COMMODITY BONDS: A POTENTIAL RISK MANAGEMENT INSTRUMENT FOR CAPITAL CONSTRAINED COMMODITY PRODUCERS

by

Richard J. Ball

A PLAN B PAPER

Submitted to

Michigan State University

in partial fulfillment of the requirements

for the degree of

MASTER OF SCIENCE

Department of Agricultural Economics

1988

ABSTRACT

The comodity-linked bond offers a potential means for producers of primary goods both to raise capital and to hedge against output price risk. Commodity bonds are distinguished from conventional bonds in that their return structure is denominated in quantities of the underlying commodity. Optimal levels of bond issue and commodity production are derived, first for producers whose only source of capital is the revenue raised by issuing bonds, and then for producers who can raise capital either by issuing bonds or by borrowing commercially. The models used are extensions of the standard models of futures market hedging. Relatively less risk averse producers are found to be unwilling to pay the premium for transferring the risk implicit in bond sales, and finance production exclusively through commercial loans. More risk averse producers, however, choose to issue bonds. For such producers, the possibility of issuing commodity bonds leads to levels of output and of producer welfare higher than if conventional loans are the sole source of financing available.

ACKNOWLEDGEMENTS

I would like to thank my major professor, Dr. Robert Myers, without whose excellent guidance I could not have written this paper. Dr. Stanley Thompson also offered valuable support as a member of my guidance committee. Dr. Jack Meyer made several important suggestions as I developed the paper, and served on my examining committee.

I owe a great debt of gratitude to Amy Purvis for her moral (and nutritional) support. Thanks also to Laura Geis, who cheerfully and patiently drew the figures, while she surely had better things to do.

Special thanks are due to my brother Larry, who (to the delight of Ma Bell) provided much sage advice throughout the course of my Master's program.

I. Introduction

Uncertain commodity prices and lack of investment capital are two problems that plague developing country exporters of primary products. A potential instrument for mitigating both of these problems is the commodity-linked bond. The issuer of a commodity bond receives a cash payment upon initial sale of the bond and, at maturity, pays the value of a specified quantity of the commodity to the bond holder. Dividend payments, if any, may also be linked to commodity values. The principal feature distinguishing commodity bonds from conventional bonds is their commodity dominated return structure (Schwartz, 1982; Lessard and Williamson, 1985).

This paper investigates whether commodity bonds can offer producers an effective means of raising capital and hedging against output price risk. Two issues are examined. First, the optimal levels of commodity production and bond issue are determined for a producer who has no initial wealth and no conventional source of investment funds. Second, the assumption of no initial wealth is maintained but producers are provided with the opportunity to take out conventional loans, as well as to issue commodity bonds. In this case interest will center not only on the output and bond issue decisions, but also on the conditions under which issuing bonds or borrowing commercially will be the dominant strategy for raising capital. The models used to investigate these issues are extensions of the standard model of futures market hedging (Danthine, 1978; Holthausen, 1979; Feder, Just and Schmitz, 1980; and Kahl, 1983). Issuing commodity bonds is similar to selling futures contracts since, in both cases, the producer transfers some of the risk of a change in commodity price to the purchaser of the contract or bond.

In the next section, existing models of hedging on futures markets are briefly reviewed. These hedging models form a basis for the commodity bond analysis that follows. The behavior of a producer facing output price uncertainty is then examined under three different scenarios. The first scenario is that commodity bonds are the only source of investment funds, the second is that conventional loans are the only source of funds, and the third is that both commodity bonds and conventional loans are available. Concluding comments summarize the potential of commodity bonds for mitigating commodity price risks and providing investment capital.

II. Models of Hedging on Futures Markets

The problem of a commodity producer using futures contracts to hedge against price risk has received considerable study. Feder, Just and Schmitz (1980), for instance, analyze the problem in an expected utility framework, and Robison and Barry (1987) discuss the problem using a mean-variance approach.

In these models, revenue is earned by selling the quantity produced, q, at a random price, $\tilde{p}$, and by selling futures contracts, b, at a known price, r. Total costs consist of production costs and the cost of buying back the futures contracts after the random price has been realized. Random profit is thus denoted

$$(1) \qquad \tilde{\pi} = rb - c(q) + \tilde{p}(q-b)$$

where c(q) is a strictly convex cost function.

Two main results have been established in the literature.

Result 1 The optimal level of output, q^*, satisfies $c'(q^*) \equiv r$.

Optimal output thus depends only on the price of the futures contract. Feder, Just and Schmitz (1980) term the independence of optimal output from all parameters other than the futures price as the "separation property."

Result 2 The optimal quantity of futures contracts sold relative to the optimal level of production depends on the degree of bias in the futures market. If $\bar{p} > (=,<)\ r$, then $b^* < (=,>)\ q^*$, where $\bar{p}$ is the expected output price and b^* is the optimal sale of futures contracts.

These results can be illustrated graphically in mean-standard deviation space using the approach of Meyer and Robison (1988). The mean, μ, and standard deviation, σ, of profits are given by

$$(2a) \quad \mu = rb - c(q) + \bar{p}(q-b)$$

$$(2b) \quad \sigma = |q - b|\sigma_p$$

where σ_p is the standard deviation of the output price. Meyer and Robison (1988) show that, in this model of hedging under output price uncertainty, the behavior of an expected utility maximizing producer is consistent with maximization of some function, $V(\mu,\sigma)$, of the mean and standard deviation of profits. The properties of $V(\mu,\sigma)$ are outlined in detail in Meyer (1987).

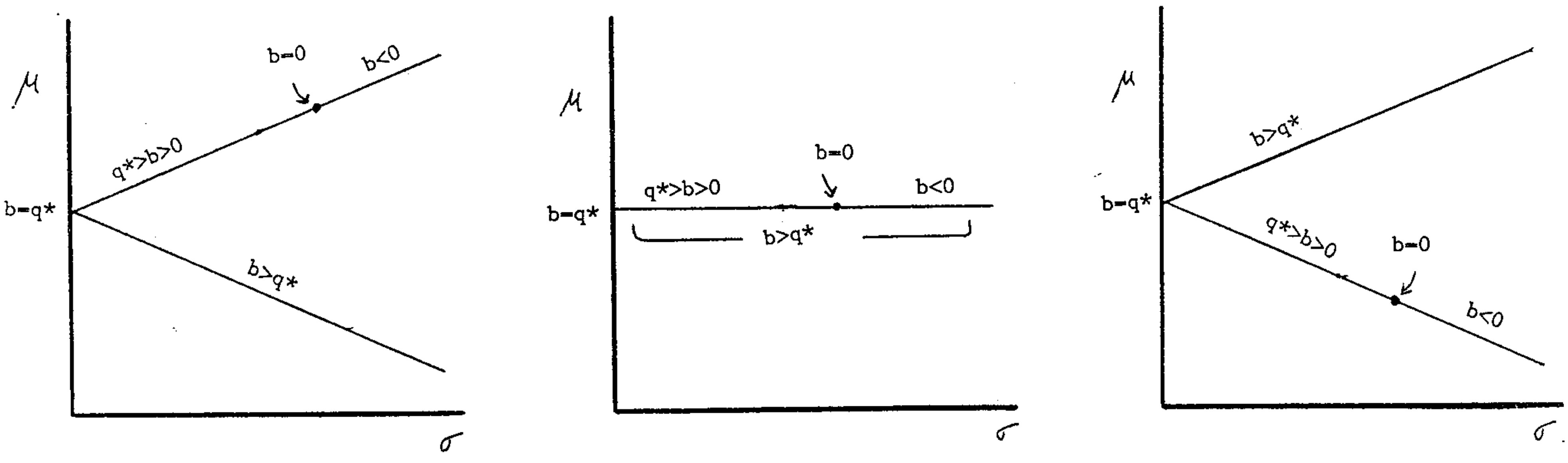
Result 1 above follows directly from differentiating $V(\mu,\sigma)$ with respect to q and b, and then manipulating the first order conditions. Result 2 is obtained by observing that the producer's opportunity set is comprised of points representing different levels of b, given that the quantity produced is the q* defined in Result 1. Thus, the slope of the opportunity set is defined

$$(3) \qquad \frac{d\mu}{d\sigma} = \pm \left|\frac{\bar{p}-r}{\sigma_p}\right|.$$

Equations (2) and (3) completely characterize the producer's opportunity set. Three cases can arise, as indicated in figure 1. Each case reflects a different direction of bias in the futures market.

First, $\bar{p}$ may be greater than r (see figure 1 part a). In this case, b=q* provides a deterministic profit so σ=0 at this point. Furthermore, as b falls below q*, the opportunity set slopes upward since the mean and standard deviation of profit both increase. As b rises above q*, the opportunity set slopes downward since higher b provides lower mean profit. With the usual positively sloped indifference curves, the optimal b will satisfy b*<q*. Second, $\bar{p}$ may equal r (see figure 1 part b). In this case, the opportunity set has zero slope since mean profit is not affected by the choice of b. Therefore, b is set equal to q* so that σ=0 and profits are known with certainty. Third, $\bar{p}$ may be less than r (see figure 1 part c). In this case, σ again equals 0 where b=q*. In contrast to the first case, however, when b falls below q*, the opportunity set slopes downward since mean profit declines. And as b rises above q*, the opportunity set slopes upward since mean profit and its standard deviation both rise. With positively sloped indifference curves, the optimal b* will therefore satisfy b*>q*.

FIGURE 1



(a) $\bar{p}>r$

(b) $\bar{p}=r$

(c) $\bar{p}<r$

III. Use of Commodity Bonds by Capital Constrained Producers

Suppose a producer has no futures market available for the commodity being produced, but output price is random at the time resource allocation decisions are made. The producer holds no initial wealth, but can raise investment capital by issuing commodity bonds, b, at some price, r, in the resource allocation period. The bonds mature at the time output is produced, at which time the producer buys them back at price p.[1] In this case the random profit of the producer can be written

$$(4a)\quad \tilde{\pi} = (1+i)[rb - c(q)] + \tilde{p}(q-b)$$

subject to

$$(4b)\quad rb \geq c(q)$$

$$(4c)\quad b \geq 0$$

where i is the interest rate and, as before, c(q) is a strictly convex cost function.

The commodity bond issuer's profit function differs in four important ways from the profit function of the futures market hedger. First, the timing of payments is different. The issuer of commodity bonds receives an immediate payment, some of which can be used to finance the cost of production, and the remainder of which (if any) can be invested risklessly at the interest rate i. This characteristic of commodity bonds is reflected in (4a) where final profit

[1]For simplicity, we suppose that commodity bonds bear no coupon before maturity.

equals the return on excess funds invested at rate i, plus the difference between the value of the commodity and the value of the bonds after the output price has been realized. Unlike issuing commodity bonds, selling futures contracts does not generate initial capital and so the producer must find an alternative way of financing production costs.

The second distinguishing characteristic of the commodity bond model is the inequality constraint (4b). This constraint implies that funds raised from selling commodity bonds are the only source of investment funds available to finance production costs. It reflects the situation of a capital constrained developing country that has exhausted conventional means of investment financing.

The third important aspect of the commodity bond problem is the non-negativity constraint (4c). Since futures contracts are standardized and traded actively on organized exchanges, producers are able either to purchase or to sell futures. Commodity bonds, however, are nonstandard contracts not traded on active markets. Producers, therefore, may not be able to purchase bonds linked to the commodity they produce.

Fourth, investors are assumed to be willing to purchase commodity bonds only if their time discounted expected value at maturity, $\bar{p}/(1+i)$, exceeds the price of a bond, r. The bond issuer effectively pays the bond holder a premium, $\bar{p}/(1+i) - r$, for bearing the risk associated with changes in output price.

The producer is assumed to have a strictly concave utility function and to maximize expected utility of profits. However, since the random profit (3a) satisfies Meyer's (1987) location and scale condition, maximizing a function, $V(\mu,\sigma)$, of the mean and standard deviation of profits is equivalent to maximizing expected utility. Meyer (1987) shows that if the utility function is increasing and strictly concave, then V is increasing in μ, decreasing in σ, and strictly concave in both its arguments. The advantage of analyzing the problem using the $V(\mu,\sigma)$ function is that graphical analysis in mean-standard deviation space provides a useful means of presenting and interpreting results.

The producer's problem is to choose q and b to maximize $V(\mu,\sigma)$ subject to

(5a) $\mu = (1+i)[rb - c(q)] + \tilde{p}(q-b)$

(5b) $\sigma = |q - b|\sigma_p$

(5c) $rb - c(q) \geq 0$

(5d) $b \geq 0;\ q \geq 0.$

The Kuhn-Tucker conditions for a solution to this inequality constrained maximization problem are given in appendix 1. Two solutions can arise. First, if the capital constraint is not binding, then the optimal levels of output, q^*, and bond issue, b^*, are identical to those defined in Results 1 and 2 above. Second, if the capital constraint is binding, the optimal level of output remains at q^* as defined in Result 1. The imposition of the capital constraint does not alter the "separation property" governing the production decision. The level of bond issue, however, is increased from that defined in Result 2 to a level, b^c, just sufficient to satisfy the capital constraint, so that $b^c = c(q^*)/r$. The capital constraint is thus satisfied by an increase in bond sales rather than a decrease in production.

In the following section, the conditions under which each of these solutions will hold are demonstrated graphically.

IV. Graphical Analysis of the Commodity Bond Problem

The opportunity set of the commodity bond issuer (see figure 2) is a modification of that of the unconstrained hedger as presented in section II. The introduction of the time discount factor means that the slope of the opportunity set is now

$$(6) \qquad \frac{d\mu}{d\sigma} = \pm \left|\frac{\bar{p} - (1+i)r}{\sigma_p}\right|.$$

Further, the assumption that the price of a bond will always be less than its time discounted expected value rules out the cases of zero or positive bias illustrated in figure 1 parts b and c. Most importantly, the capital constraint eliminates from the opportunity set all values of b less than $c(q^*)/r$. Appendix 2 demonstrates that this constraint truncates the opportunity set at some level of b satisfying $q^*>b>0$.

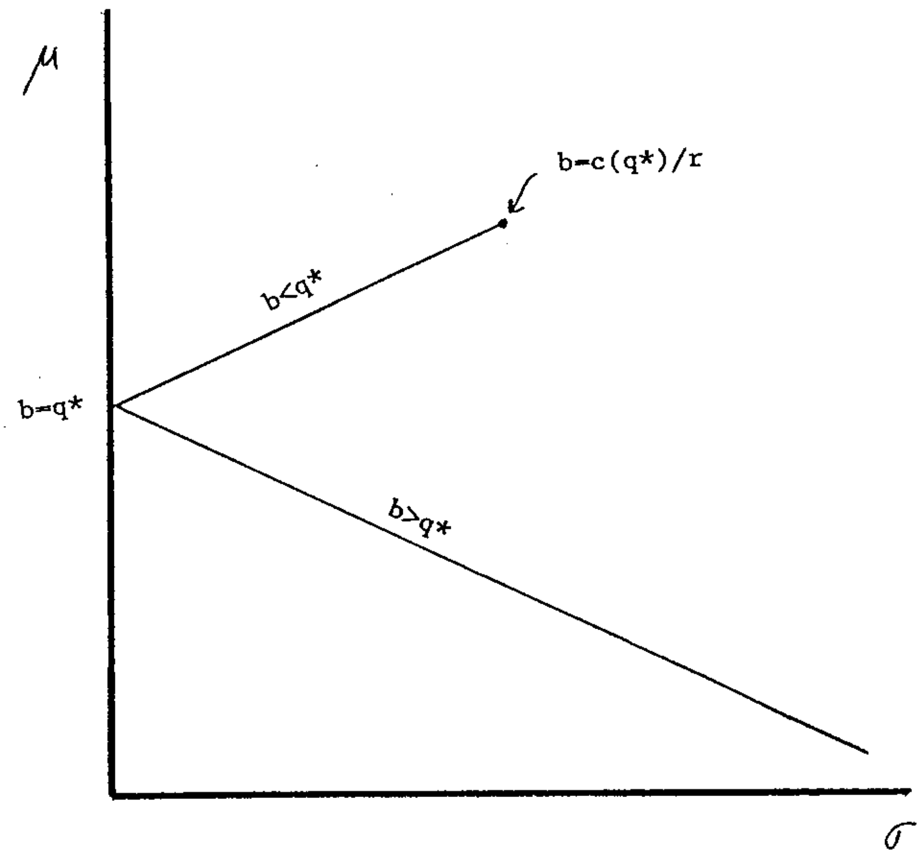
The indifference curves of an agent who maximizes the function $V(\mu,\sigma)$ will have slope $-V_\sigma/V_\mu$ where V_σ and V_μ represent partial differentiation with respect to the subscripted variable. Since $V(\mu,\sigma)$ is increasing in μ and decreasing in σ, this slope will be positive.

Whether or not the capital constraint is binding depends on the slope of the producer's indifference curves relative to the slope of the opportunity set. Appendix 1 establishes an identity (1.4) that holds when the constraint is not binding:

$$(7) \qquad \frac{\bar{p} - (1+i)r}{\sigma_p} \equiv \frac{-V_\sigma(q^*,b^*)}{V_\mu(q^*,b^*)}$$

where $V_\sigma(q^*,b^*)$ and $V_\mu(q^*,b^*)$ indicate evaluation of the partial derivatives at the point (q^*, b^*). This identity equates the slope of the opportunity set

FIGURE 2



p/(1+i)>r

with the slope of an indifference curve (see figure 3 part a). The resulting tangency occurs on a portion of the opportunity set not eliminated by the capital constraint, and represents a solution identical to that of the unconstrained case.

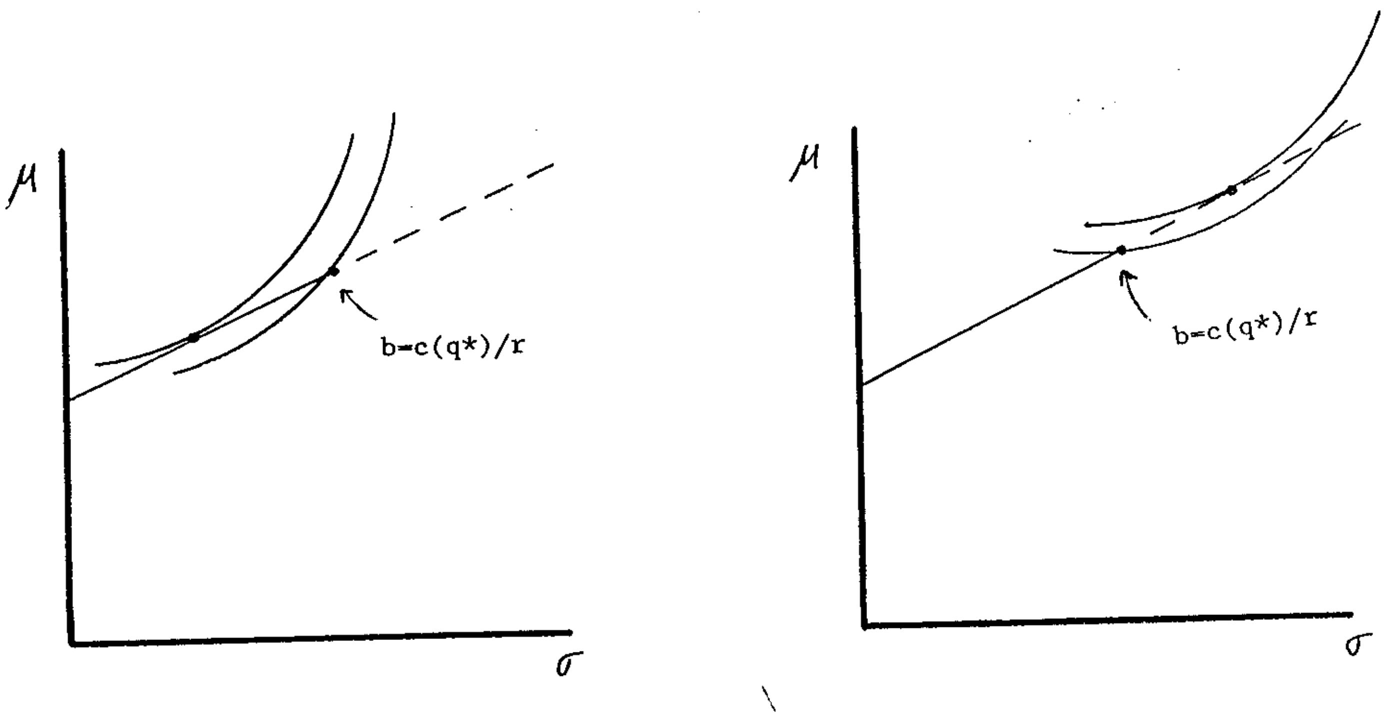
Appendix 1 also establishes an inequality (1.7) that holds when the capital constraint is binding:

$$(8) \qquad \frac{\bar{p} - (1+i)r}{\sigma_p} > \frac{-V_\sigma(q^*,b^c)}{V_\mu(q^*,b^c)}.$$

Note that b^c is the point at which the capital constraint truncates the opportunity set, since by definition $b^c=c(q^*)/r$. This inequality thus states that, at its boundary, the opportunity set is more steeply sloped than the producer's indifference curve (see figure 3, part b). In this case, the unconstrained solution tangency would occur in the region eliminated by the capital constraint. The producer must therefore increase its bond issue to b^c, and the illustrated boundary solution results.

The indifference curves in figure 3 part a are, at all points in mean-standard deviation space, steeper than the indifference curves in figure 3 part b. The flatter indifference curves in part b reflect less willingness to accept a decrease in the mean of profit for a decrease in the variability of profit. A producer with these preferences is less willing to pay the premium for transferring price risk implicit in bond sales. The capital constraint, however, obliges such a producer to issue at least enough bonds to cover production costs.

FIGURE 3



(a) (b)

Notes:

1) The downward sloping portion of the opportunity set has been omitted from theses figures because this region is irrelevant to a producer with upward sloping indifference curves.

2) The dashed region represents points eliminated by the capital constraint.

V. Comparative Statics

The optimal output, q^*, of the commodity bond issuer always satisfies $c'(q^*)=r$, where the price of a bond, r, equals its time discounted expected value less the premium paid to the bond holder for bearing the associated price risk. Denoting this risk premium as γ, we have the identity

$$(9) \qquad r \equiv p/(1+i) - \gamma.$$

Optimal output q^* therefore satisfies $c'(q^*) = \bar{p}/(1+i) - \gamma$. Comparative statics of output can thus be performed with respect to p, γ, and i.

Differentiation yields

$$(10a) \qquad \frac{\partial q^*}{\partial \bar{p}} = [(1+i)c''(q^*)]^{-1} > 0$$

$$(10b) \qquad \frac{\partial q^*}{\partial \gamma} = -[c''(q^*)]^{-1} < 0$$

$$(10c) \qquad \frac{\partial q^*}{\partial i} = -\bar{p}[(1+i)^2 c''(q^*)]^{-1} < 0.$$

These results are intuitively reasonable. An increase in $\bar{p}$, the expected price of output, leads to an increase in output; an increase in γ, the cost of hedging against price changes, leads to a decrease in output; and an increase in i, the rate of return on a riskless investment, decreases the level of production of the risky commodity.

Performing comparative statics on the optimal level of bond issue yields derivatives of ambiguous sign, both for the unconstrained solution b^* and the constrained solution b^c. Changes in the parameters change the slope of the opportunity set and generate new solution tangencies, but the assumptions made about the properties of $V(\mu,\sigma)$ are insufficient to determine how the optimal level of b will change as the opportunity set rotates.

VI. Conventional Loans

In this section, a simple model is developed in which the producer can finance the costs of production only by borrowing commercially. The results obtained are useful in the analysis of the final model in which both commodity bonds and conventional financing are available.

Suppose again that a commodity producer holds no initial wealth to invest in production, but assume that money can be borrowed commercially at interest rate i. No mechanism for transferring price risk is available. The producer's random profit function is thus

$$(11) \quad \tilde{\pi} = (1+i)[m-c(q)] + \tilde{p}q - (1+i)m$$

where m represents the amount of money borrowed and all other terms are as in the preceding model of the bond issuer. Any excess of funds borrowed over costs of production earns a rate of return equal to the rate of interest paid on borrowed capital. This is equivalent to the assumption of a perfect capital market.

Because of the perfect capital market assumption, the profit function reduces to

$$(12) \quad \tilde{\pi} = -(1+i)c(q) + \bar{p}q.$$

This is simply a time-discounted formulation of the standard problem of a producer choosing an output level under price uncertainty (see, for example, Robison and Barry, 1987).

The existence of a perfect capital market effectively eliminates the capital constraint. The producer can choose its optimal level of output, and then borrow the funds necessary to finance that level of production. Because the profit function is not affected by the value of m chosen, the producer is indifferent among all levels of borrowing.

The producer will thus choose q to maximize $V(\mu,\sigma)$ subject only to

$$\text{(13a)} \quad \mu = -(1+i)c(q) + \bar{p}q$$

$$\text{(13b)} \quad \sigma = q\sigma_p.$$

The first order condition for a maximum can be rearranged to yield an identity defining the optimal level of output q^m:

$$\text{(14)} \quad \frac{\bar{p} - (1+i)c'(q^m)}{\sigma_p} \equiv \frac{-V_\sigma(q^m)}{V_\mu(q^m)}.$$

This identity can be rewritten as

$$\text{(15)} \quad \frac{\bar{p}}{1+i} \equiv c'(q^m) - \frac{\sigma_p V_\sigma(q^m)}{(1+i)V_\mu(q^m)}.$$

The producer thus equates time discounted expected marginal revenue with the marginal cost of production, plus a term dependent on the variability of output price and on the slope of the producer's indifference curve. This term represents the implicit marginal cost to the producer of bearing the risk of uncertain output price. We will define this term to be the producer's risk premium, denoted as ω, where

$$(16) \qquad \omega \equiv - \frac{\sigma_p V_\sigma(q^m)}{(1+i)V_\mu(q^m)}.$$

VII. Graphical Analysis of Conventional Loans

These results can again be illustrated graphically in mean-standard deviation space. Solving (13b) for q as a function of σ and substituting this expression for q into (13a), we can write μ as a function of σ:

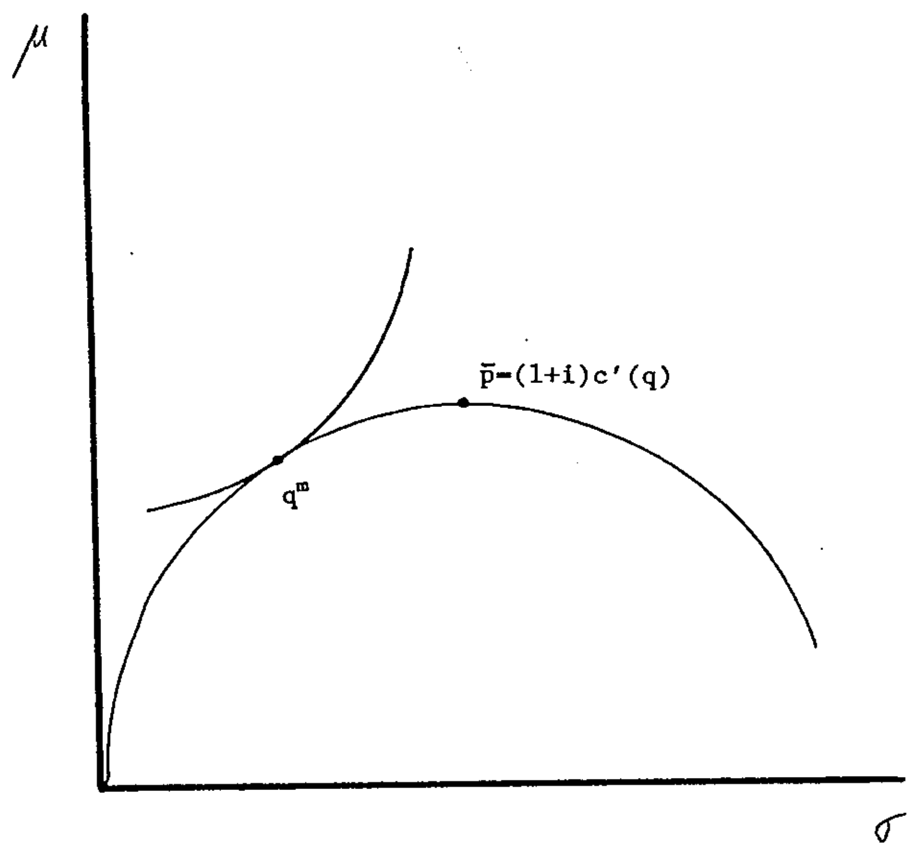
$$(17) \qquad \mu(\sigma) = \bar{p}\frac{\sigma}{\sigma_p} - (1+i)c(q).$$

Each point in the opportunity set will represent a different value of q. Since c(q) is strictly convex, $\mu(\sigma)$ is strictly concave. Furthermore, $\mu(\sigma)$ passes through the origin, and reaches a maximum at the level of q satisfying $p=(1+i)c'(q)$. The slope of the opportunity set, which varies with q, is obtained by taking

$$(18) \qquad \frac{d\mu}{d\sigma} = \frac{\bar{p} - (1+i)c'(q)}{\sigma_p}.$$

The first-order condition, (14), thus defines a tangency between the opportunity set and the producer's indifference curve (see figure 4).

FIGURE 4



VIII. Commodity Bonds and Conventional Loans

When the producer can both issue commodity bonds and borrow commercially, the random profit function is

$$(19) \qquad \tilde{\pi} = (1+i)[m + rb - c(q)] + \tilde{p}(q-b) - (1+i)m$$

where all terms are as previously defined. Money is raised initially through loans and bond sales. Any revenue thus generated in excess of costs of production earns the rate of interest i. Once production is complete and output price has been realized, the producer receives the difference between the value of the commodity produced and the value of the bonds issued. Any loans taken out must also be repaid with interest.

As in the previous section, the assumption of a perfect capital market reduces the profit function to

$$(20) \qquad \tilde{\pi} = (1+i)[rb - c(q)] + \tilde{p}(q-b)$$

and the producer is indifferent among all levels of borrowing.

The producer's problem is now to choose q, b, and m to maximize $V(\mu,\sigma)$ subject to

$$(21a) \qquad \mu = (1+i)[rb - c(q)] + \bar{p}(q-b)$$

$$(21b) \qquad \sigma = |q - b|\sigma_p$$

$$(21c) \qquad rb + m - c(q) \geq 0$$

$$(21d) \qquad b \geq 0;\ q \geq 0;\ m \geq 0.$$

This problem is identical to that of the producer who can raise capital only by issuing commodity bonds, except that in this version the capital constraint (21c) also allows the producer to borrow commercially. The Kuhn-Tucker conditions for this problem are given in appendix 3. Three solutions can arise.

In the first case, production is financed exclusively through commercial loans. When loans are the chosen source of capital, the producer effectively solves the problem of the producer whose only means of raising funds is commercial borrowing (section VI). Output is chosen at the level q^m as defined in (14), and a level of borrowing m is chosen to satisfy $m \geq c(q^m)$.

The cases in which commodity bonds are issued, either as the exclusive capital raising instrument or in conjunction with commercial loans, correspond to the problem of the producer who can finance production only by issuing bonds (section III). The solution levels of output and bond issue are identical to the q^* and b^* found in the case of the bond issuer for whom the capital constraint is not binding, whether bonds are used alone or in conjunction with borrowing. When rb^*, the revenue raised by issuing b^* bonds, is at least as great as $c(q^*)$, no loans need be taken out, so $m=0$. When $rb^*<c(q^*)$, however, the revenue raised by issuing b^* bonds is insufficient to meet the cost of producing q^*, and funds must be borrowed commercially at a level satisfying $m \geq c(q^*) - rb^*$.

In the following section, a graphical analysis is used to illustrate these three cases and to interpret the conditions under which each solution will arise.

IX. Graphical Analysis of Commodity Bonds and Commercial Loans

When issuing commodity bonds and borrowing commercially are both possible, the producer's opportunity set is comprised of two sets of points (see figure 5). One portion of the opportunity set, identical to figure 4, represents different levels of output, assuming that capital is raised exclusively through commercial loans. Another portion, corresponding to figure 2, represents different levels of bond issue. Although the capital constraint truncates the opportunity set in figure 2 at $b=c(q^*)/r$, the availability of commercial loans allows the producer to choose any non-negative bond issue and to finance excess production costs with loans. In this case, therefore, the opportunity set extends to the point at which $b=0$.

These two portions of the opportunity set meet and are tangent when $q=q^*$ in the borrowing set and $b=0$ in the bond issuing set. At these values of the choice variables, the mean and standard deviation of the bond issue profit function (5a and b) are identical to the mean and standard deviation of the commercial borrowing profit function (13a and b), and the slope of the borrowing opportunity set (18) is equal to the slope of the bond issuing opportunity set (16).

Figure 6 illustrates the three solutions found in the previous section. Whether commercial loans or bond issues are used to finance production depends on whether output q^m is greater than or less than q^*. When $q^m \geq q^*$ (see figure 6 part a), the solution tangency occurs in the region of the opportunity set representing commercial borrowing. When $q^m < q^*$, the tangency in the borrowing region is dominated by a tangency in the bond issuing region (see figure 6 parts b and c). If the bond issuing tangency occurs at a level of b such that $b > c(q^*)/r$ (figure 6 part b), then the revenue raised by issuing bonds is sufficient to meet the capital constraint, and commercial borrowing is not

FIGURE 5

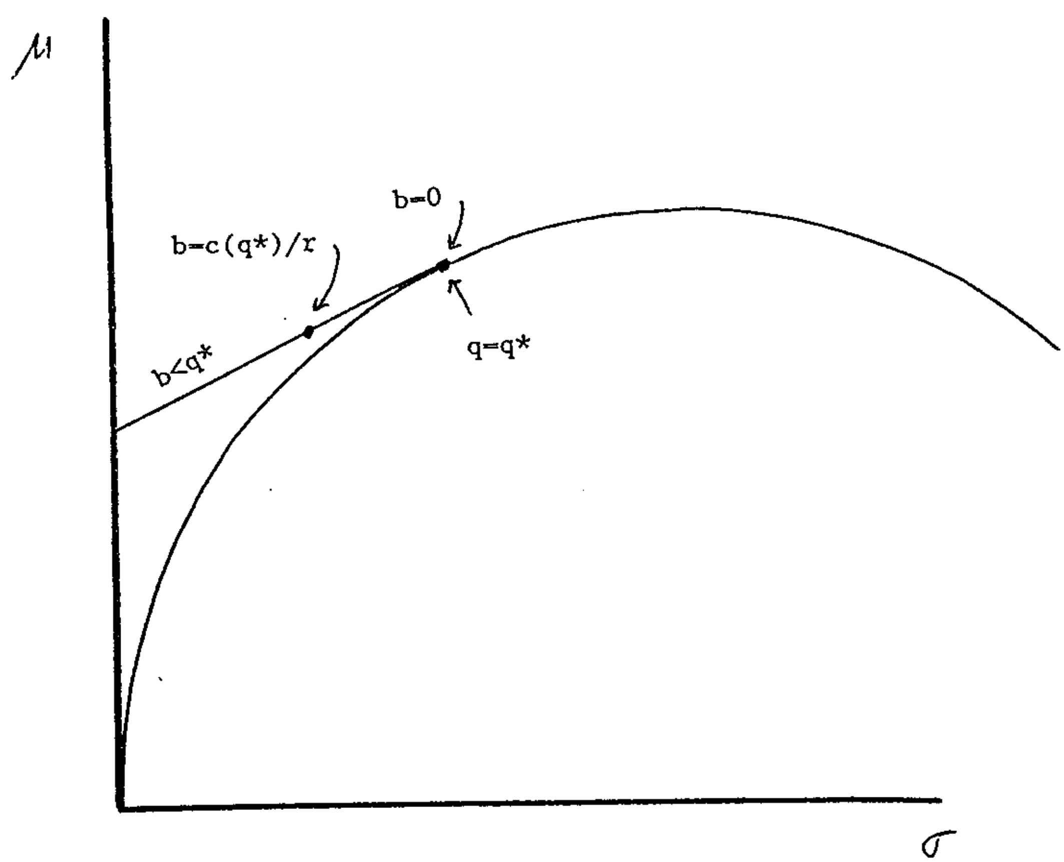
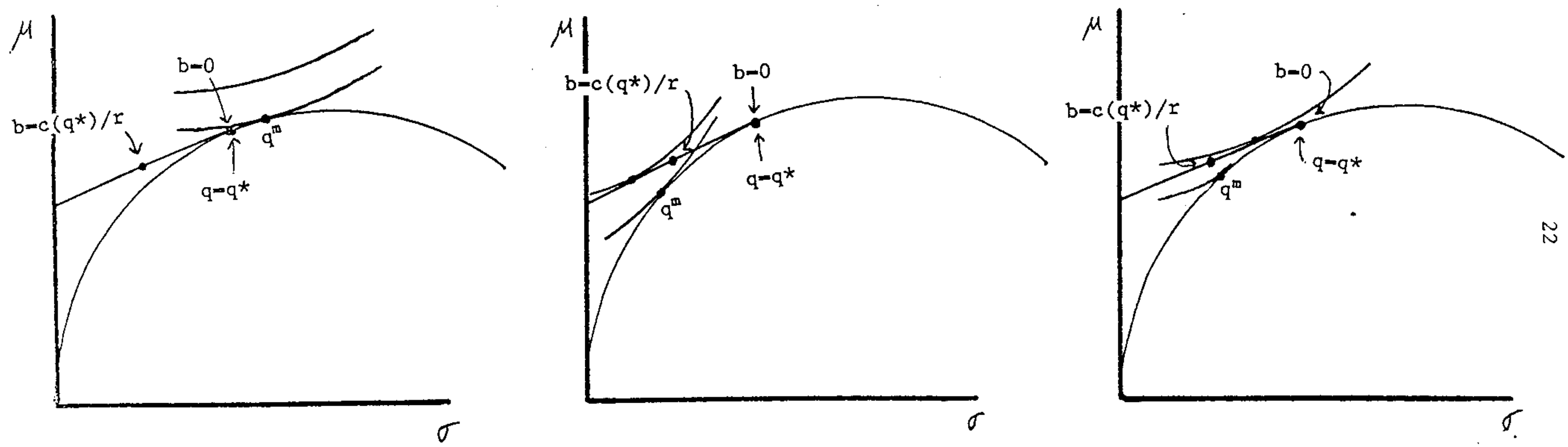


FIGURE 6



(a) Loans Used Exclusively

(b) Bonds Used Exclusively

(c) Bonds Supplemented With Loans

necessary. If, however, the bond issuing tangency occurs at a level of b not sufficient to cover production costs (figure 6 part c), then the bond issue must be supplemented with commercial loans.

Appendix 4 shows that the inequality $q^m \geq q^*$, which implies that loans will be the dominant capital raising instrument, also implies that the inequality

$$(22)\qquad \frac{\bar{p}}{1+i} - r \geq - \frac{\sigma_p V_\sigma(q^m)}{(1+i)V_\mu(q^m)}$$

will hold. Using the notation introduced for the risk premium on the bond market and the producer's risk premium, this inequality can be rewritten simply as $\gamma \geq \omega$.

This result has an obvious interpretation. When γ, the premium that the producer must pay to induce commodity bond purchasers to assume the risk of changes in commodity price, is greater than ω, the cost to the producer of bearing the price risk, the producer will choose not to transfer the risk. No bonds will be issued. All capital needed to cover the cost of production must therefore be raised through commercial loans.

The converse of this argument holds for the solutions in which $q^m < q^*$ and bonds are issued (see appendix 4). In these cases, $\gamma < \omega$, and so the producer chooses to transfer price risk to investors by issuing commodity bonds. Supplementary loans are used only if the solution level of bond issue raises capital insufficient to cover production costs.

At all points in mean-standard deviation space, the indifference curves of producers who choose to finance production exclusively through loans (see figure 6 part a) are less steeply sloped than those of producers who choose to issue commodity bonds (see figure 6 parts b and c). Commercial borrowers are thus less risk averse than bond issuers, in the sense that they are less willing to trade off expected profit for a decrease in the variability of profits. Correspondingly, steeper indifference curves imply a greater producer's risk premium, since this premium is proportional to the slope of the indifference curve at the tangency q^m. It is the producers with steeper indifference curves and greater risk premia that choose to pay bond purchasers to assume price risk.

For the producers who choose to issue bonds, the inequality $q^m<q^*$ indicates that the level of output chosen exceeds the level of output that would be chosen if commercial loans were the only available source of financing. The fact that the solution tangency in the bond issuing region of the opportunity set dominates the tangency in the borrowing region also indicates that the welfare of such producers is greater when they are able to issue commodity bonds than when they must rely exclusively on commercial loans to raise capital.

X. Summary and Conclusions

The literature on futures market hedging has established optimal levels of output and futures positions for producers using futures contracts as a hedge against price risk. Section III of this paper developed a similar problem for producers issuing commodity bonds, with the added constraint that any funds invested in production be raised through bond sales. For producers relatively unwilling to bear price risk, the optimal bond issue raised revenue sufficient to meet the cost of producing the associated solution output. These producers could use commodity bonds to hedge against price risk exactly as in the standard hedging models, and the capital constraint did not bind.

Producers less averse to bearing price risk, however, were less willing to pay the premium for transferring risk implicit in bond sales. Unconstrained, such producers would choose a level of bond issue insufficient to meet production costs. The optimal strategy for such producers was to set output at the unconstrained level, and to increase bond sales to a level just great enough to satisfy the capital constraint. The need to raise capital obliged these producers to take out a larger hedge than they would have done otherwise.

Section VIII developed a model in which both commodity bonds and conventional loans were available as capital raising instruments. Which of these instruments was chosen depended on producer attitudes towards risk bearing. Less risk averse producers were unwilling to pay the premium for transferring risk implicit in bond sales, and financed production exclusively through commercial loans. Commodity bonds, however, were used by more risk averse producers, for whom the cost of bearing risk exceeded the cost of transferring it to bond purchasers. For such producers, the possibility of issuing commodity bonds led to levels of output and of producer welfare higher than if conventional loans had been the sole source of financing available. These producers used commodity bonds to hedge against price risk in exactly the manner of the futures market hedger described in the standard models. The availability of commercial loans effectively eliminated the capital constraint.

A useful extension of this analysis would be to develop a multi-period model of a producer making sequential output and bond issue decisions. A dynamic model could be used to derive an optimal strategy over many, perhaps overlapping, cycles of bond sales and investment in production. A dynamic approach would also add flexibility by allowing the maturity of the bonds to differ from the length of the production process.

REFERENCES

Danthine, Jean-Pierre. "Information, Futures Prices and Stabilizing Speculation." Journal of Economic Theory 17(1978): 79-98.

Feder, Gershon, Richard E. Just, and Andrew Schmitz. "Futures Markets and the Theory of the Firm Under Price Uncertainty." Quarterly Journal of Economics 94 (1980): 317-28.

Holthausen, Duncan M. "Hedging and the Competitive Firm Under Price Uncertainty." American Economic Review 69(1979): 989-95.

Kahl, Kandice H. "Determination of the Recommended Hedging Ratio." American Journal of Agricultural Economics 65(1983): 603-05.

Lessard, Donald R. and John Williamson. Financial Intermediation Beyond the Debt Crisis, Washington, DC: Institute for International Economics, 1985.

Meyer, Jack. "Two-Moment Decision Models and Expected Utility Maximization." American Economic Review 77(1987): 421-30.

Meyer, Jack and Lindon J. Robison. "Hedging and Price Randomness." American Journal of Agricultural Economics 70(1988): 268-80.

Robison, Lindon J. and Peter J. Barry. 1987. The Competitive Firm's Response to Risk. New York: Macmillan Publishing Company.

Schwartz, Eduardo S. "The Pricing of Commodity-Linked Bonds." Journal of Finance 37(1982):525-41.

APPENDIX 1

The Lagrangian for the problem of the capital constrained commodity bond issuer, presented in section III, is

$$(1.1) \qquad L = V(\mu,\sigma) - \lambda[c(q) - rb]$$

where λ is the Lagrange multiplier.

The Kuhn-Tucker conditions for this problem are

$$(1.2a) \qquad \frac{\partial L}{\partial q} = V_\sigma \sigma_p + V_\mu[\bar{p} - (1+i)c'(q)] - \lambda c'(q) \le 0; \; q \ge 0; \; q\frac{\partial L}{\partial q} = 0$$

$$(1.2b) \qquad \frac{\partial L}{\partial b} = V_\sigma(-\sigma_p) + V_\mu[-\bar{p} + (1+i)r] + \lambda r \le 0; \; b \ge 0; \; b\,\frac{\partial L}{\partial b} = 0$$

$$(1.2c) \qquad \frac{\partial L}{\partial \lambda} = -c(q) + rb \ge 0; \; \lambda \ge 0; \; \lambda\frac{\partial L}{\partial \lambda} = 0$$

where V_σ and V_μ indicate partial differentiation with respect to the subscripted variable. These conditions are necessary and sufficient for a maximum since V is concave, c is convex, and the constraint qualification is satisfied.

There are two interesting cases.

<u>Solution A: Captial Constraint Not Binding($q>0$, $b>0$, $\lambda=0$)</u>

In this case the Kuhn-Tucker conditions reduce to

$$(1.3a) \qquad V_\sigma \sigma_p + V_\mu[\bar{p} - (1+i)\ c'(q)] = 0$$

$$(1.3b) \qquad V_\sigma(-\sigma_p) + V_\mu[-\bar{p} + (1+i)r] = 0$$

$$(1.3c) \qquad c(q) - rb \le 0.$$

Adding (1.3a) and (1.3b) defines the optimal output as a q^* that satisfies $c'(q^*)=r$. Rearranging (1.3b) then yields an identity defining the optimal bond issue:

$$(1.4) \qquad \frac{\bar{p} - (1+i)r}{\sigma_p} \equiv \frac{-V_\sigma(q^*,b^*)}{V_\mu(q^*,b^*)}$$

where $V_\sigma(q^*,b^*)$ and $V_\mu(q^*,b^*)$ indicate evaluation of the partial derivatives at the point (q^*, b^*).

Condition (1.3c) indicates that the capital constraint does not bind, as reflected in the zero value of the Lagrange multiplier. The solution q^* and b^* are thus identical to those of the unconstrained hedger, described in Results 1 and 2.

Solution B: Capital Constraint Binding ($q>0$, $b>0$, $\lambda>0$)

In this case, the Kuhn-Tucker conditions reduce to

$$(1.5a) \qquad V_\sigma\sigma_p + V_\mu[\bar{p} - (1+i)c'(q)] - \lambda c'(q) = 0$$

$$(1.5b) \qquad V_\sigma(-\sigma_p) + V_\mu[-\bar{p} + (1+i)r] + \lambda r = 0$$

$$(1.5c) \qquad c(q) - rb = 0.$$

Adding (1.5a) and (1.5b) again defines optimal output as q^* satisfying $c'(q^*)\equiv r$.

Condition (1.5c) indicates that the capital constraint is binding, as reflected in the positive value of the Lagrange multiplier. Optimal output b^c is thus defined by the identity

$$(1.6) \qquad b^c \equiv \frac{c(q^*)}{r}.$$

Noting that λ and r are both positive and rearranging (1.5b) establishes an inequality useful in the graphical analysis of the problem

$$(1.7) \qquad \frac{\bar{p} - (1+i)r}{\sigma_p} > \frac{-V_\sigma(q^*, b^c)}{V_\mu(q^*, b^c)}.$$

APPENDIX 2

In the model of the capital constrained commodity bond issuer presented in section III and IV, the capital constraint eliminates from the producer's opportunity set all values of b less than $c(q^*)/r$. This constraint thus truncates the opportunity set at the point at which $b=c(q^*)/r$.

Proposition The b satisfying $b=c(q^*)/r$ satisfies $0<b<q^*$.

Proof This b is clearly positive, since costs and the price of a bond are always positive. Since the cost function is convex, average cost is always less than marginal cost. Therefore, $c(q^*)<q^*c'(q^*)$. Recalling that by definition $c'(q^*)=r$ and rearranging terms, this implies that $c(q^*)<rq^*$. Thus when $b=c(q^*)<r$ we have $b<q^*$.

APPENDIX 3

The Lagrangian for the problem of the producer who can both issue bonds and borrow commercially, presented in section VIII, is

$$(3.1) \qquad L = V(\mu,\sigma) - \lambda[c(q) - rb - m].$$

The Kuhn-Tucker conditions for this problem are

$$(3.2a) \qquad \frac{\partial L}{\partial q} = V_\sigma \sigma_p + V_\mu[\bar{p} - (1+i)c'(q)] - \lambda c'(q) \leq 0;\ q \geq 0;\ q\frac{\partial L}{\partial q} = 0$$

$$(3.2b) \qquad \frac{\partial L}{\partial q} = V_\sigma(-\sigma_p) + V_\mu[-\bar{p} + (1+i)r] + \lambda r \leq 0;\ b \geq 0;\ b\,\frac{\partial L}{\partial b} = 0$$

$$(3.2c) \qquad \frac{\partial L}{\partial m} = \lambda \leq 0;\ m \geq 0;\ m\,\frac{\partial L}{\partial m} = 0$$

$$(3.2d) \qquad \frac{\partial L}{\partial \lambda} = -c(q) + m + rb \geq 0;\ \lambda \geq 0;\ \lambda\frac{\partial L}{\partial \lambda} = 0.$$

These conditions are necessary and sufficient for a maximum since V is concave, c is convex, and the constraint qualification is satisfied. There are three interesting cases.

<u>Solution A: Loans Dominate ($q>0$, $b=0$, $m>0$, $\lambda=0$)</u>

In this case, the Kuhn-Tucker conditions reduce to

$$(3.3a) \qquad V_\sigma \sigma_p + V_\mu[\bar{p} - (1+i)c'(q)] = 0$$

$$(3.3b) \qquad V_\sigma(-\sigma_p) + V_\mu[-\bar{p} + (1+i)r] \leq 0$$

$$(3.3c) \qquad \lambda = 0$$

$$(3.3d) \qquad -c(q) + m \geq 0.$$

Rearranging (3.3a) yields an identity defining the optimal level of output q^m:

$$(3.4) \qquad \frac{\bar{p} - (1+i)c'(q^m)}{\sigma_p} \equiv \frac{-V_\sigma(q^m)}{V_\mu(q^m)}.$$

This expression is identical to the first order condition (14) found in the model of the producer whose only source of capital is commercial borrowing (section VI).

(3.3d) indicates that the solution level of borrowing will be any amount sufficient to meet production costs. The zero value of the Lagrange multiplier reflects the fact that the possibility of commercial borrowing effectively eliminates the capital constraint.

Solution B: Bonds Dominate ($q>0$, $b>0$, $m=0$, $\lambda=0$)

In this case, the Kuhn-Tucker conditions reduce to

$$(3.5a) \qquad V_\sigma \sigma_p + V_\mu[\bar{p} - (1+i)c'(q)] = 0$$

$$(3.5b) \qquad V_\sigma(-\sigma_p) + V_\mu[-\bar{p} + (1+i)r] = 0$$

$$(3.5c) \qquad \lambda = 0$$

$$(3.5d) \qquad -c(q) + rb \geq 0.$$

Adding (3.5a) and (3.5b) defines optimal output as a q^* satisfying $c'(q^*)=r$. Rearranging (3.5b) then yields an identity defining the optimal bond issue b^*:

$$(3.6) \qquad \frac{\bar{p} - (1+i)r}{\sigma_p} \equiv \frac{-V_\sigma(q^*,b^*)}{V_\mu(q^*,b^*)}.$$

These definitions of q^* and b^* are identical to those found in the case of the producer who can raise capital only by issuing commodity bonds, but for whom the capital constraint is not binding (see appendix 1 solution A). Correspondingly, (3.5d) indicates that the revenue raised by issuing bonds is at least as great as production costs. Even without the possibility of commercial borrowing, therefore, the capital constraint would not be binding. The solution output and bond issue are precisely those of the unconstrained hedger in the standard futures market models.

Solution C: Bonds Used in Conjunction with Loans ($q>0$, $b>0$, $m>0$, $\lambda=0$)

In this case, the Kuhn-Tucker conditions reduce to

$$(3.7a) \qquad V_\sigma \sigma_p + V_\mu[\bar{p} - (1+i)c'(q)] = 0$$

$$(3.7b) \qquad V_\sigma(-\sigma_p) + V_\mu[-\bar{p} + (1+i)r] = 0$$

$$(3.7c) \qquad \lambda = 0$$

$$(3.7d) \qquad -c(q) + m + rb \leq 0.$$

Equation (3.7a) and (3.7b) are identical to (3.5a) and (3.5b). Optimal output, therefore, is again a q^* satisfying $c'(q^*) = r$ and the optimal bond issue, b^*, is again defined by (3.6).

In this case, however, the revenue raised through bond issues need not be sufficient to meet production costs. Equation (3.7d) allows the producer to use commercial loans to finance any production costs in excess of the revenue raised by selling bonds.

APPENDIX 4

<u>**Proposition**</u> If $q^m \geq q^*$ then $\frac{\bar{p}}{1+i} - r \geq - \frac{\sigma_p V_\sigma(q^m)}{(1+i)V_\mu(q^m)}$.

<u>**Proof**</u> Notice that q^* is defined by $c'(q^*) \equiv r$ and q^m is defined by

$$(4.1) \qquad \frac{\bar{p} - (1+i)c'(q^m)}{\sigma_p} \equiv \frac{-V_\sigma(q^m)}{V_\mu(q^m)}.$$

Suppose $q^m \geq q^*$. Then by convexity of $c(q)$, $c'(q^m) \geq c'(q^*)$.

By definition of q^*, this implies that $c'(q^m) \geq r$. Simple algebra then shows that

$$(4.2) \qquad \frac{\bar{p} - (1+i)r}{\sigma_p} \geq \frac{\bar{p} - (1+i)c'(q^m)}{\sigma_p}.$$

Then by definition of q^m,

$$(4.3) \qquad \frac{\bar{p} - (1+i)r}{\sigma_p} \geq \frac{-V_\sigma(q^m)}{V_\mu(q^m)}.$$

Rearranging this inequality yields

$$(4.4) \qquad \frac{\bar{p}}{1+i} - r \geq - \frac{\sigma_p}{(1+i)} \frac{V_\sigma(q^m)}{V_\mu(q^m)}.$$

An analogous argument, with the inequalities reversed, shows that $q^m < q^*$ implies that

$$(4.5) \qquad \frac{\bar{p}}{1+i} - r < - \frac{\sigma_p}{(1+i)} \frac{V_\sigma(q^m)}{V_\mu(q^m)}.$$