} \left(\frac{\partial \Pi}{\partial w_k}\right) - d_k D_k (w_k)^{d_k - 1} (\Pi)^{-d_k} - (-1) \sum_{k=1}^{n} d_k D_k (w_k)^{d_k} (\Pi)^{-d_k - 1} \left(\frac{\partial \Pi}{\partial w_k}\right) = 0$$

$$\Leftrightarrow (-1)\left(\frac{\partial \Pi}{\partial w_k}\right) \Pi^{-1} \left[\sum_{i=1}^{m} b_i B_i \left(\frac{p_i}{\Pi}\right)^{b_i} - \sum_{k=1}^{n} D_k \left(\frac{w_k}{\Pi}\right)^{d_k}\right] = d_k D_k (w_k)^{d_k - 1} (\Pi)^{-d_k - 1} \Pi \qquad [9]$$

Re-arranging expression [9] and expressing x_k as a function of other terms and parameters yields the following relationship defining the demand function for input *k*:

$$x_k = -\frac{\partial \Pi}{\partial w_k} = -\frac{d_k D_k \left(\frac{w_k}{\Pi}\right)^{d_k - 1}}{K} \qquad \text{with } K \text{ as given in [7].} \qquad [10a]$$

The demand function for input *k* can also be expressed in a profit-share form:

$$\delta_k = -\frac{w_k x_k}{\Pi} = \frac{d_k D_k \left(\frac{w_k}{\Pi}\right)^{d_k}}{K} = \frac{w_k \left(\frac{\partial \Pi}{\partial w_k}\right)}{\Pi} = \left(\frac{\partial \Pi}{\partial w_k}\right)\left(\frac{w_k}{\Pi}\right) = \frac{\frac{\partial \Pi}{\Pi}}{\frac{\partial w_k}{w_k}} = \frac{\partial \ln(\Pi)}{\partial \ln(w_k)} \qquad [10b]$$

Note that the profit share δ for input *k* is on the one hand negative but also equal to the logarithmic derivative of profit with respect to the price of input k. Then the following identity is respected by all output and input profit shares.

$$\sum_{i=1}^{m} \delta_i + \sum_{k=1}^{n} \delta_k = 1 \qquad [11]$$

Before we go into the derivation of price elasticities of output supply and input demand, it is worth analyzing in more detail the possible values that can be taken by the parameters[1] of the CDE profit function, and that have already been stated in connection with expression [4].
Let us first examine the parameters b_i. These parameters associated with each commodity *i* are greater than 1. Concerning the parameters d_k, we know that they must be smaller than 1. However, Hanoch does not say much about the possible ranges of values than can be taken by these parameters d_k. Indeed by stating that they should be smaller than one, he does not make the same distinction in the CDE profit function as he did for the CDE cost function where the corresponding parameters could either take negative values or positive values smaller than one. This situation is not discussed for the CDE profit function[2]. For ease of simplification, let us assume that the parameters d_k are positive and smaller than one. It is straightforward to observe that the parameters B_i must be positive because they are of the same sign as the parameters b_i.The same reasoning applies for the parameters D_k. In this case, if we assume that the parameters d_k are positive and smaller than one, then the parameters D_k must be positive.

2.2 **Derivation of Own- and Cross Price Elasticities of Output Supply and Input Demand**[3]

Given the results from the previous section, we can now derive the own-price elasticities of output supply for commodity *i,* the cross-price elasticities of commodity *i* with respect to the price of commodity *j*, the input demand elasticity with respect to input price w_k and the input demand with respect to output price p_i.

Note that for [12] – [17] always $ROT = \sum_{i=1}^{m} b_i \delta_i + \sum_{k=1}^{n} d_k \delta_k$

The own-price elasticity of supply for commodity *i* is derived as follows:

$$\varepsilon_{ii} = \frac{\partial \ln(y_i)}{\partial \ln(p_i)} = (b_i - 1) - (b_i - 1)\frac{\partial \ln(\Pi)}{\partial \ln(p_1)} - \left(\frac{\partial K}{\partial p_i}\right)\left(\frac{p_i}{K}\right) = -2b_i\delta_i + (b_i - 1) + \delta_i + \delta_i\, ROT \qquad [12]$$

The cross-price elasticity of supply for commodity *i* with respect to the price of output *j* is:

$$\varepsilon_{ij} = \frac{\partial \ln(y_i)}{\partial \ln(p_j)} = -(b_i - 1)\frac{\partial \ln(\Pi)}{\partial \ln(p_j)} - \frac{\partial \ln(K)}{\partial \ln(p_j)} = -(b_i - 1)\delta_j - \left(\frac{\partial K}{\partial p_j}\right)\left(\frac{p_j}{K}\right) = -b_i\delta_j - b_j\delta_j + \delta_j + \delta_j ROT \qquad [13]$$

The cross-price elasticity of supply for commodity *i* with respect to the price of input *k*:

$$\varepsilon_{ik} = \frac{\partial \ln(y_i)}{\partial \ln(w_k)} = -(b_i - 1)\frac{\partial \ln(\Pi)}{\partial \ln(w_k)} - \frac{\partial \ln(K)}{\partial \ln(w_k)} = -(b_i - 1)\delta_k - \left(\frac{\partial K}{\partial w_k}\right)\left(\frac{w_k}{K}\right) = -b_i\delta_k - d_k\delta_k + \delta_k + \delta_k ROT \qquad [14]$$

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[1] Note that the notation in Hanoch (1978a, 1978b) uses $\alpha_i = 1 - b_i$ and $\alpha_k = 1 - d_k$ which leads to more parsimonious expressions than working with the parameters b_i and d_k directly. For the sake of transparency of our abbreviated derivations in the next section and, also with respect to the econometric application presented in section 3, however, we will stick to b_i and d_k.
[2] It is interesting to note that Hanoch did for the CDE profit function not discuss the possibility for local regularity conditions, as he did for the CDE cost function (Hanoch, 1975).
[3] Hanoch does not provide the derivations of elasticities developed in this section in great detail but only states some of the final expressions defining these elasticities. A more detailed version of this section is therefore available from the authors upon request.

Furthermore, the own-price elasticity of the demand for input *k* is defined as follows:

$$\varepsilon_{kk} = \frac{\partial \ln(x_k)}{\partial \ln(w_k)} = (d_k - 1) - (d_k - 1)\frac{\partial \ln(\Pi)}{\partial \ln(w_k)} - \frac{\partial \ln(K)}{\partial \ln(w_k)} = -2d_k\delta_k + (d_k - 1) + \delta_k + \delta_k\, ROT \qquad [15]$$

And the cross-price elasticity of demand for input *k* with respect to the price of input *l*:

$$\varepsilon_{kl} = \frac{\partial \ln(x_k)}{\partial \ln(w_l)} = -(d_k - 1)\frac{\partial \ln(\Pi)}{\partial \ln(w_l)} - \frac{\partial \ln(K)}{\partial \ln(w_l)} = -(d_k - 1)\delta_l - (d_l\delta_l - \delta_l ROT) \qquad [16]$$

Finally, the elasticity of the demand for input k with respect to the price of output *i*:

$$\varepsilon_{ki} = \frac{\partial \ln(x_k)}{\partial \ln(p_i)} = -(d_k - 1)\frac{\partial \ln(\Pi)}{\partial \ln(p_i)} - \frac{\partial \ln(K)}{\partial \ln(w_k)} = -(d_k - 1)\delta_i - (b_i\delta_i - \delta_i ROT) \qquad [17]$$

The homogeneity condition states that the sum of price elasticities of the supply for commodity *i* and the sum of price elasticities of the demand for input *k* must be equal to zero, which implies that:

$$\varepsilon_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{m} \varepsilon_{ij} + \sum_{k=1}^{n} \varepsilon_{ik} = 0 \quad \text{and} \quad \varepsilon_{kk} + \sum_{i=1}^{m} \varepsilon_{ki} + \sum_{\substack{l=1 \\ k\neq l}}^{n} \varepsilon_{kl} = 0 \qquad [18]$$

2.3 **The Restricted CDET Profit Function**

The CDET model represented by [6] requires observed prices, which is usually the case for most agricultural outputs, yet not for all inputs in the same way: Especially land prices are usually difficult to obtain and at best of dubious quality, if not in part biased due to taxation issues. Furthermore, the state of a given technology, as well as production quotas, are examples of other potentially fixed factors for which price data are difficult to observe. For such cases, Hanoch (1978b, p.129) briefly addresses the option to express the CDET model as a restricted profit function; however, given the restriction that *z* has to be separable with respect to variable inputs and outputs. As a restricted CDE profit function expression [6] would take one of the two following general forms:

$$\sum_{i=1}^{m} B_i\left(\frac{p_i}{\Pi}, z_0\right)^{b_i} - \sum_{k=1}^{n} D_k\left(\frac{w_k}{\Pi}, z_0\right)^{d_k} = 0 \qquad [19]$$

$$\sum_{i=1}^{m} B_i\left(\frac{p_i}{\Pi}\right)^{b_i} - \sum_{k=1}^{n} D_k\left(\frac{w_k}{\Pi}\right)^{d_k} = \frac{1}{F(z)} \qquad [20]$$

However, with respect to empirical applications it should be difficult to identify cases under which the fixed elements *z* are completely separable from the variable elements. If such a case can be identified, [20] can be homogeneous of any degree in *F(z)* (Bergmann 1997).

3. Empirical application: Regional Agricultural Accounts of Switzerland

In order to estimate the CDE profit function econometrically, several conditions implied by equation [6] as well as by the identity [11] must be satisfied: *i)* Profits must be positive, *ii)* observed input- and output quantities must be optimal given the corresponding set of prices, and *iii)* input- and output-profit shares must some to one.

These conditions show that empirical applications of the CDE are best suited for data that come from a relatively homogeneous set of firms or farms, and that include price observations which are standardized in such a way that they have the same relative meaning for the optimization decision underlying each observation. Furthermore, profits must be positive, which is not necessarily the case for farm-level observations. Instead, profit as defined in equation [6] itself takes the role of a residual, such as the rent going to a non-separable fixed factor. Indeed, rents to land or quotas would be promising candidates to serve as observed profit for empirical specifications. A different approach is to choose value added, as long as that is positive.

We therefore consider annual agricultural accounts, as they are compiled by many national statistical agencies to be potential candidates to reflect these assumptions. Furthermore, especially the available regional sub-aggregation of those agricultural accounts provides the required number of observations for the estimation of a larger system of equations. Finally, representative price data for outputs and inputs are usually published together with such statistical accounts and therefore are readily available in a consistent manner.

We use regional agricultural account data for the agricultural sector of Switzerland disaggregated at the level of each *Kanton* and for the years 1999 to 2008 (Statistik Schweiz 2009). The Swiss data are provided in a wide range of possible aggregations and therefore allow for convenient estimation of different specifications. The regional agricultural accounts for Switzerland are organized as follows:

$$\sum_{i=1}^{11} p_{i,r,t} y_{i,r,t} - w_{k,r,t} x_{k,r,t} = GVA_{r,t} - Depreciation_{r,t} - Labor_{r,t} - Fees_{r,t} + Subsidies_{r,t} + netRents_{r,t} = NVA_{r,t} \qquad [21]$$

where GVA= Gross Value added and NVA = Net Value added, or net profit of the representative regional farm in *Kanton r* and year *t*.

According to [21], GVA qualifies as a potential candidate for positive profits, as assumed by the CDE profit function. In contrast, net value added (NVA) cannot be used because further inseparable expenses and transfers to the regional account [see 21] would violate the assumption profit shares for inputs and outputs summing to one (identity [11]). Furthermore, due to the non-separable nature of the CDE profit function, we have to aggregate all inputs into one aggregate, that we link to the corresponding input price index; all prices for inputs and outputs in these data are relative prices with respect to the base year 2000. Similar regional accounts exist for many other European countries and for the United States. Therefore, it should in principle be possible to estimate the CDE profit function also for many other regions, or potentially even for one comprehensive large European dataset. However, we experienced that the corresponding price indices that are issued by EUROSTAT still contained missing values or are otherwise not yet fully available in the way that would provide enough comparable observations in order to estimate a system whit multiple several equations.

The agricultural accounts of Switzerland include not only the value of crop products, animals and products of animal origin, but also wine, fruit and horticulture. In addition, agricultural services such as contracting work provided to other farmers, artificial insemination and sheep shearing are represented. This output category "services" also includes the rental of dairy quota.

Furthermore, due to the multifunctional character of Swiss agriculture, that is also often involved in direct marketing and agro tourism, regional agricultural accounts capture and measure the value of non- agricultural activities, as far as they are inseparable from the main agricultural enterprises. Such activities include on-farm processing of fruit and dairy products, services provided to firms outside agriculture (e.g. snow removal), and agrotourism involving tourists who camp on the farm site or sleep in the straw (Statistik Schweiz 2009). In few instances values were missing or zero in specific regions and for individual products[4].

[4] In order to be able to express the dependent variable in logarithms, these zero values have been replaced by the sample been, which induces the effect that the profit shares according to equation [11] do not exactly sum to one, but instead to 1.018.

4. Estimation Strategy

Output supply- and input demand equations according to expression [8] and [10], respectively, would require the nonlinear estimation of a system of more than ten equations. However, as outlined in Surry (1993), the derivation of estimable output supply- and input demand equations is done by diving each equation by a *numeraire* such as y_1 and applying the logarithmic transformation. This yields in the cases of [8] and [10b], respectively, the following estimable equations:

$$\log\left(\frac{y_j\, p_j}{y_1\, p_1}\right) = \log\left(\frac{B_j b_j}{B_1 b_1}\right) + b_j \log\left(\frac{p_j}{\Pi}\right) - b_1\left(\frac{p_1}{\Pi}\right) + c_r Kanton_r + t_{year} + u_{i,j,r,t} \quad [22]$$

$$\log\left(\frac{x_i w_i}{y_1 p_1}\right) = \log\left(\frac{D_i d_i}{B_1 b_1}\right) + d_i \log\left(\frac{w_i}{\Pi}\right) - b_1\left(\frac{p_1}{\Pi}\right) + c_r Kanton_r + t_{year} + u_{i,j,r,t} \quad [23]$$

We have set the output value of vegetables as the numeraire (p_1 in [22] and [23]), and we added fixed effects for each region (*Kanton*) of Switzerland, and a time trend. According to equation [10] we take the negative of the dependent variable of equation [24] when estimating the full system of 11 equations using 3SLS based on the software by Hennigsen and Hamann (2007). The only restriction that we impose is that $b_{vegetables}$ have to be equal across all equations.

Table 1: 3SLS Estimation Results and Diagnostics

	DF	**SSR**	**OLS-R^2**	**McElroy-R^2**	**Estimated Coefficient of b_i or d_k**	**Standard Error**	**Pr(>|t|)**
System:	2420	320.71	0.96	0.99			
Equation:	**DF**	**SSR**	**RMSE**	**Adj.R^2**			
Crops	222	22.446	0.318	0.980	1.008	0.1552	0.0000
Feed	222	15.370	0.263	0.964	1.242	0.0662	0.0000
Fruit (insignificant)	214	21.864	0.320	0.928	0.654	0.4443	0.1409
Wine	222	144.882	0.808	0.882	1.785	0.8108	0.0277
Dairy & Cattle	214	14.788	0.263	0.975	1.413	0.1852	0.0000
Pigs	214	14.336	0.259	0.981	2.483	0.3151	0.0000
Poultry & Eggs	222	20.941	0.307	0.961	1.627	0.1744	0.0000
Sheep & Goats	222	17.445	0.280	0.961	1.646	0.1529	0.0000
Services	222	17.816	0.283	0.934	1.494	0.1658	0.0000
Non Agric. Activities	222	17.545	0.281	0.958	1.325	0.1590	0.0000
Inputs	214	13.278	0.249	0.957	0.921	0.5514	0.0949
Vegetables (Numeraire)					-1.281	0.2860	0.0000

Furthermore, initial estimations with the instruments[5] indicate that not all coefficients appear to fulfill the theoretical conditions $b_j>1,\ 0 \le d_i < 1$. In these cases, we have added dummy variables instead of a time trend for each different year, which led all coefficients except from the price of fruits to fulfill the theoretical conditions. Table 1 indicates for which prices the $b_j>1,\ 0 \le d_i < 1$ condition has initially not been met: for these equations the number of degrees of freedom is 214 instead of 222 due to the additional fixed effects for eight years.

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[5] Instruments used for all 11 equations: *log(*Priceindex of Crops*), log(*Priceindex Feed*), log(*Priceindex Pigs*), log(*Priceindex Inputs*), log(*Priceindex Vegetables*), log(*Priceindex Services*), log(*Priceindex inseparable Non-Agricultural Activities*), log(*Priceindex Poultry & Eggs*)*, Value of labor input, *Kanton r=1…25, Year t=1999,…,2008*; Net. Profit in 1000 CHF, hectares per fulltime farm in *Kanton r*.

Table 1 also indicates that the CDET model fits the data overall very well according to the coefficient of determination, which has already been suspected by Hanoch (1975) based on the theoretical properties of the CDE. Furthermore, as one would expect from the non-separability property of the CDET, estimations failed to match the $b_j>1$, $0 \leq d_i < 1$ conditions if more than one input is introduced.

5. Results for Switzerland

Table 1 shows that most estimated price coefficients are statistically significant at 5% or better, exceptions are the price of inputs (significant at 10%), and the price of fruit (insignificant). This estimated coefficient also remains the only parameter for which the condition that $b_j>1$ is not satisfied. With respect to the global regularity conditions we find two eigenvalues associated with the Hessian of the CDE profit function calculated the sample means to be negative. Thus the global regularity conditions did not exactly match the theoretical expectations. However, based on the estimated coefficients presented in Table 1 and on the relationships derived in the previous section we present the calculated corresponding own- and cross-price elasticities in Table 2, and correspondingly in Table 3 the compensated own- and cross-price elasticities[6]. All these elasticities are calculated at the sample mean.

Tables 2 and 3 also show the elasticities of input demand with respect to a change in output price *i* (last row), and the elasticities of a output supply with respect to a change of input price (last column). Furthermore, the own-price elasticity of demand for the aggregate input is -2.18, and zero for the compensated case (Table 3)[7]. The estimated elasticities presented in Table 2 appear to be within plausible ranges; all except the insignificant own-price elasticities for fruit are positive. However, for different products the range of estimated own-price elasticities varies from the most inelastic effect estimated for crops to the most elastic effect for pigs. None of the estimated own-price elasticities is substantially greater than one, however many cross-price effects appear to be positive, which indicates that many agricultural outputs appear to be complements. From an empirical point of view this appears plausible because crops, feed, dairy, sheep and goats certainly relate to each other, and are often part of the same multi-output farm, given the family-farm based structures which are typical for Switzerland. Thus, complementary output changes seem to reflect the corresponding price effects in years with a relatively high overall price level for producer prices.

Recently, Jansson and Heckelei (2011) have estimated own- and cross-price elasticities for the French crop sector, using similar regional accounting data with a Bayesian estimation approach. Most of their own-price elasticities for cereals and oilseeds are slightly greater than one, and cross-price effects are either negative or zero. Our aggregated elasticitiy $\varepsilon_{i,i}$ for crops in Switzerland (Table 2) is much smaller, which is intuitively plausible because elasticities of supply (or demand) of aggregated products should *ceteris paribus* appear less elastic than their individual components. Furthermore, structural differences between Switzerland and France have to be considered when comparing output supply response.

With respect to our results for the pig sector in Switzerland, most cross-price elasticities are according to Table 2 negative. This could be explained by the fact that pig production takes place in relatively specialized farms, implying true output substitution effects relative to most other outputs along with

[6] The derivations of the compensated elasticities are developed using the formulations proposed by Lopez (1984) on the relationships between profit function on one side, and cost and revenues functions on the other.

[7] This result is not surprising since there is only one defined aggregate input involved in the estimation of the CDE profit function.

ongoing structural change (note that the compensated elasticities can also be interpreted as the short run effect). Indeed, a comparison of Table 2 and 3 in this respect reveals that the pig sector in Switzerland exhibits only very small output expansion effects. Instead, uncompensated and compensated own-price elasticities are very similar. Accordingly, cross-price elasticities for pigs with respect to other outputs are relatively similar between the compensated- and uncompensated representation. We interpret this as a reflection of structural change in the Swiss pig sector that induces farms to specialize the scope of their operations on the expense of other outputs. Especially between dairy & cattle on the one hand, and pig production on the other appear these compensated substitution effects to be strong.
Furthermore, it has to be considered that piglets constitute both an important output and also an important input of the pig sector. Due to the hog cycle, prices for piglets tend to be high when the price of pork is also high, and according to our estimates of the CDET, this effect could over-compensate true substitution effects among inputs within pig production.

Table 2: Estimated *uncompensated* own and cross-price elasticities of sector *i* (row) with respect to price *i,j,k (column)* evaluated at the sample mean.

	Crops	*Feed*	*Fruit*	*Wine*	*Dairy & Cattle*	*Pigs*	*Poultry & Eggs*	*Sheep & Goats*	*Services*	*Non-Agric. Act.*	*Vege-tables*	*Inputs (price)*
Crops	**0.273**	0.299	0.111	0.075	0.668	-0.061	0.052	0.011	0.100	0.065	0.291	-1.883
Feed	0.214	**0.470**	0.095	0.035	0.475	-0.117	0.032	0.006	0.068	0.048	0.219	-1.545
Fruit	0.342	0.407	**-0.209**	0.135	0.958	0.023	0.082	0.017	0.148	0.090	0.400	-2.394
Wine	0.096	0.063	0.056	**0.728**	0.029	-0.246	-0.015	-0.004	-0.006	0.009	0.051	-0.761
Dairy & Cattle	0.177	0.176	0.083	0.006	**0.748**	-0.158	0.017	0.003	0.044	0.036	0.166	-1.298
Pigs	-0.056	-0.149	0.007	-0.176	-0.545	**1.071**	-0.076	-0.016	-0.101	-0.041	-0.163	0.246
Poultry & Eggs	0.130	0.111	0.067	-0.030	0.159	-0.208	**0.626**	-0.001	0.015	0.020	0.100	-0.989
Sheep & Goats	0.126	0.105	0.066	-0.034	0.143	-0.213	-0.003	**0.646**	0.013	0.019	0.094	-0.961
Services	0.159	0.151	0.077	-0.008	0.268	-0.177	0.010	0.002	**0.528**	0.030	0.141	-1.182
Non-Agric. Act.	0.196	0.203	0.089	0.021	0.408	-0.136	0.024	0.005	0.057	**0.367**	0.193	-1.426
Vegetables	0.205	0.216	0.092	0.028	0.443	-0.126	0.028	0.006	0.062	0.045	**0.488**	-1.489
Inputs	0.284	0.326	0.117	0.090	0.739	-0.041	0.059	0.012	0.112	0.071	0.318	-2.087

Source: own calculations based on the estimations in equation [22] and [23].

In contrast, agricultural services and non-agricultural activities such as agro-tourism and on-farm processing of food products exhibit relatively large uncompensated cross-price elasticities with respect to crops, feed, dairy and pigs, but very small corresponding compensated cross price elasticities. With respect to the role of farm operations for the economy of rural areas and the potentially 'multifunctional' character of agriculture in Switzerland, this may imply that additional earnings from agricultural production are partly invested into farm-level diversification with respect to these 'secondary' activities, however, without affecting the direction of structural change.
The last column in Table 3 presents the calculated expansion effects of inputs with respect to a change in the corresponding output. As expected are these effects positive, except for the pig sector: Input savings (e.g. labor) due to technological change dominate the 'true' input expansion effect.

In general, results presented in Table 2 and 3 are in line with Hanoch's prediction that the theoretical properties of the CDE profit function would tend to restrict substitution effects while favoring expansion effects, and that elasticities of transformation can be of either sign, depending on the relative magnitudes of b_i , b_j , d_i and the corresponding profit shares (Hanoch 1978a, p.307).

Table 3: Estimated *compensated* own and cross-price elasticities of sector *i* (row) with respect to price *i,j,k (column)* evaluated at the sample mean.

	Crops	*Feed*	*Fruit*	*Wine*	*Dairy & Cattle*	*Pigs*	*Poultry & Eggs*	*Sheep & Goats*	*Services*	*Non-Agric. Act.*	*Vege-tables*	*Inputs (quantities)*
Crops	**0.017**	0.005	0.005	-0.006	0.001	-0.025	-0.002	0.000	-0.001	0.001	0.004	0.902
Feed	0.004	**0.229**	0.008	-0.031	-0.072	-0.087	-0.012	-0.003	-0.015	-0.005	-0.016	0.740
Fruit	0.016	0.033	**-0.344**	0.032	0.111	0.069	0.014	0.003	0.020	0.009	0.036	1.147
Wine	-0.008	-0.056	0.013	**0.696**	-0.241	-0.231	-0.037	-0.008	-0.047	-0.017	-0.064	0.364
Dairy & Cattle	0.000	-0.027	0.010	-0.050	**0.288**	-0.132	-0.020	-0.004	-0.025	-0.009	-0.031	0.622
Pigs	-0.023	-0.111	0.021	-0.166	-0.457	**1.067**	-0.069	-0.015	-0.088	-0.033	-0.126	-0.118
Poultry & Eggs	-0.004	-0.043	0.012	-0.073	-0.191	-0.189	**0.597**	-0.007	-0.037	-0.013	-0.050	0.474
Sheep & Goats	-0.005	-0.045	0.012	-0.075	-0.198	-0.194	-0.031	**0.640**	-0.039	-0.014	-0.052	0.460
Services	-0.001	-0.033	0.010	-0.059	-0.150	-0.154	-0.024	-0.005	**0.464**	-0.010	-0.039	0.566
Non-Agric. Act.	0.002	-0.020	0.009	-0.040	-0.097	-0.109	-0.016	-0.004	-0.020	**0.318**	-0.024	0.684
Vegetables	0.003	-0.016	0.008	-0.036	-0.084	-0.097	-0.014	-0.003	-0.017	-0.006	**0.262**	0.713
Inputs	0.151	0.211	0.049	0.118	0.569	0.165	0.060	0.013	0.094	0.050	0.213	0

Source: own calculations based on the estimations in equation [22] and [23].

6. Discussion and conclusion

We have provided the first empirical application of the CDET profit function that Hanoch (1978a) has derived theoretically from a dual relationship between the CRES and CDE function. We argue in this paper that the CDET functional form is potentially better suited to capture sector specific substitution- and expansion effects than less flexible functional forms such as the CES or CET. On the other hand, a relatively small number of parameters needs to be estimated compared to the requirements of fully flexible functional forms, and this can be of advantage when the estimation of a large number of outputs is the goal. We found the regional agricultural accounts of Switzerland to be well suited for an empirical application, because the number of regions provides us with a substantial number of observations over a relatively limited time horizon, such that the corresponding system of equations could be estimated. Our estimates empirically confirm (overall) the theoretical properties of the CDE profit function.

On the other hand, we have so far been unable to apply the CDE profit function to other -similar- datasets of longer time periods, regardless the specification of the time trend, lagged variables, or first differences, etc. Furthermore, it should be mentioned that estimation of our system of 11 equations according to the 3SLS procedure has been more sensitive with respect to the choice of appropriate instruments than we would have expected from similar applications of alternative functional forms. We interpret these preliminary findings as evidence for the fact that the theoretical structure of the CDET profit function, namely a multiple output frontier with positive profits and optimal input- and output quantities, must be exactly reflected in the data (compare our findings with respect to the pig sector in Switzerland). Thus, pooling observations over a longer time horizon and across very heterogeneous regions may imply that not one and the same frontier is reflected in the data, but due to technical change and other varying regional conditions effectively several frontiers might be present, and this could be the reason for unsuccessful econometric applications of the CDET.

Furthermore, we have not tested whether our estimation approach could potentially be biased due to Jensen's inequality (compare Santos Silva and Tenreyro, 2006). Future research needs to clarify under which conditions this model in principle can be estimated successfully, and to what extent more recent econometric techniques, such as non-linear estimation of systems of equations may provide an alternative to our ‘conventional’ log-linearization. In addition, it would be valuable to undertake some further in-depth analysis of the CDE profit function using simulated data in connection with the approaches of Perroni and Rutherford (1998) and Barnett, Lee and Wolfe (1987) for other functional forms, in order to check the validation domain for which the CDET fulfills global regularity conditions.

In closing we emphasize that the empirical opportunities provided by the CDET profit function as a parsimonious agricultural sector model have not yet been exploited, and might only begin to emerge as consistent sectoral account data are increasingly becoming available, and econometric techniques improve.

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