A DYNAMIC INVESTMENT MODEL FOR
TREE CROP AGRICULTURE

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ABSTRACT

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The objective of this paper is to develop a dynamic model that may be useful in the analysis of investment in long term tree crop agriculture. The model is based on linear programming as modified by fixed assets theory(propounded by Dr. Glenn Johnson and incorporated in a linear programming model by Peter Hildebrand).

The starting point is the Heady-Loftsgard dynamic linear programming model restated in a stochastic form and imbedded in a decision theoretic construct to give a stochastic mixed integer programming model.

Specific examples of how it might be used is not explored in this paper, as this is the subject of another being developed as a possible doctoral dissertation. In the final section, the paper goes into a discussion of problems associated with agricultural data collection in Nigeria -- on the conviction that specific use of any model can only come after the data

problem has been reasonably solved.

The rationale for such an elaborate model arises from:

(1) the proliferation of descriptive rather than prescriptive models for analyzing investment behaviors of tree crop farmers in Nigeria

(2) absence of studies, specifically centered on large scale cocoa or rubber producing enterprises to which such models might be applicable.

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CHAPTER I

INTRODUCTION

A body of literature has accumulated in recent years on the economics of cocoa production in West Africa. Most of the papers have been largely descriptive in nature, though there were scattered attempts at more analytical behavior of the mass of peasant farmers who constitute almost the whole source of cocoa output in Nigeria and Ghana. This paper is an attempt to add to an analytical aspect of cocoa production -- the development of a dynamic investment model which can be used to establish optimum investment criteria for large scale cocoa producers in Nigeria.

Motivation for the study

Motivation for this type of study stems from a number of shortcomings one can point out in the existing body of literature on economics of cocoa production in Nigeria. Some of these reasons form the point of departure for this study:

(I) Most studies on cocoa production by Nigerian and foreign economists have been largely historical in nature; and they are invariably aggregative -- lumping both large and small scale producers together to explain past investment behavior.

(II) The studies have subordinated investment analyses to aggregate supply response analyses.

(III) Such investment analyses as undertaken so far have been based on theories of investment not taking into consideration asset fixity induced by differences in acquisition and salvage prices.

(IV) Distinction, with respect to constraints faced by each type of producer have not been pointedly drawn between private and publicly-owned large scale cocoa producers.

(V) Results (in terms of cultural practices and output) of large government expenditures on research into and development of better strains of cocoa plants can better be analyzed by focusing on large producers who are likely to take advantage of these services more readily than smaller producers.

(VI) Rough estimates made since the 1952/53 cocoa economics study (1)* have shown an increasing trend in the cost of labor which is the largest cost element in cocoa plantation. At the rate wage level is rising, cocoa production may become uneconomic soon (2).

(VII) The tax structure faced by cocoa farmers is so regressive that some authors (3) conclude it introduces an

*References (in parentheses) are found at the end of the thesis.

undesirable difference between privately and socially optimum adjustments.[1]

(VIII) As the economy grows, there will exist alternative outlets for cocoa producers' investment funds, hence the need for the type of model envisaged (in this paper) which will show in reasonably precise terms what amount of acreage and output per acre is absolutely necessary for economic rationality in cocoa production.

(IX) The world cocoa market is one of the most unstable, thus Nigeria may some day find itself in an overproduction trap (in terms of optimal export level).

(X) Recent studies on costs of production, rate of return on investments (however measured) and amount of public investment have been simple aggregates and averages so that one cannot draw any valid economic or statistical comparisons between the small peasant and the large scale producers (that is the private and public plantation owners).

(XI) Laurent et al (4) have concluded that well-managed investments in large scale perennial tree crop agriculture offers the best method of meeting the multiple objectives of maximum use of labor, available capital, increased income (to labor and entrepreneurship) and foreign exchange earnings for Nigeria.

[1]Export produce farmers in Nigeria are subject to two main taxes - a fixed amount per ton of produce (paid to the state government) and a flat head tax paid to the local government. Both taxes are regressive in an economic sense.

(XII) There is need for a model which can help in pinpointing more specific investment adjustments in each class of farms in light of specific variations in the constraints faced by each producer.

(XIII) Failure of public investments in Nigerian agriculture call for a method where results can act as standards against which to measure current and future performances.

In view of these reasons, a better overall picture of the problems in cocoa production (on the economic and managerial side) may be gained by a less aggregative analysis of producers' investment behavior. This understanding could be achieved by constructing a dynamic linear programming model as modified by fixed assets theory and stochastic behavior of some important variables to obtain a normative investment program that better approximates reality.

Although supply management may be the ultimate goal of any study and policy recommendations, yet one has to know the investment decisions that result in these supplies -- this is where investment studies have their place to show the real opportunity costs of supplying what amount of cocoa to the world market. Before turning to the main task of the paper, an overview of Nigeria's cocoa situation and problems may help in better appreciating the motivation for the exercise envisioned in this paper.

An overview of cocoa industry in Nigeria

In light of intensive studies by the Food and Agricultural Organization of the United Nations (5), the Consortium for the study of Nigeria's Rural Development (6) and other scholars, one can abstract the following as the most important problems facing Nigeria's cocoa industry currently:

1) shifts in the relatively inelastic world demand for cocoa.
2) shifts in the relatively more elastic supply of cocoa from producers.
3) an almost static technological improvement in cocoa production techniques.[1]
4) atomistic structure of farming system and farmer holdings.
5) inadequate knowledge on the part of producers (farmers) regarding probable level of producer price in the succeeding marketing season -- at the time of highest level of labor input or time of new planting decisions (i.e., development of a new farm).
6) Rising levels of production in Nigeria and other major producing countries -- and the concurrent instability in the world market for cocoa.

[1]There has been little change in the type of implements, mode of planting, harvesting and preparation for sale of cocoa beans. The only new inputs have been pesticides to control the spread of known cocoa-tree diseases.

7) lack of international agreement on cocoa with respect to prices and output or levels of exportation and importation.

8) Government control of producer prices -- which may not reflect the actual trend of world prices to the farmers.

9) absence of information on weather trend and other climatic factors relevant for production decisions.

10) possible trade-offs between export and food crops.

The magnitude of the problem can be assessed from a short history of cocoa producers' response to prices over the past half century. Helleiner (7) stated, "From 1905 on, production rose rapidly .... By 1915, exports had increased nearly twenty-fold to 9,000 tons. There followed a period of steady and substantial growth, interrupted periodically by bad weather and blights which brought exports to 114,000 tons by 1939." He further mentioned that this growth was maintained in spite of fluctuating world and hence producer prices. Finally, even though, the statutory marketing boards which were created after World War II "held producer prices well below those which would have been justified by the prices they received for their export produce, the price increases which they did pass along were sufficient to produce a revival of the fortunes of the cocoa growers and a resumption of the expansion of cocoa acreage. Between 1940 and 1962, acreage under cocoa in the Western Region is estimated to have risen by about 40%."

The only periods of slowdown in acreage expansion and maintenance efforts were in the depression years (especially maintenance efforts) and the subsequent war years where very low prices were forced on the world by the war situation and shortage of shipping space resulted in forced production control.

In the 1950's and 1960's, acreage expansion has shown marked decline because of mass infection by diseases in the older section of cocoa growing area and more importantly the consistently low prices farmers have been receiving. The pricing scheme of the marketing boards has dampened farmer enthusiasm for acreage expansion either through new plantings or rehabilitation of old ones. Thus, it would be interesting to project what could be expected in case farmers are allowed full value of the returns to their investment in labor and other resources.

The objective of this paper is, as stated earlier, to develop a dynamic investment model (based on linear programming method) to analyse what should be the best investment behavior of large scale cocoa producers. This objective can be seen to follow from the shortcomings of aforementioned studies concerning the problems faced by cocoa producers (in general) with special regard to optimum input combination in terms of both level and time period. More formally, the paper attempts to determine how rational decisions could be taken, by large cocoa farmers, so as to:

1) maximize the present value of their farm holdings by varying levels of replacement and net investment outlays over time, and

2) maximize total revenue in the short run (annually) based on:

a) expected price trend of output (over orchard life) and variations therefrom

b) yield pattern of cocoa plants (over orchard life) and variations therefrom

c) operator's management ability

d) other relevant objectives or variables.

Results from such an analysis would help in advising appropriate Nigerian governments and their agencies about future policies on:

1) realistic proportions of prices to pass on to farmers in periods of high, medium or low prices.

2) subsidizing inputs.

3) minimizing marketing administration costs so as to:

(a) maximize cocoa farming efficiency

(b) maximize returns to farmers through specialization or diversification in farming efforts as may be called for

(c) bridge the existing difference between current private overproduction and social underproduction resulting in low returns to producers (in cocoa farming) and the consuming Nigerian public (in terms of social returns to investment outlays in cocoa production).

(d) lessen dependence of economic growth programs on just a subsector of the agricultural economy (especially at the state government level).

To complete this introductory chapter, we give more substance to the topic of paper by defining what is large scale cocoa enterprise in Nigeria and discuss briefly the general features of such enterprises (plantation) - private and publicly owned.

Defining a plantation

For the purposes of this paper, the definition of a plantation will be in abstract terms only -- just for the purpose of constructing the model. The unit of definition is the manager and a plantation is a tract or set of tracts of cocoa orchard measuring at least 100 acres under the supervision of a manager. This definition is necessary in that many operators have more than one tract where each tract may be less than 100 acres but collectively all can add up to more than the management unit as defined.

In further description of a cocoa plantation in Nigeria, one should mention that not all operators are farmers as such. To some cocoa farming is the sole source of income -- these are the farmers. To most others, their plantations are secondary sources of income, even though this income may be far higher than their primary sources; this group includes government officials -- elected and career; merchants, traders, teachers and other professionals. The last category are the government-owned plantations. This obviously is not the primary or a very important source of income to the government.

Organizationally, cocoa plantation enterprises are relatively simple in structure, the simplest structure is that of the farmer's plantations. In such cases the farmer is the entrepreneur, the manager and supervisor of the enterprise. He hires and fires all the workers, decides on what other inputs to buy and in what combination or at what level. For merchants and other professionals, they act mainly as entrepreneurs and hire supervisors who oversee the daily routines of the enterprise. Where the holdings are very large or very scattered, some plantation owners have managers who coordinate the work of the unit supervisors. At the head of organizational complexity is the government owned plantations where the structure is determined by the enabling legislative acts. Usually the enterprise is structured along existing governmental concerns and thus one would expect to find more layers of well defined power (decision making) levels. In general, there would be a farm (plantation) manager in overall charge of the enterprise. Then there would be an accountant (accounts manager), personnel manager, plant and building maintenance superintendent and a host of other submanagers and foremen -- a veritable source of bureaucratic red tape. Some of these posts are necessary though, since most workers are housed in those estates which are far from the towns -- this is a problem private entrepreneurs can avoid since their holdings are generally located near village sites.

The definition, therefore, has bearing on the model construction and analysis in two ways:

(1) it designates a management unit

(2) it gives an indication as to what activities would enter into public plantation LP format vis-a-vis the private holdings format -- in the analysis phase.

This second function of the definition is pursued further in the following section where the major differences between the two forms of plantations are explored in a little more detail. The main differences to be discussed between private and public cocoa plantations necessarily focus on their goals, investment behaviors and organizational flexibility.

The objectives faced by private and public enterprises generally differ -- and very widely in most respects. The private entrepreneur is lured into a business by profits prospects in that particular line, whether in the long or short run. There may be other objectives which may be very important, but most Nigerian plantation owners have demonstrated over and again the importance of cocoa production as an investment project (source of long term income) be it as a primary or important secondary source of income. This is also the stated objective of the public plantations but an important corollary is always the provision of more employment and the capturing of economics of large scale production which is supposed to be associated with large single block establishments.[1] While this may be true regarding the use of heavy equipments, this has to be offset by other fixed capital investment outlays in houses, roads and other social amenities that go towards setting up a colony of workers.

Secondly labor costs have been shown to be the largest cost element in cocoa production and by paying the government wage rate which is higher than open market rate paid by private employees, public plantations open themselves to higher level of costs even without adding the large overhead administrative and plant maintenance costs.

Investment decisions of public plantations are subject to more than economic forces -- more often than not, there are political and legislative constraints they have to take account of. Investment in particular items may require legislative approval (which may result in a delay) or the purchase has to go through certain governmental agencies.

[1]Saylor and Eicher (3) hold the contrary view - that is, there is no evidence of economics of scale in plantation agriculture in West Africa.

Workers cannot be easily dismissed in cases of inefficiency and sources of credit may be highly restricted by legislative or administrative directives. The civil service structure of their employment environment makes for some degree of inflexibility. This contrasts sharply with private employers who are not subject to any higher authorities and therefore have greater flexibility in their investment and employment decisions. This same argument covers the decision about where and when to locate as well as the purchase and sale of unproductive assets or enterprises. So, it would appear that government plantations are more insulated from market signals than private enterprises because of their less flexible structure in economic decision making. Under these circumstances comparisons between both sets of enterprises with respect to efficiency of investment assets will show the real costs of providing the extra "services" public plantations make to society. However, whether the returns are worth the cost is a question for welfare economists.

CHAPTER II

A SURVEY OF DECISION THEORY

Some preliminary concepts on decisions and the decision problem

Decision making as an art is as old as man himself, but the application of quantitative methods for scientific decision making is rather new and such innovation in the business and managerial fields is newer still. Mathematical economists of the last century and early part of the present relied mostly on analogies and theories (mathematical) in the physical sciences as bases for their own work (8)[1] (9)[2] (10)[3]. Since the 1940's, however, the branch of applied mathematics dealing with information processing and decision making has made very remarkable

[1]According to Jevons "the theory of economy ... presents a close analogy to the science of statical mechanics and the laws of exchange are found to resemble the laws of equilibrium of a lever as determined by the principle of virtual velocities."

[2]Fisher claims "the principle underlying the equilibrium of a pendulum or any mechanical equilibrium ... is: that configuration will be assumed which will minimize the potential. So also the supreme principle in economic equilibrium is: that arrangement will be assumed which will maximize utility."

[3]According to Marshall, demand and supply tools in economics represent "an engine of analysis".

progress. This progress followed from theoretical works in the fields of game theory, mathematic programming, systems analysis, simulation methods and other areas of operations research techniques. The development of analog and digital computing machines facilitated numerical analysis of such research findings. The use of the computer dictates a lesser reliance on the intuitive understanding of human intelligence and a more explicit description of the conditions and circumstances surrounding the business, which used to be tacitly assumed as the "environment" (over which control was taken to be unachievable and hence ignored in the analysis). This new approach implies the development of formal frameworks more specific and encompassing than the older, narrower, classical optimization techniques. The importance of these information processing and decision making theories to economic problems was quickly realized by the theoretically inclined economists and some of the problems in the theory of decision making, especially in the area of mathematical programming were formulated and developed to deal with economic problems (11). Game theory, on the other hand, has had limited success mainly because appropriate decision rules have been difficult to devise for games involving more than two persons.

Before considering specific solutions suggested in various fields for the resolution of the decision problem, it is desirable to define what is involved in the concept of decision. Considerable research, analyses and verification have been done to discover the nature of decisions, the theory behind and the actual process of decision making. Philosophers, mathematicians, economists and psychologists have been involved in these exercises. Philosophers have occupied their time defining what a decision is. Decision theory as a field has been dominated by statisticians, game theorists and management scientists while economists and other social scientists monopolize the area of decision making processes. It will be impossible to enter into the question of particular schools of thought on the definition of decision, hence a simple workable sketch (from an economic point of view) would be given -- mainly as an introduction to a survey of decision theory and some solutions (to the decision problem) proposed by a number of disciplines.

A decision may be defined as a judgment on a set of alternatives, based on analysis of relevant (irrelevant) prior information (complete, incomplete or nonexisting). Thus, a decision may be rational or irrational. From the definition, one sees that a decision is different from an action. An action is an attempt to actualize a particular situation -- subsequent to a particular decision. In formal economic language, a decision is rational if arrived at by a rational process where the process is rational if the cost of information gathering and analysis (of relevant information) is less than or equal to expected returns from the decision arrived at; otherwise the process is irrational. A decision process may also be irrational where irrelevant, inadequate or no information is used in the process.

This is not to discount the role of past experience or intuition or habits. Actually, one can subsume past experience as forms of information latent in the decision environment; the same also goes for habits. In most real situations, many decision makers rely on habit or past experience to carry them through -- more often than not, such decisions have been as highly remunerative as sophisticated analyses would have led them to do. Intuition is in a different class. It is difficult to analyze as to what its nature is, its sources and character, but those who believe in it have always claimed it is better than reason. This claim is based (most likely) on the observation that intuition is the most commonly used method by decision makers either in the absence of any formal procedures or when the situation is so complex that no elegant analytical technique would work (not even simulation approach).

The foregoing definition provides a setting for a discussion of the decision problem. Following Raiffa and Schlaiffer (11), a decision problem can be formalized as:

Given E, Z, A, Θ, U and $P_{\Theta,Z|e}$

where E = space of experiments (to acquire information):

$$E = \{e\}$$

Z = sample space, i.e., set of experimental outcomes:

$Z = \{z\}$

A = space of terminal acts, i.e., $A = \{a\}$

Θ = state space -- $\Theta = \{\theta\}$ i.e., set of states of nature

U = utility measure (assigned to each particular e)

P = joint probability measure (in the probability space)

The question to be solved is, how should the decision maker choose an e, and then having observed z, choose an a so as to maximize his expected utility. The solution of the problem consists of four moves (since there are four strategy spaces) which proceed as:

(1) the decision maker selects an e in E

(2) nature selects a z in Z according to the measure $P_{Z|e}$

(3) the decision maker chooses an a in A

(4) nature selects a θ in Θ according to the measure $P_{\Theta|Z}$

The game closes with the decision maker receiving u (e, a, A, Θ). It is assumed the decision maker has full control over his choice of e and a, but he has neither perfect control nor perfect knowledge of the choices of Z and Θ which will be made by nature. However, one usually assumes that he is able one way or another to assign probability measures over these choices.

In a less abstract sense one defines a decision problem as:

(1) specifying possible courses of action and their consequences,

(2) specifying the objectives of the participant(s) and the nature of the variables (random?) that create the problem,

(3) specifying theories to solve such problems, which theories act as guides in formulation of optimal strategies to obtain best results (that is, theories help in evaluating and making comparisons of the alternative courses of action open to the decision maker in terms of the goals he desires to attain).

Typically, a decision problem can be divided into four components (12):

(1) a model expressing a set of assumed empirical relations among the set of variables.

(2) a specified subset of decision variables whose values are to be chosen by the firm or other decision making entity.

(3) an objective function of the variables, formulated in such a way that the higher its value, the better the situation is from the point of view of the decision maker.

(4) procedures for analyzing the effect on the objective function of alternative values of the decision variables.

Needless to say, the most important of these four components is the model since it is a starting point for the analysis -- thus, the better it is, the more desirable the solution one is likely to obtain. Secondly, a good model is a setting for solutions to problems with the same structure and similar essential features as the problem for which it is originally designed. The complexity of most decision problems forces some classification for rigorous analyses and solutions. These classifications help in deciding which specific solution method will be most appropriate for the particular problem at hand. One's classification, however, depends on the subset of solutions being considered. The subset of solutions being considered in this paper are those of games theory, programming techniques, systems analysis, simulation methods and statistical decision theory. In general, one may classify decision problems as static, dynamic, deterministic, stochastic, simple, compound, deferred, etc.

The first and most basic distinction is that dependent on time, that is static and dynamic decision problems, the first in which time variables are not involved and the latter in which they are -- functionally, "The principal technical difference between dynamic (multistage) and static models (of decision problems) is the degree of complexity in describing a given strategy or procedure. In a static situation, a strategy is selected once and for all to be carried out directly, whereas the strategies available in the dynamic situation are usually complicated functions of information received and actions undertaken in the preceding stages"(13). In other words, a static situation is a stationary state of a dynamic situation and similarly, a dynamic situation may be regarded as static when the same variables enter the decision problem at successive time periods as new variables. The most notable contributions to the dynamic (sequential) approach to the decision problem are Wald (sequential analysis of sampling of statistical data) (14) and Bellman (dynamic programming technique)(15).

Secondly, there are the deterministic and stochastic cases in which the overriding distinction is the absence or presence of uncertainties. Deterministic cases are easier to handle, but are rather unrealistic since most real life decision situations necessarily result from the uncertainties and variabilities of human nature and other uncontrollable variables.

Another distinction involves the complexity of the decision involved. The decision may be of a simple nature in which the decision space is highly restricted (finite dimensional, especially one, two or three dimensional). Compound decision problems can exist in any decision space of dimensionality greater than one -- thus finite dimensionality does not imply the existence of simple decision situations. The more difficult infinite decision space problem has received little attention so far, because of the difficulty of analysis and, the impossibility of solutions in most cases.

In terms of game theory solution, one can classify the decision problem as one, two or more persons, zero- or nonzero-sum game. In the one person game, absence of uncertainties reduces the problem to one of optimization of the objective function of the model subject to natural constraints. On the other hand, presence of uncertainties turns the problem into a game against nature in which the decision maker chooses his strategy in response to explicit natural conditions (nature's moves). Nature is considered an unconscious adversary, that is, an opponent who shows no intention of doing in the decision maker. In the two-person game, both players are conscious opponents and each tries pure or mixed strategies to maximize his returns and minimize his opponent's intake. This is the area that has been subject to the most intensive mathematical treatment -- theorems and criteria abound for mechanical solutions, although efforts to generalize these to many-person games have not met with the same level of success. It is in the two-person game also where the payoffs sum to zero or not as to whether one party's gain is equivalent to the other's loss or not.

Finally, one might also consider the distinction based on solution type. This distinction is rather fluid and is essentially dependent on the type of information available to the decision maker. Statistical data presuppose a statistical decision problem; the existence of competing parties for a given payoff indicates a game theory decision problem. Similarly, a single decision maker facing nature without uncertainties gives rise to a classical economic decision problem while the presence of uncertainties and a large number of constraints define a problem in mathematical programming.

These are the distinctions delineated by the limited number of solutions which are standard so far. More extensive and finer distinctions may be found in books and other publications specifically devoted to these subjects, that is, those on statistical decision theory, mathematical programming theory and methods and theory of games.

Investment as a decision problem

Traditionally, investment or disinvestment analysis has been treated as a mechanical effort based on intuitive analyses of the enterprise manager. Later, theoretically inclined economists started examining these phenomena more systematically and various rules of thumb were derived to help in determining at what point investment or disinvestment in particular assets are desirable. Among the well known criteria derived were the present value criterion and its modifications, the marginal efficiency of capital (or investment) both of which are based on simple discount factors and rates of return. Most modern economists reject these naive models and considerable effort has gone into developing more sophisticated models that take into account specific types of investments such as (1) point input, point output; (2) point input, continuous output; (3) continuous input, point output, and (4) continuous input, continuous output and other minor variants.[1]

These and other related concepts are used to derive optimal levels and lengths of lives of given investment undertakings. Whatever the formulation, the idea of the discount rate is fundamental to investment analyses and they are all treated in the strict supply-demand framework of classical economics. This strict understanding of investment analysis leads to unrealistic conclusions because it failed to consider acquisition and salvage values as upper and lower limits, respectively, to the range within which investment and disinvestment decisions can be made. Secondly, it failed to indicate that a decline in price of output to less than minimum average cost will not lead to automatic demise of an enterprise -- so long as the price of such outputs cover more than the salvage price of

[1]A fuller discussion of these distinctions will be found on pages 70-72.

fixed inputs and acquisition price of variable inputs used in producing them. In fact, when product price falls below salvage value of inputs, it may not be advisable to dispose of all inputs in case some of them still experience returns greater than their salvage values. Still in other cases, say, for reorganization, it is possible for entrepreneurs to simultaneously buy and sell (or rent out) given assets for optimum resource use.

This short discussion, indicates that an investment analysis exercise is not a cut and dried issue -- most modern economists (especially those of the behavioral and operations research schools) consider such exercises as an aspect of the general decision problem. This follows from the fact that there are many variables -- both as objectives and constraints to be taken into account before arriving at a rational investment decision. In particular, an investment (disinvestment) decision may be regarded as a game situation in which the investor is involved in a game against nature (an unconscious adversary).

The decision process:

Central to the notion of management function is decision making. This is attested to by a definition of management as the controlling decision making unit in an enterprise -- separate from the production enterprises, per se, but related to it organizationally. Since the decision made by management has far-reaching implications for an enterprise, considerable research and analyses have gone into the formalization of the processes involved in arriving at decisions by the management units in various situations. According to White (16): "an essential prerequisite of an occurrence of decision is the existence of a motivating state of ambuguity." This ambiguity is identified by a set of alternative actions (which can be taken to resolve the ambiguity) and the resolution (of the ambiguity) constitutes the decision process and culminates in the decision. Thus, logical decision requires a linking of the state of ambiguity to the act of selection by a set of identifiable nonambiguous cognitive operations (which must be unambiguous themselves)" (16). Hence, these operations must be cognitive if one would be able to claim that the decision maker is actually deciding. In passing, it might be noted that there are levels of problems and therefore, levels of decisions -- we have primary problems whose resolution requires the solution of secondary or tertiary problems and so on for lower order problems.

On the basis of the above, White formalizes the resolution of the decision problem by stating that an ambiguity A, in a state of knowledge K is resolved by a pure decision process if there is an identifiable unambiguous cognitive operation Θ such that:

$$(A, K) \xrightarrow{\Theta} a \quad A$$

that is, we use Θ to identify a particular type of ambiguity "a", contained in the space of ambiguities A.

In operational form, the decision process involves going through the five steps of:

(1) Data collection: to confirm and determine the level of divergence between the present state of a system and the standard of the ideal specified by the decision maker.

(2) Establishment of alternatives: based on information gathered, the decision maker tries to identify what possible decisions can be taken to move him towards his objective.

(3) Assignment of measures of utility to each alternative on the basis of a given decision rule (policy). These are weights that determine how attractive each alternative is.[1]

(4) Selection of an alternative: This is the end result of the previous steps -- a decision is taken.

(5) Implementation of the chosen alternative. This is where the rationality or feasibility of the

[1]As example, the firm may wish to maximize earnings; but the question is whether the maximization should be in the long or short run. These alternatives may be achieved by sales maximization, acquisition of market leadership or outright monopoly power. Whichever alternative or method of achieving this alternative is desired will depend on how much utility the decision makers attach to the chosen alternative.

chosen alternative is put to test. Thus, the good decision maker is one whose chosen alternatives (decisions) prove to be consistently workable and capable of bearing results.

This decision making process, though is subsequent to a problem identification which dictates the collection of data. And, the process is precedent to responsibility bearing which differentiates the decision maker from an employee or an adviser.

By the nature of things, very few investment assets can be procured instantaneously, also most products, especially in agriculture take anything between a month and several years to be transformed into saleable forms. In light of this, producers have to plan beforehand for an uncertain future when their products would be ready for disposal. The preplanning and subsequent waiting have stimulated economists into studying expectations and uncertainties as they affect decision making within different economic entities.

Uncertainty, as an integral part of economics was formalized in Frank Knight's pathbreaking book, "Risk, Uncertainty and Profits", but the modern, more rigorous definition of this concept is found to have very wide application and is usually expressed in terms of theory of games or statistical decision theory. In a very general sense, one may say that uncertainty exists when an individual in a game against nature or an intelligent opponent knows only that the probability of any action by the opponent is neither zero nor unity (17). This definition is at an extreme from the certainty situation when one can make a perfect prediction of the outcome of a given action. Following from Knight (18) and Wald (19), the discipline of management science has distinguished several points along the spectrum between the two extremes of certainty and uncertainty. However, Knight's trichotomy of the basic knowledge situations relevant in a decision making situation is still the best introduction. In an abstract sense, one distinguishes structured and nonstructured uncertainty, risk and certainty. Starting with a system which can assume certain conditions from the present instant, t_0 one may designate the degrees of knowledge thus (20):

(1) Nonstructured uncertainty: This is the situation when the states of the system are unknown at any time t ($> t_0$). This may be exemplified by the number of acres farmers will put under a newly introduced crop in any given future year.

(2) Structured uncertainty: When one knows the states of the system now but lacks information as to the state at any time $t > t_0$.

(3) Risk: A situation when the states of the system as well as the laws of probability governing the system are known (within a given range of probability) for any time $t > t_0$. If

the laws are invariant with respect to t, the chance is termed stationary, otherwise it is nonstationary.

(4) Certainty: Presupposes a knowledge of the state of the system now and for all time $t > t_0$.

Thus, a situation can be classified into any of these categories according to the amount of information available. A more sophisticated but practical approach, however, has been taken by agricultural economists, who have done considerable work on the economies of information within the last two decades. Johnson et al. (21) in their Interstate Management Survey report have identified five categories in the information-decision structure. Their approach focuses more on the individual decision maker than an abstract system. They showed that with each decision-action is associated some knowledge situation which will determine whether one would go through all the five steps of the decision-action process. The knowledge situations as identified are:

(1) Subjective certainty: Which exists when the decision maker feels that he knows enough to be able to take action without further observations. In other words, he feels he can correctly predict all outcomes with the amount of information (knowledge) at his disposal.

(2) Risk action: The decision maker acts as if he is certain of the outcome of his actions. This differs from the previous situation in that action here is based on a probability distribution of outcomes. The decision maker feels that the probability is high enough that his actions would elicit the desired outcomes.

(3) Inaction: A situation when the individual believes that the evidence in his possession is not enough to permit him to commit himself one way or another.

(4) Forced action: Such action occurs when some external influence forces the manager to act, even though he would like to acquire more information or think longer before taking a decision.

(5) Learning: Occurs when available information is inadequate for a decision, but the value of acquiring more knowledge is greater than its cost. Such learning, however, may be voluntary or involuntary.

There is a missing link, though, in all the foregoing discussion in that no mention had been made as to whether a decision maker has anything to turn to as a guide towards an effective control of his information-decision structure. With a given amount of information, a decision maker needs to know certain principles which he may follow to achieve the necessary control. Following are some such principles which have been found to be very useful(as given in (22)):

(1) Decisions are always unique: There should be no opportunity for a given decision to be made in more than one organizational entity (to eliminate duplication and confusion).

(2) Information is not unique; it can be used in several decisions. However, there should be no opportunity for a given information component to be originated more than once. Also, information is not perishable, when used, it is not consumed. It is still available for other uses and other decisions.

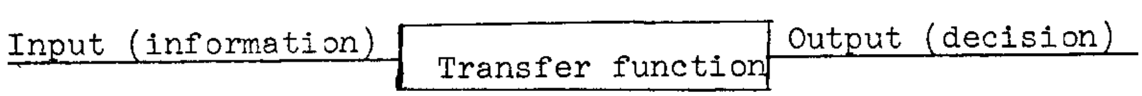
(3) Information should be processed to the maximum extent possible at the same time for all information-decision subsystems.

(4) At least two components of information are required for each decision. This is so because it is recognized that any decision requires the processing of information. The simplest way to present information is to scale it (that is, multiply it by a constant). The information being processed and the scaling factor are two different information components, the former being a variable and the latter, a parameter. Thus, each decision requires at least two

information components.

(5) Any pair of decisions cannot require the same exact set of information components. At least one component required by one of the decisions cannot be in the set required for the other. A single set of information components (both parameters and variables) can result in only one decision.[1]

These principles can be integrated into the decision process through the concept of the transfer function; which function is very useful in describing decision processes, mathematical or otherwise in a control system. Conceptually, one may regard a decision process as consisting of three basic elements in an input-output space where the input is the information available and the decision required is the output. Thus, the transfer function is the process by which the input is converted into a decision. Graphically we have:



This is a simple crude representation of the actual process in that feedbacks, subsystem interrelationships

[1] Substantiation of these principles can be found in various sections of the IMS study of Johnson et al. in reference 21; especially chapters 2, 4, 5, 6 and 8.

and adjustments in system state are not presented. But, it serves the purpose of representing a simple information decision structure and one can make the following observations:

(1) The input may consist of new data feedback from operations, a previous decision or parameters.

(2) The output may become input to another decision process.

(3) The transfer function may be of many forms such as:

(a) A mathematical expression

(b) A mathematical programming model

(c) A statistical analysis procedure

(d) A decision rule

(e) A simulation or other computer model

(f) A heuristic procedure

(g) Human judgment

(h) A tabular procedure

(i) Any combination of the above

(4) The transfer function concept is applicable to any decision rule.

(5) The transfer function agrees with the earlier five-step discussion of decision making process since the function assigns utility measures to the alternatives having the largest utility measure.

However, this simple transfer function is incomplete in the sense that:

(1) It does not account for repeated performance of the transfer function.

(2) Information is not distinguished as to type or source.

Any serious discussion of the transfer function will not fail to distinguish between the two broad categories of information -- parameter and variable type information. Parameter type information are those which are relatively constant in the system within a specified length of time (such as standard processing times or performance criteria). The second (variable) type are information components subject to frequent changes. The latter type can be further classified into system status data which describes the current status of the operating system (such as stock on hand); and the results of other decisions which may be other transfer functions that provide results required as information.

So, putting the pieces together, one solves the decision problem, in terms of model formulation (23) by:

(1) Formulating the problem: This implies a preliminary specification of admissible alternatives.

(2) Constructing a model (mathematical or otherwise) to represent the system under study.

(3) Deriving a solution from the model.

(4) Testing the model and solution derived from it.

(5) Establishing controls over the solution.

Some solutions to the decision problem

The earliest solutions to the decision problems in a scientific form were the classical statistical tests of hypotheses in which decisions to accept or reject a given situation were based on tests of hypotheses against some alternatives. Such tests may be simple hypotheses against simple alternatives or composite hypotheses against several alternatives. Basic to such statistical tests are certain assumptions as listed by Ferguson (24):

(1) The decision maker must select an action from a certain number of available actions (however, the number can be augmented as more information is accumulated). This is the learning situation.

(2) The appropriate action depends upon an unknown parameter Θ which determines the density function $f(x; \Theta)$ of the population to be sampled.

(3) If Θ were known, the density function would be known and so would the appropriate action.

To solve the problem, one selects a random X_i, $i=1, 2 \ldots n$ from $f(X, \Theta)$ and decides on an action on the basis of the sample values. After defining the parameter space, Ω, which is the set of all possible values Θ can take, he then examines A, the set of all possible actions that can be associated with the particular problems. Finally, a decision function, d, based on a,(a subset of A)is chosen; in other words, letting $a = d(X_1, X_2 \ldots X_n)$, one decides to take action a if the designated random sample X_i, $i=1, 2 \ldots n$ is observed.

Naturally, there are very many (sometimes infinite) different decision functions which could be associated with a given problem, hence one needs a theory to facilitate the evaluation of decision functions for rational choices. Such an evaluation would reasonably be preceded by an examination of the consequences of the terminal actions, A. This introduces the concept of the loss function $L=f(a, \Theta)$, a real valued function (non-negative) "which reflects the loss in taking a particular action a, when Θ is the parameter," (25) and L will be zero whenever a is the best action for Θ. In most statistical problems, Θ is unknown, hence, it is not always possible to specify a; thus one applies a strategy (decision function) d which yields $a = d(X_1 \ldots X_n)$ such that $f(a, \Theta) = f(d(X_i); \Theta)$ where f is a function of the sample values, that is, f is also a random variable. An easy way to solve for f is to take its expected value E(f) so that $R = E(f)$ is a risk function for d when the parameter Θ is as defined earlier. Thus, R is a function of d, f and Θ but does not depend on the particular random sample chosen.

Weiss (26) has criticized this classical statistical approach to the decision problem as being unrealistic in the sense that solutions obtained are based only on the assumed prior distribution. He favors solutions based on the joint prior (assumed) and actual posterior distributions. He argued that in statistical decision theory, the loss depends on the decision chosen and on the true distribution, but in many cases one will only learn exactly which distribution is the true one at the time the loss is actually paid. This is so because the decision maker knows the decision he takes, and once he knows the loss that must be paid, he can solve for the true distribution as the one that yields the given loss in combination with the known decision. However, it is difficult to imagine what mecha- would make the true distribution known beforehand. So, it seems that the loss actually incurred cannot be a function of the true distribution $X_1 \ldots X_m$ (the unknown true distribution) alone but will depend also on random variables $Y_1 \ldots Y_n$ (the observed distribution). Even then, the joint distribution of $X_1 \ldots X_m$, $Y_1 \ldots Y_n$ is not known for certainty, but is known to be one of a given class of distributions.

He concluded that making the loss depend upon random variables which will be observed after the decision is taken rather than upon the distribution of the random variables on which the decision is based does not change the analysis in any way, rather, it does put the problem in a more realistic perspective.

The narrowness in the classical statistical solution of the decision problem is rectified by A. Wald's sequential analysis. The narrowness is in the sense that such solution assumes only one piece of information and only one alternative available to the decision maker. This obviously is unsatisfactory to students and practitioners in decision making because

(1) They have to contend with several pieces of information and as many alternatives in decisions whose consequences are spread over time.

(2) In many cases, the decision maker undertakes not just one decision (a once and for all) but acts in sequences (stages) over time. At each given time, one must take into account the effect of the decision one chooses on the whole future duration of a problem. One cannot just simply choose the decision that works best for the immediate future, for such a decision may result in serious losses in the more distant future (27).

(3) Studies of sequential aspects of decision making developed from the fact that for any action taken, there are several alternatives based on several pieces of information and criteria.

(4) In real life, uncertainties introduce complexities which require that more than a simple decision be made before a project is terminated.

As Hadley (13) pointed out, "decisions which have to be made after a preceding decision, must in general, be delayed for as long as possible when uncertainty is important if the best possible decision is to be made. The role of uncertainty is to allow several possible alternatives with different outcomes in a decision making situation." Hence, the sequential decision situation in which the problem is one in which two or more decisions have to be taken, the decision taken at any stage will depend on the previous decision and the state of nature subsequent to such previous decision. This, however, is not strictly an n-stage decision structure since the number of decisions to be taken is not fixed but depends on previous decisions and the states of nature resulting. This is an interesting area and is very rich in terms of logic and mathematical sophistication and depth that have gone into its development. However, a simple solution (one of many) based on minimax Wald sequential rule is abstracted for this paper. The chosen solution is equivalent to a sequential test of a simple hypothesis against a simple alternative. Proof of the rule is lengthy, therefore, only a statement of the rule will be given. (a proof is found in Mood and Graybill (25)).

<u>Problem</u>: Given two decision rules $r(1, t_b)$, $r(2, t_b)$,

find values of A and B which make $r(1, t_b) = r(2, t_b)$ so that t_b is minimax.

Actually, exact values of A and B cannot be found owing to the complexity of computations involved, but good approximations are obtained by use of likelihood sequential ratio test where:

$$A = \frac{1 - \beta}{\alpha}, \quad = \text{probability of type I error}$$

$$B = \frac{\beta}{1 - \alpha}, \quad = \text{probability of type II error}$$

This simplifies the actual performance of a sequential test since no sampling distribution theory is involved; one only selects X and arbitrarily and computes A and B, and then just proceeds to the test

Rule: test $H_0 : \theta = \theta_0$ in the density $f(X; \theta)$

vs $H_1 : \theta = \theta_1$

with type I error probability of and type II error probability of and compute:

(1) $A = 1-\beta/\alpha$

(2) $B = \beta/1-\alpha$

(3) Take an observation X, at random from $f(X; \theta)$ and compute $\lambda_1 = \frac{f(X_1;\theta_1)}{f(X_1;\theta_0)}$

(4) If $\lambda_1 \leq B$, accept H_0

(5) If $\lambda_1 \geq A$, reject H_0

(6) If $B < \lambda < A$, then take another observation at

random from $f(X;\ \Theta)$ and compute

$$\lambda_2 = \frac{f(X_1;\Theta_1)\quad f(X_2;\Theta_1)}{f(X_1:\Theta_0)\quad f(X_2;\Theta_0)}$$

(7) Repeat (4) and (5) replacing λ_1 by λ_2

(8) Continue taking observations till either (4) or (5) is satisfied for some λ_m

This rule applies to a sequential composite hypothesis testing, though with larger but still not serious error. Proofs exist (28) to show that on the average, a sequential sampling process involves only half the samples needed for fixed sample tests.

Game theory solution

Games decision problems arise when there are antagonistic participants involved in a decision situation -- the antagonists may be nature (market forces, weather, etc.) human beings (military conflicts, poker games, etc.) or other enterprises (oligopolistic rivals) against the decision maker. As indicated earlier, it is only the one and two person games that have lent themselves to rigorous mathematical analyses, hence the few solutions (criteria) to be described are limited to the one person game situation, which also applies in most cases to the two person games (conscious adversaries). The cases to be considered also presuppose the existence of uncertainty since this is more interesting from theoretical and analytical viewpoints. Lastly, the games are assumed to possess saddle points so that pure strategies are the only ones employed.

Fundamental to the game theory concepts are the notions of strategies and payoffs. A strategy may be defined as a complete enumeration of all actions a player will take for every contingency that may arise, whether the contingency be one of chance or one created by a move of the opponent. Payoff is a rule that shows how much each player may be expected to win from the other by following any particular strategies from his whole set of strategies. Thus, the payoff is the link between the sets of strategies open to both players.

A full digression on games theory and methods is impossible in this paper, so only brief descriptions of the most quoted criteria standard so far, will be attempted.

The two most popular criteria frequently mentioned are minimax and maximin solutions where in the first criterion one minimizes his maximum losses, while in the second he maximizes his minimum gains. Actually, in games with saddle points, the two criteria yield identical payoffs -- which are equivalent to the saddle point. This can be illustrated by the following example from Karlin (12).

Consider X and Y as strategy spaces for players I and II. Define K(x, y) as the payoff and v as the value of the game. Choose particular strategies x_0 from X and

y_0 from Y. If V is a real number, such that

$K(x_0,y) \geq v$ for all y contained in Y

and $K(x,y_0) \leq v$ for all x contained in X

then $\bar{v} = \underset{y}{\text{mini}}\underset{x}{\text{max}}\, K(x,y) = v = \underset{x}{\text{maxi}}\underset{y}{\text{min}}\, K(x,y) = \underline{v}$

and conversely;

where $K(x,y) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_i a_{ij} y_j$

$\|a_{ij}\|$ = matrix of expected payoffs.

$\bar{v}$ = upper value of the game

$\underline{v}$ = lower value of the game

v = value of the game

The proof is found in (12). One other criterion closely related to the above is Savage minimax regret criterion. In this criterion, the (negative) regret measures the difference between the payoff actually obtained and the payoff which could have been obtained had the true state of nature been known. If one applies Wald's minimax criterion to the regret matrix, one obtains

$$r_{ij} = a_{ij} - \bar{v}$$

$$= a_{ij} - \underset{y}{\min}\, \underset{x}{\max}\, K(x,y)$$

or

$$r_{ij} = a_{ij} - \sum_{j=1}^{m} \sum_{i=1}^{n} x_i a_{ij} y_j$$

where r_{ij} = regret

a_{ij} = payoff actually received

$$\sum_{j=1}^{m} \sum_{i=1}^{n} a_{ij} x_i y_j = \text{upper value of the game}$$

From the same basic results obtained above, one can derive solutions based on the LaPlace (Bayes) and Hurwicz criteria.

LaPlace (Bayes) criterion is premised on the assumption of lack of knowledge of the different possible states of nature. In such situations, one may assume that the probabilities are all equal. Thus, if a player chooses any given row of the game matrix, his expected payoff is

$$K(x,y) = \frac{1}{m + n} \sum_{i=1}^{m} \sum_{i=1}^{n} x_i a_{ij} y_j$$

and he should choose a row for which this average is maximized.

Hurwicz criterion may be looked upon as a direct opposite of Savage criterion in that here one selects a constant which measures the player's optimism. In other words, one adopts a strategy that gives relevant weights to one's degree of optimism and hence, pessimism (since the constant lies between 0 and 1). If the most pessimistic payoff is assigned a weight λ, then the most optimistic weight has a value $1 - \lambda$. Thus, one chooses a strategy that gives the highest weighted outcome. The payoff, then, to both players is given by

$K(\ x_1 + (1-\lambda)\ x_2,\ )$

and $K(x, \lambda y_1 + (1-\lambda)y_2)$ respectively

where $0 \leq \lambda \leq 1$. $\lambda = 0$ gives Wald's minimax solution.

An example may help fix ideas regarding the situations in which each criterion may be preferred in a given game situation:

A_i \ θ_i	θ_1	θ_2	θ_3	θ_4	Preferred criterion
A_1	4	4	0	1	LaPlace
A_2	2	2	2	2	Wald's minimax
A_3	0	8	0	0	Hurwicz (for $\lambda > 1/4$)
A_4	2	6	0	0	Savage minimax regret

where:

A_i = action space (i.e. the set of all possible moves in the game).

θ_i = states of nature (i.e. nature's "response" to the player's move).

and the numbers represent expected payoffs in the game.

Other operations research methods

Closely related to games solutions of economic problems (at least theoretically) are other operations research methods which have also been developed in the last three decades. A large number of these techniques have been developed thus far, but they are all, in a sense, search techniques -- employed principally to seek optimal solutions to various decision situations. The most widely known are the mathematical programming and gradient methods used in handling relatively "simple" problems which possess certain convexity properties and fairly well-defined objective functions. Where the problem is so complex that it cannot be put in such programmed form, one resorts to system analysis and simulation procedures as aid in the resolution of the decision problem using the OR methods as solution techniques in the simulation procedure.

Within mathematical programming, there are the subheadings of linear, nonlinear, dynamic, geometric, quadiatic, convex programming methods and their variants. The main uniting feature of all these techniques are the sophisticated level of mathematics involved in their formulation, not necessarily in their interpretation (29). These techniques correct for the impracticality of classical optimization methods (differential calculus, say) in solving complex economic problems. The classical techniques can work well in the physical sciences where experiments can be rigidly controlled so that constrained maximization solutions can be obtained easily. This is not so in economics because apart from the variability in human behavior, resource limitations are ever present, hence the large number of variables to contend with.

In the general mathematical programming format, the problem may be formulated as:

$$Z = f(x_1 \ . \ . \ . \ x_n) = \max \ (\min) \tag{1}$$

$$\text{subject to } g(x_1 \ . \ . \ . \ x_n) \gtreqless b_i \tag{2}$$

$$i = 1,2 \ . \ . \ . \ m$$

where (1) is the objective function (such as profits, costs, etc.) and (2) are the constraints (resources, say). In more formal language, one desires to find (determine) values for the n variables $x_1 \ . \ . \ . \ x_n$ which satisfy the m inequalities or equations in (2), given the objective function implied in (1). In (2), the $g_i(x_1 \ . \ . \ . \ x_n)$ are assumed to be specified functions and the b_i's are known constants (assumed). Further, one and only one of the signs $\geq$, $=$, $\leq$ holds for each constraint, but the sign may vary from one constraint to the other. It is not necessary that m and n be equal, actually m may be zero, so that (2) can be specified as if no constraints exist at all. Generally, one assumes that some or all of the constraints are subject to non-negativity conditions, so one can interpret the formulation (1) and (2) as a problem in which it is desired to find numerical values for the variables $x_1 \ . \ . \ . \ x_n$ which optimize (1) subject to (2), and any non-negativity and/or integrability conditions. In some cases the x_j might be functions of one or more parameters and hence the problem would be one of determining a set of functions rather than a set of variables (30).

The simplex algorithm developed in 1947 (31) to solve the general programming problem gave the impetus for rapid growth in interest and application of programming techniques. Thus, if one restates the above formulation as:

$$\text{optimize } f(x_1 \ . \ . \ . \ x_n) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } g_i(x_1 \ . \ . \ . \ x_n) = \sum_{j=1}^{n} a_{ij} x_j$$

a_j, c_j known constants;

the programming problem is said to be linear provided there exist no other restrictions except the nonnegativity conditions on some or all of the variables. The nonnegativity condition of each variable in the general linear programming problem is usually specified:

$$x_j \geq 0; \quad j = 1, 2 \ . \ . \ . \ n$$

because of the convenience this affords in numerical computations. The addition of slack variables (activities) converts a formulation in which some or all of the variables are unrestricted in sign to one in which the nonnegativity conditions are satisfied. Thus, a linear programming problem is linear if and only if one seeks to

$$\text{optimize } Z = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } b_i \gtreqless \sum_{j=1}^{n} a_{kj}x_j \quad , \quad x_j \geq 0$$

without further restrictions than indicated in this formulation.[1] A simple additional restriction $x_j = (x)$, where (x) is the greatest integer function makes the problem nonlinear (i.e., the integer programming problem). All formulations not linear in the sense of the above formulation, are, therefore, nonlinear.

The nonlinear programming problem is a variant of the general programming problem stated earlier -- the distinguishing mark from the linear subset is the added restriction regarding the nonlinearity of the variables of the objective functions or the constraints; that is, the n-tuple variables x_i are elements of X (X a convex subset of the n-dimensional Euclidean space, satisfying the nonnegativity conditions as above). According to many authors, the most widely studied areas of nonlinear programming are those in which only the variables of the objective function are nonlinear while those of the constraints are linear and the integer programming problem where the results are expected to be strictly integers. In mathematic form, the nonlinear programming problem would be stated as:

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[1] Full listing of assumptions of L.P. will be found in Chapter III pages 76-78.

$$\max Z = g_i(x_1 \ . \ . \ . \ x_n)$$

$$\text{subject to } f_j(x) \gtrless b_i \quad i = 1, \ 2 \ . \ . \ . \ m$$

$$x \in X$$

$$X \subset E^n$$

Nonlinear programming techniques have found wide applications in the field of investment analysis (portfolio selection) where the quadratic programming aspect has been shown to work well (32). Production and inventory studies have also made extensive use of nonlinear programming methods (33). Unlike linear programming, though, there is no standard algorithm such as the simplex approach to solving the nonlinear programming problem -- each has to be solved in its particular way, depending on the nature of the problem and the solution desired.

Lastly, dynamic programming as a subset of the general programming technique finds its unique characteristics in the type of problems it solves. Such problems involve a large number of (usually infinite) decisions made in the course of time, that is, dynamic programming deals with multistage decision problems. In particular, each decision depends on all preceding decisions and in turn, it affects all future system states. This is in agreement with the postulates of the fixed assets theory in which the current state of an enterprise is a result of the accumulated mistakes made by the decision maker in the past and any adjustment attempted is going to affect (determine) the enterprise's future situation. Hadley (30) notes that dynamic programming technique is both a computational and an analytic device since it can be used in the form of a recurrence equation to solve a nonlinear programming problem in which time is of importance or in the analysis of a broad class of functional equations arising in control (engineering and economic) problems. Thus, one can consider dynamic programming technique as an aspect of the general programming problem where distinct stages (possibly sequential) have to be recognized and decisions made successively over the stages. Since a multistage process can be characterized by the initial state of the system and by the length of the process, one can write a dynamic programming problem as

$$f_N(x) = \max_{p \in P} \; R_N(x,p) + f_{N-1}(x'\ (N,x,p))$$

where $f_N(x)$ = total return from an N-stage process starting in state x, where an optimal policy is used

P = set of admissible policies

$R_N(x,p)$ = return from the first stage of a process of length N starting in state x, and using decision p p, (p is a subset of P.)

$x'(N,x,p)$ = the new state resulting from decision p.

In words, the equation states that the total return from an N-stage process is the sum of the first stage return plus the optimal return from the (N-1) stage decision process, where the decision p (a subset of P) is chosen so as to maximize the sum of all returns. At the heart of dynamic programming is the principle of optimality (which links the N stage to the (N-1) stage decision process so as to ensure optimal returns). The principle states that an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (34).

The remaining two methods of solving the decision problem to be considered in this paper are systems analysis and the technique of simulation. Both are closely related and could be treated under a single general discussion. Substantial effort has gone into the development of these two techniques since World War II; like the other methods discussed earlier, a vast amount of publications have appeared on these subjects. Hence, as before, only a very short overview can be attempted. They are being mentioned because they play a very important role in the solution of large scale complex decision problems ranging from cattle feedlot management to space vehicle performance, to analysis of large scale watershed programs. Systems analysis is premised on the theory that each object to be studied is composed of several subparts which can be viewed as components of the given system (object); hence optimization of the system involves integrating the components in a particular way to achieve one's objective (35). Suboptimizations of components may be attempted in cases where the whole system is too large or too complex (say, a whole national economy). Actually, in most situations, the decision maker knows little of the techniques of systems analysis; so models designed to solve such large scale decision problems would invariably be beyond the understanding of such a layman. Thus, the decision maker is an outsider to the model. He only states his objectives for the use of the analyst in the model design, and he makes his decisions later on, on the basis of results of the analyses returned to him.

There are several methods used in systems optimization techniques, and these include the differential calculus, search techniques, different programming techniques, calculus of variation, the maximum (minimum) principle (especially in control problems), classical matrix analysis and simulation methods. There is no single way to formulate a problem in systems analysis; each formulation is peculiar to each problem, but solutions sought to problems usually revolve around systems performance, reliability, profit maximization or cost minimization. However, none of these criteria can encompass all the desirable features of a system, hence the choice of any criterion to optimize should be based on that objective which includes as many system significant factors as possible (36).

Simulation as an aid in decision making is well suited to studying large systems which are so complex that a maximization formulation of the problem becomes almost impossible. In such a case, a model of the particular situation is developed and is tested using facts from real life conditions as an aid towards an understanding and possible solution (not necessarily optimal) of the problem. Such analysis may be static or dynamic or heuristic. In dynamic simulation analysis, the solution to the problem relies on sequential tests of a system model where each sequence in the procedure results in an improvement in the system design. Thus, dynamic simulation models are structured on a recursive relationship and rely on numerical analysis for design solutions (37). Dynamic simulation models rely on the specification of

(1) initial conditions

(2) system state

(3) operational relationships, and

(4) system feedback mechanisms.

On the other hand, static simulation analysis, relative to the dynamic approach bears the same relation that static systems bear to dynamic ones (that is the static can be viewed as stationary state of the dynamic for each point of time). Heuristic simulation technique may be static or dynamic but it recognizes the subjective biases of the decision maker, hence it is also strictly nonoptimizing, rather, heuristic simulation analyses tend to solve the practical -- from the viewpoint of the decision maker.

Before discussing the close relation between simulation and search techniques (in the broadest sense) which are used in simulation, one may briefly mention that the usefulness or otherwise of a given simulation model depends in a crucial way on certain criteria. In other words, one tests the validity of the simulation model by comparing the performance of the simulator under historical conditions with the actual performance of the system -- the closeness of these two is a measure of its validity. Below are the main points:

(1) Internal validity: questions whether the simulation has a low variance of outputs when replicated with all exogenous inputs held constant.

(2) Face validity: tests the reasonability of the model.

(3) Variable parameter: another way for sensitivity testing where one tests whether the variables and parameters compare with their

assumed counterparts in the observable world of reality?

(4) Hypothesis validity: this is a test of the validity of the models of the subsystems making up the whole system being simulated.

(5) Event validity: this seeks to ask in a scientific sense whether the model predicts observable events, events patterns or variations therefrom as closely as one would expect from such a model.

Simulation, as an aid to decision making, is treated last because it is not a technique as such, but an approach to aid in decision making. This means that one can employ any (and others) of the aforementioned techniques in a simulation procedure to obtain the needed help in the decision problem. There are many definitions of what simulation is, but only one is stated, as several others can be found in appropriate publications. The one given here is found in (38) where simulation is defined as " . . . a model of some situation in which the elements of the situation are represented by arithmetic and logical processes that can be executed on a computer to predict the dynamic processes of the situation." Thus, the model (simulation) is a procedure which expressed the dynamic relationships that are hypothesized to exist in the real situation by means of a series of elementary operations on the appropriate variables. These last two statements taken together imply that simulation (and other models that do not utilize an explicit mathematical calculus -- calculus in the widest sense of definition) depends on search processes or numerical methods to find optimum combination of the controllable variables of the model. This is where the use of operations research methods come in useful in simulation procedures. Search techniques have been used in simulation procedures in two ways:

(1) as a way of selecting the best system operating conditions,

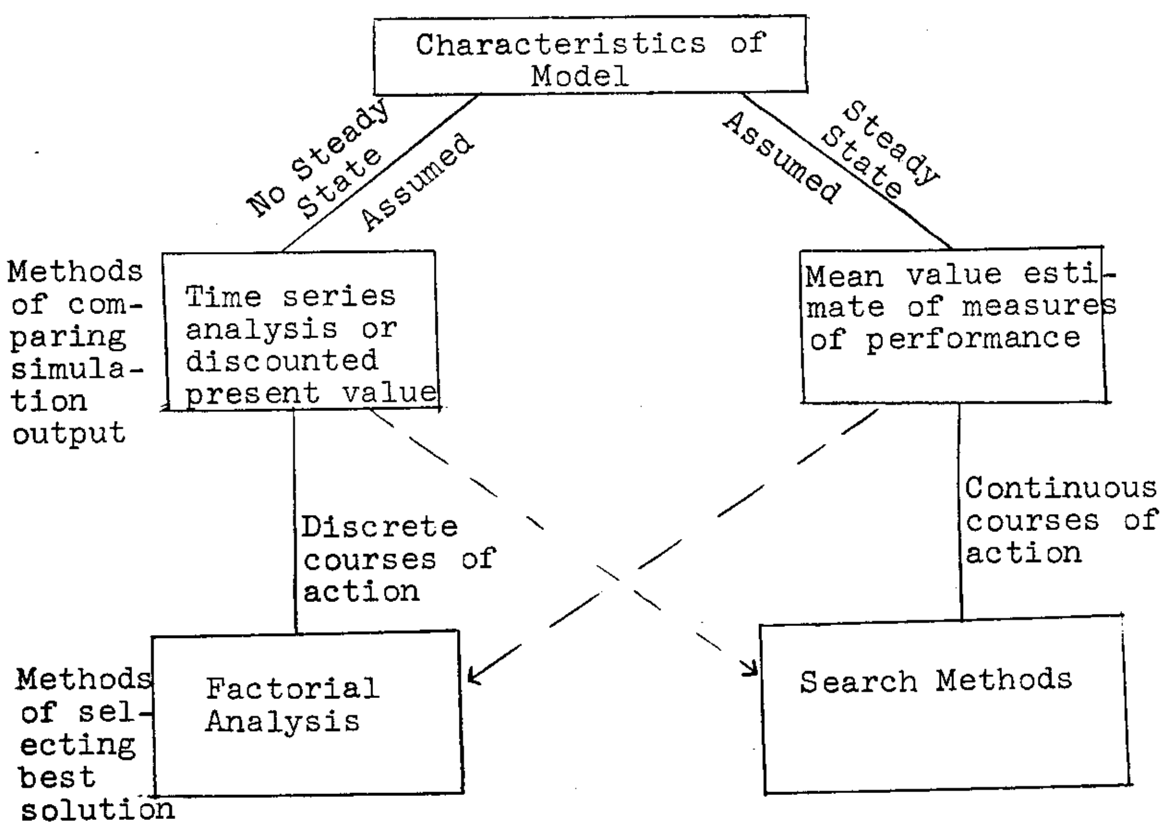
(2) to estimate model parameters when no data on them are available.

The main purpose of search techniques is to find the best setting (in terms of stated objective of the system) of controllable variables of the decision problem in order to obtain the best solution to the problems formulated for the simulation analysis. The controllable (decision) variables may be subject to continuous or discrete changes. When the variables can change continuously, the problem of finding the optimum is analogous to finding the peak of a hill, but usually in a multidimensional surface -- and if there is only one optimum point, such problem is unimodal. Otherwise it is multimodal. In the multimodal case, one has to make a random search over the space, find the local peak from each point, use smoothing process or replication at each setting (local peak) and make comparisons to determine whether a global optimum has been achieved or not. For the simpler unimodal situation, the general method of achieving the highest peak is to search for higher points from each peak (L.P. technique is a search technique on a unimodal surface). These are the so-called hill climbing or gradient search techniques.

On the other hand, one uses factorial analysis or zero-one programming method to study the effects of changes in discrete variables.

A diagram to connect ideas is also given by reference (38):

Methods for Finding "Optima" with Simulation



The biggest problem in designing a search procedure has to do with its efficiency. A useful search procedure has to be efficient over a wide range of response surfaces. There is, at present, no general solution to deal with surfaces with multimodal maxima, for most procedures, pertubations are applied at different local maxima, but the only true way to confirm that one has obtained the global maxima is to evaluate all local maxima and choose the largest, but the expense of such procedure is usually prohibitive. A method suggested to overcome this is to use the search procedure several times to estimate the distribution of the local optima plot against the search number, identify each local search which produces a response greater than any previous response and fit a smooth curve through the response points. This estimated effectiveness curve (the smooth fitted curve) can then be used to project the estimated response that will be achieved from one more search -- this continues until the value of the estimated improvement in the solution is less than the cost of completing one additional search. This is the Las Vegas rule to solve the multimodality problem; further methods can be found in Wilde (39).

The relation between simulation and search techniques are, however, not one-way. One can use operations research methods in solution of those parts of simulation procedure that can be cast in strict optimization models. For example, in plant location studies, the overall process would normally be handled by simulation while within each trial plant

The biggest problem in designing a search procedure has to do could be used to find the least cost transportation patterns. On the other hand, one may run into L.P. problems with non-finite solutions, and if parametrization of some variables can be carried out, simulation may be used to solve the problem in a finite number of iterations. Such situations arise when dealing with optimization problems in policy (strategy) spaces so that there are infinitely many strategy combinations.

A general critique of the various solutions as guides to investment decisions

In discussing critiques of each solution (to the decision problem) considered above, one may note that statistical decisions are solved with the risk function as penalty for following a particular course of action. In most cases, it is very difficult to explicitly formulate what the loss function is or to place precise costs on such risk functions (this is also true of simulation methods). This is to say that the constraints considered in statistical decision functions cannot be used in handling complex decision problems where such constraints cannot be rigorously controlled and, therefore, cannot be held as constants. It is only when constraints can be treated as constants (i.e., the coefficients of the equations of the system under consideration are stable over the period of analysis) such that the situation is nonstochastic that classical statistical solutions have any chance of being treated with reasonable confidence. This is the situation rectified by Wald's sequential analysis but here the problem is the uncertainty attaching to how much cost will be involved in a particular decision problem solved by the sequential sampling approach. Hence, in some cases, fixed sized sampling methods may make more sense in terms of dollars -- especially in a situation where uncertainty is of little significance and resources are severely limited.

Games theory solution to the decision problem came into the limelight with the publication of Von Neumann and Morgenstern's landmark (40) but practical applications have had limited successes in real economic decision making. Game theory has succeedded only in one person games (games against nature) and in enterprises where assumptions of perfect competition are nearly met (such as agriculture). Two person games, especially those involving monopolies have had severely restricted successes even though this is where such decision solutions could be most useful. Many-person games solutions only help in the analyses of possible collusion patterns - which brings us back, full circle to few person games.[1] The fact that most criteria proposed so far are only relevant to one- or two-person games confirm these criticisms.

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[1]A particularly simple discussion of this idea is contained in Economic Theory and Operations Analysis by W.J. Baumol (Prentice Hall Inc., Englewoods Cliff N.J. 2nd edition) pp. 544-549.

Programming techniques, just as games analyses, suffer from their being prescriptive tools,[1] hence decision makers have to temper their resulting decisions (from such analyses) with some elements of objectivity to achieve the best balance between what is normatively desireable and what is objectively attainable. Linear programming methods are the most widely exploited of all mathematical programming techniques and the glaring shortcomings of these methods are seen in the observed aggregation problem the method ignores when used to derive normative supply functions in regional or nationwide studies. Also, the question of salvage and acquisition prices as lower and upper bounds to supply function estimates are usually overlooked except in Smith's earlier works (41) (42) (43) so that one sees smooth monotonic curves resulting from such studies, rather than step function supply curves that experience and intuition would indicate. Smith's works just cited confirm this last conclusion.

Systems analysis and simulation approaches appear to offer the best methods so far for solution of the decision problem. This, because they can accomodate both normative and nonnormative positions of the decision maker in specifying goals for action to solve problems, especially those that:

[1]Fuller explanations are given on pp. 72-76 where the nature of L.P. and its variants are explained.

(1) cannot be solved in the market place

(2) are not amenable to solution by maximization.

But, then the complexity of specification, analysis and interpretation of simulation and systems analysis models may outweigh any advantages they have (especially for small or medium sized concerns).

The foregoing observations are not to say that the solutions suggested are worthless. They are very useful; apart from helping in solutions of most of the (vexing) existing problems, they have pointed out ways of going about more and more complex ones arising in day to day experiences.

References made to normativist and positivist positions of decision makers are very fundamental to certain schools of thought regarding the usefulness of the normativist versus positivist solution of the decision problem. This is a standard distinction in model building in economics. Positivist or predictive models are used to refer to what "is" or "actual" while normative refers to "what ought to be". The optimizing solutions considered above are prescriptive in that there is no compulsion as to what should be; rather the implication is what could be if the decision maker were to follow the solutions indicated. The prescriptive models, however, must possess some predictive power if they are to be meaningful since their prediction must have some relevance to reality if and when they are to be used (44, 45). But such predictions need not necessarily conform to what is actually observed at any particular point of time. The purely predictive models of economics and other decision sciences are designed for these situations.

In summary, one may assert that, although the predictive power of normative models are not generally known, such models are elegant and flexible, hence they should not be regarded as worthless for predictive purposes . Earl Heady et.al. (46) among others, have derived regional supply curves for some important agricultural products by different techniques of mathematical programming during the past two decades. The results have induced a reevaluation of the use of prescriptive models for prediction. In other words, it may be concluded that to the extent that the various norms (profit maximization, income maintenance, etc.) and other assumptions of the models are met, such models will be predictive in a conditional sense.

In a more fundamental sense, most analyses in general economics have always assumed equality between acquisition and salvage prices; so it may be argued that results and policy recommendations from such analyses will be at variance with observed reality where inequality between acquisition and salvage prices is the rule rather than the exception.

Johnson's fixed assets theory (investment-disinvestment theory) was originally espoused by G. L. Johnson (47, 48, 49) and formalized and extended by him and some of his students. This will not be a full review as a number of theses and dissertations expounding the ideas and applications of the theory are readily available (50, 51, 52). The main motivating force for the fixed assets theory is the arbitrary definition of certain factors as fixed or variable in classical and neoclassical production analyses without regard to the consequences of such designation in economic works. Johnson's contention is that rather than predetermine (before analysis) which factors are fixed and which not, one should make the fixity or nonfixity of each asset a function of the analysis for more meaningful results from analyses of economic organization of enterprises. To this end, he introduced the concept of two prices for an asset:

(1) acquisition price which is the price per unit of a factor of production which an entrepreneur will have to pay to acquire another unit of that factor,

(2) salvage value which is the price per unit for a factor that the entrepreneur would receive if he were to dispose of some of the factors he has on hand.

The neoclassical production function for the short run (or intermediate run) is given by

$$Y = f(x_1 \ . \ . \ . \ x_d \mid x_{d+1} \ . \ . \ . \ x_n) \qquad (1)$$

where:

$x_1 \ . \ . \ . \ x_d$ = variable factors of production

$x_{d+1} \ . \ . \ . \ x_n$ = fixed factors of production.

Now let p^A_{xi} be the acquisition price per unit for each factor X_i, i = 1 . . . n and p^s_{xi} = salvage value per unit of factor X_i, then for $X_i \ . \ . \ . \ X_d$ variable factors

$$\infty > p^a_{xi} = p^s_{xi} > 0 \qquad (2)$$

in the neoclassical analysis and for $X_{d+1} \ . \ . \ . \ X_n$, fixed factors

$$\infty = p^a_{xi} > p^s_{xi} = 0 \qquad (3)$$

Thus each factor is predetermined (in neoclassical analysis) as to whether it is variable or fixed. This is what Johnson set out to correct by arguing that fixity or nonfixity is not to be predetermined but must be endogenous to the enterprise organization such that his exposition allows for

$$\infty > p^a_{xi} > MVP_{xi(y)} > p^s_{xi} > 0 \qquad (4)$$

and $MVP_{xi(y)} > p^a_{xi}$ would imply acquisition of more of the relevant x_i, i.e., invest in such x_i, while $MVP_{xi(y)} < p^s_{xi}$ would indicate disposal of such x_i, i.e., disinvest in such x_i.

One implication of this analysis is that "once the enterprise is organized at the HPP (high profit point) for a given set of prices in three categories (factor acquisition, factor salvage and product) changes in the levels of any of these three categories of prices may cause a reorganization of the firm. Furthermore, a reorganization in response to price changes upward may be different than reorganizations in response to price changes downward" (53).

This theory has wide applications in the field of supply analysis, income distribution, resource immobility and intersectoral analysis, it is really an investment-disinvestment theory (as it is now called) and this is confirmed by its use in explanation of resource allocation and farm reorganization in United States Agriculture over the past half century (52, 53, 54, 55).

CHAPTER III

DEVELOPMENT OF THE MODEL

Motivation for the model

Any exercise in investment analysis is necessarily dynamic in nature since time is involved in an essential way. As the objective of this paper is to develop a dynamic investment model, certain notions need to be explained for clarity of subsequent exposition.

The first notion to be discussed is that of dynamics. Capital using production techniques are based on the principle of roundaboutness whereby products emerge following a series of transformation processes. Dynamism in economics has occupied the attention of scholars for some time, but the most quoted contributors are Hicks and Samuelson. Both agree on basic principles, but they diverge in the senses of their definitions and stability conditions of dynamic economic processes. In the Hicksian sense, "we call . . . economic dynamics those parts where every quantity must be dated" (56). Samuelson on the other hand, considers a truly dynamic situation as one in which time is involved in a functional sense; that is ". . . we reserve the designation dynamics for systems which involve economically significant variables at different points of time in an irreversible way . . . " (57).

Whichever position one takes, the essential point is that in a dynamic analysis, an activity produced in time period t_0 is considered to be different from the same activity undertaken in period t_1. Thus, so long as a positive value can be attached to time, such activities are not considered comparable. An example is a stream of incomes produced over a time period whereby comparability among different incomes is achieved by discounting each income to present value by using the discount factor $(\frac{1}{1+r})t$, t=0, 1, 2 . . . and r is the discount rate.

This discount rate is the link between types of investment and the notion of dynamics in economics. The principal types of investment structures will be distinguished because of the conceptual and analytical differences the various forms (of investment) give rise to. The different forms have been outlined earlier, but not explained. Following is a short description of each as generally stated in the literature:

(1) Point input-point output: Such investment outlay is concentrated within one single period of time, the consumption of the product of the investment is also concentrated within a single period of time subsequent to the first. Such investment include timber tree production, wine aging and annual cropping undertakings in agriculture.

(2) Continuous input-point output: In this type, investment expenditures are spread over time while the investment product is concentrated within a single period of time. An example is industrial process which involve successive operations upon raw materials to obtain a finished product (assembly line products).

(3) Point input-continuous output: Here investment expenditure is concentrated within a single period of time while the product of the investment is spread over a more or less lengthy period. Investment in productive durable equipments (blast furnace,farm buildings) are leading examples.

(4) Continuous input-continuous output: This typifies a situation where investment outlays and product from same are both spread over time. This, really is the rule rather than the exception, especially if rates of utilization and the resultant expenditures are regarded as investment outlays. Analysis of this type of investment is more complex than the others above and so it has excited less interest.

Other distinctions, no less important are:

(1) Replacement investment: used to replace obsolete (economic, technical) or worn out assets.

(2) Expansion (net) investment: The type that allows enterprises to meet a growing demand in the more-dynamic sectors of the economy -- such expansion investment may be quantitative or qualitative (increased capacity utilization, research and development investment, etc.).

(3) Modernization investment: This type is essentially designed to reduce costs. Sometimes, types (2) and (3) may coincide.

With these distinctions in mind, the next section focuses on a particular method which has been found to be very flexible in the analysis, among others, of the investment problem. The particular technique is linear programming (L.P.) and its variants.

L.P. as a tool of investment analysis

As indicated earlier, linear programming is a subset of the apparatus used in solving the decision problem under which investment analysis can readily be subsumed. This claim (58) is premised on the validly observed phenomena of a broad range in which long term marginal cost -- which we have to deal with in investment problems -- is practically constant. The implication is that in this range, long term total cost is practically a linear function of output. This range corresponds to the case in which the expansion of output comes about by the addition of identical assets.

Other relevant reasons (of interest to economists) include:

(1) Linear programming analyses can help show both short and long run optima. An investment with low immediate cost (which is thus very attractive) may turn out to be very costly in the long run (in terms of discounted costs).

(2) Linear programming analyses facilitate the adoption of solutions combining different types of assets intelligently.

(3) The analysis can help in identifying and avoiding bottlenecks especially where there is investment fund ceiling. This is achieved by formulating the problem in terms of inputs and outputs at given prices, where inputs and outputs are determined so as to maximize the entrepreneur's profits (useful in small scale farming).

(4) Linear programming algorithms have been developed to work out variations in the assumptions that will be compatible with the solutions obtained, but the method cannot usurp the position of the decision maker.

At a more abstract level, linear programming possesses some philosophical and mathematical properties that have attracted able mathematicians to the study of this particular technique. Among these properties listed by Dantzig (59) are:

(1) L.P. has a philosophy to model building that has application to a broad class of decision problems in government, industry, economics and engineering.

(2) L.P. possesses a simple mathematical structure which can be used to solve the practical scheduling problems associated with areas mentioned in the immediately preceding statement.

(3) L.P. is concerned with the study of the behavior of systems. It focuses on the combination of resources in total as an economic process, that is, in describing the interrelations of the components of a system.

(4) The theory of L.P. is concerned with scientific (i.e., systematic) procedures for arriving at the best design of a system -- given the technology, the required specifications and the stated objectives.

(5) All answers to the question of resource allocation can be found in the dual to the L.P. primal problem -- sometimes it is easier to solve the dual than the primal -- an aid to investment analysts.

Useful extensions of the L.P. have been made in the following directions:

(1) <u>Network theory</u>: To solve transportation or network flow problems since their extreme point

solutions are integers -- a key factor in a theory linking certain combinational (discrete) problems of topology with the continuous process of network theory.

(2) Convex programming: Arises when the linear part of the inequality constraints and the objectives are replaced by convex functions; of special interest has been work on quadratic programming (to solve problems with quadratic objective functions) which have been used in portfolio selection and milk supply analyses.

(3) Integer programming: This type of programming technique can be used for 'all-or-nothing' situations or situations where production has to be at discrete levels only. Though mixed integer (discrete and marginal continuous levels of investment assets) techniques have been developed, most attention has been on the pure integer problem solutions.

(4) Linear programming under uncertainty: This is ordinary linear programming but the coefficients of the objective function or the technology or the constraints are stochastic (probabilistic) in nature.

(5) Dynamic linear programming: As the name suggests, dynamic L.P. is a programming technique in which

the L.P. is solved sequentially from one period to another. These sequential L.P. problems are connected by interperiod capital surpluses or other entities. In general, dynamic L.P. problems are characterized by

(a) A set of linear restrictions: Equations which are repetitive in nature, hence, certain coefficients will be the same from one time period to another.

(b) The sparseness of the coefficient matrix, that is, the very small number of nonzero elements which appear in the matrix.

The economic interpretation of D.L.P. models is that they help in connecting up maximization over time with some kind of market mechanism. The shadow price duality relations can only be interpreted by economists as discount factors, interest rates or marginal value productivities. Thus, one may assert that this programming approach casts new light on some unsettled questions of classical and neoclassical capital theory.

From the foregoing discussion, it is apparent that linear programming methods can be used to:

(1) Handle single objective, single constraint or single objective multiple constraints or

problems in production or investment analyses.[1]

(2) L.P. methods can be used to determine optimum programs for combining inputs which will minimize the enterprise costs for a given production program. This is the most frequent form assumed in investment analyses.

(3) The technique can be employed in determining, simultaneously, the optimum combination of inputs and optimum production program which maximize the firm's profits.[2]

All these results can be expected so long as the problem under consideration satisfies the basic L.P. assumptions that:

(1) There is no substitution within an activity, that is, the technology coefficients are given and invariant.

(2) Proportionality between inputs and outputs are maintained at all activities, thus for the x_i level of activity, the output (or input) of good j is $a_{ij}x_i$ and the profit for activity i is $c_i x_i$.

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[1] The single objective that L.P. and other decision models can handle may be simple functions or they may be complex functions of several variables - that is, some single objectives are actually a composite of several related objectives.

[2] This view is based on the fact that one can read the primal and dual solutions to a given problem from the same program.

(3) The additivity assumption is satisfied with respect to inputs, outputs and profits so that the solution guarantees the existence of $\Sigma a_{ij}x_i$ and $\Sigma_i c_i x_i$ (that is, output and profits are summable over all activities). This presupposes the independence of each activity from any other activities.

(4) Divisibility is assured, in other words, activity levels x must be able to vary continuously.

(5) The problem guarantees the convexity of the entire set of admissible vectors $\langle x \rangle$ such that if x^0 and x^1 are feasible with respect to the constraints, the same must hold for $x^0 + (1-\lambda)x^1$ where λ is an arbitrary number between 0 and 1. This is what many economists regard as the phenomenon of constant returns to scale.

(6) Nonnegativity condition is satisfied, that is, the vectors representing the various activities are carried on at a positive or zero level.

(7) Single-valued expectation of the decision maker, that is, the manager has only one objective, to maximize.

These are the main characteristics and assumptions behind linear programming. Now, one may connect ideas by mentioning the principal theorems underlying the technique.

No proofs are attempted as these are available in many texts and journals. They are being mentioned only because of the insight they provided in the resolution of the valuation problem which is at the heart of investment analysis. Many formulations and proofs of these theorems exist, but one which appears straightforward and adequate for the purpose of this paper is abstracted from (60). With these theorems of L.P., one can derive equilibrium values for assets (shadow prices) without going through the market for observations and confirmation of such investment decisions as L.P. may be used to solve.

Following are the theorems:

Theorem 1: A vector $\langle \bar{x} \rangle$ is a solution to the primal L.P. problem if and only if there exists a vector $\langle \bar{y} \rangle$ such that $\{\bar{x}, \bar{y}\}$ is a saddle point of the Lagrangian form

$$\phi(x,y) = (c,x) + (y, b - Ax)$$

where $x = \langle x_1 \; . \; . \; . \; x_n \rangle \geq 0$

$y = \langle y_1 \; . \; . \; . \; y_m \rangle \geq 0$

(c,x), $(y, b - Ax)$ are inner products of the vectors concerned

A = matrix of input-output coefficients of appropriate dimensions

b_i = available stock of ith resource

c_j = value of output achieved by operating the jth activity at unit intensity

x_j = level of intensity of the jth activity to be undertaken

y_i = shadow (internal) prices of unit of resources

Theorem 2: (Duality theorem): If the primal of a linear programming problem has a solution $\langle \bar{x} \rangle$, then the dual of the problem has a solution $\langle \bar{y} \rangle$ and

$$(c, \bar{x}) = (\bar{y}, b)$$

Theorem 3: (Existence theorem): If both the primal and the dual of an L.P. problem have feasible vectors, then both have solutions. All such pairs of solutions $\langle \bar{x} \rangle$ and $\langle \bar{y} \rangle$ satisfy

$$(c, \bar{x}) = (\bar{y}, b)$$

Corollary:

If x^* and y^* are feasible for the primal and dual of the L.P. problems respectively, and $(c, x^*) = (y^*, b)$ then x^* and y^* constitute a pair of solutions and conversely.

Theorem 3 guarantees the existence of solution to L.P. problems whether the solution is bounded or not.

Theorem 4: If $\langle \bar{x} \rangle$ and $\langle \bar{y} \rangle$ are solutions to the primal and dual of the L.P. problems respectively, then

$(A\bar{x})_j < b_j$ -- which implies $\bar{y}_j = 0$

and $(\bar{y}A)_i > c_i$ -- which implies $\bar{x}_i = 0$

These results are a property of the solutions $\{\bar{x}, \bar{y}\}$ implicitly expressed in Theorem 3. These properties are very significant in the economic interpretation of LP results; they are also very valuable in any computational algorithm.

In economic language, the first result expresses the fact that if $\langle \bar{x} \rangle$ represents an optimal program that does not utilize all of the jth resource, then such a resource is overabundant; hence, it has no money value in a competitive market (at equilibrium). The second result asserts that if the money value of the ith activity is less than its operating costs, then that activity is not undertaken (i.e., its level in the program is zero).

Proofs to the preceding four theorems rests on a certain fundamental lemma which is also stated without proof.

Lemma:

Let S be a closed convex polyhedral set in the m + n Euclidean vector space E^{m+n} ($m \geq 1$) which satisfies the following two properties:

(I) If $Z = \langle x_1 \ldots x_n, y_1 \ldots y_m \rangle = \binom{x}{y}$ contained in S and $x_i \geq 0$, $i = 1, 2 \ldots n$, then $\langle y \rangle \ngtr 0$ (if $y \neq 0$, then, y cannot have all components y_j, $1 \leq j \leq m$ nonnegative).

(II) S contains at least one point $\binom{x^o}{y^o}$, $x^o \geq 0$, $y \geq 0$($y^o \equiv 0$ by hypothesis)

Then there are vectors $\langle u \rangle$ (of n components) and $\langle v \rangle$ (of m components) such that:

(a) $u^o \geq 0$, $v^o \geq 0$

and (b) $(u^o, x) + (v^o, y) \leq 0$ for all $\binom{x}{y}$ contained in S

Hildebrand's modification of the L.P., based on fixed assets theory

Peter Hildebrand, in a thesis at Michigan State (61) advanced some modifications in the standard L.P. formulation to avoid generating unrealistic solutions in enterprise resource utilization analysis by linear programming techniques. A discussion closely following Hildebrand will be given before the relevant sections are incorporated in the model of this paper.

The basic assumption of Hildebrand's model concerns initial resource fixity which was that the supply schedule for spendable funds is the only fixed resource. Secondly, he took land as partially fixed, while other resources are variable such that they present no limit to production. In this paper, though, land is assumed variable. The program from his model emphasizes the simple most profitable activity (implicitly assumed to be cocoa production in this paper) relative to the use of capital surpluses. The level of this activity will expand to the point at which the cost of obtaining additional factors of production -- a function of the increasing cost of credit -- exceed the marginal value productivity (MVP) of the factors in this one activity or to the limit of a resource whose MVP lies between acquisition and salvage values and is therefore fixed. He emphasized that this process of enterprise expansion can create idle services from some of the resources during the periods in which they are not used and these might be profitably used in other enterprises or activities. Hence, these idle services have become fixed for the firm as a by-product of the expansion in resources to produce the most profitable product. Any increase in the proportion of services which are thus fixed would tend to create seasonal complementarity (or supplementarity) between enterprises such that the program will select the next most profitable activity (food and fiber production in Nigeria) to make fuller use of the endogenously fixed stock of resources. Such endogenous resource fixity in an L.P. model requires acquisition and salvage activities for all durable resources. This is a result of the stock-flow problem arising from the nature of durable assets because the use of value of such assets during a given time period is derived from the flow of services available from the stock of the resource on hand. The activities are defined in terms of acquisition and salvage values associated with a resource in use in the time period. The acquisition cost of an additional unit of a durable asset for a time period (say a year) consists of annual depreciation, interest and taxes. This, rather than the market value, is the annual marginal factor cost to the firm, of acquiring the asset. The corresponding annual salvage value consists of the same elements, but this time based on the salvage price at the time of sale. In this wise, he concluded that the imputed value of resources given in a model incorporating endogenous fixity will be

1) annual cost of acquisition for all resources increased in quantity
2) annual salvage value for all resources decreased in quantity
3) annual value-in-use for all resources fixed at the original quantity and neither purchased or sold.

Therefore, every durable asset in Hildebrand's model receives an imputed value based on the annual flow of resources from it.

This approach is to correct the errors arising from the predetermined fixity of resources at initial levels in standard L.P. formulation without regard to MVP of such resources in the enterprises within which they are currently being employed. Briefly, these errors include:

1) A resource fixed in abundance can be utilized to the point where its MVP is equal to zero, indicating that salvage value equals zero when in fact it may be greater than zero.
2) A resource fixed in short supply will indicate MVP as being greater than marginal factor cost of another unit.

These two errors will result in nonoptimal resource allocation as the program will tend to choose inefficient technologies with respect to both really scarce or abundant factors. Therefore, if adjustments cannot be based on MVP of factors when there are no real causes for acting otherwise, less desirable solutions will result.

Lastly, the use of predetermined fixed quantities of resources in optimizing a farm organization results in the undesirable characteristic that the capital surpluses and credit were used for cash expenses instead of being converted into profit-earning resources.

To activate the program, some specialized equations were defined for cash and credit as well as double acquisition activities -- all directed towards making the program make the maximum use of resources. This corrects for standard L.P. program where cash throwoffs may in some cases not be converted into productive uses (investment).

In his model, a production activity can, but need not enter the solution when a non-money resource becomes limiting. If the productivity of the factor is such that more of the asset should be purchased, an acquisition activity will replace the slack activity. Hence, a production process (activity) may not be obtained in the solution to replace a stock resource activity unless one of the resources is just exactly used up and no more acquired, i.e., the resource has been endogenously fixed at the initial level. However, since spendable funds are limited in amount, at least one production activity will enter so long as the solution indicates any production at all (the only alternative would be to sell out). Other possibilities for production processes to enter into the solution would be when any of the specialized equations (defined in the model) is an exact equality and the slack activity drops out. Thus, if it is profitable for the firm to diversify, the program mechanically is capable of arriving at such a solution. The final section of his thesis to be used has to do with determination of discrete investment levels: To overcome the unrealistic assumption of infinite divisibility assumed in standard L.P., especially with respect to expensive durable items, an arbitrary method is incorporated in the model to find the most profitable discrete level of investment for the important expensive durable items. That is, the problem is one of determining the most profitable discrete level at which an asset should be fixed -- the method depending upon the concept of resource fixity. The degree of fixity for assets are determined individually, beginning with the one most subject to fixity. It is contended that the variations of the other assets will be less likely to cause the MVP of the fixed assets to shift beyond the bounds of fixity if the one with the greatest differential between acquisition and salvage values is the first to be fixed in the solution.

The method for determining discrete investment levels is first to obtain an optimal solution with all assets assumed to be infinitely divisible. Then choose from among the assets in which investment occurred, the one most subject to fixity. This particular asset is then fixed at the next higher and next lower discrete level by changing the initial restrictions by the amount of the coefficients in the acquisition activity multiplied by the level of the activity for each case and removing the acquisition and salvage activities for the asset from the matrix. This process, it is claimed, may, however, result in negative values for the restrictions in some equations, particularly the cash equations, so that manipulation of some other activity levels may be necessary to increase the negative values to some non-negative or zero level.

Model development

In time dependent analyses, there are three prescriptive decision models (in the mathematical programming subfield) that may be attempted for a resolution of the decision problem. These are dynamic programming, dynamic linear programming (polyperiod type) and recursive programming models. Earlier reference to dynamic programming in this paper (pp 50-53) listed some of the advantages and shortcomings of the model. However, this method of analysis is not considered to be very appropriate to the problems on hand because a large amount of data is required for computation in such a formulation and also because the method imposes severe limitations on the number of state variables that can be considered in a single problem.

Up to date, the method can handle no more than three variables -- even in such a case, a Lagrangian multiplier technique is needed to fix one of the variables. Recursive programming, on the other hand, focuses on a single period, even though solution for any particular period depends on solutions of the preceding periods as constraints. Hence, recursive programming method centers on very short run objectives (whatever it may be) whereas an investment undertaking in tree crop agriculture necessarily considers the long run.

On the basis of the foregoing discussion, a DLP (i.e., a polyperiod type) is considered desirable for fulfilling the objectives of this paper. Apart from the advantages the method shares with standard (static) LP method, there are certain features peculiar to DLP (62):

(1) it allows production timing and capital acquisition in the model formulation

(2) it gives a decision rule whereby a given solution in any period has impact on subsequent decisions in later periods

(3) it allows visualizing levels of investment and expansion over time

(4) it can answer questions concerning growth through data generated

(5) though most formulations are dynamic in a dating sense, modification can be introduced

to handle uncertainty and resource and price variations. This modification will help in resolving questions of length of planning, horizon and size of farm (if need be).

(6) its most desirable asset is that "instead of a plan for some point of time in the future, we may readily solve for the best plan in a series of years; with the optimum for any one year depending on the optimum in other years, on the availability of and returns on capital in other years, on the need for household consumption, etc. Based on resource supplies and optimum use in previous years, these procedures also can specify the plans for transitional years required to get to "the future point in time, as most budgets are made up." (63)

(7) alternatives for trading capital equipments in later periods can be included so that the model could be used to study machinery replacement models.

(8) DLP techniques allow the objective of the model to be defined as maximizing present value or net worth at some future time period.

Before giving a stochastic reformulation of the Heady-Loftsgard DLP model, the original model and relevant concepts in stochastic linear programming will be discussed to put matter in proper perspective.

The basic model being considered in this paper is the Heady-Loftsgard formulation of the dynamic linear program (63); this, however, is being modified by Hildebrand's important contribution which enables an LP model to determine resource fixity endogenously. The basic model is:

$$\text{maximize } Z = \sum_{i=1}^{m} \left(\frac{1}{1+r}\right)^{i} \sum_{j=1}^{n} c_j^k \; x_j^i$$

$$\text{subject to} \quad \sum_{i=1}^{m} b_i^k - \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^k \; x_j^k$$

$$x_j^k \geq 0$$

where Z = objective function to be maximized

$\frac{1}{1+r}$ = discount factor to equalize income among periods

c_j^k = returns to activity j in year k (discounted)

x_j^k = activity j (production process) in year k

b_i^k = resource i available in year k

a_{ij}^k = input-output coefficient in year k

In more explicit form, we have

$$\begin{aligned} Z = {} & c_1^1 x_1 + c_2^1 x_2 + \ldots + c_j^1 x_j^1 + \ldots + c_n^1 x_n^1 \\ & + c_2^2 x_2^2 + \ldots + c_j^2 x_j^2 + \ldots + c_n^2 x_n^2 + \ldots \\ & + \ldots + c_1^k x_1^k + \ldots + c_j^k x_j^k + \ldots + c_n^k x_n^k + \ldots \\ & + c_1^+ x_1^+ + \ldots + c_j^t x_j^t + \ldots + c_n^t x_n^t \end{aligned} \qquad (1)$$

where $c_j^k = (\frac{1}{1+r})^k \bar{c}_j^k$

$\bar{c}_j^k$ = undiscounted net return for the jth activity in the kth year.

and the first dynamic linear programming (DLP) equation can be expressed as:

$$b_1^1 = a_{11}^1 x_1^1 + \dots + a_{2j}^1 x_j^1 + \dots + a_{1n}^1 + x_n^1 + a_{11}^2 x_2^2 + \dots + a_{2j}^2 x_j^2 + \dots + a_{1n}^2 x_n^2 + a_{11}^k x_1^k + a_{12}^k x_2^k + \dots a_{1j}^k x_j^k + \dots + a_{1n}^k x_n^k + \dots + a_{11}^t x_1^t + a_{12}^t x_2^t + \dots + a_{2j}^t x_j^t \dots + a_{1n}^t x_n^t \quad (2)$$

Equation (2) is a complete expression for b_1^1 (the resource restriction in year 1). This is an equation rather than an inequality since one can view potential activities in year k - 2 as "slacks." However, for $k \neq 1$, all a_{ij}^k $(k \neq 1) = 0$ except those representing inter-year capital flows, since activities for year k - 2 will not use resource supplies from b_1^1. Hence, the relevant terms for (2) become:

$$b_1^1 - a_{11}^1 x_1^1 + a_{12}^1 x_2^1 + \dots x_{2j}^1 x_j^1 + \dots + a_{1n}^1 x_n^1 \quad (3)$$

This restriction does not hold for $b_i^k (k = 2 \dots t)$ because transfers of net income occur from one year to operating capital of the next year for years 2 . . . t. In this respect, the supply of operating capital is increased each year by the difference between the net income of the previous year and the fixed costs and household withdrawals of the previous year. This capital transfer in this model is accomplished by giving a positive coefficient to an activity produced in year k and a negative coefficient to the same (activity) in year k + 1. Thus, the total supply of operating capital so accumulated in year k is:

$$b_1^k = \sum_{j-1}^{n-1} (c_j^{k-1} - a_{ij}^{k-1} x_j^{k-1}) - a_{1n}^k x_n^k \qquad (4)$$

where s_1^k = first resource supply (capital)

x_n = fixed cost family living activity

Hence, in terms of the model, the set of equations for year one can be expressed as:

$$\sum_{i=1}^{m} b_1^1 - \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^1 x_j^1 \qquad (5)$$

where b_i^1 = capital supply in year 1

$\sum_{i=2}^{m-1} b_i^1$ = other resource restrictions

b_m^1 = fixed cost including household consumption

$a_{mj}^1 = 0$ for $j \neq n$

In view of previous discussion, equations for year k = 2 . . . t are larger in the b_1^k row because of the capital transfer process:

$$b_1^k \geq - a_{11}^{k-1} x_1^{k-1} - a_{12}^{k-1} x_2^{k-1} - \dots - a_{ij}^{k-1} x_j^{k-1} - a_{1n}^{k-1} x_n^{k-1} + a_{11}^k x_1^k + a_{12}^k x_2^k + \dots + a_{ij}^k x_j^k + \dots + a_{1n}^k x_n^k \qquad (6)$$

$$\begin{aligned} b_2^k &\geq a_{21}^k x_2^k + a_{22}^k x_2^k + \dots + a_{2j}^k x_2^k + \dots + a_{2n}^k x_n^k \\ &\dots \\ b_m^k &\geq a_{m1}^k x_1^k + a_{m2}^k x_2^k + \dots + a_{mj}^k x_j^k + \dots + a_{mn}^k x_n^k \end{aligned} \qquad (7)$$

as before b_1^k = supply of capital in year k

$\sum_{i=1}^{m-1} b_i^k$ = other types of resources having no inter-year transfers

b_m^k = total fixed cost, including household consumption

a_{ij}^{k-1} = all negative for b_1^k

a_{mj}^k = 0 for $j \neq n$

This is the Heady-Loftsgard (H - L) model as abstracted from reference (58). Next, to discuss relevant concepts in stochastic linear programming and justification (with respect to stability and optimal solutions) for such modification. The following discussion follows from an analyses of two principal contributors in the field of stochastic programming as applied to agriculture -- G. Tintner and Van Moseke.

Stochastic formulation of the linear programming problem is a response to observations that nothing is really ever certain; more so the future, hence in planning an investment or future production, one can only be content with projections which may prove to be true or not. Foremost among writers on the subject are Charnes and Cooper (64) in the industrial field and Tintner (65) in the agricultural area.

In stochastic programming, parameters of the L.P. become random variables so that one only knows or assumes a particular distribution for the variables. There are two main subtypes of stochastic L.P.; the here and now problem (or passive approach according to Tintner) and the wait and see (or active approach). In the passive approach, one approximates the distribution of the variable of the objective function and bases his decisions on this distribution. In the active approach, however, the amount of resources to be allocated to various activities are the decision variables. In short, an L.P. problem is stochastic if the data of the problem are not known, but are random variables.

Starting from the standard L.P. problem,

$\max_{x} Z = c'x$, Z = vector of objective function

c' = vector of returns

S.T. $A_x \leq b$

x = vector of activities

$x \geq 0$; A = matrix of input-output coefficients

b = vector of resources available.

and assume that the probability distribution of the elements of the vectors b and c and the matrix A is given by

$f(A, b, c)$

Further assume that optimal activities x are selected for all possible configuration of the random variables, then one can specify the stochastic L.P. problem as

$$\max_{x} c'x = f(A, b, c)$$

$$\text{S.T.} \quad Ax \leq b$$

$$x \geq 0$$

This is the passive approach to solving the stochastic L.P. problem. Under this condition, it is possible to derive from the probability distribution f, the distribution $Q(f)$

of the linear form p to be maximized.

Solutions through the passive approach may not be satisfactory, in which case one uses the active approach by converting the L.P. problem into a decision problem. First define (u) as the matrix of the decision variables u_{ij} (i.e., the proportion of resource i to be devoted to activity j) where all resources are to be completely used. Secondly, one assumes the probability distribution $f(A, b, c)$ to be known; then the problem can be formulated as:

$$\max_{x} Z = c' x$$

$$S \cdot T \cdot \quad w_{ij}x_j = b_i u_{ij}, \quad i = 1 \ldots m, \; j = 1, 2 \ldots n$$

$$0 \leq u_{ij} \leq 1$$

$$\sum_{j=1}^{m} u_{ij} = 1, \; i = 1, 2 \ldots n$$

where Z, c' and x are as defined above

w_{ij} = weights attached to each u_{ij}

u_{ij} = elements of the decision matrix.

With the assumptions of the transformed problem, it is possible to derive the probability distribution of the expected net revenue Z (especially if f is well behaved, say normal or uniform distribution) as

$R(Z; u)$ which depends now on the choice of the elements of (u). Hence if one defines a utility functional

$$F = f(R(Z; u))$$

one maximizes F with respect to the elements of (u).

The foregoing has been rather general. Practical solutions can be obtained if one makes specific assumptions regarding the distribution of the variables (of the model) which are random and designating a particular decision rule to use. In the following paragraphs, we follow a development of stochastic linear program (S.L.P.) due to Van Moseke (66).

$$\max_{x \in X} (f(x) = cx) \qquad (1)$$

$$X = \left\{ x \middle| Ax \leq b, \; x \geq 0 \right\} \qquad (2)$$

where c, A, b are parameter matrices over the real number system with dimensions lxn, mxn and mxl respectively

x = real n-tuple of variables (i.e. vector of outputs for all x in X)

$X \subset R^n_+$ = set of feasible actions (restricted to positive values)

c = vector of c_j (i.e., net returns per activity level)

b = available quantities b_i of resources r (r = m-tuple)

A = input-output coefficients a_{ij} of the vector $x = x(r)$ of production functions

Following Debreu (67) one defines the economic environment agent in terms of the values c, A, b; imperfect knowledge (of the environment) is interpreted in the sense that some or all of the parameters are stochastic. Thus (1) and (2) define a stochastic L.P. problem.

Define $p = (c_1 \ldots c_n, a_{11} \ldots a_{mn}, b_1 \ldots b_m)$

and let $P = \{p\}$, a parameter space where p = state of nature; then in the stochastic case, $P \subset R^{n + mn + m}$.
If the distribution of the components of p is known and p is stochastic, the maximand of (1) can be written:

$$f(x; p) \; ; \quad f: X \cdot P \rightarrow R \qquad (3)$$

Hence, allocation under imperfect knowledge consists of choosing an x and letting nature select a $p \in P$ and observing the outcome $f(x|p)$, but by (2) $X = X(p)$, so an arbitrary x is feasible if and only if

$x \in X\ (p)$

This is the stochastic feasibility problem which points up the fact that not only is the relation between actions and results (outputs) imprecise, but even the space of actions that limited resources will allow is not known accurately. A way out of the problem is the development of a stochastic linear program based on the truncated minimax with the following assumptions:

1) Only c is random so that the stochastic feasibility problem does not arise.
2) The components of c are jointly normal so that the distributions $Df(x)$ of the minimand are normal for all x and hence completely characterized by their expectations and standard deviations.
3) Actions x are ordered by means of a criterion that weights the expectations and standard deviations of the outcomes $f(x)$.
4) The weighting reflects the psychology of the decision maker.
5) The criterion, derived from the minimax rule has a confidence limit interpretation.
6) The matrices A, b are assumed deterministic so that the feasibility issue is obviated; $x(p)$ is constant, hence indeterminacy of the weights

in the maximand is the dominant cause of variability.

In terms of the S.L.P. above, the minimax criterion of Wald would read

$$\max_{x \in X} \min_{p \in P} \quad f(x, p) \qquad (4)$$

and Bernoullis expected value criterion is:

$$\max_{x \in X} \left(Ef(x) = \int_{p} f(x, p)\, d\wp\right) \quad , \wp = \text{the distribution notation}$$

Bernoullis criterion has been criticized as inappropriate for decisions involving significant potential loss hence the suggestion that other characteristics of the distribution be taken into consideration. This leads to Markowitz's E, V criterion:

Only efficient decisions should be considered, $\bar{x} \in X$ is efficient if it satisfies both of the following conditions:

1. $Vf(\bar{x}) \leq Vf(x)$ for all $x \in X$ such that $Ef(x) \geq Ef(\bar{x})$; V = variance function
   E = expected value function
2. $Ef(\bar{x}) \geq Ef(x)$ for all $x \in X$ such that $Vf(x) \leq Vf(\bar{x})$.

This, however, still leaves further choice among efficient decisions to one's relative valuation of risk versus returns, hence, the specific assumption:

$Df(x)$ is normal for all $x \in X$

so that the distributions Df(x) are fully characterized by their first and second moments; therefore, one can construct a risk preference functional in terms of Ef(x) and σf(x) that possesses a confidence limit interpretation:

$$\phi f(x) = Ef(x) + m\sigma f(x) \ , \ m \in R \qquad (5)$$

on which to base the truncated minimax criterion

$$\max_{x \in X} \ f(x) \qquad (6)$$

The indeterminancy in the maximand of the S.L.P. arises if one applies (3) to f(x,p) when Df(x) is normal since this results in

$$\min_{p \in P} \ f(x,p) = -\infty \text{ for all } x \in X \text{ so that}$$

$$\max_{x \in X} \min_{p \in P} \ f(x,p) \text{ is indeterminate}$$

The decision rule (5) is truncated minimax in the sense that the entrepreneur does not consider the whole real line over which the normal distribution is defined, but rather truncates this by specifying a particular confidence limit beyond which he is indifferent as to what nature's response is (however unfavorable). This is what happens in real life where imperfect knowledge is the rule rather than the exception. As a particular example, if one considers m = 1.96 and let $X = \{X_1^*, X_2\}$ and the graph of the densities ($D' = dDf(x)/df(x)$) of f(x) for $x = x_1$ and $x = x_2$, the table of normal distribution

$$(2\pi)^{-1/2} \bullet \int_{-\infty}^{m} e^{(-t^2/2)}\, dt = \alpha \qquad (7)$$

lists $\alpha = .025$ as the confidence limit corresponding to 1.96. Thus, maximizing $\phi f(x) = Ef(x) - 1.96\,\sigma f(x)$ is equivalent to comparing the lower .025 confidence limit of the competing distributions. This maximization corresponds to applying the regular minimax criterion to the distributions after truncating them at their .025 confidence limits; then clearly

$$\min_{p \in P} f(x,p) = f(x) \qquad (8)$$

$$\max_{x \in X} \phi f(x) = \max_{x \in X} \min_{p \in P} f(x,p) \qquad (9)$$

Thus, one can characterize m as the risk preference parameter of the truncated minimax criterion. Because of the normality assumption made earlier, it is a matter of indifference whether one interprets ϕ in terms of confidence limits or is considered as a functional weighting mathematical expectations and standard deviations of outcomes. The latter interpretation (ϕ, a relative weight) holds even if non-normality prevails, since the probability statement expressed by the Bienayme-Tchebycheff inequality (68)

$$pr\left\{\left|f(x) - Ef(x)\right| \geq |m|\sigma f(x)\right\} \leq \frac{1}{m^2} \qquad (10)$$

can always be made.

From all this, it is clear that values of m, the risk preference parameter corresponds to different attitudes of the decision maker toward risk:

$m > 0$ implies risk seeking so that one can therefore explain $\max_{x \in X} f(x)$ as a truncated maximax criterion.

$m = 0$ implies the expected value criterion and

$m < 0$ implies risk aversion. Now, letting

$$f(x,p) = cx$$

$$Ec_i = \int c_i dp \quad , \quad 1 = i \ . \ . \ . \ n$$

$$V = (\sigma_{ij}) = (\int(c_i - Ec_i)(c_j - Ec_j)\, dt)\ i, j = 1, 2 \ . \ . \ . \ n$$

where p is now the joint distribution of the coordinates of c(A, b being given). Then the expectation and standard deviation of f(x) are given by $Ef(x) = E(cx)$ and

$\sigma f(x) = (xVx)^{1/2}$ so that

$$\phi f(x) = (Ec)x + m(xVx)^{1/2} \qquad (11)$$

Therefore, if c is expressed per unit of x, so also is $\phi f(x)$ and hence $\sigma f(x)$. This is why $\sigma f(x)$ is preferred to $\sigma^2 f(x)$ since it is hard to interpret $\$^2$ in any meaningful way (when dealing with money returns).

Recalling an earlier assumption that only the components of c are random and the choice of the truncated minimax criterion as a decision rule, the SLP can be written:

$$\max_{x \in X} \phi f(x) = (Ec)x + m(xVx)^{1/2} \qquad (12)$$

$$X = \left\{ x \middle| Ax \leq b \quad x \geq 0 \right\} \qquad (13)$$

where all the terms are as defined above.

In terms of the typical production problem, agricultural enterprises conform more or less closely to the assumption of (12) and (13). This is attested to by the fact that in crop farming b(resources) are usually known, A (input coefficients) are nearly constant per acre but yields per acre and market prices of output fluctuate violently. It can be shown by certain lemmas and theorems in convex analysis (69) that (12) and (13) yield optimal solutions.

However, the model of this paper is stochastic, so by primal and dual theorems of convex homogenous programming (Kuhn and Tucker (70)) one is guaranteed optimal solution to the stochastic formulation of LP (1) and (2) as well as to the deterministic LP problem (12) and (13). The theorems are stated, some without proofs:

<u>Theorem</u>: To solve the programming problem $\max_{x \in X} \phi f(x)$ for $m \leq 0$, it is sufficient to determine a local maximum of $\phi f(x)$ on the boundary δx of X. This local maximum exists and is a maximum.

<u>Theorem</u>: If the risk preference is decreasing the variance of the optimal program is nonincreasing.

Proof: let $m_1 < m_0 \leq 0$

and x^0 = maximizer of $\phi f(x|m_0)$ on X

x^1 = maximizer of $\phi f(x|m_1)$ on X

$\sigma_0 = \sigma f(x^0)$; $\sigma_1 = \sigma f(x^1)$

then

$$(Ec)x + m_1\sigma_1 \geq (Ec)x^0 + m_1\sigma_0 \qquad (14)$$

let $Ec = \bar{c}$

$$\bar{c}x^0 + m_0\sigma_0 \geq \bar{c}x^1 + m_0\sigma_1 \qquad (15)$$

define $\Delta m = m_1 - m_0$, $\Delta\sigma = \sigma_1 - \sigma_0$ and $\Delta x = x^1 - x^0$,

therefore we have

$$m_1\Delta\sigma \geq - \bar{c}\ x$$

$$-m_0\Delta\sigma \geq \bar{c}\ x$$

and addition gives

$$\Delta m\Delta\sigma \geq 0$$

But it is still possible that x^0, the optimal decision for $m = m_0$ can be abandoned for another maximizer as m decreases but resumed as m further decreases to m_1, hence, we have the following:

Theorem: If the same program x^0 is optimal for distinct values of the risk preference, then it is optimal for all intermediate values

Proof: by assumption, for all x X

$$\bar{c}x^0 - m_0\sigma_0 \geq \bar{c}x - m_0\sigma f(x) \qquad (17)$$

$$\bar{c}x^0 - m_1\sigma_0 \geq \bar{c}x - m_1\sigma f(x) \qquad (18)$$

let $\sigma f(x) = \sigma(x)$

multiply (17) by λ and (18) by $(1-\lambda)$ and add, then we have for all λ, $0 < \lambda < 1$

$$ix^0 - (\lambda m_0 + (1-\lambda)m_1)\sigma_0 \geq \bar{c}x - (\lambda m_0 + (1-\lambda)m_1)\ \sigma(x) \quad (19)$$

<u>Theorem</u>: (duality theorem of convex homogenous programming)

(I) Primal (convex-homogenous programming problem)

$$\max_{x \in X} F(x),\ X = \left\{x \middle| h(x) \leq b,\ x \geq 0\right\} \tag{20}$$

and its dual (II): $\min_{v \in V} vb$ (21)

$$V \bigcup_{x \in X} V(x)\ ;\ V(x) = \left\{V \middle| Vh_x \geq F_x,\ V \geq 0\right\} \tag{22}$$

and the dual pair of linear programming problems (III) and (IV) where x* indicates a solution to (I):

$$\text{(III)} \max_{x \in X^*} F_{x^*}x;\quad X^* = \left\{x \middle| h_{x^*}X \leq b,\ x \geq 0\right\} \tag{23}$$

$$\text{(IV)} \min_{v \in V(x^*)} vb;\ V(x^*) = \left\{v \middle| vh_{x^*} \geq F_{x^*}, V \geq 0\right\} \tag{24}$$

then the duality theorem of the convex homogenous programming states that the feasible programs x* and v* solve problems (I) and (II) respectively if and only if

$$F(x^*) = v^*b \tag{25}$$

Lastly, we have

<u>Theorem</u>: Denote by x* any solution to (I). If v* solves (IV) it solves (III) as well

<u>Proof</u>: Let $\bar{x}$ be a solution to (III)

By the duality theorem of LP

$$v^*b = F_{x^*}\bar{x} \tag{26}$$

By the theorem of the Lagrangean $(\bar{x}, v^*)$ is a saddle point of (III) and is also a saddle point of (I). Hence $\bar{x}$ is an x* and (26) becomes

$$v^*b = F_{x^*}x^* \tag{27}$$

Because of homogenity, (27) implies (25). Hence by duality theorem of convex homogenous programming stated above, v* solves (II).

These results can now be applied to risk (stochastic) programming problem

$$\max_{x \in X} \phi(x)$$

$h(x) = A(x)$, and (IV) therefore becomes

$$\min_{v \in V} vb;\ V(x^*) = \left\{ v \,\middle|\, vA \geq \phi_{x^*},\ v \geq 0 \right\}$$

$$\text{where } \phi_x = \frac{\partial}{\partial x}\left(\bar{c}x + m(xVx)^{1/2}\right) = \bar{c} + \left(m/\sigma(x)\right) Vx \qquad (28)$$

Finally, the duality theorem is relevant to the treatment of the stochastic feasibility problem of the SLP where A and b are considered random in (2). Following Charnes and Cooper (71), specify for each constraint, a probability $(1-\gamma)$ with which the decision maker wants the constraints to be satisfied.

If the ith row vector a^i of the coefficients of A is random, and the components $a_{ij} (j = 1 \ldots n)$ of a^i are jointly normal, with covariance matrix V_i, the ith constraint becomes

$$a^i x - m_i (xVx)^{1/2} - b_i \qquad (29)$$

where γ and m_i are related by (7) and by the convexity of $\phi(x)$ for $m \leq 0$, (29) is convex (i.e., $m_i \leq 0$). Hence, if X is defined in terms of constraints such as (29) one has again a convex-homogenous programming problem and the duality theorem applies.

In view of the exposition of preceding several paragraphs, the original Heady-Loftsgard model can be reformulated as:

$$\max_{x \in X} \phi f(x) = \sum_{i=1}^{m} \left(\frac{1}{1+r}\right)^{i} \bar{c}_j^{k} x_j^{k} + m(x_j^{k} V x_j^{k})^{1/2} \qquad (30)$$

$$X = \left\{ x \,\middle|\, \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^{k} x_j^{k} - \sum_{i=1}^{m} b_j^{k} , x_j^{k} \geq 0 \right\} \qquad (31)$$

where one would solve the program for each period on basis of the assumed distribution and the given decision rule.

Now we have a model that gives the expected value of how much we have to invest (annual capital surpluses). In the computation stage one would introduce Hildebrand's modification of the standard L.P. model so that the model of this paper can determine resource fixity endogenously for more rational economic decisions regarding when to acquire or dispose of any given asset in the farm organization. But the model, thus far, can only determine purely integer solutions. This is undesirable in that some inputs can enter the enterprise at both discrete and continuous (marginal) levels while others can enter only at discrete (lumpy) levels. So to make the model more realistic, it would be necessary to present it in a mixed integer programming form. This form would allow the model to simultaneously handle both discrete and nondiscrete levels of investment of the various inputs in the program. A method of mixed integer programming model due to Clark Edwards (72) allows the model to be rewritten as:

$$\max_{x \in X} \; f(x) = \sum_{i=1}^{m} \left(\frac{1}{1+r}\right)^{i} \bar{c}_j^k x_j^k + m(x_j^k \, V x_j^k)^{1/2} \qquad (32)$$

$$X = \left\{ x \,\middle|\, -\sum_{l=1}^{m} \sum_{j=1}^{n} a_{ij}^k \, x_j^k + \sum_{l=1}^{m} (b_i^k) - \sum_{l=1}^{m} x_p = \sum_{i=1}^{m} \delta_i \right\} \qquad (33)$$

$$x_j \geq 0 \qquad (34)$$

where $\phi f(x)$ = the objective function maximized for each x (activity) over the entire set of all X (vector of activities)

$\frac{1}{1+r}$ = discount factor

$\bar{c}_j^k$ = vector of expected value of c (net returns per activity)

v_j^k = vector of activities j for each year k.

V = variance function

a_{ij}^k = vector of input-output coefficients for each year k.

(b_i^k) = the greatest integer function of the vector of available resourcei for each year k.

x_p = activity in the current basis assigned the value b_i (b_i is the resource restriction under examination)

$\delta_i = b_i - (b_i)$, the fractional part of an integer variable which assumed a noninteger value in the current basis.

The foregoing explanation of components of the model clearly indicates it is a computational model which, with the underlying existence and optimality theorems should be amenable to manipulations in investment analyses. Specifically, it is a stochastic, mixed integer programming model, specified in a dynamic form so that investment in long term cocoa enterprise can be analyzed in a single program.[1]

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[1] Clark Edward's method is one of the simplest of the branch and bound (tree searching) methods of solving the mixed integer programming model. Essentially, 'tree searching' involves enumeration -- starting from the L.P. solution of the problem with the integer requirements replaced by 0 - $_i$ - 1. All the steps involved in transforming the model of equations (30) and (31) to that of (32), (33) and (34) are found in reference 72 pages 53-60.

CHAPTER IV

EPILOGUE

Observations on the model: Possible application to large scale cocoa enterprises in Nigeria

The first section of this final chapter will focus on general observations on the model and hence its possible applications. But before this, it is necessary to examine some problems associated with data collection since any analysis ultimately rests on availability of data. The examination will be concluded by the design of a sampling procedure to be used in data collection.

One of the most serious problems that can vitiate the efforts of a research worker from making any useful contribution in his field of research is lack of adequate and reliable data (statistical) from which he can draw any meaningful conclusions from whatever model he might be working with. This is very true of the situation in Nigeria, especially with respect to privately owned farms. A number of reasons can be advanced for this, the most important being inadequate knowledge about large private farms (by Nigerian standards) by the tax authorities.[1] But the main concern of this paper is not to

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[1]The inadequate knowledge about private holdings is a consequence of the tax structure. Many owners of private plantations would not expose themselves so that they would be subjected to no more than the flat head tax collected by the local governments.

discuss the why of lack of data but to identify where the absence occurs and what problems this poses for a research worker. The lack of adequate data on agriculture is very acute in certain areas of the industry such as land use, crop acreage, production level (per tree or acre). This is a result of small peasant farmers not keeping records on any of the listed items. Also, most holdings of such farmers are irregularly shaped, crops are unevenly spaced and intercropping is widespread so to collect data on agriculture in Nigeria one has to contend (in most cases) with:

1. Absence of accounting record -- To get anything worthwhile, one may have to design his own accounting procedure or make do with mere guesses.
2. Absence of productivity indices regarding land, labor or capital assets. This follows from 1, so that what remedies lack of accounting practices would aid in eliminating absence of productivity indices.
3. Inadequate record on equipment use: Tractor and equipment loan service exist in certain parts of Nigeria, but the accuracy of data from the government agencies operating the system would be difficult to verify since those who hire the equipments usually keep no records of such transactions.

4. Most of the large scale producers of export crops have scattered plots but the organization of the farming enterprise has no formal structure so that it is not very easy to nail down such owners for better classification of farms in many subsectors of the agricultural industry.
5. There is absence of trust from most farmers towards interviewers. Though this is a universal problem, it is rather serious in Nigeria because data collectors have failed to achieve honest rapport with farmers. Good data have been obtained where and when the interviewers have succeeded in convincing the farmer of their true intentions.
6. Communications and computational facilities as well as coordination of data are not yet at the level at which a research worker can avoid expending considerable time and resources on data transformation.[1] The situation is improving but is it a long journey to what is hoped for.

[1]As example, the result of the cocoa study conducted in the years 1952/53 (reference 1) was not published till 1956. Even then this feat could be accomplished only because the computations and analyses of the data collected were done in England.

7. With all the foregoing, it becomes difficult to validate results from a time series analysis with cross sectional data and vice versa because of the time lag that may be involved in data collection and transformation.

The cocoa producing area of Nigeria is confined to the southern part of the country. The major producing area is in the western part of the south -- this area accounting for about 95 percent of total Nigerian output. This area is contained in the Western State and parts of Midwestern Kwara and Lagos states. The other section is in the eastern area of southern Nigeria and is contained in the east central and southeastern states of the Federation of Nigeria.

To obtain data for the model described above, it is necessary to have a sampling plan. There is no special technique needed for the government plantations. These are few in number and can be sampled hundred percent. Apart from the small number involved, these publicly owned plantations are those likely to keep the most tolerable account of their activities -- their organization structure will facilitate reasonable definition of activities in the L.P. formulation of the problem.

Sampling for the private plantations present a very great problem because in this case, the sampling is from an unknown population. A way towards an educated guess of the size of such population is through the licensed buying agents of the various state marketing boards, who act as intermediaries between the boards and the cocoa producers. From the licensed agents' books, one can guess at large scale producers among the cocoa farmers. On collecting such information, it would be possible to use probability methods in obtaining the subsample of private plantation owners.

For this subsample, it is proposed that the cocoa growing area be divided into five zones, each zone serving as a stratum or substratum. The three strata are:

1. Kwara State zone
2. Eastern states zone
3. the Western zone

The Western zone will be divided into three substrata of

(a) Ibadan-Egba-Ijebu and Lagos state substratum

(b) Ife-Ilesha substratum, and

(c) Ondo-Midwest substratum

The strata and substrata are based on a combination of political, geographical and economic considerations. The Kwara state was part of the old Northern Region of Nigeria while East Central and South Eastern are parts of the old Eastern Region. The rest of the cocoa area are integral parts of the old Western Region. Western and Kwara zones are geographical contiguous, but the Eastern zone is separated by a few hundred miles from the rest.

The Western zone exhibits some differences in the age and size of holdings. The first substratum in the Western sector is the oldest cocoa growing area in Nigeria -- where farms tend to be very small, old and subject to mass infections of cocoa diseases. The Ife-Ilesha sub-stratum represents a mature cocoa zone -- not yet subject to mass casualty as the older area. The Ondo-Midwest zone as well as the Eastern zone are newest and farms here generally exhibit newness and increase in size over the older areas.

This stratification, it is hoped, will allow proportional sampling for inter-zone sampling, that is, the percentage of total subsample from each zone will depend on how many large producers can be estimated as coming from each zone. The last problem to be cleared is in regard to sampling within each stratum or substratum. This, really, is where the problem of sampling from unknown population emerges. The subsample size is not going to be fixed. What would be attempted are:

1. obtaining sampling units from each stratum or substratum.
2. obtaining sampling units from each area where government plantations are established (to make program comparisons as precise as possible).

Thus, depending on the data obtainable from licensed agents, it would be possible to determine whether a sample is coming from any given stratum or substratum. After knowing from which strata and substrata samples are coming, a simple random sampling procedure will be employed to select the ultimate sampling units. But the selection will be sequential in the sense that sampling over all strata will continue until enough units with reliable records are obtained. This is a crucial point, because it is difficult to guess at the beginning which farmers keep records -- and of those who keep, which ones are going to allow an inspection for required data. Also, the ratio of samples between private and government owned plantations cannot be precisely determined until one knows how many such private plantations exist. Thus, the whole sampling procedure rests on the location and number of private plantations over the cocoa zone as well as the reliability of data obtained from those who can be located.[1]

The discussion so far has concentrated on how to obtain the needed sample, but the raison d'etre for the

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[1]The sample of record keeping farmers will not be entirely representative of the universe being sampled but extrapolations can be made for non-record keeping farmers since record keepers are likely to be leaders in their communities. Thus the actions of non-record keepers would approximate the actions of the leaders.

sample selection is to be able to make comparisons among the various groups of private plantation holdings on one hand and between the private and publicly owned plantation groups on the other. For this delineation of different groups of private plantations, we could follow Orlan Buller (73) who suggested using net worth of farms, age of operators, desire and ability for technological improvement as more economically rational criteria for designation into relevant groups for the purpose of collecting samples for an L.P. analysis of farm reorganization.

We would not be able to follow these steps in this paper because (as stated earlier) we are sampling essentially from an unknown population. Things are turned around so that we shall have our sample (as outlined above) first and then group the units for the D.L.P. analysis on the basis of

1. annual capital surpluses each firm has experienced historically
2. type of operator (full or part time)
3. ability of operator to achieve technological improvement.

The age of operators is not considered as a criterion for grouping because both old and young operators regard cocoa holdings as long term investments which they can always pass on as inheritance to their offspring.

After this comparison among private plantations on their efficiency of resource use, an aggregative position of all private holdings can be established (either on an actual or projected basis) for comparison with the publicly-owned holdings. Designation of public plantations into categories need not be made since the number of such holdings is so small that such designated groups may not contain more than one orchard each.

Critique of the model

Like any techniques of analysis, linear programming has its own share of criticism. The most frequently referred to are:

1. Discontinuous nature -- it often happens that small variations in the data produces considerable variations in the structure of the model. This, however, is remedied by the development of paramatic linear programming technique where one focuses on solutions in the neighborhood of the optimum by varying one or more parameters in the data (61).
2. One of the early criticisms against linear programming is that it is static in nature, but dynamic L.P. was developed to take advantage of repetitions in investment problems based on the fact that certain structural relations recur

from one period to another.

3. Linearity -- it is contended that linearity in the norms posted and constraint variables may not lead to the best solutions every time. But, such assumptions can be abandoned either in the objective function (based on the nature of costs, revenues or profits) so as to take into account the phenomena of diminishing returns or utility; or in both the objective function and the constraints, which later leads to nonlinear programming techniques.
4. Divisibility assumption -- integer programming was developed in answer to the justified criticism that the nature of most investment projects preclude infinite divisibility with respect to their acquisition or operation. This, because most industrial and agricultural inputs are lumpy in nature. Thus, divisibility is an imperfect simplification of reality when each enterprise is dependent on a particular location (e.g., rich soil) even though it allows blocking out of main lines of investment so that one can obtain approximate solutions to a problem (by treating each asset as an entity). Thus, while its drawback is that it tends to compartmentalize the various categories (of assets) it does

have the valuable advantage of permitting discrete variations which makes possible avoidance of seriôus errors that may arise from the extrapolations of a marginal approach.

These are basic criticisms of any linear programming analysis of a given problem. There are some others, which are very specific to the stochastic dynamic linear programming model developed in this paper. The first and foremost is that the model is dynamic only in the dating sense. A more realistic formulation would be in Samuelson's sense of dynamism where functional relationship is taken into account, but at present, data for such a formulation may be very hard to come by except at a highly aggregated level. Secondly, only maximization of net revenue (in the short run) together with discounted present value (in the long run) are the main focus of this paper. This is not necessarily so, especially in the case of government establishments where political influence and social goals (employment, farmer education) may be very important in the location of such plantations. The third serious problem concerns the accuracy of data expected for running the model, especially in regard to private plantations. Even when one has succeeded in convincing the owners of his intentions, there are serious doubts as to whether one would get the very true picture of the whole enterprise. The fear is always lurking in their minds that government agents can always use the published data to revise tax calculations. Accuracy of data is needed for successful handling of the stochastic nature of the model. A way out of the situation would involve a promise that the source of whatever data is given will not be divulged under any condition. The quality and quantity of data will also dictate what activities to be defined, the nature of the constraints, the size of the format and the number of periods over which the program can be run. This also is true of what statistics to use as variables and constraints in the format. Enough of care and diligence would lessen if not eliminate most of these problems so as to be able to obviate a degenerate or infeasible program. The modifications of Hildebrand and mixed integer programming technique ensure endogenous fixity of economically fixed assets as well as investment solutions consistent with reality. Thus, the inbuilt postulates of fixed assets theory fixes the lower and upper bounds (endogenously) to solutions derivable from the program of the model.

To summarize, this paper has attempted to derive a model from which investment behaviors of large scale cocoa producers in Nigeria could be studied. The basic model has been applied in the United States. It is the opinion in this paper that such a model, enhanced by the modifications introduced, can be applied in Nigeria also.

This claim derives from the universal conclusions that farmers everywhere are not very far from the economic man, be they American or African farmers (74).

The strengths and shortcomings of the model were listed, and it is still the conclusion that the imperfection of the model should not preclude its being used. Farmers are not expected to follow any model results to the letter -- such results are only guides to rational behavior. The exercise here is expected to be a beginning of more intensive micro study of investment behavior in different sub-sectors of Nigerian agriculture.

It is to be hoped that refinements or more likely, simplifications in the model will be carried out as it is applied, and suspected as well as its hidden weaknesses are discovered and verified.[1]

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[1]A fuller discussion of this will be detailed in a follow-up work which will be undertaken very soon in Nigeria. The discussion is being delayed since the actual nature of data to be used in verifying the model is not yet known.

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