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OF PUBLIC INTERMEDIATE GOODS

by

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ANRE Working Paper WP:85-2



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INTRODUCTION

The general theory of second best is powerful for providing insights into a variety of public policy problems. Its basic provision is that an economy constrained away from any condition necessary to achieving its Pareto optimum will also be required to violate the other Pareto conditions if a second best optimum is to be achieved.¹ This powerful and elegant result shows the futility of piecemeal policy making aimed at preserving as many as possible of the optimality conditions when policy prescriptions or actual conditions require violating one or more of them. The theory has applications in almost all of economic planning and policy making, even in economies nearly free from price-making firms or government regulation.

However, applications of the theory of second best have primarily focused on regulation of existing industries, particularly natural monopolies, and creation of taxation schema for existing political-economic systems. Previous research has focused mainly on creation of optimal pricing mechanisms for the regulation of public enterprises and monopolies. Feldstein (1972a, 1972b) considered the case of publicly produced factors being sold downstream both for final consumption and as factor inputs, taking account of equity and distribution issues. He treated the case where downstream firms are competitive. Spencer and Brander (1983) extended the analysis by considering the case

¹Lipsey and Lancaster, (1956).

where the factor is sold downstream both to price-making firms and for final consumption. But the question of optimal pricing rules as treated by these authors and others presumes the existence of a given industrial structure, including the investment in place. In those applications, the primary departure from marginal cost pricing is induced by the rate of return constraint that ordinarily appears in applications of the theory of second best, especially the "Ramsey pricing" variant as applied to public utility regulation. The violation of Pareto conditions follows from the existence of natural monopoly and the decision to have the regulated industry be self-sustaining through requiring that revenue be at least equal to some proportion of cost.

The focus of our research is the decision that precedes price regulation and taxation schema, namely, how much investment to put into a particular public enterprise. In the case we examine the departure from marginal cost pricing is brought about by the decision to produce a factor input by a public enterprise. Others have also considered this topic, the case of public education being a case in point, but not from the point of view of the general theory of second best.² Optimal second best prices and taxation schema will not necessarily remain optimal if the level of investment in productive capacity is changed, especially for lump-sum investments made by non-competitive suppliers. We find that, unlike some previous results focusing

²See Atkinson and Stiglitz, (1980).

on revenue adequacy constraints, optimal investment decisions in a second-best world may dictate pricing an input factor below marginal cost. Indeed, this is one of the primary insights provided by applying the theory of second best to the question of optimal public investment in factor production.

Of course, both Pareto and second best solutions are always conditional in the sense that they depend upon the flexibility of the other economic variables in the decision problem. For example, the oft-cited applications of Ramsey pricing depend on the flexibility of capital investment to attain optimal investment levels and efficient production systems. In order to be truly optimal a second best pricing solution depends on the flexibility of capital expenditure. In this paper we develop conditions for optimal departures from the competitive solution to investment in production of factor inputs as induced by public investment in factor production.

But investigation of optimal levels of investment in public production has another contribution to make. It has recently been recognized that economics offers only weak explanations of capital formation. Peter F. Drucker has noted that this is one of the most pressing needs in economics, deserving primary attention³. There does not appear to be an adequate theory explaining the relationship between private and public investment. Neither does this paper provide one. But it does begin (albeit modestly) to provide some theory of optimal public

³Peter F. Drucker, (1980).

investment in supplying certain factors of production. Thus, it makes a start with one component of a theory of investment in a real world.

THE PROBLEM SETTING

The issue upon which we focus is: What is the optimal investment for production of a publicly-supplied intermediate good? An important corollary issue is also investigated: What are the optimum pricing policies associated with the production of the factor at the optimal investment level? The specific application that has led to the investigation concerns finding the optimal investment into irrigation systems by public agencies. Given a stock of capital available to the public agency for investment and the rates of return to various alternative investments, how much irrigation system should be developed? Similar questions could be asked with respect to investment in other intermediate factors of production, including public education, supply of energy sources, transportation facilities and equipment, research and development, energy exploration and development, and land reclamation.

Entry barriers of various kinds may prevent an economy from achieving a Pareto optimum in factor production capacity. Entry barrier constraints ordinarily derive from practical circumstances, so that removal of them would be more costly than dealing with them. A combination of physical conditions and technological constraints may prevent entry of perfectly competitive

firms. An irrigation system offers a case in point. Indeed, the physical system almost demands that there is a single producer supplying irrigation to a given sector. For many combinations of terrain, water sources, land ownership, and land use patterns, it would be unthinkable to have more than one water delivery system. Further, private firms usually lack powers of eminent domain, thus preventing them from obtaining rights-of-way required for investment. In other cases, government restrictions prevent entry. Few governments allow more than one electric power company to serve a given geographical area. Some investment projects require a minimum threshold level of investment, the sheer size of which prevents private firms, or even cooperatives, from undertaking them. These entry barriers may prevent an economy from achieving the Pareto optimum level of investment.

Other conditions besides entry barriers may prevent achievement of Pareto optima. One of these is a cost structure that induces natural monopoly. Another is the existence and provision of public goods. An irrigation system will not ordinarily be a natural monopoly in the strict sense that marginal cost is falling over the relevant range of output. Nor is it necessarily non-exclusive as required for a public good. But it is usually supplied most efficiently by a single producer. This fact alone will lead to market conditions that deny the Pareto optimal solution provided by competitive markets supplying the factors of production to water users. Irrigation systems are not unique in this respect. There are several factors of production that must

be supplied under non-competitive conditions, especially in developing nations.

Considerations besides physical efficiency may require that irrigation systems and other publicly-produced intermediate factors be produced in second-best conditions. Grants to developing nations are often made with specific designations for their uses. In such cases there will be lump-sum investments into various public production projects. Such projects will enter or create markets that are not competitive in the sense required for optimal solution, but which nonetheless offer essential services in the economy. Some desirable projects, such as irrigation systems, public communications and transportation, supply of energy commodities, and education, may be too large for single firms, cooperatives, or small governmental units to finance. Or, as in the case of education, the direct or immediate cash flow may be too low to induce private-sector suppliers to enter the market. It may be necessary and desirable, as well as efficient, to produce them through public agencies. Public investment decisions can benefit from application of the theory of second best in setting investment levels first and thereafter setting pricing policies consistent with the general welfare. Each of these conditions may require departure from the conditions that produce Pareto optimal solutions to actual problems.

However, the Pareto optimal conditions do provide a baseline against which to measure second best solutions. An optimum

position would be attained in the economy if all goods were produced by competitive firms and sold to competitive firms or consumers. But there are certain factors and commodities that are not produced by competitive firms or sold to competitive buyers. These departures from the optimal market solutions occur for a variety of sound practical reasons. The general theory of second best provides insights into the consequences of supplying resource factors through public investment.

The problem treated here focuses upon publicly-produced intermediate goods. These are not public goods in the sense of being non-exclusive. They are factors of production produced by public investment. It is not necessary that they be natural monopolies, although they may be. Often they will be monopolies in the sense that efficient allocation of resources will dictate only one investment project, even though marginal costs may be either falling or rising.

Increasing General Welfare

Investment in factors of production will ordinarily increase the supply of the final output. Such an increase changes both consumers' and producers' surpluses. The optimum investment in publicly-produced intermediate goods is the investment that maximizes the sum of changes in producers' and consumers' surpluses. It is recognized that any increase in supply of the final output will increase consumers' surplus if the demand schedule is normal. But increasing supply of a factor of

production may increase or decrease the producers' surplus, depending upon the elasticity of demand for the final output.⁴ Thus, the optimand is the change in the sum of consumers' and producers' surpluses. The maximum of this change will indicate the optimal level of investment in publicly-produced intermediate goods.

The optimal level of investment in public intermediate goods is constrained by two factors. First, it is required that the final output market not be forced into disequilibrium by the public investment. Second, the investment in the intermediate good shall not yield a rate of return less than its opportunity cost. Taken together, these two constraints limit the amount of investment in public production of intermediate goods.

The equilibrium constraint requires that the output market clears after adjustment to the newly available input factor. To provide an input factor that forces the output market into permanent disequilibrium will reduce the efficiency of resource use. In fact, it may be a solution less desirable than continuing without the input factor at all. How could this occur? Suppose that the introduction of the factor changes the shape of the supply surface, including relative elasticities in such a way that the output market is destabilized. Then the search for equilibrium will put the market into a corner solution. It may

⁴David H. Richardson has provided an example of a public investment project where the producers' surplus is negative. See Richardson, (1984), for an ex post empirical analysis of a specific instance of the general problem we are addressing here.

change economies of scale, scope, or density. There are a variety of realistic effects upon the supply side of a market that could prevent the market from clearing and re-establishing equilibrium after the introduction of the factor. Thus, one of the constraints imposed is that the final output market clears upon adding or increasing the publicly produced factor input.

The constraint on the rate of return actually performs two tasks. It ensures that the rate of return on public investment will not fall below its opportunity cost. This is required for achievement of an efficient resource allocation. But it also ensures that the so-called revenue adequacy constraint is met as long as the opportunity cost is positive. Therefore, the solution to the problem should provide a second-best solution that also assures investment quantities and prices for the publicly-produced factor at levels that cover costs and normal profits.

MATHEMATICAL FORMULATION

The optimand in this problem is the change in the sum of consumers' and producers' surpluses that follows a change in the investment in public production of the intermediate good. This change is named $D(q,p,w)$, denoting that the change is a function of the quantity of final output, q ; the price of final output, p ; and the supply of the input factor, which we shall name water, w . The demand function for the final product is written as $h(q)$ and the supply function is $g(q,w)$. We will write the derived

demand for water as $P_w(q, w)$, the cost of water as $C(w)$, and the capital investment expenditure function for producing water as $f(w)$. The cost of water measured as $C(w)$ does not include the opportunity cost of capital investment. The opportunity cost component is the product of the investment expenditure, $f(w)$, and the opportunity cost expressed as a rate of return, which we label 'b'. Thus, complete cost is $C(w) + bf(w)$.

Using y_1 and y_2 as Lagrange multipliers, the problem is

(1) Maximize $D(q, p, w)$

subject to (i) $[h(q) - g(q, w)] = 0^5$

(ii) $[P_w(q, w)w - C(w) - b f(w)] \geq 0$.

The market clearing requirement is in constraint (i), while constraint (ii) incorporates the rate of return constraint. The rate of return constraint expresses the requirement that total revenue shall not be less than total cost measured as the sum of operating costs and the opportunity cost of invested capital. Thus, the problem is stated as: choose w so as to

(2) maximize $D(q, p, w) + y_1[h(q) - g(q, w)] - y_2[P_w(q, w)w - C(w) - bf(w)]$.

⁵We have written the market clearing constraint subject to the final adjustment of the output market to the introduction of the public production, or increased production, of the factor. It could also be written as an inequality constraint, thus recognizing that there may be an iterative adjustment to introduction of the factor input.

Since the optimization problem contains an inequality constraint we use the Kuhn-Tucker theorem to obtain the conditions for the second-best optimum. The conditions required to maximize (2) are as follows⁶:

$$(i) \quad P_W - MC_W \geq Y_2^{-1}(\partial D/\partial w) + (Y_1/Y_2)[E^{-1} - K^{-1}](p/q)(\partial q/\partial w) \\ - Y_1(\partial g/\partial w) - w[\frac{\partial P_W}{\partial w} - \frac{\partial P_W}{\partial Q} \frac{\partial Q}{\partial w}] + b(\partial f/\partial w),$$

where E and K are own price and supply elasticities for final output;

$$(ii) \quad w^* \geq 0 \quad (\text{superscript } * \text{ indicates value at optimum}),$$

$$(iii) \quad y^* \geq 0$$

$$(iv) \quad h(q) - g(q, w) = 0$$

$$(v) \quad P_W(q, w)w - C(w) - bf(w) \geq 0$$

$$(vi) \quad y^*[P_W(q, w^*)w^* - C(w^*) - bf(w^*)] = 0$$

$$(vii) \quad w(P_W - MC_W) = w\{Y_1/Y_2)(\partial D/\partial w) + (Y_1/Y_2)[E^{-1} - K^{-1}](p/q)/ \\ (\partial g/\partial w) - w[\frac{\partial P_W}{\partial w} - \frac{\partial P_W}{\partial Q} \frac{\partial Q}{\partial w}] + b(\partial f/\partial w)\}.$$

These conditions for optimum second-best solutions reveal several important aspects of the problem. Condition (i) indicates that marginal cost pricing is not necessarily optimal. In fact, the optimum may require that the factor be priced below its

⁶See, e.g., Intriligator, (1971), chapter 4.

marginal cost. The first RHS term is the change in the sum of producers' and consumers' surpluses associated with a change in supply of the input factor multiplied by the inverse of the Lagrangian multiplier. It must be non-negative. The Lagrangian multiplier may be interpreted here as the change in the objective function due to a change in the constraint value of the opportunity cost of public investment, b . Then the first RHS term is the ratio

$$\frac{\text{rate of change of objective function wrt } w.}{\text{rate of change of objective function wrt } b}$$

The second RHS term is always negative so long as the final output commodity is normal for both producers and consumers. All components of this term are positive except the own-price elasticity. That being negative makes the term in square brackets negative so that the entire term is negative.

The third RHS term is negative due to the sign at the front since both components are positive. This term measures the rate of change in production of final output with respect to availability of the factor input. It is the change in marginal cost of the final output with respect to factor input.

The fourth term is positive if the input factor is normal, since the term is the product of the quantity and own-price elasticity for the factor preceded by a negative sign.

Finally, the last RHS term is positive since the cost of the project is assumed to increase with the magnitude of the optimal level of factor supply. It is a measure of the change in

required return to investment with respect to a change in the quantity of factor input supplied.

The second RHS term of condition (i) calls to mind the so-called Ramsey Rule⁷ since the price - marginal cost relationship is inversely proportional to the own-price and supply elasticities. Here price and marginal cost depend on the sum of the inverses of the own-price and supply elasticities, as well as on the slope of the derived factor demand schedule. These are weighted by the multipliers and augmented additively by terms for changes in the objective and cost functions.

In summary, optimal investment in publicly produced factors can lead to pricing those factors below their marginal costs. Later in this paper we give an instance where pricing irrigation water below marginal cost is required for attainment of an optimal solution. This result contrasts with some of the existing literature based only on the rate of return constraint, where pricing above marginal cost is required in certain cases.⁸ Of course, the original work by Lipsey and Lancaster (1956) recognized the possibility that pricing below marginal cost may be necessary, but did not give actual instances. Spencer and Brander (1983) show that below-marginal-cost pricing may be required if a publicly produced factor is sold both downstream to price-making firms and for final output.

⁷See Ramsey (1927) or Baumol and Bradford (1970).

⁸See Baumol and Bradford, (1970).

Optimal Investment

While conditions (i) - (vii) are required for existence of a solution to the optimization problem, they may not actually specify the optimal level of investment or the prices that follow upon its achievement. They will provide an exact solution if all of the constraints are actually in force, so that the inequalities in the conditions become equalities. Otherwise, a gradient solution method may be invoked in order to actually specify the optimal level of investment and associated prices. A conceptual algorithm for solution providing the optimal level is shown both in the following diagrammatic analysis and in the actual problem presented in this paper. What the conditions do show clearly is that marginal cost pricing is not necessarily optimal. Prices may be required to be either above or below marginal cost in order to reach a second-best solution.

DIAGRAMS OF THE PROBLEM AND SOLUTION

Several aspects of the problem are illustrated by study of the graphs in Figures 1, 2, and 3. The figures present the comparative static analysis for an increase in water supply from w_0 to w_1 . Figure 1 shows the market for irrigation water, Figure 2 shows the final output market, and Figure 3 shows an irrigation water market under decreasing costs. The analysis begins with water supply at w_0 and derived demand for water at D_{w0} as in Figure 1. The price of water is at p_{w0} . These conditions agree with the price and quantity of final output determined at p_0 and

Q_0 in Figure 2. Reference to Figure 1 allows comparison of the price of water, p_{w0} , with the marginal cost of water, MC_w . In the beginning period p_{w0} exceeds MC_{w0} .

The boundary conditions associated with the rate of return constraint are shown in Figures 1 and 2. Figure 1 shows the maximum rightward shift in the supply of irrigation water that is consistent with the constraint. This maximum occurs at the intersection of the long-run average cost curve, with the opportunity cost of investment included in the average cost, and the derived demand schedule for irrigation water. It is labeled BB. If the factor is produced in an increasing cost system, then marginal cost of the factor will exceed its price at this boundary. The boundary BB implies a maximum increase in the supply curve of agricultural output, ceteris paribus. This boundary is labeled as line bb in Figure 2.

Now let the irrigation water supply be increased to w_1 . This increase maximizes the change in consumers' and producers' surpluses as it shifts the supply of agricultural commodities to S_1S_1 (shown in Figure 2)⁹. The derived factor demand curve moves to D_{w1} , thus establishing water price at p_{w1} . The corresponding marginal cost is indicated at MC_{w1} in Figure 1. Comparing p_{w1}

⁹The shifted curves in Figures 1, 2, and 3 are drawn presuming that intermediate adjustments toward equilibrium have already occurred. For example, the initial shift in the output supply curve is likely to be followed by further shifts toward the old position in response to the change in the price of the factor. These movements are likely to proceed iteratively until the new equilibrium is reached. That it is reached is one of the constraints.

with MC_{w1} , we see that price of irrigation water is now below marginal cost, whereas p_{w0} was above MC_{w0} in the earlier period.

Figure 3 is a counterpart to Figure 1, showing the cost curves in the market if the factor is subject to decreasing costs. These two figures allow comparison of the pricing results in this paper with the previous results. If the factor is produced in a decreasing cost environment, the result will lead to setting prices above marginal costs, as shown in Figure 3.¹⁰ This is simply a result of marginal cost falling below average cost in the decreasing cost industry. But, in an increasing cost industry, the price may be above, equal to, or below marginal cost. The price - marginal cost relationship depends on the optimal level of investment.

¹⁰The market shown in Figure 3 is thought to be stable, in the sense that the demand curve intersects the cost curves from below. The market would apparently be unstable if derived demand intersected cost curves from above.

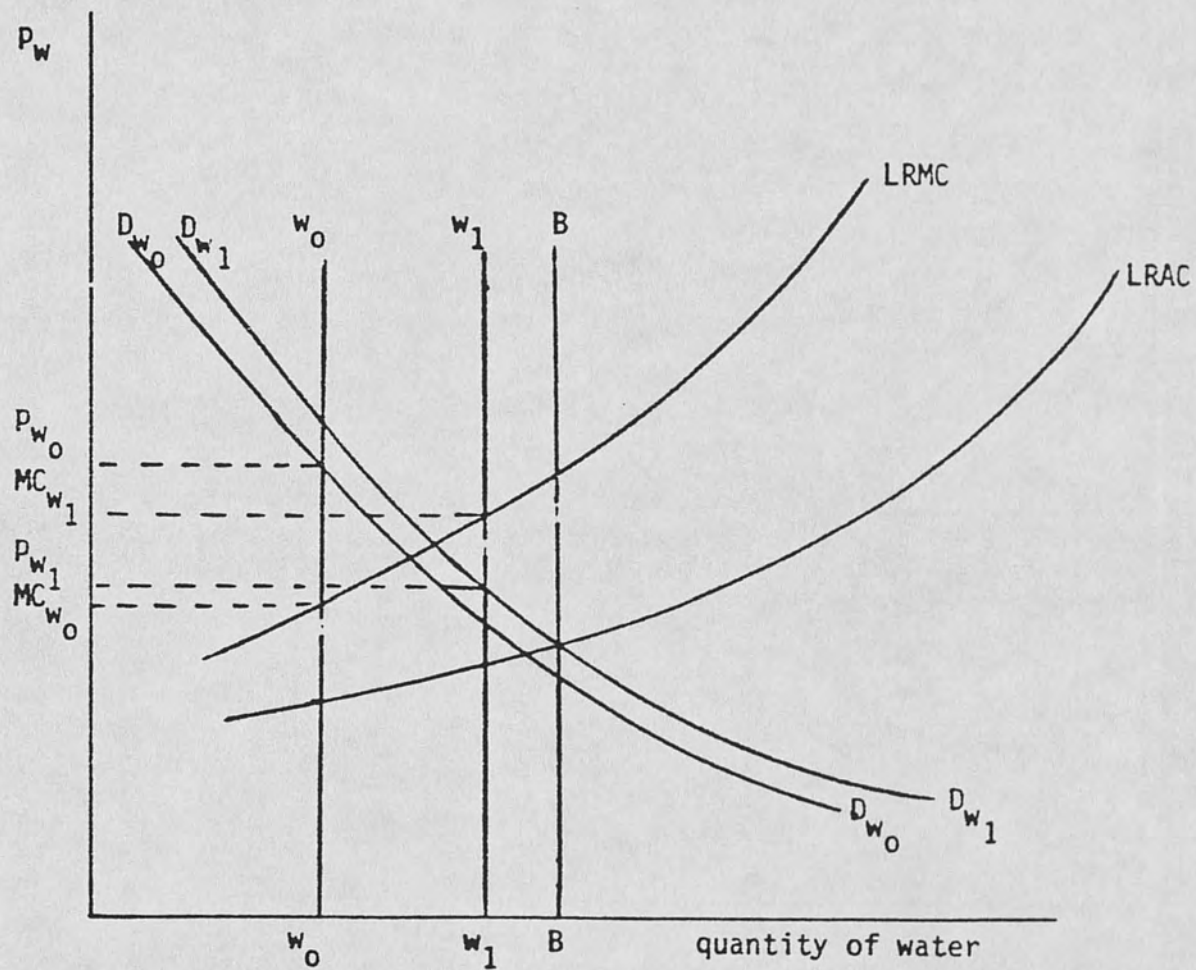


Figure 1

Irrigation Water Market

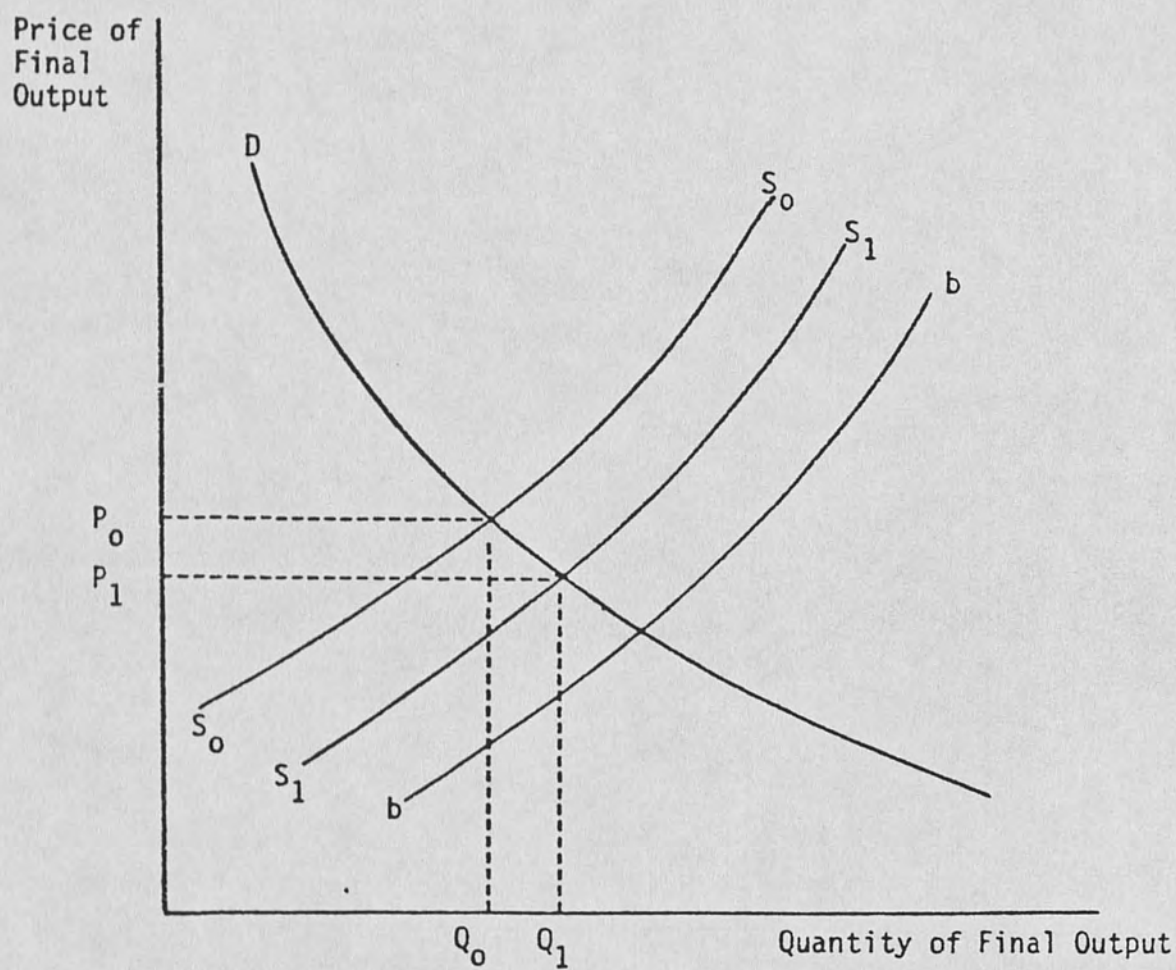


Figure 2

Final Output Market

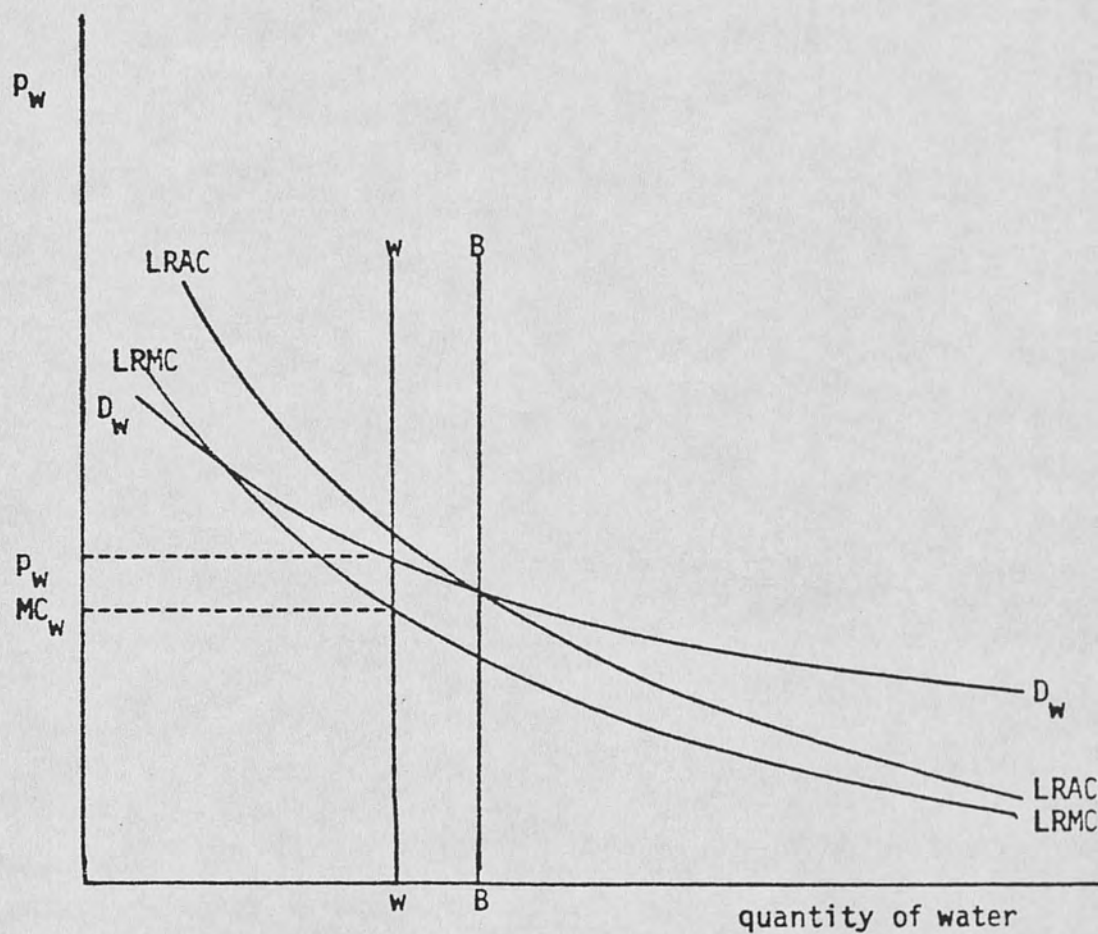


Figure 3
Irrigation Water Market with
Decreasing Costs

A SPECIFIC MODEL

The aim of our research is to provide both a general theory and models for specific application in the optimal public investment problem. In this section we present a class of models for determining optimal investment and pricing for an irrigation system. The models provide solutions that apply at the boundary conditions, thus indicating specific solutions for aggregate agricultural output markets characterized by Cobb-Douglas production and demand systems. The class of models is derived through implicit solution to the problem and therefore maps the orderly heuristic solution shown in the preceding graphical exposition. These solutions shown are not restricted to water systems, but apply to any factor input characterized by Cobb-Douglas models.

Assume that every government irrigation investment brings an increase in the availability of irrigation water leading either to increases in the quantity of water available to each farm or increases in the level of irrigated area, or at least to improved dependability of the irrigation systems. Whatever the nature of the investment, its ultimate impact upon irrigation development results in rightward shifts in the supply curves of agricultural products.

Since increased investment in irrigation will lead to development of costlier and more difficult projects, the relation between the level of irrigation investment and the quantity of irrigation water supplied will be subject to diminishing returns to scale. That is, the marginal cost of irrigation water will be

an increasing function of the quantity of water supplied. Let W be the quantity of irrigation water, I the investment expenditure for irrigation, and δ a constant. Then the investment expenditure function is

$$(3) \quad W = I^{\delta} \quad 0 < \delta < 1$$

Thus as the level of investment increases, the quantity of irrigation water supplied also increases, but at a decreasing rate.

Setting the opportunity cost of public investment equal to ' r ' and imposing the constraint that at the margin the rate of return to investment in irrigation equals ' r ' gives the total cost of irrigation development as

$$(4) \quad TC_w = rI = r W^{\frac{1}{\delta}} = r W^{\gamma},$$

by setting $\gamma = 1/\delta$ and using (3).

Then the marginal cost curve of water is

$$(5) \quad MC_w = \frac{\partial TC_w}{\partial w} = \gamma r w^{\gamma-1} > 0$$

and the MC_w curve is an increasing function of w , which is evident from

$$(6) \quad \frac{\partial^2 TC_w}{\partial w^2} = (\gamma-1) \gamma r w^{\gamma-2} > 0,$$

since $\gamma > 1$.

To complete the picture, we derive the social welfare function and the derived input demand function for irrigation water. Since irrigation water is strictly an intermediate product, the derived demand comes from the producers' side only.

Let us represent the equilibrium quantity of demand and supply for agricultural products by

$$(7) \quad Q_0 = D_0 = a p_0^\alpha \text{ with } \alpha < 0$$

$$(8) \quad Q_0 = S_0 = b p_0^\beta \text{ with } \beta > 0$$

Since in equilibrium $D_0 = S_0$, we have

$$(9) \quad p = \left(\frac{b}{a}\right)^{\frac{1}{\alpha-\beta}}$$

$$(10) \quad a = \frac{Q_0}{p_0^\alpha}$$

and

$$(11) \quad b = \frac{Q_0}{p_0^\beta}$$

The relation between the supply function (the mc curve of the agricultural products) and the level of irrigation investment is captured by the supply shift factor k (>1). That is, with governments' newer investment

$$(12) \quad S = k b p^\beta$$

(13) Where $k = f(w) = w^\lambda$ with $0 < \lambda <$

Thus

$$(14) \quad \frac{\partial k}{\partial w} = \lambda w^{\lambda-1} > 0$$

$$(15) \quad \frac{\partial^2 k}{\partial w^2} = (\lambda-1)\lambda w^{\lambda-2} \leq 0$$

Thus, every increase in the level of investment will shift the supply curve to the right, though the degree of the shift will decline as the level of investment goes up. Further, as the quantity of irrigation water supplied increases, the marginal cost of irrigation supply increases. With the demand function remaining the same, the new equilibrium quantity at the new equilibrium price (P_1) will be

$$(16) \quad D_1 = a P_1^\alpha,$$

with

$$(17) \quad S_1 = k b P_1^\beta = a P_1^\alpha,$$

and

$$(18) \quad P_1 = \left(k \frac{b}{a}\right)^{\frac{1}{\alpha-\beta}}$$

$$(19) \quad P_1 = \left[k \frac{Q_0}{P_0^\beta} \frac{P_0}{Q_0} \right]^{\frac{1}{\alpha-\beta}} = k^{\frac{1}{\alpha-\beta}} P_0$$

Since we assumed the social welfare (TS) as the sum of consumers' (CS) and producers' (PS) surplus,

$$(20) \quad \Delta TS = \Delta CS + \Delta PS$$

This can be derived, as in [Sampath (1983)], as

$$(21) \quad \Delta CS = P_0 Q_0 \frac{1}{1+\alpha} \left[1 - k^{\frac{1+\alpha}{\alpha-\beta}} \right]$$

$$(22) \quad \Delta PS = P_0 Q_0 \frac{1}{1+\beta} \left[k^{\frac{1+\alpha}{\alpha-\beta}} - 1 \right]$$

$$(23) \quad \Delta TS = P_0 Q_0 \left[\frac{1}{1+\alpha} - \frac{1}{1+\beta} \right] \left[1 - k^{\frac{1+\alpha}{\alpha-\beta}} \right]$$

Now the governments' objective is to maximize the total social welfare subject to the marginal return from irrigation investment being equal to the opportunity cost, 'r.' That is, maximize

$$(24) \quad Z = \Delta TS = P_0 Q_0 \left[\frac{1}{1+\alpha} - \frac{1}{1+\beta} \right] \left[1 - k^{\frac{1+\alpha}{\alpha-\beta}} \right]$$

$$= P_0 Q_0 \left[\frac{1}{1+\alpha} - \frac{1}{1+\beta} \right] \left[1 - w^{\lambda \frac{1+\alpha}{\alpha-\beta}} \right]$$

Substituting (13) for k above, maximizing (24) gives

$$(25) \quad \frac{\partial Z}{\partial W} = - \lambda \frac{1+\alpha}{\alpha-\beta} P_0 Q_0 \left[\frac{1}{1+\alpha} - \frac{1}{1+\beta} \right] W^{\lambda \frac{1+\alpha}{\alpha-\beta} - 1} \\ = MCW = \gamma r W^{\gamma-1}$$

Substituting (25) for optimum supply of 'w' gives

$$(26) \quad W_s^* = \left[\frac{\lambda P_0 Q_0}{\gamma r} \frac{1}{1+\beta} \right]^{\frac{\alpha-\beta}{\gamma(\alpha-\beta) - \gamma(1+\alpha)}}$$

W^* is the social welfare maximizing irrigation supply. Now let us derive the derived input demand for water. Though irrigation water benefits both consumers and producers, it is directly purchased and consumed only by the producers. The objective of the producers is to maximize their surplus or net income. Thus, if the price of water is P_w , then the producers will set the marginal productivity of water equal to P_w . That is

$$(27) \quad \frac{\partial PS}{\partial W} = \frac{\partial \left[P_0 Q_0 \frac{1}{1+\beta} \right]}{\partial W} \left[W^{\lambda \frac{1+\alpha}{\alpha-\beta} - 1} \right] \\ = \lambda \frac{1+\alpha}{\alpha-\beta} P_0 Q_0 \frac{1}{1+\beta} \left[W^{\lambda \frac{1+\alpha}{\alpha-\beta} - 1} - 1 \right] = P_w$$

Solving (27) for 'w,' we get the producers' optimum demand for 'w' as

$$(28) \quad W_d^* = \left[\frac{P_w (\alpha-\beta) (1+\beta)}{\lambda (1+\alpha) P_0 Q_0} \right]^{\frac{\alpha-\beta}{\lambda (1+\alpha) + (\beta-\alpha)}}$$

Here it should be noted that the demand for irrigation water will be positive only if the absolute elasticity of demand for agricultural products is greater than unity. Since our purpose here is only to show that under certain conditions marginal cost pricing is not desirable even if there are no budgetary and other constraints, it does not matter if our conclusion does not hold under inelastic demand conditions. Thus, assuming the elasticity of demand to be greater than unity, to equate demand with supply, the public enterprise has to set the price of irrigation water (or for that matter the price of any other intermediate output it produces) such that

$$\text{RHS of (26)} = \text{RHS of (28)}$$

That is

$$(29) \quad W_s^* = W_d^* \\ \left[\frac{P_0 Q_0}{\gamma r} \frac{1}{1+\beta} \right]^{\frac{\alpha-\beta}{\gamma(\alpha-\beta) - \lambda(1+\alpha)}} = \left[\frac{P_w (\alpha-\beta)(1+\beta)}{\lambda(1+\alpha) P_0 Q_0} \right]^{\frac{\alpha-\beta}{(1+\alpha) + (\beta-\alpha)}}$$

Solving (29) for P_w , we get the optimum price of water (P_w^*) which maximizes social welfare as

$$(30) \quad P_w^* = \left[\frac{P_0 Q_0}{\lambda(1+\beta)} \right]^{\frac{\lambda(1+\alpha) + (\beta-\alpha)}{\gamma(\alpha-\beta) - \lambda(1+\alpha)} + 1} (\gamma r)^{-\frac{\alpha-\beta}{\gamma(\alpha-\beta) - \lambda(1+\alpha)}} \left[\frac{1+\alpha}{\alpha-\beta} \right]$$

Now the question is whether P_w^* will be greater than, less than, or equal to the marginal cost of W_s^* which is

$$(31) \quad MC_{W_s^*} = \gamma r \left[\frac{P_0 Q_0}{\lambda(1+\beta)} \right]^{\frac{\alpha-\beta}{\gamma(\alpha-\beta) - \lambda(1+\alpha)}} (\gamma-1)$$

Substituting (26) for W_s^* in the $MC_{W_s^*}$ equation (5):

That is,

$$(32) \quad \text{RHS (30)} \begin{matrix} > \\ < \end{matrix} \text{RHS (31)}$$

Simplifying (32) we get

$$\left[\frac{\lambda P_0 Q_0}{(1+\beta) \gamma r} \right]^{\frac{\lambda(1+\alpha) + (\beta-\alpha)}{\gamma(\alpha-\beta) - \lambda(1+\alpha)} + 1} \left[\frac{1+\alpha}{\alpha-\beta} \right] \begin{matrix} > \\ < \end{matrix} \left[\frac{\lambda P_0 Q_0}{(1+\beta) \gamma r} \right]^{\frac{\alpha-\beta}{\gamma(\alpha-\beta) - \lambda(1+\alpha)} (\gamma-1)}$$

That is

$$(33) \quad \frac{1+\alpha}{\alpha-\beta} \begin{matrix} > \\ < \end{matrix} 1$$

If $|\alpha|$ is assumed to be greater than unity, it is clear (33) will always be less than unity. That is, if the government wants to maximize social welfare, then it should set the price, at which it will sell the intermediate product to the producers, below the marginal cost of production under elastic demand conditions.

The ratio of price to MC will equal:

$$(34) \quad R_1 = \frac{\text{RHS of (30)}}{\text{RHS of (31)}} = \frac{1+\alpha}{\alpha-\beta} < 1$$

Now let us see the relationship between the elasticity of supply and demand on the one hand and the deviation of the price from the marginal cost:

$$(35) \quad \frac{\partial R_1}{\partial \alpha} = \frac{(\alpha - \beta) - (1 + \alpha)}{(\alpha - \beta)^2} = \frac{1 + \beta}{(\alpha - \beta)^2} < 0$$

That is, as the absolute value of α increases, the gap between the price and marginal cost increases at an increasing rate since

$$(36) \quad \frac{\partial^2 R_1}{\partial \alpha^2} = \frac{(1 + \beta) [2(\alpha - \beta)]}{(\alpha - \beta)^4} < 0$$

Further, since (for $|\alpha| > 1$),

$$(37) \quad \frac{\partial R_1}{\partial \beta} = \frac{-(1 + \alpha)(-1)}{(\alpha - \beta)^2} = \frac{1 + \alpha}{(\alpha - \beta)^2} < 0,$$

and

$$(38) \quad \frac{\partial^2 R_1}{\partial \beta^2} = \frac{(1 + \alpha) 2(\alpha - \beta)}{(\alpha - \beta)^4} > 0$$

as the supply elasticity goes up, the gap between the price and the marginal cost widens at a decreasing rate.

An interesting question that arises at this juncture is whether below marginal cost pricing will cover the public cost or not. That is

$$(39) \quad r \frac{W_s^*}{s} \stackrel{?}{>} \frac{P^* W_s^*}{w s}$$

Substitution of (26) for W_s^* and (30) for P_w^* and simplifying (39) we get

$$(40) \quad r \stackrel{?}{>} \frac{1 + \alpha}{\alpha - \beta}$$

In other words, whenever the opportunity cost of irrigation investment exceeds RHS of (40), we will have revenue falling short of costs resulting in deficits. Whenever 'r' is less than RHS of (40), we will have revenues exceeding costs resulting in surplus and when 'r' equals RHS of (40), we have a balanced budget.

CONCLUSIONS

Application of the general theory of second best shows that optimal public investment for producing intermediate factors can lead to pricing factors below marginal cost¹¹. It is essential to recognize that lump-sum investments may take an economy away from Pareto optimal conditions both through the investment level and through a non-competitive pricing structure. This may lead to second best pricing structures that set input factor prices below their marginal costs if they are produced through public investment. In applying the general theory of second best it is necessary to first set the optimal level of public investment and thereafter to seek the optimal second best pricing scheme. Applications of the theory to existing markets without regard to optimal investment levels may be thought of as conditional second best results. Or, perhaps more appropriately, as short run second best positions. By first setting the optimal level of investment in accordance with the second best theorem it is possible to achieve a true second best, one that would be precluded if the investment level is fixed without this consideration.

In our work we have derived the conditions required for optimal investment in publicly produced factors and shown that

¹¹Lipsey and Lancaster (1956) note that such a result is possible, but they do not actually include it in their results. It has been reached in other works that have focused on already existing systems without noting the necessity of investing the optimal amounts in production of publicly produced commodities. See for example Spencer and Brander (1983), Ebrill and Slutsky (1983).

prices for publicly produced factors may be optimally priced below marginal cost if maximizing consumers' and producers' surpluses is the aim sought. We have also applied the results, through implicit solution, to an irrigation water and agricultural system characterized by Cobb-Douglas type demand and cost functions. What we have found is that below marginal cost pricing may not be rare, as one might expect from reading the extant literature applying second best pricing to existing natural monopolies.

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