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**Strategic Network  
Interdiction**

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# Climate Change and Sustainable Development Series

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### Strategic Network Interdiction

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#### Summary

We develop a strategic model of network interdiction in a non-cooperative game of flow. An adversary, endowed with a bounded quantity of bads, chooses a flow specifying a plan for carrying bads through a network from a base to a target. Simultaneously, an agency chooses a blockage specifying a plan for blocking the transport of bads through arcs in the network. The bads carried to the target cause a target loss while the blocked arcs cause a network loss. The adversary earns and the agency loses from both target loss and network loss. The adversary incurs the expense of carrying bads. In this model we study Nash equilibria and find a power law relation between the probability and the extent of the target loss. Our model contributes to the literature of game theory by introducing non-cooperative behavior into a Kalai-Zemel (cooperative) game of flow. Our research also advances models and results on network interdiction.

**Keywords:** Network Interdiction, Noncooperative Game of Flow, Nash Equilibrium, Power Law, Kalai-Zemel Game of Flow

**JEL Classification:** C72, D85, H56

*This paper is written under the supervision of Professor Myrna Wooders and has been presented at the Stony Brook International Conference on Game Theory, July 2009, the Newport Beach Behavioral and Quantitative Game Theory Conference, May 2010, and the Second Brazilian Workshop of the Game Theory Society, August 2010. The author gratefully acknowledges comments of the participants and also comments of Professors Andrew Daughety, Jennifer Reinganum, Jacob Sagi, and John Weymark.*

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# Strategic Network Interdiction\*

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## Abstract

We develop a strategic model of network interdiction in a non-cooperative game of flow. An adversary, endowed with a bounded quantity of bads, chooses a flow specifying a plan for carrying bads through a network from a base to a target. Simultaneously, an agency chooses a blockage specifying a plan for blocking the transport of bads through arcs in the network. The bads carried to the target cause a target loss while the blocked arcs cause a network loss. The adversary earns and the agency loses from both target loss and network loss. The adversary incurs the expense of carrying bads. In this model we study Nash equilibria and find a power law relation between the probability and the extent of the target loss. Our model contributes to the literature of game theory by introducing non-cooperative behavior into a Kalai-Zemel (cooperative) game of flow. Our research also advances models and results on network interdiction.

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# 1 Introduction

We live in a connected world. We produce and consume goods and services that have gone through networks. Airlines, railroads, computer networks, and social networks are a few examples. However, those goods and services are not always beneficial to everyone. For example, firms may earn less profits because the products of their rival firms are brought to markets through supply chains. Revenue services may earn less tax revenue because taxable assets are transferred through financial networks and can be concealed in other countries. Websites and network service providers may suffer from malicious software sent through the Internet. Countries and their citizens may suffer from hazardous materials carried through transportation systems. In these contexts, competing products, concealed assets, malicious software, and hazardous materials are viewed as bads, as opposed to goods, because they are harmful to some economic agents. The possibility of bads being carried through networks motivates our model of network interdiction.

There are two players, say an adversary and an agency, interacting strategically in a given network. The adversary is given a bounded quantity of bads at a base node and plans to carry bads to a target node. The adversary chooses a flow of bads that specifies a plan for carrying bads through the network from the base to the target. The agency is operating the network and wishes to stop the transport of bads to the target. The agency chooses a blockage of arcs that specifies a plan for stopping the transport of bads through the network. The bads carried to the target cause a target loss while the blocked arcs cause a network loss. The adversary earns and the agency loses from both target loss and network loss. The adversary incurs the expense of carrying bads.

In this model we analyze the equilibrium behavior of the players. If the bounded quantity of bads is small, there are pure strategy Nash equilibria. In these equilibria, the adversary carries bads up to the bounded quantity in a dispersed way through the network, but the agency does not block any arcs. If the bounded quantity of bads is either intermediate or large, there are mixed strategy Nash equilibria in which each player chooses only two pure strategies with positive probability. In these equilibria, the adversary carries no bads or carries a positive amount of bads to the target. Meanwhile, the agency blocks no arcs or blocks all the arcs necessary to make the target unreachable through the network. From this analysis we learn which arcs the agency blocks and how often she blocks them. We also learn how the adversary carries bads through the network and how often he does.

In these Nash equilibria, the adversary successfully carries bads to the target if and only if the adversary carries a positive amount of bads to the target and the agency does not block any arcs. By computing the probability of this joint event, we calculate the equilibrium probability of the target loss. If the bounded quantity of bads is either intermediate or large, there is a power law relation between the probability and the extent of the target loss. This theoretical finding is consistent with empirical evidence.<sup>1</sup>

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<sup>1</sup>In empirical research Bohorquez et al. [5] and Clauset et al. [6] show that the fatality distribution of terrorist events follows a power law.

This paper contributes to the game theory literature by introducing noncooperative behavior into a Kalai-Zemel network flow model. Kalai and Zemel [14] define a (transferable utility) cooperative game, called a flow game, where the worth of a coalition is defined as the value of a maximum flow in the network restricted to the members of the coalition.<sup>2</sup> Their main result is that a cooperative flow game is totally balanced and thus has a nonempty core (that is, there are distributions of the total payoff of the game that are stable against the formation of coalitions). The core of a flow game depends on the structure of a network and the ownership of arcs in the network. Our framework differs in that players interact strategically. The agency owns and operates all arcs in a network while the adversary abuses the network.

This paper also contributes to the literature on network interdiction. Washburn and Wood [18] introduce a zero-sum game, where an evader chooses a path to move through a network and an interdicator chooses an arc at which to set up an inspection site. If the evader traverses a path that includes the inspected arc, the evader is detected with some exogenously given positive probability. Otherwise, the evader is not detected. Both players are allowed to choose mixed strategies. Given a mixed strategy profile, the interdiction probability is defined to be the average probability of the evader being detected. The evader aims to minimize the interdiction probability by choosing a path-selection mixed strategy, while the interdicator aims to maximize the interdiction probability by choosing an arc-inspection mixed strategy. By using linear programming and network flow techniques, Washburn and Wood [18] study the Nash equilibria of this game. Kodialam and Lakshman [15] also introduce a related game of network interdiction in the context of network security.<sup>3</sup>

Our model differs from the existing models on network interdiction in four aspects:

- (i) The definition of a network is different in that each arc has a capacity.
- (ii) The adversary is endowed with a bounded quantity of bads, which may, in equilibrium, be binding.
- (iii) Both players have larger sets of strategies. The adversary chooses a flow rather than a path. If there are multiple paths in a network, the adversary can use them all at once. The agency chooses a blockage rather than an arc. That is, the agency can block multiple arcs at once.
- (iv) Our network interdiction game is not a zero-sum game nor even a strictly competitive game.

Because of (i), we do not need to take the detection probability as given. In our model this probability is determined endogenously. By virtue of (ii), we can analyze how the adversary's resource constraint affects the adversary's and the agency's equilibrium behavior. By virtue of (iii), our model creates a more tractable environment and gives sharper results on equilibrium behavior. Because of (iv), we need to use a different

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<sup>2</sup>For other studies on cooperative flow games, see Kalai and Zemel [13], Granot and Granot [10], Potters et al. [16], and Reijnierse et al. [17].

<sup>3</sup>Other than these papers, most of the literature on network interdiction deals with an interdicator's optimization problem subject to some budget constraints. See Cormican et al. [7], Israeli and Wood [11], and Wood [19].

solution technique to find equilibria. We exploit the idea that in any Nash equilibrium each player is indifferent between the pure strategies played with positive probability.

Security in network games has attracted significant interest. For example, Ballester et al. [4] study the interaction between players whose payoffs depend on a network. They obtain a proportional relationship between how much effort a player exerts and how central the player’s position is in the network. Baccara and Bar-Isaac [2] study the formation of networks between criminals and terrorists and find optimal policies for law enforcement agencies. Baccara and Bar-Isaac [3] further study how the choice of interrogation methods affects the formation of terrorist networks. Goyal and Vigier [9] study the design and protection of networks robust to attacks from outside on the networks’ nodes.<sup>4</sup>

The remainder of this paper is organized as follows. Section 2 develops a game-theoretic model of network interdiction. Section 3 studies the Nash equilibria of the model. Section 4 discusses our theoretical finding, together with empirical evidence, and also discusses future research topics.

## 2 The Model

Two players, player 1 and player 2, strategically interact with each other in a given network. Players can be thought of as firms in the context of market competition, as a taxpayer and a revenue service in the context of tax evasion, as a malicious hacker and a network operator in the context of network security, or as a terrorist group and a security agency in the context of national security. Having these security applications in mind, we call player 1 an *adversary* and player 2 an *agency*.

### 2.1 Networks

We first introduce the definition of networks. A network consists of a set of nodes,  $N$ , a set of arcs,  $A \subset N \times N$ , and a (row) vector of arc capacities,  $c := (c_{ij})_{(i,j) \in A}$ . Each arc is an ordered pair of distinct nodes and has a positive capacity. For each  $i, j \in N$  with  $i \neq j$ , if  $(i, j) \in A$ , node  $i$  is connected to node  $j$  through arc  $(i, j)$  with capacity  $c_{ij} > 0$ . Formally a *network* is defined as a collection  $(N, A, c)$ .

### 2.2 Strategies

We now introduce the set of strategies for each player.

Player 1, the adversary, is given a *bound quantity*  $q > 0$  of bads at a node. This node is called *base*  $s$ . Player 1 plans to carry bads to another node. This node is called *target*  $t$ . Player 1 chooses a flow of bads specifying a plan for carrying bads through network  $(N, A, c)$  from base  $s$  to target  $t$ .

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<sup>4</sup>For a survey on other literature on networks, see Jackson [12].

For each  $j \in N$ , we denote by  $IA(j) := \{(i, j) : (i, j) \in A\}$  the set of the arcs coming into node  $j$  and by  $OA(j) := \{(j, i) : (j, i) \in A\}$  the set of the arcs going out from node  $j$ .

Formally a *flow* of bads from base  $s$  to target  $t$  with bound quantity  $q$  in network  $(N, A, c)$  is a (column) vector  $f := (f_{ij})'_{(i,j) \in A}$  satisfying the following constraints:

$$0 \leq f_{ij} \leq c_{ij} \quad \text{for each } (i, j) \in A, \quad (1)$$

$$f_{is} = 0 \quad \text{for each } (i, s) \in IA(s), \quad (2)$$

$$\sum_{(s,i) \in OA(s)} f_{si} \leq q \quad \text{and} \quad (3)$$

$$\sum_{(i,j) \in IA(j)} f_{ij} - \sum_{(j,i) \in OA(j)} f_{ji} = 0 \quad \text{for each } j \in N \setminus \{s, t\}. \quad (4)$$

Constraint (1) says that each arc flow is at least zero and at most the arc capacity. Constraint (2) says that each incoming flow to the base is zero. Constraint (3) says that the total outgoing flow from the base does not exceed the bound quantity. Constraint (4) says that at each node, except for the base and the target, the total incoming flow equals to the total outgoing flow. We denote by  $F(s, t, q, N, A, c)$  the set of all flows of bads from base  $s$  to target  $t$  with bound quantity  $q$  in network  $(N, A, c)$ . When there is no ambiguity, we write  $F$  instead of  $F(s, t, q, N, A, c)$ . Then the set of pure strategies for player 1 is denoted by  $F$ . By choosing a flow  $f = (f_{ij})'_{(i,j) \in A} \in F$ , player 1 carries  $f_{ij}$  amount of bads through arc  $(i, j)$ .

The *value* of a flow is defined as the total incoming flow to the target less the total outgoing flow from the target. Thus, the value of a flow shows how much bads player 1 carries to the target. Let  $v := (v_{ij})_{(i,j) \in A}$  be a (row) vector with  $v_{it} = 1$  for each  $(i, t) \in IA(t)$ ,  $v_{ti} = -1$  for each  $(t, i) \in OA(t)$ , and  $v_{ij} = 0$  for each  $(i, j) \notin IA(t) \cup OA(t)$ . Then the value of a flow  $f \in F$  is calculated as

$$v \cdot f = \sum_{(i,t) \in IA(t)} f_{it} - \sum_{(t,i) \in OA(t)} f_{ti}. \quad (5)$$

Constraints (1) through (4) imply that the value of a flow is non-negative and constrained by the bound quantity. That is, for each  $f \in F$ , we have

$$0 \leq v \cdot f \leq q. \quad (6)$$

We present examples of strategies for player 1. A flow  $f^o \in F$  is the *zero flow* if  $f^o$  is the vector of zeros. A flow  $f^\tau \in F$  is *trivial* if  $v \cdot f^\tau = 0$ . Notice that the zero flow  $f^o$  is trivial. A flow  $f^* \in F$  is a *maximum flow* if for each  $f \in F$ , we have  $v \cdot f^* \geq v \cdot f$ . Notice that the value of a maximum flow is constrained by the bound quantity.

Player 2, the agency, wishes to stop the transport of bads to the target. Player 2 chooses a blockage of arcs specifying a plan for stopping the transport of bads through network  $(N, A, c)$  to target  $t$ . Formally a *blockage* of arcs in network  $(N, A, c)$  is a (column) vector  $b := (b_{ij})'_{(i,j) \in A}$  with  $b_{ij} \in \{0, 1\}$  for each  $(i, j) \in A$ . We denote by  $B(N, A, c)$  the set of all blockages of arcs in network  $(N, A, c)$ . When there is no ambiguity, we write  $B$  instead of  $B(N, A, c)$ . Then the set of pure strategies for player 2 is denoted by  $B$ . By choosing

a blockage  $b = (b_{ij})'_{(i,j) \in A} \in B$ , if  $b_{ij} = 1$ , player 2 blocks arc  $(i, j)$ , and if  $b_{ij} = 0$ , player 2 does not block the arc. For each  $b \in B$ , we denote by  $A^b := \{(i, j) \in A : b = (b_{ij})'_{(i,j) \in A} \text{ and } b_{ij} = 1\}$  the set of all blocked arcs.

The *capacity* of a blockage is defined as the total capacity of the blocked arcs. Thus, the capacity of a blockage shows how much total arc capacity player 2 blocks in the network. The capacity of a blockage  $b \in B$  is calculated as

$$c \cdot b = \sum_{(i,j) \in A} c_{ij} b_{ij}. \quad (7)$$

A *cut*  $(C, \overline{C})$  in network  $(N, A, c)$  is a partition of the node set  $N$  with  $s \in C$  and  $t \in \overline{C}$ . For each cut  $(C, \overline{C})$ , an arc  $(i, j) \in A$  is a *cut arc* if  $i \in C$  and  $j \in \overline{C}$ . That is, through a cut arc  $(i, j)$ , node  $i$  in  $C$  is connected to node  $j$  in  $\overline{C}$ . For each cut  $(C, \overline{C})$ , we denote by  $A(C, \overline{C}) := \{(i, j) \in A : i \in C \text{ and } j \in \overline{C}\}$  the set of all cut arcs.

We present examples of strategies for player 2. A blockage  $b^o \in B$  is the *zero blockage* if  $b^o$  is the vector of zeros. A blockage  $b \in B$  is a *cut blockage* if there is a cut  $(C, \overline{C})$  such that  $A(C, \overline{C}) = A^b$ . A blockage  $b^* \in B$  is a *minimum cut blockage* if for each cut blockage  $b$ , we have  $c \cdot b^* \leq c \cdot b$ .

Players are allowed to choose mixed strategies. The set of mixed strategies for player 1 is denoted by  $\Delta(F)$  and the set of mixed strategies for player 2 is denoted by  $\Delta(B)$ .

## 2.3 Net Flows

Here we want to know how much bads player 1 *successfully* carries to the target when player 1 chooses a flow of bads and player 2 chooses a blockage of arcs. To answer this question we introduce the definition of net flows. For each flow of bads and each blockage of arcs, the net flow of bads to the target is obtained by (i) decomposing the flow of bads into cycle flows and path flows and (ii) removing all the cycle flows and all the path flows with blocked arcs. To introduce net flows formally we need the following definitions and notations.

An  $s - t$  *path* in network  $(N, A, c)$  is a sequence of distinct nodes  $i_1, \dots, i_K$  such that  $(i_k, i_{k+1}) \in A$  for each  $k \in \{1, \dots, K-1\}$  with  $i_1 = s$  and  $i_K = t$ . In this case we say that the  $s - t$  path *includes* arcs  $(i_1, i_2), \dots, (i_{K-1}, i_K)$ . A *cycle* in network  $(N, A, c)$  is a sequence of distinct nodes  $i_1, \dots, i_K$  such that  $(i_k, i_{k+1}) \in A$  for each  $k \in \{1, \dots, K-1\}$  with  $(i_K, i_1) \in A$ . In this case we say that the cycle *includes* arcs  $(i_1, i_2), \dots, (i_{K-1}, i_K)$ , and  $(i_K, i_1)$ . We denote by  $H$  the set of all  $s - t$  paths and cycles in network  $(N, A, c)$ .

The arc-path-cycle *incidence matrix* of network  $(N, A, c)$  is a matrix  $M := (m_{ah})_{a \in A, h \in H}$  with

$$m_{ah} = \begin{cases} 1 & \text{if } h \in H \text{ includes } a \in A \\ 0 & \text{otherwise.} \end{cases}$$

A *cycle flow* is a flow of bads along a cycle. A *path flow* is a flow of bads along an  $s - t$  path. By the *flow decomposition algorithm*, which will be presented in Appendix A, we can decompose a flow of bads into



cycle flows and path flows.<sup>5</sup> Formally, for each  $f \in F$ , we find a (column) vector  $x := (x_h)'_{h \in H}$  such that  $f = Mx$ . That is, either along a cycle  $h \in H$ , or along an  $s - t$  path  $h \in H$ , player 1 carries  $x_h$  amount of bads.

For such vector  $x$  and each blockage  $b$ , let  $x^b := (x_h^b)'_{h \in H}$  be a (column) vector with

$$x_h^b = \begin{cases} x_h & \text{if } h \text{ is an } s - t \text{ path including no blocked arcs} \\ 0 & \text{otherwise.} \end{cases}$$

That is, only along an  $s - t$  path  $h \in H$  with no blocked arcs, player 1 successfully carries  $x_h^b = x_h$  amount of bads to the target.

We are ready to define net flows. For each  $f \in F$  and each  $b \in B$ , the *net flow* of bads to target  $t$  under flow  $f$  and blockage  $b$  is a (column) vector  $f^b := (f_{ij}^b)'_{(i,j) \in A}$  such that  $Mx^b = f^b$ . Then the value of the net flow  $f^b$  is calculated as  $v \cdot f^b$ , which shows how much bads player 1 successfully carries to the target.

Notice that the net flow  $f^b$  under a flow  $f$  and a blockage  $b$  contains no cycle flows. Furthermore, the net flow  $f^{b^\circ}$  under a flow  $f$  and the zero blockage  $b^\circ$  contains all path flows but no cycle flows. We say that a flow  $f \in F$  is *acyclic* if  $f = f^{b^\circ}$ . Also notice that the net flow  $f^b$  under a flow  $f$  and a cut blockage  $b$  is the zero flow  $f^\circ$ . That is, if  $b$  is a cut blockage, for each  $f \in F$ , we have  $f^b = f^\circ$ .

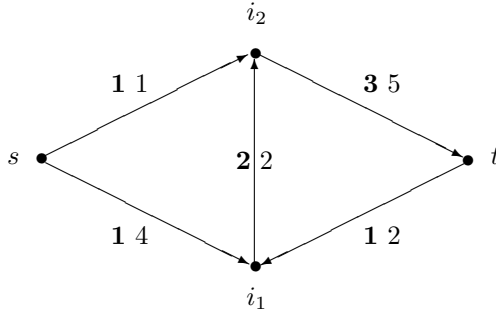
The following example shows how to find net flows.

**Example 1** Suppose that a network is given as  $(N, A, c)$ , where  $N = \{s, i_1, i_2, t\}$  is the set of nodes,  $A = \{(s, i_1), (s, i_2), (i_1, i_2), (i_2, t), (t, i_1)\}$  is the set of arcs, and  $c = (c_{si_1}, c_{si_2}, c_{i_1i_2}, c_{i_2t}, c_{ti_1}) = (4, 1, 2, 5, 2)$  is the vector of arc capacities. A bound quantity is given as  $q = 3$ . Suppose that player 1 chooses a flow  $f = (f_{si_1}, f_{si_2}, f_{i_1i_2}, f_{i_2t}, f_{ti_1})' = (1, 1, 2, 3, 1)'$ . See Figure 1. In network  $(N, A, c)$  there are two  $s - t$  paths  $s, i_1, i_2, t$  and  $s, i_2, t$  and one cycle  $i_1, i_2, t, i_1$ . The arc-path-cycle incidence matrix of network  $(N, A, c)$  is

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

where the first column corresponds to path  $s, i_1, i_2, t$ , the second column corresponds to path  $s, i_2, t$ , and the third column corresponds to cycle  $i_1, i_2, t, i_1$ . By using the flow decomposition algorithm, we find a vector  $x = (1, 1, 1)'$  such that  $f = Mx$ . Each entry of the vector  $x$  shows the amount of bads player 1 carries along path  $s, i_1, i_2, t$ , path  $s, i_2, t$ , and cycle  $i_1, i_2, t, i_1$ , respectively. Now suppose that player 2 chooses a blockage  $b = (b_{si_1}, b_{si_2}, b_{i_1i_2}, b_{i_2t}, b_{ti_1})' = (0, 1, 0, 0, 0)'$ . Then path  $s, i_1, i_2, t$  is the only  $s - t$  path with no blocked arcs. Thus,  $x^b = (1, 0, 0)'$ . Therefore, the net flow of bads to target  $t$  under flow  $f$  and blockage  $b$  is  $f^b = Mx^b = (1, 0, 1, 1, 0)'$  and the value of this net flow is  $v \cdot f^b = 1$ .  $\square$

<sup>5</sup>The flow decomposition algorithm is developed by Ford and Fulkerson [8]. For a discussion see Ahuja et al. [1]. In our model we use this algorithm to find net flows.



**Figure 1** Each solid circle indicates a node; each arrow indicates an arc; in each pair of numbers the first bold number indicates an arc flow and the second light number indicates the arc capacity.

## 2.4 Payoff Functions

We introduce the payoff function of each player.

The bads carried to the target cause a target loss. This target loss is determined by the value of the net flow of bads,  $v \cdot f^b$ , as well as by the *marginal target loss*,  $\ell_t > 0$ . For each  $f \in F$  and each  $b \in B$ , the target loss amounts to  $\ell_t(v \cdot f^b)$ . Player 1 earns  $\ell_t(v \cdot f^b)$  and player 2 loses the same amount from the target loss.

The blocked arcs cause a network loss. This network loss is determined by the capacity of the blockage of arcs,  $c \cdot b$ , as well as by the *marginal network loss*,  $\ell_k > 0$ . For each  $b \in B$ , the network loss amounts to  $\ell_k(c \cdot b)$ . Player 1 earns  $\ell_k(c \cdot b)$  and player 2 loses the same amount from the network loss.

Player 1 incurs the expense of carrying bads. This expense is determined by the value of the flow of bads,  $v \cdot f$ , as well as by the *marginal expense of carrying bads*,  $e > 0$ . For each  $f \in F$ , the expense of carrying bads amounts to  $e(v \cdot f)$ .

Player 2 earns a constant worth of operating the network,  $w$ .

For each  $(f, b) \in F \times B$ , the *payoff function* of player 1 is defined as

$$u_1(f, b) = \ell_t(v \cdot f^b) + \ell_k(c \cdot b) - e(v \cdot f),$$

and the *payoff function* of player 2 is defined as

$$u_2(f, b) = w - \ell_t(v \cdot f^b) - \ell_k(c \cdot b).$$

For each  $\sigma = (\sigma_1, \sigma_2) \in \Delta(F) \times \Delta(B)$ , the expected payoff functions of the players are

$$u_1(\sigma_1, \sigma_2) = E_\sigma[u_1(f, b)] \text{ and } u_2(\sigma_1, \sigma_2) = E_\sigma[u_2(f, b)].$$

**Remark 1** Since expected payoff functions are unique up to an affine transformation, without loss of generality, we assume that the marginal network loss equals to one, that is,  $\ell_k = 1$ .

### 3 Results

We analyze the equilibrium behavior of the players in the model. In a Nash equilibrium each player has no incentive to change his or her strategy.

**Definition 1** A strategy profile  $(\sigma_1, \sigma_2) \in \Delta(F) \times \Delta(B)$  is a Nash equilibrium if for each  $\sigma'_1 \in \Delta(F)$  and each  $\sigma'_2 \in \Delta(B)$ , we have  $u_1(\sigma_1, \sigma_2) \geq u_1(\sigma'_1, \sigma_2)$  and  $u_2(\sigma_1, \sigma_2) \geq u_2(\sigma_1, \sigma'_2)$ .

We suppose that the marginal target loss is greater than the marginal expense of carrying bads.<sup>6</sup> Then the adversary has an incentive to carry bads through the network. Given this, we want to answer the following questions: Does the agency have any incentive to block arcs in the network? Which arcs does the agency have to block? And how often does the agency have to block the arcs? To answer these questions we divide our analysis into three cases depending on the bound quantity. We say that the bound quantity  $q$  is *small* if  $q \leq (1/\ell_t)c \cdot b^*$ , *intermediate* if  $(1/\ell_t)c \cdot b^* < q \leq c \cdot b^*$ , and *large* if  $c \cdot b^* < q$ .

For our analysis we need the following definitions and notations. We denote by  $f^\alpha$  an *acyclic maximum flow* with large bound quantity  $q$  in network  $(N, A, c)$ . Because  $f^\alpha$  is acyclic,

$$f^\alpha = (f^\alpha)^{b^o}. \quad (8)$$

Because  $q$  is large,

$$v \cdot f^\alpha = c \cdot b^*. \quad (9)$$

That is, the value of an acyclic maximum flow equals to the capacity of a minimum cut blockage. This equality is called the *max-flow min-cut theorem*.<sup>7</sup>

A flow  $f^\beta \in F$  is a *binding flow* if  $f^\beta = (q/(c \cdot b^*))f^\alpha$ . Because  $f^\alpha$  is acyclic,  $f^\beta$  is also acyclic. That is,

$$f^\beta = (f^\beta)^{b^o}. \quad (10)$$

In addition the max-flow min-cut theorem (9) implies that

$$v \cdot f^\beta = q. \quad (11)$$

That is, the value of a binding flow equals to the bound quantity.

We are ready to start our equilibrium analysis. First we suppose that the bound quantity is small.

We call  $(f^\beta, b^o)$  a *binding-flow zero-blockage strategy profile*. In any binding-flow zero-blockage strategy profile, if the bound quantity is small, each player has no incentive to change his or her strategy. Thus, we have the following proposition.

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<sup>6</sup>If the marginal target loss is no greater than the marginal expense of carrying bads, the adversary has no incentive to carry bads through the network from the base to the target. Given this, the agency has no incentive to block arcs in the network. Thus, any trivial-flow zero-blockage strategy profile  $(f^\tau, b^o)$  is a Nash equilibrium.

<sup>7</sup>Ford and Fulkerson [8] introduce the maximum flow problem in networks and show the max-flow min-cut theorem. For a detailed discussion see Ahuja et al. [1].

**Proposition 1** *If the bound quantity is small, that is, if  $q \leq (1/\ell_t)c \cdot b^*$ , then any binding-flow zero-blockage strategy profile  $(f^\beta, b^\circ)$  is a Nash equilibrium.*

The proof of Proposition 1 is presented in Appendix B.

In any binding-flow zero-blockage Nash equilibrium, player 1 carries bads up to the bound quantity *in a dispersed way* through the network, but player 2 does not block any arcs in the network. We provide an example of this equilibrium.

**Example 2** *Consider network  $(N, A, c)$  in Example 1. Notice that  $f^\alpha = (2, 1, 2, 3, 0)'$  is the only acyclic maximum flow and  $b^* = (0, 1, 1, 0, 0)'$  is the only minimum cut blockage. Also notice that the capacity of the minimum cut blockage is  $c \cdot b^* = 3$ . Suppose that the marginal target loss is  $\ell_t = 2$ , the marginal expense of carrying bads is  $e = 1$ , and the bound quantity is  $q = 1$ . Then the binding flow is  $f^\beta = (2/3, 1/3, 2/3, 1, 0)'$ . Because the bound quantity is small, the binding-flow zero-blockage strategy profile  $(f^\beta, b^\circ)$  is a Nash equilibrium. However, if player 1 carries bads up to the bound quantity only through arcs  $(s, i_2)$  and  $(i_2, t)$ , player 2 has the incentive to block arc  $(s, i_2)$ .  $\square$*

If the bound quantity is small and player 1 carries bads in a dispersed way through the network, then player 2 has no incentive to block arcs. However, if the bound quantity is *not* small, that is, if the bound quantity is either intermediate or large, then player 2 has an incentive to block arcs in the network. Now we want to know which arcs player 2 must block and how often she blocks them. Consider the following example.

**Example 3** *Consider network  $(N, A, c)$  in Example 1. Suppose that player 2 chooses a cut blockage  $b = (1, 1, 0, 0, 0)'$ , that is, suppose that player 2 blocks all the arcs from the base. Then player 2 incurs a network loss of 5. However, if player 2 chooses the minimum cut blockage  $b^* = (0, 1, 1, 0, 0)'$ , she incurs a network loss of 3. Since both  $b$  and  $b^*$  are cut blockages, for each flow  $f$ , the net flow is the zero flow, that is,  $f^b = f^\circ$  and  $f^{b^*} = f^\circ$ . Thus, the target loss is zero. Therefore,  $b$  is a dominated strategy for player 2.  $\square$*

In general, if  $b$  is a cut blockage but not a minimum cut blockage and  $b^*$  is a minimum cut blockage, then  $b$  is dominated by  $b^*$  for player 2, that is, for each  $f \in F$ ,  $u_2(f, b) < u_2(f, b^*)$ . Thus, it may be a dominated strategy for player 2 to block all the arcs from the base or to block all the arcs into the target. If player 2 blocks arcs, she must block minimum cut arcs in the network.

Now imagine that player 2 blocks minimum cut arcs in the network with probability 1. Then player 1 has no incentive to carry bads through the network because he always fails to reach the target. If player 1 carries no bads to the target with probability 1, player 2 has no incentive to block the arcs. This is because she wants to avoid the network loss if there is no threat to the target. In turn, if player 2 blocks no arcs with probability 1, player 1 has an incentive to carry bads. If player 1 carries bads with probability 1, player 2 has an incentive to block arcs. In general, if the bound quantity is either intermediate or large, there is no pure strategy Nash equilibrium.

To study how often to block minimum cut arcs, we examine the mixed strategy Nash equilibria of the model. Now we suppose that the bound quantity is large.

A mixed strategy  $\sigma_1^\lambda \in \Delta(F)$  is a  $\lambda$ -scaled *max-flow strategy*, or simply a  $\lambda$ -flow strategy, for player 1 if for some  $\lambda \in [1/\ell_t, 1]$ ,  $\sigma_1^\lambda(f^\tau) = 1 - 1/\lambda\ell_t$  and  $\sigma_1^\lambda(\lambda f^\alpha) = 1/\lambda\ell_t$ . By choosing a  $\lambda$ -flow strategy player 1 chooses a trivial flow  $f^\tau$  with probability  $1 - 1/\lambda\ell_t$  and a  $\lambda$ -scaled acyclic maximum flow  $\lambda f^\alpha$  with probability  $1/\lambda\ell_t$ . For example, if  $\lambda = 1$ , player 1 carries no bads from the base to the target with probability  $1 - 1/\ell_t$  and carries the maximum possible amount of bads through the network with probability  $1/\ell_t$ . Here  $\lambda$  is a scale to adjust the probability and the amount of bads.

A mixed strategy  $\sigma_2^* \in \Delta(B)$  is a *min-cut strategy* for player 2 if  $\sigma_2^*(b^o) = e/\ell_t$  and  $\sigma_2^*(b^*) = 1 - e/\ell_t$ . By choosing a min-cut strategy player 2 chooses the zero blockage  $b^o$  with probability  $e/\ell_t$  and a minimum cut blockage  $b^*$  with probability  $1 - e/\ell_t$ . That is, player 2 blocks no arcs with probability  $e/\ell_t$  and blocks minimum cut arcs with probability  $1 - e/\ell_t$ .

We call  $(\sigma_1^\lambda, \sigma_2^*)$  a  $\lambda$ -flow *min-cut strategy profile*.

Notice that player 1 chooses only two pure strategies  $f^\tau$  and  $\lambda f^\alpha$  with positive probability. Given that player 2 chooses a min-cut strategy  $\sigma_2^*$ , by choosing a trivial flow  $f^\tau$ , player 1 earns an expected payoff of

$$\begin{aligned} u_1(f^\tau, \sigma_2^*) &= \sigma_2^*(b^o)u_1(f^\tau, b^o) + \sigma_2^*(b^*)u_1(f^\tau, b^*) \\ &= (1 - e/\ell_t)(c \cdot b^*), \end{aligned}$$

because player 1 earns  $u_1(f^\tau, b^o) = 0$  with probability  $\sigma_2^*(b^o) = e/\ell_t$  and earns  $u_1(f^\tau, b^*) = c \cdot b^*$  with probability  $\sigma_2^*(b^*) = 1 - e/\ell_t$ . Given a min-cut strategy  $\sigma_2^*$ , by choosing a  $\lambda$ -scaled acyclic maximum flow  $\lambda f^\alpha$ , player 1 earns an expected payoff of

$$\begin{aligned} u_1(\lambda f^\alpha, \sigma_2^*) &= \sigma_2^*(b^o)u_1(\lambda f^\alpha, b^o) + \sigma_2^*(b^*)u_1(\lambda f^\alpha, b^*) \\ &= (1 - e/\ell_t)(c \cdot b^*), \end{aligned} \tag{12}$$

because player 1 earns  $u_1(\lambda f^\alpha, b^o) = (\ell_t - e)(v \cdot \lambda f^\alpha)$  with probability  $\sigma_2^*(b^o) = e/\ell_t$  and earns  $u_1(\lambda f^\alpha, b^*) = c \cdot b^* - e(v \cdot \lambda f^\alpha)$  with probability  $\sigma_2^*(b^*) = 1 - e/\ell_t$ . Thus,  $u_1(f^\tau, \sigma_2^*) = u_1(\lambda f^\alpha, \sigma_2^*)$ . By choosing a min-cut strategy  $\sigma_2^*$ , player 2 makes player 1 indifferent between the two pure strategies  $f^\tau$  and  $\lambda f^\alpha$ .

Now notice that player 2 chooses only two pure strategies  $b^o$  and  $b^*$  with positive probability. Given that player 1 chooses a  $\lambda$ -flow strategy  $\sigma_1^\lambda$ , by choosing the zero blockage  $b^o$ , player 2 earns an expected payoff of

$$\begin{aligned} u_2(\sigma_1^\lambda, b^o) &= \sigma_1^\lambda(f^\tau)u_2(f^\tau, b^o) + \sigma_1^\lambda(\lambda f^\alpha)u_2(\lambda f^\alpha, b^o) \\ &= w - v \cdot f^\alpha, \end{aligned}$$

because player 2 earns  $u_2(f^\tau, b^o) = w$  with probability  $\sigma_1^\lambda(f^\tau) = 1 - 1/\lambda\ell_t$  and  $u_2(\lambda f^\alpha, b^o) = w - \lambda\ell_t(v \cdot f^\alpha)$  with probability  $\sigma_1^\lambda(\lambda f^\alpha) = 1/\lambda\ell_t$ . Given a  $\lambda$ -flow strategy  $\sigma_1^\lambda$ , by choosing a minimum cut blockage  $b^*$ , player 2 earns an expected payoff of

$$u_2(\sigma_1^\lambda, b^*) = w - c \cdot b^*, \tag{13}$$

because player 2 earns  $w - c \cdot b^*$  whichever strategy player 1 chooses. Thus, the max-flow min-cut theorem (9) implies that  $u_2(\sigma_1^\lambda, b^o) = u_2(\sigma_1^\lambda, b^*)$ . By choosing a  $\lambda$ -flow strategy  $\sigma_1^\lambda$ , player 1 makes player 2 indifferent between the two pure strategies  $b^o$  and  $b^*$ .

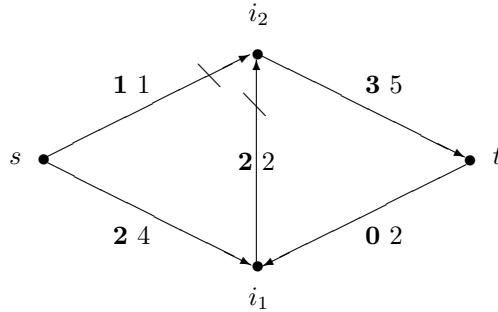
In addition, we can show that for each player, these pure strategies are at least as good as any other pure strategies. Thus, in any  $\lambda$ -flow min-cut strategy profile, each player has no incentive to change his or her strategy. Therefore, we obtain the following proposition.

**Proposition 2** *If the bound quantity is large, that is, if  $c \cdot b^* < q$ , then any  $\lambda$ -flow min-cut strategy profile  $(\sigma_1^\lambda, \sigma_2^*)$  is a Nash equilibrium.*

The proof of Proposition 2 is presented in Appendix B. We provide an example of  $\lambda$ -flow min-cut Nash equilibria.

**Example 4** *Consider network  $(N, A, c)$  in Example 1. Recall that  $f^\alpha = (2, 1, 2, 3, 0)'$  is the acyclic maximum flow and  $b^* = (0, 1, 1, 0, 0)'$  is the minimum cut blockage. Suppose that  $\ell_t = 4$ ,  $e = 1$ , and  $q = 5$ . Because the bound quantity is large, any  $\lambda$ -flow min-cut strategy profile  $(\sigma_1^\lambda, \sigma_2^*)$  is a Nash equilibrium. For instance, in a  $\lambda$ -flow min-cut Nash equilibrium with  $\lambda = 1$ , player 1 chooses the zero flow  $f^o$  with probability  $\sigma_1^\lambda(f^o) = 3/4$  and the acyclic maximum flow  $f^\alpha$  with probability  $\sigma_1^\lambda(f^\alpha) = 1/4$ , and player 2 chooses the zero blockage  $b^o$  with probability  $\sigma_2^*(b^o) = 1/4$  and the minimum cut blockage  $b^*$  with probability  $\sigma_2^*(b^*) = 3/4$ . See Figure 2.*

□



**Figure 2** The bold numbers indicate the acyclic maximum flow; the line segments indicate the minimum cut blockage.

In any  $\lambda$ -flow min-cut Nash equilibrium there is a power law relation between the probability and the extent of the target loss. In this equilibrium, player 1 successfully carries bads to the target if and only if player 1 chooses a  $\lambda$ -scaled acyclic maximum flow  $\lambda f^\alpha$  and player 2 chooses the zero blockage  $b^o$ . This joint event takes place with probability  $(1/\lambda \ell_t)(e/\ell_t) = (1/\lambda)(e)\ell_t^{-2}$ . Thus, with this probability, the bads carried to the target cause the target loss. Therefore, in any  $\lambda$ -flow min-cut Nash equilibrium, the *target loss probability* is  $p_\lambda = (1/\lambda)(e)\ell_t^{-2}$ .

In any  $\lambda$ -flow min-cut Nash equilibrium, if player 1 successfully carries bads to the target, the *target loss* amounts to  $TL_\lambda = (\lambda \ell_t)(c \cdot b^*)$ . Because  $p_\lambda = (1/\lambda)(e)\ell_t^{-2}$  and  $\ell_t = (1/\lambda)(1/(c \cdot b^*))TL_\lambda$ , we have

$$p_\lambda = (\lambda)(e)(c \cdot b^*)^2(TL_\lambda)^{-2} \quad (14)$$

where  $\lambda \in (1/\ell_t, 1]$ . Furthermore, if  $\lambda = (\ell_t)^{-\theta}$  for some  $\theta \in [0, 1)$ , equality (14) can be rewritten as

$$p_\lambda = (e)(c \cdot b^*)^{\frac{\theta-2}{\theta-1}}(TL_\lambda)^{-\frac{\theta-2}{\theta-1}} \quad (15)$$

because  $p_\lambda = (e)(\ell_t)^{\theta-2}$  and  $\ell_t = (c \cdot b^*)^{\frac{1}{\theta-1}}(TL_\lambda)^{-\frac{1}{\theta-1}}$ . Thus, in any  $\lambda$ -flow min-cut Nash equilibrium with  $\lambda = (\ell_t)^{-\theta}$  for some  $\theta \in [0, 1)$ , the target loss probability  $p_\lambda$  is a *negative power function* of the target loss  $TL_\lambda$ . However, if  $\lambda = 1/\ell_t$ , we have  $p_\lambda = (e)\ell_t^{-1}$  and  $TL_\lambda = c \cdot b^*$ . Thus, if  $\lambda = 1/\ell_t$ , the equilibrium probability  $p_\lambda$  is independent of the target loss  $TL_\lambda$ .

Finally we suppose that the bound quantity is intermediate.

A mixed strategy  $\sigma_1^\mu \in \Delta(F)$  is a  $\mu$ -scaled *binding-flow strategy*, or simply a  $\mu$ -flow *strategy*, for player 1 if for some  $\mu \in [(1/\ell_t)(1/q)(c \cdot b^*), 1]$ ,  $\sigma_1^\mu(f^\tau) = 1 - (1/\mu\ell_t)(1/q)(c \cdot b^*)$  and  $\sigma_1^\mu(\mu f^\beta) = (1/\mu\ell_t)(1/q)(c \cdot b^*)$ . By choosing a  $\mu$ -flow strategy player 1 chooses a trivial flow  $f^\tau$  with probability  $1 - (1/\mu\ell_t)(1/q)(c \cdot b^*)$  and a  $\mu$ -scaled binding flow  $\mu f^\beta$  with probability  $(1/\mu\ell_t)(1/q)(c \cdot b^*)$ . For example, if  $\mu = 1$ , player 1 carries no bads from the base to the target with probability  $1 - (1/\ell_t)(1/q)(c \cdot b^*)$  and carries bads up to the bound quantity with probability  $(1/\ell_t)(1/q)(c \cdot b^*)$ . Here  $\mu$  is a scale to adjust the probability and the amount of bads.

We call  $(\sigma_1^\mu, \sigma_2^*)$  a  $\mu$ -flow *min-cut strategy profile*.

Notice that player 1 chooses only two pure strategies  $f^\tau$  and  $\mu f^\beta$  with positive probability. Given a min-cut strategy  $\sigma_2^*$ , by choosing a trivial flow  $f^\tau$ , player 1 earns an expected payoff of  $u_1(f^\tau, \sigma_2^*) = (1 - e/\ell_t)(c \cdot b^*)$ . Given a min-cut strategy  $\sigma_2^*$ , by choosing a  $\mu$ -scaled binding flow  $\mu f^\beta$ , player 1 earns an expected payoff of

$$\begin{aligned} u_1(\mu f^\beta, \sigma_2^*) &= \sigma_2^*(b^o)u_1(\mu f^\beta, b^o) + \sigma_2^*(b^*)u_1(\mu f^\beta, b^*) \\ &= (1 - e/\ell_t)(c \cdot b^*), \end{aligned} \quad (16)$$

because player 1 earns  $u_1(\mu f^\beta, b^o) = (\ell_t - e)(v \cdot \mu f^\beta)$  with probability  $\sigma_2^*(b^o) = e/\ell_t$  and earns  $u_1(\mu f^\beta, b^*) = c \cdot b^* - e(v \cdot \mu f^\beta)$  with probability  $\sigma_2^*(b^*) = 1 - e/\ell_t$ . Thus,  $u_1(f^\tau, \sigma_2^*) = u_1(\mu f^\beta, \sigma_2^*)$ . By choosing a min-cut strategy  $\sigma_2^*$ , player 2 makes player 1 indifferent between the two pure strategies  $f^\tau$  and  $\mu f^\beta$ .

Now notice that player 2 chooses only two pure strategies  $b^o$  and  $b^*$  with positive probability. Given a  $\mu$ -flow strategy  $\sigma_1^\mu$ , by choosing the zero blockage  $b^o$ , player 2 earns an expected payoff of

$$\begin{aligned} u_2(\sigma_1^\mu, b^o) &= \sigma_1^\mu(f^\tau)u_2(f^\tau, b^o) + \sigma_1^\mu(\mu f^\beta)u_2(\mu f^\beta, b^o) \\ &= w - c \cdot b^*, \end{aligned}$$

because player 2 earns  $u_2(f^\tau, b^o) = w$  with probability  $\sigma_1^\mu(f^\tau) = 1 - (1/\mu\ell_t)(1/q)(c \cdot b^*)$  and earns  $u_2(\mu f^\beta, b^o) = w - (\mu\ell_t)q$  with probability  $\sigma_1^\mu(\mu f^\beta) = (1/\mu\ell_t)(1/q)(c \cdot b^*)$ . Given a  $\mu$ -flow strategy  $\sigma_1^\mu$ , by choosing a mini-

mum cut blockage  $b^*$ , player 2 earns an expected payoff of

$$u_2(\sigma_1^\mu, b^*) = w - c \cdot b^*, \quad (17)$$

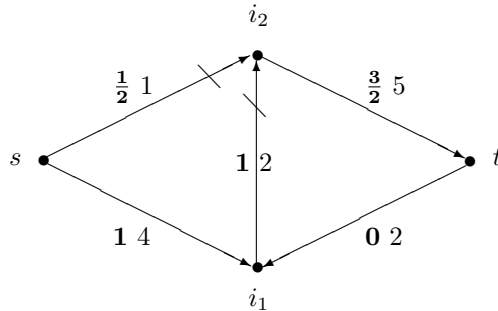
because player 2 earns  $w - c \cdot b^*$  whichever strategy player 1 chooses. Thus,  $u_2(\sigma_1^\mu, b^o) = u_2(\sigma_1^\mu, b^*)$ . By choosing a  $\mu$ -flow strategy  $\sigma_1^\mu$ , player 1 makes player 2 indifferent between the two pure strategies  $b^o$  and  $b^*$ .

In addition, we can show that for each player, these pure strategies are at least as good as any other pure strategies. Thus, in any  $\mu$ -flow min-cut strategy profile, each player has no incentive to change his or her strategy. Therefore, we establish the following proposition.

**Proposition 3** *If the bound quantity is intermediate, that is, if  $(1/\ell_t)c \cdot b^* < q \leq c \cdot b^*$ , then any  $\mu$ -flow min-cut strategy profile  $(\sigma_1^\mu, \sigma_2^*)$  is a Nash equilibrium.*

The proof of Proposition 3 is presented in Appendix B. We provide an example of  $\mu$ -flow min-cut Nash equilibria.

**Example 5** *Consider network  $(N, A, c)$  in Example 1. Suppose that  $\ell_t = 4$ ,  $e = 1$ , and  $q = 3/2$ . Notice that the binding flow is  $f^\beta = (1, 1/2, 1, 3/2, 0)'$ . Because the bound quantity is intermediate, any  $\mu$ -flow min-cut strategy profile  $(\sigma_1^\mu, \sigma_2^*)$  is a Nash equilibrium. For instance, in a  $\mu$ -flow min-cut Nash equilibrium with  $\mu = 1$ , player 1 chooses the zero flow  $f^o$  with probability  $\sigma_1^\mu(f^o) = 1/2$  and the binding flow  $f^\beta$  with probability  $\sigma_1^\mu(f^\beta) = 1/2$ , and player 2 chooses the zero blockage  $b^o$  with probability  $\sigma_2^*(b^o) = 1/4$  and the minimum cut blockage  $b^*$  with probability  $\sigma_2^*(b^*) = 3/4$ . See Figure 3.  $\square$*



**Figure 3** The bold numbers indicate the binding flow; the line segments indicate the minimum cut blockage.

In any  $\mu$ -flow min-cut Nash equilibrium the probability and the extent of the target loss show a power law relation. In this equilibrium, player 1 successfully carries bads to the target if and only if player 1 chooses a  $\mu$ -scaled binding flow  $\mu f^\beta$  and player 2 chooses the zero blockage  $b^o$ . This joint event takes place with probability  $(1/\mu\ell_t)(1/q)(c \cdot b^*)(e/\ell_t) = (1/\mu)(1/q)(c \cdot b^*)(e)\ell_t^{-2}$ . Thus, in any  $\mu$ -flow min-cut Nash equilibrium, the *target loss probability* is  $p_\mu = (1/\mu)(1/q)(c \cdot b^*)(e)\ell_t^{-2}$ .



In any  $\mu$ -flow min-cut Nash equilibrium, if player 1 successfully carries bads to the target, the *target loss* amounts to  $TL_\mu = (\mu\ell_t)q$ . Because  $p_\mu = (1/\mu)(1/q)(c \cdot b^*)(e)\ell_t^{-2}$  and  $\ell_t = (1/\mu)(1/q)TL_\mu$ , we have

$$p_\mu = (\mu)(e)(q)(c \cdot b^*)(TL_\mu)^{-2} \quad (18)$$

where  $\mu \in ((1/\ell_t)(1/q)(c \cdot b^*), 1]$ . Furthermore, if  $\mu = (q)^{-\theta}(c \cdot b^*)^\theta(\ell_t)^{-\theta}$  for some  $\theta \in [0, 1)$ , equality (18) can be rewritten as

$$p_\mu = (e)(q)(c \cdot b^*)^{-\frac{1}{\theta-1}}(TL_\mu)^{-\frac{\theta-2}{\theta-1}} \quad (19)$$

because  $p_\mu = (e)(q)^{\theta-1}(c \cdot b^*)^{1-\theta}(\ell_t)^{\theta-2}$  and  $\ell_t = (q)^{-1}(c \cdot b^*)^{\frac{\theta}{\theta-1}}(TL_\mu)^{-\frac{1}{\theta-1}}$ . Thus, in any  $\mu$ -flow min-cut Nash equilibrium with  $\mu = (q)^{-\theta}(c \cdot b^*)^\theta(\ell_t)^{-\theta}$  for some  $\theta \in [0, 1)$ , the target loss probability  $p_\mu$  is a *negative power function* of the target loss  $TL_\mu$ . However, if  $\mu = (1/\ell_t)(1/q)(c \cdot b^*)$ , we have  $p_\mu = (e)\ell_t^{-1}$  and  $TL_\mu = c \cdot b^*$ . Thus, if  $\mu = (1/\ell_t)(1/q)(c \cdot b^*)$ , the equilibrium probability  $p_\mu$  is independent of the target loss  $TL_\mu$ .

## 4 Discussion

We first relate our results to some empirical studies of terrorist events and then discuss related research in progress and further directions.

### 4.1 Fatality Distribution of Terrorist Events

Let  $z$  denote the number of fatalities in a terrorist event and let  $p(z)$  denote the frequency of a terrorist event in which the number of fatalities is  $z$ . The fatality distribution of terrorist events follows a *power law* if for each  $z \geq z_{\min}$ ,

$$p(z) \propto z^{-\gamma}$$

where  $z_{\min}$  and  $\gamma$  are the parameters of the distribution. The estimates of the parameters are derived from data and denoted by  $\hat{z}_{\min}$  and  $\hat{\gamma}$ .

Recent empirical studies show that the fatality distribution of terrorist events follows a power law. Clauset et al. [6] use the database of National Memorial Institute for the Prevention of Terrorism (MIPT) and conclude that the fatality distribution follows a power law. The estimate of the scaling parameter is  $\hat{\gamma} = 2.38$ . Bohorquez et al. [5] construct a data set on insurgent wars and conclude that for each insurgent war the fatality distribution follows a power law. The estimates of the scaling parameter are clustered around 2.5.

Recall that in any  $\lambda$ -flow min-cut Nash equilibrium with  $\lambda = (\ell_t)^{-\theta}$  for some  $\theta \in [0, 1)$ , the target loss probability  $p_\lambda$  is a negative power function of the target loss  $TL_\lambda$ . Precisely, from equality (15), we have

$$p_\lambda = (e)(c \cdot b^*)^{\frac{\theta-2}{\theta-1}}(TL_\lambda)^{-\frac{\theta-2}{\theta-1}},$$

which can be rewritten as

$$p_\lambda(TL_\lambda) \propto (TL_\lambda)^{-\frac{\hat{\theta}-2}{\hat{\theta}-1}}.$$

To link this theoretical finding and empirical evidence we make two additional assumptions. Suppose that the target loss is measured by the number of fatalities and that the target loss probability is proportional to the frequency of a terrorist event.

Now suppose that the estimate of the scaling parameter,  $\hat{\gamma} \geq 2$ , is derived from data. By setting  $\hat{\gamma} = \frac{\hat{\theta}-2}{\hat{\theta}-1}$  and solving for  $\hat{\theta}$ , we have  $\hat{\theta} = \frac{\hat{\gamma}-2}{\hat{\gamma}-1}$ . Notice that  $\hat{\theta} \in [0, 1)$ . Therefore, in the  $\lambda$ -flow min-cut Nash equilibrium with  $\lambda = (\ell_t)^{-\hat{\theta}}$ , the fatality distribution is predicted to be

$$p_\lambda(TL_\lambda) \propto (TL_\lambda)^{-\hat{\gamma}}$$

and is consistent with data. Similarly, in the  $\mu$ -flow min-cut Nash equilibrium with  $\mu = (q)^{-\hat{\theta}}(c \cdot b^*)^{\hat{\theta}}(\ell_t)^{-\hat{\theta}}$ , the predicted fatality distribution,  $p_\mu(TL_\mu) \propto (TL_\mu)^{-\hat{\gamma}}$ , is consistent with data.

## 4.2 Further Research

This paper presents a strategic model of network interdiction where two players have complete information and simultaneously choose their strategies. Building on this research we can study a model with incomplete information where players may not know each other's type. For example, a security agency may not know the strategies and payoffs of an adversary. This extension to incomplete information is, in our view, of clear importance. We can also study a model where players sequentially choose their strategies. For example, a security agency may observe an adversary's plots and choose her own strategy conditional on this observation or, alternatively, the agency may move first in setting up a security system. Both these approaches are subjects of our current and future planned research.

## Appendix A

In this appendix we provide the flow decomposition algorithm.<sup>8</sup> A network is given as  $(N, A, c)$ . For each  $f \in F$ , we find a vector  $x = (x_h)'_{h \in H}$  such that  $f = Mx$ . Initially we are given a flow  $f$  and the zero vector  $x$ . At each step we construct a sequence of distinct nodes, and obtain either an  $s - t$  path or a cycle. We then modify vector  $x$  and flow  $f$ . This algorithm terminates when the modified flow is the zero flow.

### Algorithm 1 Flow Decomposition

Let  $f = (f_{ij})'_{(i,j) \in A} \in F$  be given. Let  $x = (x_h)'_{h \in H}$  be the vector of zeros.

At Step  $k = 1, 2, \dots$ , if  $f$  is the zero flow, this algorithm terminates and yields vector  $x$ . If  $f$  is not the zero flow, there is an arc  $(i, j) \in A$  with  $f_{ij} > 0$ .

(i) We start from base  $s$ . If there is  $(i_1, i_2) \in A$  with  $i_1 = s$  and  $f_{i_1 i_2} > 0$ , we begin the construction of a sequence of distinct nodes with the two nodes  $i_1$  and  $i_2$ . If there is  $(i_2, i_3) \in A$  with  $f_{i_2 i_3} > 0$ , we add node  $i_3$  to the sequence. Repeat this until we add target  $t$  or a previously added node to the sequence. In the former case, an  $s - t$  path is obtained and, in the latter case, a cycle is obtained. We denote the outcome by  $h \in H$ . We replace  $x_h = 0$  with the minimum flow of the arcs included in  $h$ . We then replace  $f_{ij}$  with  $f_{ij} - x_h$  if  $h$  includes  $(i, j)$ . We proceed to the next step.

(ii) If there is no  $(i_1, i_2) \in A$  with  $i_1 = s$  and  $f_{i_1 i_2} > 0$ , we find another arc  $(i, j)$  with  $f_{ij} > 0$ . We start from node  $i$ . By applying the argument in (i), we obtain a cycle and modify vector  $x$  and flow  $f$ . We proceed to the next step.  $\square$

## Appendix B

We first establish the following lemmas.

**Lemma 1** For each  $(f, b) \in F \times B$ , we have  $v \cdot f^{b^\circ} - v \cdot f^b \leq c \cdot b$ .

**Proof.** Let  $f \in F$  be any flow. Because  $f^{b^\circ}$  is the net flow of bads to the target under flow  $f$  and the zero blockage  $b^\circ$ , for each  $(i, j) \in A$ , we have  $f_{ij}^{b^\circ} \leq c_{ij}$ . Thus, blocking arc  $(i, j)$  decreases the value of the net flow by at most  $c_{ij}$ . Therefore, for each  $b = (b_{ij})'_{(i,j) \in A} \in B$ , we have  $v \cdot f^{b^\circ} - v \cdot f^b \leq \sum_{(i,j) \in A} c_{ij} b_{ij}$ .  $\square$

**Lemma 2** If  $f^\alpha$  is an acyclic maximum flow with large bound quantity  $q$  in network  $(N, A, c)$ , for each  $b \in B$ , we have  $v \cdot f^\alpha - v \cdot (f^\alpha)^b \leq c \cdot b$ . Furthermore, if  $q \leq (1/\ell_t)c \cdot b^*$  and  $f^\beta$  is a binding flow, for each  $b \in B$ , we have  $\ell_t(v \cdot f^\beta) - \ell_t(v \cdot (f^\beta)^b) \leq c \cdot b$ .

<sup>8</sup>See Ahuja et al. [1] and Ford and Fulkerson [8] for reference.

**Proof.** Lemma 1 implies that for each  $b \in B$ ,  $v \cdot (f^\alpha)^{b^\circ} - v \cdot (f^\alpha)^b \leq c \cdot b$ . Because  $f^\alpha = (f^\alpha)^{b^\circ}$  from equality (8), we have  $v \cdot f^\alpha - v \cdot (f^\alpha)^b \leq c \cdot b$ . Now multiplying both sides by  $(\ell_t)(q/(c \cdot b^*))$ , we have  $(\ell_t)(q/(c \cdot b^*))(v \cdot f^\alpha - v \cdot (f^\alpha)^b) \leq (\ell_t)(q/(c \cdot b^*))(c \cdot b)$ . Because  $f^\beta$  is a binding flow and  $f^\beta = (q/(c \cdot b^*))f^\alpha$ , we have  $(\ell_t)(q/(c \cdot b^*))(v \cdot f^\alpha - v \cdot (f^\alpha)^b) = \ell_t(v \cdot f^\beta) - \ell_t(v \cdot (f^\beta)^b)$ . Because  $q \leq (1/\ell_t)c \cdot b^*$ , we have  $(\ell_t)(q/(c \cdot b^*))(c \cdot b) \leq c \cdot b$ . Thus, for each  $b \in B$ , we have  $\ell_t(v \cdot f^\beta) - \ell_t(v \cdot (f^\beta)^b) \leq c \cdot b$ .  $\square$

We now present the proofs of the propositions.

**Proof of Proposition 1.** Suppose that  $q \leq (1/\ell_t)c \cdot b^*$ . We show that in any binding-flow zero-blockage strategy profile  $(f^\beta, b^\circ)$  each player has no incentive to change his or her strategy. Since  $(f^\beta)^{b^\circ} = f^\beta$  from equality (10) and  $v \cdot f^\beta = q$  from equality (11), we have  $u_1(f^\beta, b^\circ) = (\ell_t - e)q$ . Suppose that player 1 chooses any flow  $f$ . Since  $v \cdot f^{b^\circ} \leq v \cdot f$  and  $v \cdot f \leq q$ ,

$$\begin{aligned} u_1(f, b^\circ) &= \ell_t(v \cdot f^{b^\circ}) + c \cdot b^\circ - e(v \cdot f) \\ &\leq \ell_t(v \cdot f) - e(v \cdot f) \\ &\leq (\ell_t - e)q. \end{aligned}$$

Thus, player 1 has no incentive to change his strategy. Since  $(f^\beta)^{b^\circ} = f^\beta$  from equality (10) and  $v \cdot f^\beta = q$  from equality (11), we have  $u_2(f^\beta, b^\circ) = w - (\ell_t)q$ . Suppose that player 2 chooses any blockage  $b$ . Since  $\ell_t(v \cdot f^\beta) - \ell_t(v \cdot (f^\beta)^b) \leq c \cdot b$  from Lemma 2 and  $v \cdot f^\beta = q$  from equality (11),

$$\begin{aligned} u_2(f^\beta, b) &= w - \ell_t(v \cdot (f^\beta)^b) - c \cdot b \\ &\leq w - \ell_t(v \cdot f^\beta) \\ &= w - (\ell_t)q. \end{aligned}$$

Thus, player 2 has no incentive to change her strategy. Therefore,  $(f^\beta, b^\circ)$  is a Nash equilibrium.  $\square$

**Proof of Proposition 2.** Suppose that  $c \cdot b^* < q$ . In any  $\lambda$ -flow min-cut strategy profile  $(\sigma_1^\lambda, \sigma_2^*)$  player 1 chooses only two pure strategies  $f^\tau$  and  $\lambda f^\alpha$  with positive probability and player 2 chooses only two pure strategies  $b^\circ$  and  $b^*$  with positive probability. In addition each player is indifferent between the two pure strategies played with positive probability. Thus, to show that  $(\sigma_1^\lambda, \sigma_2^*)$  is a Nash equilibrium, it suffices to show that (i) for each  $f \in F$ ,  $u_1(\lambda f^\alpha, \sigma_2^*) \geq u_1(f, \sigma_2^*)$  and (ii) for each  $b \in B$ ,  $u_2(\sigma_1^\lambda, b^*) \geq u_2(\sigma_1^\lambda, b)$ .

(i) We show that for each  $f \in F$ ,  $u_1(\lambda f^\alpha, \sigma_2^*) \geq u_1(f, \sigma_2^*)$ . Let  $f \in F$  be any flow. Calculate player 1's payoffs. Since  $v \cdot f^{b^\circ} \leq v \cdot f$ ,

$$\begin{aligned} u_1(f, b^\circ) &= \ell_t(v \cdot f^{b^\circ}) + c \cdot b^\circ - e(v \cdot f) \\ &\leq (\ell_t - e)(v \cdot f). \end{aligned}$$

Since  $v \cdot f^{b^*} = 0$ ,

$$\begin{aligned} u_1(f, b^*) &= \ell_t(v \cdot f^{b^*}) + c \cdot b^* - e(v \cdot f) \\ &= c \cdot b^* - e(v \cdot f). \end{aligned}$$

Since  $\sigma_2^*(b^o) = e/\ell_t$  and  $\sigma_2^*(b^*) = 1 - e/\ell_t$ ,

$$\begin{aligned} u_1(f, \sigma_2^*) &= \sigma_2^*(b^o)u_1(f, b^o) + \sigma_2^*(b^*)u_1(f, b^*) \\ &\leq (e/\ell_t)(\ell_t - e)(v \cdot f) + (1 - e/\ell_t)(c \cdot b^* - e(v \cdot f)) \\ &= (1 - e/\ell_t)(c \cdot b^*). \end{aligned}$$

From (12) we know that  $u_1(\lambda f^\alpha, \sigma_2^*) = (1 - e/\ell_t)(c \cdot b^*)$ . Thus, for each  $f \in F$ ,  $u_1(\lambda f^\alpha, \sigma_2^*) \geq u_1(f, \sigma_2^*)$ .

(ii) We show that for each  $b \in B$ ,  $u_2(\sigma_1^\lambda, b^*) \geq u_2(\sigma_1^\lambda, b)$ . Let  $b \in B$  be any blockage. Calculate player 2's payoffs. Since  $v \cdot (f^\tau)^b = 0$ ,

$$\begin{aligned} u_2(f^\tau, b) &= w - \ell_t(v \cdot (f^\tau)^b) - c \cdot b \\ &= w - c \cdot b. \end{aligned}$$

Since  $v \cdot (\lambda f^\alpha)^b = \lambda(v \cdot (f^\alpha)^b)$ ,

$$\begin{aligned} u_2(\lambda f^\alpha, b) &= w - \ell_t(v \cdot (\lambda f^\alpha)^b) - c \cdot b \\ &= w - \lambda \ell_t(v \cdot (f^\alpha)^b) - c \cdot b. \end{aligned}$$

Since  $\sigma_1^\lambda(f^\tau) = 1 - 1/\lambda \ell_t$ ,  $\sigma_1^\lambda(\lambda f^\alpha) = 1/\lambda \ell_t$ , and  $v \cdot f^\alpha - v \cdot (f^\alpha)^b \leq c \cdot b$  from Lemma 2,

$$\begin{aligned} u_2(\sigma_1^\lambda, b) &= \sigma_1^\lambda(f^\tau)u_2(f^\tau, b) + \sigma_1^\lambda(\lambda f^\alpha)u_2(\lambda f^\alpha, b) \\ &= (1 - 1/\lambda \ell_t)(w - c \cdot b) + (1/\lambda \ell_t)(w - \lambda \ell_t(v \cdot (f^\alpha)^b) - c \cdot b) \\ &= w - c \cdot b - v \cdot (f^\alpha)^b \\ &\leq w - v \cdot f^\alpha. \end{aligned}$$

Then the max-flow min-cut theorem (9) implies that  $u_2(\sigma_1^\lambda, b) \leq w - c \cdot b^*$ . From (13) we know that  $u_2(\sigma_1^\lambda, b^*) = w - c \cdot b^*$ . Thus, for each  $b \in B$ ,  $u_2(\sigma_1^\lambda, b^*) \geq u_2(\sigma_1^\lambda, b)$ .

Therefore,  $(\sigma_1^\lambda, \sigma_2^*)$  is a Nash equilibrium.  $\square$

**Proof of Proposition 3.** Suppose that  $(1/\ell_t)c \cdot b^* < q \leq c \cdot b^*$ . In any  $\mu$ -flow min-cut strategy profile  $(\sigma_1^\mu, \sigma_2^*)$  player 1 chooses only two pure strategies  $f^\tau$  and  $\mu f^\beta$  with positive probability and player 2 chooses only two pure strategies  $b^o$  and  $b^*$  with positive probability. In addition each player is indifferent between the two pure strategies played with positive probability. Thus, to show that  $(\sigma_1^\mu, \sigma_2^*)$  is a Nash equilibrium, it suffices to show that (i) for each  $f \in F$ ,  $u_1(\mu f^\beta, \sigma_2^*) \geq u_1(f, \sigma_2^*)$  and (ii) for each  $b \in B$ ,  $u_2(\sigma_1^\mu, b^*) \geq u_2(\sigma_1^\mu, b)$ .

(i) We show that for each  $f \in F$ ,  $u_1(\mu f^\beta, \sigma_2^*) \geq u_1(f, \sigma_2^*)$ . Let  $f \in F$  be any flow. Calculate player 1's payoffs. Since  $v \cdot f^{b^o} \leq v \cdot f$ ,

$$\begin{aligned} u_1(f, b^o) &= \ell_t(v \cdot f^{b^o}) + c \cdot b^o - e(v \cdot f) \\ &\leq (\ell_t - e)(v \cdot f). \end{aligned}$$

Since  $v \cdot f^{b^*} = 0$ ,

$$\begin{aligned} u_1(f, b^*) &= \ell_t(v \cdot f^{b^*}) + c \cdot b^* - e(v \cdot f) \\ &= c \cdot b^* - e(v \cdot f). \end{aligned}$$

Since  $\sigma_2^*(b^o) = e/\ell_t$  and  $\sigma_2^*(b^*) = 1 - e/\ell_t$ ,

$$\begin{aligned} u_1(f, \sigma_2^*) &= \sigma_2^*(b^o)u_1(f, b^o) + \sigma_2^*(b^*)u_1(f, b^*) \\ &\leq (e/\ell_t)(\ell_t - e)(v \cdot f) + (1 - e/\ell_t)(c \cdot b^* - e(v \cdot f)) \\ &= (1 - e/\ell_t)(c \cdot b^*). \end{aligned}$$

From (16) we know that  $u_1(\mu f^\beta, \sigma_2^*) = (1 - e/\ell_t)(c \cdot b^*)$ . Thus, for each  $f \in F$ ,  $u_1(\mu f^\beta, \sigma_2^*) \geq u_1(f, \sigma_2^*)$ .

(ii) We show that for each  $b \in B$ ,  $u_2(\sigma_1^\mu, b^*) \geq u_2(\sigma_1^\mu, b)$ . Let  $b \in B$  be any blockage. Calculate player 2's payoffs. Since  $v \cdot (f^\tau)^b = 0$ ,

$$\begin{aligned} u_2(f^\tau, b) &= w - \ell_t(v \cdot (f^\tau)^b) - c \cdot b \\ &= w - c \cdot b. \end{aligned}$$

Since  $v \cdot (\mu f^\beta)^b = \mu(v \cdot (f^\beta)^b)$ ,

$$\begin{aligned} u_2(\mu f^\beta, b) &= w - \ell_t(v \cdot (\mu f^\beta)^b) - c \cdot b \\ &= w - \mu \ell_t(v \cdot (f^\beta)^b) - c \cdot b. \end{aligned}$$

Since  $\sigma_1^\mu(f^\tau) = 1 - (1/\mu \ell_t)(1/q)(c \cdot b^*)$ ,  $\sigma_1^\mu(\mu f^\beta) = (1/\mu \ell_t)(1/q)(c \cdot b^*)$ , and  $v \cdot (f^\beta)^b = (q/(c \cdot b^*))(v \cdot (f^\alpha)^b)$ ,

$$\begin{aligned} u_2(\sigma_1^\mu, b) &= \sigma_1^\mu(f^\tau)u_2(f^\tau, b) + \sigma_1^\mu(\mu f^\beta)u_2(\mu f^\beta, b) \\ &= (1 - (1/\mu \ell_t)(1/q)(c \cdot b^*))(w - c \cdot b) + (1/\mu \ell_t)(1/q)(c \cdot b^*)(w - \mu \ell_t(v \cdot (f^\beta)^b) - c \cdot b) \\ &= w - c \cdot b - (1/q)(c \cdot b^*)(v \cdot (f^\beta)^b) \\ &= w - c \cdot b - (1/q)(c \cdot b^*)(q/(c \cdot b^*))(v \cdot (f^\alpha)^b) \\ &= w - c \cdot b - v \cdot (f^\alpha)^b \\ &\leq w - v \cdot f^\alpha. \end{aligned}$$

The last inequality comes from Lemma 2. Then the max-flow min-cut theorem (9) implies that  $u_2(\sigma_1^\mu, b) \leq w - c \cdot b^*$ . From (17) we know that  $u_2(\sigma_1^\mu, b^*) = w - c \cdot b^*$ . Thus, for each  $b \in B$ ,  $u_2(\sigma_1^\mu, b^*) \geq u_2(\sigma_1^\mu, b)$ .

Therefore,  $(\sigma_1^\mu, \sigma_2^*)$  is a Nash equilibrium.  $\square$

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