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**Can hydro-economic river basis models  
simulate water shadow prices under asymmetric access?**

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## Abstract

Hydro-economic river basin models (HERBM) based on mathematical programming are conventionally formulated as explicit ‘aggregate optimization’ problems with a single, aggregate objective function. Often unintended, this format implicitly assumes that decisions on water allocation are made via central planning or functioning markets such as to maximize social welfare. In the absence of perfect water markets, however, individually optimal decisions by water users will differ from the social optimum. Classical aggregate HERBMs cannot simulate that situation and thus might be unable to describe existing institutions governing access to water and produce biased results for alternative ones. We propose a new solution format for HERBMs, based on Mixed Complementarity Programming (MCP), where modified shadow price relations express spatial externalities resulting from asymmetric access to water use. This new problem format, as opposed to commonly used linear (LP) or non-linear programming (NLP) approaches, enables the simultaneous simulation of numerous ‘independent optimization’ decisions by multiple water users while maintaining physical interdependences based on water use and flow in the river basin. We show that the alternative problem format allows formulating HERBMs that yield more realistic results when comparing different water management institutions.

## Keywords

Hydro-economic river basin model, mixed complementarity programming, water institutions, externalities

## 1 Introduction

There is growing awareness that availability of water and its efficient management will become one of the key questions in the 21<sup>st</sup> century (Chartres and Varma 2010). An often discussed issue in that context is that of an appropriate institutional design (Livingston 1995) to improve resource allocation resulting from unregulated use of water or inefficient assignments of water use rights. The economic assessment of water management institutions is often carried out on the basis simulation with hydro-economic river basin models (HERBMs) that are based on mathematical programming. This paper proposes a specific formulation of a mixed complementarity problem (MCP) which allows to depict the multiplicity of individual (i.e. economically independent) but still physically coupled optimization problems which are difficult or simply impossible to implement in a standard NLP or LP format based on optimizing a social welfare criterion. The rest of the paper is organized as follows. Section 2 discusses the current state-of-the-art in river basin modeling. The section 3 presents our problem setting – several firms positioned along a water distribution system competing for water use - and presents algebraic formulations and solution algorithms to evaluate a range of typical institutional designs related to water use proposed and analyzed in the literature. Subsequently, we develop a small numerical example and apply the proposed solutions to highlight differences in the results between the two principal optimization formats. We then expand the example to cover several hundred users to show the applicability in large-scale modeling exercises before we conclude and summarize.

## 2 State of the art and pitfalls in river basin modeling

Current hydro-economic river basin models (HERBMs) that are used to evaluate water management tools such as water pricing, water use rights, water trade, planned water allocation, and physical infrastructure offer hydrological and bio-physical relations in rich detail. Recent examples are integrated river basin models for the Maipo river in Chile (Rosegrant et al. 2000), the Jordan Valley (Doppler et al. 2002, water pricing), and the Drâa Valley in Southern Morocco (Heidecke and Kuhn 2008), where water pricing options under conjunctive use of water resources are evaluated. These models are typically formulated as optimization problems (LP, NLP)<sup>1</sup>, allocating water among users, uses, locations, and points in time such that an *aggregate* social welfare criterion is maximized (we will further call this format ‘*aggregate optimization*’ AO). The welfare measure is typically based on profits or utility of the different water users. This model formulation thus tacitly assumes that agents will allocate the water between them such as to maximize their aggregate welfare.

In reality, however, each agent optimizes her individual welfare independently while being influenced by other users’ decisions who withdraw water from the same common resource. Upstream users, to give an example, will continue to extract water as long as this is feasible, permitted, and increases their individual welfare. But their - perhaps excessive - water use may reduce aggregate welfare, because water availability for economically more efficient downstream users will be reduced. Achieving the social

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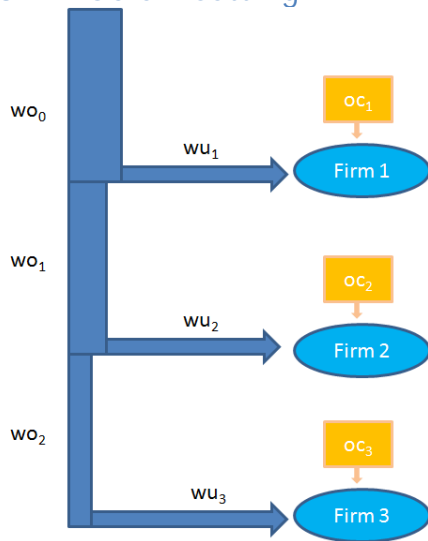
<sup>1</sup> Conradie and Hoag (2004) provide an overview on mathematical programming models used for the estimation of irrigation water prices.

optimum would require that agents either are forced to a more efficient water allocation, or agree to it, in the latter case asking for compensating losers out of the gains of winners. The costs to achieve the socially optimal solution against the background of the institutional setting simulated (such as unregulated water use, assignments of water rights etc.) will be underestimated when resource allocation in the model is driven by an aggregate welfare function. Basically, this problem format implicitly assumes that basic institutional deficiencies often found at the root of water allocation problems are already solved. Indeed, the only institutional settings where aggregate welfare maximization always leads to the same water allocation as with independent optimization is that of a perfectly functioning water market, i.e. one with zero transaction costs. A HERBM formulated as a social welfare optimization problem is hence not only unsuitable to correctly simulate the reference situation (such as unregulated water use), but also unable to spell out the true reaction of agents to alternative institutional settings and the related welfare gains or losses, both for individual agents and the aggregate.

The allocation of water in a river basin with ‘upstream-downstream problems’ is the outcome of a multiplicity of individual (i.e. economically independent) but still physically coupled optimization problems which are difficult or simply impossible to implement in a standard NLP format with aggregate optimization. Modeling the use of water and other resources in such a context of asymmetric access requires a differently structured optimization format. In that setting, coupled inter-spatial and/or inter-temporal physical balances have to hold, but not the complementary shadow price relations (i.e. economic scarcities) evolving from maximizing aggregate welfare. But the modeler cannot control independently both physical balances and complementary shadow prices when using an LP/NLP format. We will therefore in the following propose and apply a different solution based on a specific Mixed Complementarity Programming (MCP) format.

### 3 Algebraic formulations capturing institutional designs of water use

#### 3.1 Problem setting



Assume  $n$  water using firms positioned along a water distribution system such as a natural river or an irrigation system. Firms are indexed according to their positions, the first one being situated in the uppermost upstream position.

Each firm is characterized by the following attributes (see also figure 1): (1) its position in the water distribution system expressed by its index  $i$ , (2) its operating capacity  $oc_i$  and (3) its variable production costs per unit of output  $vc_i$  and finally, (4) the water available  $w0_i$  (see also graphic). The decision variable of the firm is its output quantity  $q_i$  which depends on its water use  $wu_i$ .

Furthermore, a Leontief technology is assumed; for notational simplicity one unit of output requires one

Figure 1: Illustration of the problem setting

unit of water and one unit of operating capacity. The latter is treated as a firm-specific fixed factor and could, e.g., capture the firm's endowment with capital, land and production rights or its management capacity.

### 3.2 Water allocation through 'aggregated optimization'

The AO formulation using the above setting allocates water quantities to the individual firms such that the *sum* of all firms' profits reaches its maximum. The solution of this problem requires a single central planner. However, we assume that firms do not exchange operating capacity, e.g. due to the fact that it cannot be transported. AO would hence solve the following problem::

$$\begin{aligned}
 \max_{q_i, wo_i} \quad & \pi = \sum_i q_i \bar{p} - q_i \bar{vc}_i \\
 s.t. \quad & q_i \leq \bar{oc}_i [\lambda_i] \\
 (1) \quad & q_i = wu_i \\
 & wo_{i-1} = wu_i + wo_i [\mu_i] \\
 & q_i \geq 0, wo_i \geq 0, wu_i \geq 0
 \end{aligned}$$

The related KKT conditions are:

$$\begin{aligned}
 & \bar{vc}_i + \lambda_i + \mu_i \geq \bar{p} \perp q_i \geq 0 \\
 & \bar{oc}_i \geq q_i \perp \lambda_i \geq 0 \\
 (2) \quad & wu_i \geq q_i \perp wu_i \geq 0 \\
 & wo_{i-1} = wo_i + wu_i \perp \mu_i \\
 & \mu_i \geq \mu_{i+1} \perp wo_i \geq 0
 \end{aligned}$$

The first line in (2) expresses that firms will only operate if their marginal production costs do not exceed marginal revenues per unit of output, the latter being equal to the price  $p$ . The marginal production costs consist of the variable costs  $vc$  and marginal resource use costs related to operating capacity  $oc$  and water use  $wu$ . The Lagrange multiplier  $\lambda$  captures marginal economic returns to operating capacity. It can be interpreted as the maximum price per unit of operating capacity that the firm would be willing to pay to expand its operating capacity. The third line is a water production function in its simplest form. The water resource constraint is defined by the last two equations which links the firms in the river basin by describing the physical flows of water from firm to firm and its use in the distribution system. It expresses that the sum of water use  $wu$  and outflow  $wo$  at location  $i$  of the firm (i.e. the section of the river where the firm withdraws water) must be equal to water inflow; the latter is the water not used by its upstream neighbor. Accordingly,  $\mu$  is the maximal willingness to pay of the firm for additional water. These physical balances must hold for any institutional setting. Consequently,  $wo_{i=0}$  denotes the (fixed) inflow into the system. The last line states that the firm will only use all the water at its disposal if its marginal returns to water use exceed that of its next downstream neighbor. That is the usual shadow price conditions for a social welfare optimum: marginal returns to a fixed resource must be equilibrated between competing users. The basic problem in water allocation, to find an institutional design which achieves socially optimal returns by the individual agents, is already part of

the solution format itself. Equal economic returns to water in setting (1) implies that upstream users are willing to reduce their water use and profits to make room for efficient users downstream, thus increasing basin-wide profits. But if we stick to the basic assumption of profit maximization by *individual* firms, why should upstream users behave that way if they are not compensated for their profits foregone? But such compensation is not comprised in (1). Consequently, AO should not be used to analyze such institutions if some users have asymmetric access to water. What we would need is an approach which reflects the individually optimal solution which we describe next.

### 3.2 Water allocation through ‘independent optimization’

We assume that firms maximize *independently* their profits at a given uniform output price  $p$ :

$$\begin{aligned}
 \max_{q_i, wo_i} \quad & \pi_i = q_i \bar{p} - q_i \bar{vc}_i \\
 \text{s.t.} \quad & q_i \leq \bar{oc}_i [\lambda_i] \\
 (3) \quad & q_i = wu_i \\
 & wo_{i-1} = wu_i + wo_i [\mu_i] \\
 & q_i \geq 0, wo_i \geq 0, wu_i \geq 0
 \end{aligned}$$

The difference to (1) above is that the summation sign of the profits is missing. Each firm is “socially myopic” – it takes the water inflow from its upstream neighbor as given (as it is not a decision variable, but exogenous), and, based on this, decides on its own profit-maximizing water use. The Karush-Kuhn-Tucker (KKT) conditions related to (3) are:

$$\begin{aligned}
 \bar{vc}_i + \lambda_i + \mu_i &\geq \bar{p} \perp q_i \geq 0 \\
 \bar{oc}_i &\geq q_i \perp \lambda_i \geq 0 \\
 (4) \quad wu_i &= q_i \perp wu_i \geq 0 \\
 wo_{i-1} &= wu_i + wo_i \perp \mu_i \\
 wo_i &\geq 0 \perp \mu_i \geq 0
 \end{aligned}$$

The only difference to the KKT in (2) from the AO problem is the last line. Each firm will continue to use the water at its disposal as long it increases its profit. An outflow to the next neighbor will only appear if any additional profits from water are exhausted; the water shadow prices must drop to *zero* then. What we need is to combine the decision logic underlying (4) with the hydrologic relations linking the users in the basin. Obviously, we cannot do that based on explicit optimization – we end up with AO. However, Mixed Complementary Programming (MCP, see Ferris & Munson 2000) offers a format to solve problems formulated in KKT-form. We can simply stack equations in (4) together, so that all  $wo$  (besides the inflow in the basis) are endogenous variables. The shadow price of each agent is allowed to drop to zero, independent from the shadow price of each neighbor, while the water flow from node to node is correctly accounted for. There exists no equivalent explicit optimization problem to that MCP formulation. The small change in the complementarity condition linking outflow and shadow price of water at each node from (2) to (4) now allows us to analyze different institutions

governing the basin without implicitly assuming that all agents voluntarily maximize aggregate welfare. We have constructed the core of a true agent-based model which can be solved simultaneously.

In the following, we aim at analyzing in an elegant way different institutions to manage the available water, i.e. which govern the water use of individual firms. With elegant we mean algebraic formulations and computable solutions which (a) are compact and transparent, (b) require only slight changes in the overall layout of the problem to model different institutions, and (c) avoid switching solution algorithms or sequential solution strategies. Specifically, we compare *two fundamentally different approaches*: (I) our newly proposed approach describing *independent optimization* of water allocation, i.e. rational, utility maximizing agents as usually assumed in standard micro-economics, (II) the classical *aggregate optimization* format where water allocation is simulated as it would be achieved by letting a fully informed social planner decide about each firm's production program (= socially optimal allocation). These two approaches are both expressed as MCP problems as explained above in (2) and (4).

In order to show the ability of our new approach to handle different institutions and support our claim that the aggregate programming format is not suitable to model these correctly, we apply both approaches to *four water management options* which themselves can be said to be institutional designs, namely (i) unregulated access to water, (ii) water pricing by a water agency with the policy objective of tax revenue generation, (iii) assignment of equitable water rights to improve basin-wide income distribution, and finally (iv) assignment of tradable water rights under consideration of transaction costs, i.e. the establishment of water markets to achieve an economically efficient allocation of water.

### 3.3 Capturing different institutions governing access to water

In the following paragraphs we will outline the water management institutions we want to compare. We start with water pricing and continue with non-tradable and tradable water use rights. The unregulated access case was already introduced above.

#### 3.3.1 Water pricing

It is commonly proposed to use water pricing to reduce inefficient or excessive use of water (Tsur 2005). Assuming a suitable control and enforcement strategy, firms will thus face a common water price. Under water pricing, firms will only use water as long as marginal economic returns per unit of water do not fall below the water price. The water pricing case can hence be described by the following KKT condition in (5), where the only difference to both (2) and (4) is that the exogenously set water price  $wp$  is added to the left-hand side of the first line:

$$(5) \quad \overline{vc}_i + \lambda_i + \mu_i + \overline{wp} \geq \overline{p} \perp q_i \geq 0$$

The change expressed in (5) guarantees that the firms' marginal returns to water use are at least as high as the price charged for water. The marginal returns might still exceed the water price once all available water is used.



### 3.3.2 Assignments of water rights

The second institution analyzed assigns non-tradable water rights  $wr$  to each firm. Each firm's maximal water use is now bounded both by either the property rights to use a certain amount of water, or by available water inflows. The marginal costs of the firm now additionally comprise the shadow price  $v$  related to the firm's water use right (first line in (6)) which is complementary to water use being constrained by the water right (last line in (6)):

$$\begin{aligned}
 (6) \quad & \overline{vc}_i + \lambda_i + \mu_i + v_i \geq \overline{p} \perp q_i \geq 0 \\
 & \overline{oc}_i \geq q_i \perp \lambda_i \geq 0 \\
 & wu_i \geq q_i \perp wu_i \geq 0 \\
 & wo_{i-1} = wo_i + wu_i \perp \mu_i \\
 & \mu_i \geq 0 \perp wo_i \geq 0 \text{ (IO)} \quad \text{or} \quad \mu_i \geq \mu_{i+1} \perp wo_i \geq 0 \text{ (AO)} \\
 & \overline{wr}_i \geq wu_i \perp v_i \geq 0
 \end{aligned}$$

Under decentralized water use (expressed through IO), even an equitable assignment of water rights does not necessarily abolish asymmetric access to the resource. Assume, for instance, that water rights refer to average water availability in the network and are not adjusted dynamically to the actual inflow: if total inflow is lower than the sum of water rights, upstream firms still benefit from their position.

### 3.3.3 Tradable water rights

We finally expand our framework by allowing the firms to trade their water rights. Now water use must not only be measured and over-use punished, as in the case of water pricing and assignments of water rights, but also a trading system for the water rights has to be introduced:

$$\begin{aligned}
 (7) \quad & \overline{vc}_i + \lambda_i + \mu_i + v_i \geq \overline{p} \perp q_i \geq 0 \\
 & \overline{oc}_i \geq q_i \perp \lambda_i \geq 0 \\
 & wu_i \geq q_i \perp wu_i \geq 0 \\
 & wo_{i-1} = wo_i + wu_i \perp \mu_i \\
 & \mu_i \geq 0 \perp wo_i \geq 0 \text{ (IO)} \quad \text{or} \quad \mu_i \geq \mu_{i+1} \perp wo_i \geq 0 \text{ (AO)} \\
 & \overline{wr}_i + \sum_j wt_{ji} \geq wu_i + \sum_j wt_{ij} \perp v_i \geq 0 \\
 & tc \geq \mu_j + v_j - \mu_i - v_i \perp wt_{ij} \geq 0
 \end{aligned}$$

The constraint related to water use and water rights from (6) is now expanded to allow for bi-lateral water right exchanges  $wt$  (see second last line). The total water rights of the firm shown on the left-hand-side consist of its original assignment plus any rights bought from other firms. They must exceed water use plus selling of water rights to other firms. Accordingly, the last line adds the typical arbitrage conditions for trade: price differences cannot exceed the per unit transaction costs; no trade will occur when price differences are too small. The total WTP for water use rights is composed of the water shadow price  $\mu$  plus the additional price  $v$  for the water use right.

Allowing the firms to trade their water rights will lead to mutual benefits both for firms with marginal economic returns to water above and below the median. Firms with low returns can sell part of their rights receiving additional revenues whereas firms with high returns will buy rights as long the related profit increase per unit of water exceeds the price of the water right. Water trade should theoretically eliminate the difference between unregulated and socially planned water use regarding economic efficiency, at least at zero transaction costs.

## 4 Two illustrative examples

We encoded the basic model described above in GAMS (General Algebraic Modeling System) and used the PATH solver to declare and solve the different MCPs. The declarative approach in GAMS to equations and models and built-in MCP functionalities support a smooth translation of the MCP problems.

### 4.1 A didactic example with three firms

For our first illustrative example, we only simulate three firms. We choose a total water inflow into the system of 120 units and an output price  $p$  of 100 per unit. Each firm has an operating capacity  $oc$  of 50 units, which means that total operating capacity in the basin exceeds the amount of water available. Variable costs  $vc$  for the three firms are 60, 50, and 40 per unit from upstream to downstream: the more efficient a firm is, the less privileged it is regarding its access to water. This guarantees substantial welfare losses under decentralized water allocation without further management measures. This setting allows highlighting differences between the two optimization formats in our didactic example.

We apply four management options (i) to (iv) to the two fundamental modeling formats, i.e. *independent optimization* IO and *aggregated optimization* AO, resulting in 8 experiments. The results are shown in table 1 below. The management options were designed as follows:

- i. *No management*: i.e. unregulated access: firms can use any water available at their node without costs.
- ii. *Water pricing* (see 3.3.1): a water price  $wp$  is charged by a water agency; the price level is the water shadow price of the pure ‘social planner’ solution (AO applied to i.) minus 1 (i.e.  $40 - 1 = 39$ ).
- iii. *Non-tradable water rights* (see 3.3.2): a water use right  $wr$  of 45 units is granted to each firm. This is more than the total available water, but less than the operating capacity of each firm.
- iv. *Tradable water rights* (see 3.3.3): the water use rights of 45 for each firm can be traded among the firms at transaction costs  $tc$  of 1 for each unit of  $wr$  traded. Firms can now buy water rights to fully exploit their operating capacity.

Under the parameterization chosen for these experiments, all results differ more or less strongly between IO and AO. The first two columns (‘*no management*’) compare the effect of IO versus AO on water allocation without specific management options, i.e. under unregulated access to water. The difference in basin-wide profits is large, as under IO the two most inefficient firms are located upstream and use water up to their operating capacity, leaving only 20 units to the most efficient firm 3 downstream. Under AO, as usually applied in existing HERBMs based on mathematical programming, water is first

allocated to the two most efficient firms (3 and 2). The distribution of profits among firms is thus dramatically different, leaving firm 1 with only 40 % of the profits under IO, while firms 3 can increase profits by 150 %. While the water shadow prices are 40 for all firms under AO, they differ markedly under IO, being zero for firm 1 and 2, and 60 for firm 3.

**Table 1: Results of the simulation experiments**

		No management		Water pricing		Non-tradable rights		Tradable rights	
		IO	AO	IO	AO	IO	AO	IO	AO
Water use <i>wu</i>	Firm 1	50	20	50	20	45	30	20	20
	Firm 2	50	50	50	50	45	45	50	50
	Firm 3	20	50	20	50	30	45	50	50
Outflows <i>wo</i>	Firm 1	70	100	70	100	75	90	100	100
	Firm 2	20	50	20	50	30	45	50	50
	Firm 3								
Water rights (when tradable, after trade) <i>wr</i>	Firm 1					45	45	20	35
	Firm 2					45	45	50	50
	Firm 3					45	45	65	50
Shadow prices $\mu$ for water use <i>wu</i>	Firm 1		40		1		40		40
	Firm 2		40		1		40		40
	Firm 3	60	40	21	1	60	40	41	40
Shadow prices $\nu$ for water use right <i>wr</i>	Firm 1					40		40	
	Firm 2					50	10	41	1
	Firm 3						20		1
Shadow prices $\lambda$ for operating capacity <i>oc</i>	Firm 1	40		1					
	Firm 2	50	10	11	10			9	9
	Firm 3		20		20			19	19
Profits of firms	Firm 1	2000	800	50	20	1800	1200	1800	1200
	Firm 2	2500	2500	550	550	2250	2250	2295	2295
	Firm 3	1200	3000	420	1050	1800	2700	2180	2795
Total profits		5700	6300	1020	1620	5850	6150	6275	6290
Water pricing revenues				4680	4680				
Total profits plus revenues		5700	6300	5700	6300	5850	6150	6275	6290

Under ‘*water pricing*’ at 39 per unit, the water use patterns found under ‘no management’ will not change: the water price does not exceed the variable costs of the least efficient firm (40 in the case of firm 1). At first glance, this result appears to contradict the common belief that water pricing helps improving the efficiency of water use compared to unregulated access. The later would only be the case if the water production function is strictly concave, or alternative uses of water within one firm enable substitution effects, as for instance a shift to more water-efficient cropping patterns by irrigators. Water

pricing in our restricted representation, if it were carried out by a public agency, only serves the purpose of raising tax revenues, as long as the price is low enough to let all firms use water. The reader should note that the AO problem format again produces a fundamentally different solution in our example.

Alternatively, the public agency may assign fixed (i.e. non-tradable) *water use rights* to improve access of downstream firms, and, depending on the ratio of use rights to average stream flow, to ensure a minimum basin outflow for, e.g., ecological reasons. In our example, water use rights of 45 are assigned to each firm, being below the operating capacity of 50 for each firm, but exceeding in sum the inflow in the river basis, which means that water rights will not be fully exploitable for all firms. The results show that granting water rights exceeding total water availability is not suitable to totally eliminate allocative inefficiencies: firms 1 and 2 can still exploit their locational advantage, leaving only 30 units of water to firm 3, as simulated by IO. But firm 3 is better off than under a situation without water rights as it expands its water use from 20 to 30 units, being able to profit from the reduced water use of upstream firms which are now constrained by the assigned water use right.

The AO solution again differs, as the inherent social planner ensures that only the two most efficient firms fully exploit their water rights, leaving firm 1 at a water use of 30 units, which is hardly plausible if the real world is characterized by independent firms. As can be expected, the *tradability of water rights* eliminates allocative inefficiencies. This would not be the case if the transaction costs of water trade (1 per unit traded in the example above) would exceed the smallest difference in variable costs between two firms participating in trade (10 in our case). However, as buying firms have to carry the costs of trade, tradable water rights are not able to fully achieve the maximal profit possible in the basin (as simulated by maximizing social welfare, i.e. AO under “no management”). Perhaps surprisingly at first glance, the distribution of profits between IO and AO differs markedly despite very small differences in aggregate profits and the same allocation of water to the firms. The reason is that trade in water rights has to be higher under IO in order to prevent firm 1 from still exploiting its locational advantage. Under AO, firm 1 is prevented by the inherent social planner to fully use its remaining water rights (35 units) after having sold sufficient rights to firm 2 and 3 (5 units each, so that the latter can fully use their operating capacity). Under IO, firm 3 must buy additional 15 units of water use rights from firm 1 to prevent the latter from exercising its remaining water rights. This strongly diminishes the profit of firm 3, while firm 1 enjoys profits from producing 20 units, and receives revenues from selling 25 units of water rights to its downstream neighbors.

## 4.2 An example with 1000 firms

We use the very same settings as above, with the sole exemption that the variable costs for each firm are randomly drawn from a uniform distribution in the range of [20,80]. Accordingly, upstream and downstream firms are equally efficient on average; the only difference is their position in the water network. Differences between IO and AO are hence not due to an additional assumption that downstream users are more efficient, they are solely based on asymmetric access and its inherent decision logic. At the same time, the application shows that the MCP format can indeed be applied to a large number of firms. To introduce a further realistic feature and keep the problem at a manageable size, we assume the firms do only trade water use rights with their nearest 20 neighbors.

The graph below shows the outflow at each node under the no-management for IO and AO, the axis shows positions from upstream to downstream, underlining how water availability in the basin drops. It can be clearly seen that AO again “saves” upstream water by taking it away from the inefficient firms in the upper part of the basin and assigns it to downstream firms. We just remind the reader that there is no reason to assume that such a water allocation – preferable from societal viewpoint as the same output is produced at lower costs – will come about in a non-managed basin. The IO solution however shows what we would expect without a water management institution: upstream users take advantage of their location and use the water at their disposal, not caring about their downstream neighbors. As the operation capacity of the industry exceeds the inflow by 20%, the users at the end of the basin have no water at all at their disposal, independently of how efficient they are.

A societal planner can increase the industry profits in the basin by about 10% (from 2.12 Mio Units under IO to 2.33 Mio Units under AO). That difference is the maximal welfare gain which would be achieved if costless water trade could be introduced, which is clearly impossible. To assess different institutions, their costs would need to be quantified as well. In the case of water pricing, we would need to introduce a chosen water price in the model, simulate with IO the industry profit plus water tax revenues and deduct the costs for installing, maintaining and controlling water meters and enforcing payments of water taxes.

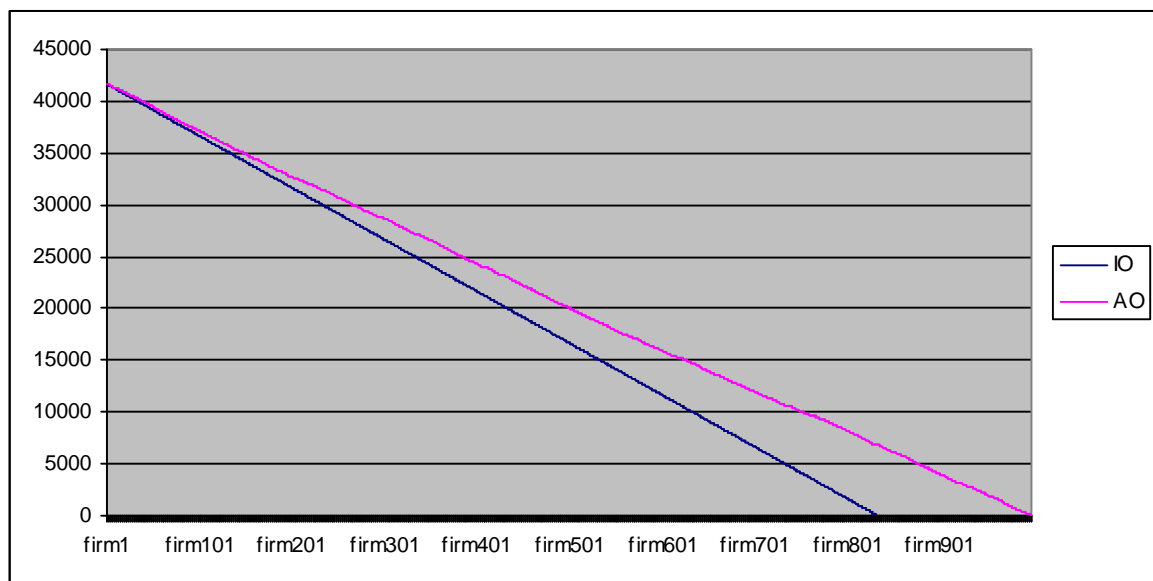


Figure 2: Outflow at each node in the basin under no management, 1000 water users

## 6 Summary and conclusion

Our paper contributes twofold to the analysis of resource institutions. First, it demonstrates how a specific problem formulation based on Mixed Complementary Programming (MCP) can be used to simulate different institutions (unregulated access, water rights, water right trading, water pricing) for a theoretically sound representation of a number of individual, decentralized decision makers and their interactions. The main difference to existing applications in the literature is that we are now able to correctly

model simultaneously non-coordinated decisions of individual water users while reflecting the position of each water user in a water distribution system along with further interdependences between users emerging e.g. from input and output markets. The usual aggregate programming format used in river basin modeling which maximizes social welfare fails to properly express the consequences of isolated utility maximization and its resulting externalities, and is hence not well suited to evaluate different institutional settings governing access to water.

Using the MCP problem format allows the modeler to take full control of both physical and co-state variables and to model real-world problems in resource use which can in some cases not even be expressed as a single optimization problem. Moreover, our approach opens new perspectives for the design of agent-based models that are algebraically simple, theoretically sound, and easier to solve as iterative procedures to reflect spatial and market interactions are no longer required.

As demonstrated in our example, our framework allows analyzing the interaction between asymmetric access and different institutions such as resource pricing. We conclude that MCP is indeed a concise, elegant and highly flexible format to model these institutions. As physical water flow balances are integrated in the system, the MCP framework can be easily linked to complex hydrological and bio-economic tools. Our presentation of a system of independent, utility-maximizing decision units linked via resource and other interdependences in MCP can thus be used as a nucleus for a new class of agent-based models and opens new opportunities, for instance, to simulate the dynamic emergence of institutions between independent spatial agents in a concise and theory-consistent manner.

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