Accumulative Pollution, "Clean Technology," and Policy Design

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Abstract

Environmental policymakers must address the adverse effects of a number of pollutants that accumulate in the environment. Goals for the regulation of these damages often involve holding long-term emissions below a level deemed to be "dangerous," or outright banning of offending products or processes along with subsidization of more "green" alternatives. This paper builds upon previous studies by Keeler, Spence, and Zeckhauser (1971) and Tahvonen and Withagen (1996) in addressing the optimal long-term management of an accumulative but assimilatable pollutant through policies that restrict more damaging production processes and thereby induce more benign alternatives. Using a simple general equilibrium approach, we consider the possibility that the assimilative capacity of the environment is diminished and eventually exhausted by pollution accumulation. In this case there is a nonconvexity in the problem that gives rise to multiple potential optima, complicating the characterization of the optimal path and the determination of decentralized policies that can support an optimal outcome. In particular, environmental quality may be preserved or completely degraded in the long term. This makes the question of whether polluting processes or products should be banned more complicated and more interesting. We characterize the circumstances under which a banning policy is consistent with an intertemporally optimizing path, we investigate the sensitivity of optimal solutions to the cost of a clean backstop technology, and we discuss more generally the design of price-based and quantity-based policies for supporting an optimal solution.

Key Words: stock externalities, nonconvexities, sustainable development

JEL Classification Numbers: Q20, Q28, D62

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ACCUMULATIVE POLLUTION, "CLEAN TECHNOLOGY," AND POLICY DESIGN

Cees Withagen and Michael A. Toman[*]

1. INTRODUCTION

Environmental policymakers must address the adverse effects of a number of pollutants that accumulate in the environment. Examples include toxic substances like PCBs and heavy metals, radioactive contamination, biological contaminants in water that require time to break down, water acidification, stratospheric ozone depletion, and accumulation of greenhouse gases. For these substances, damages to ecological systems and human interests depend on the concentration of pollution, and thus in turn on the accumulation of nondegraded emissions, not just the current emissions flow.

Goals for the regulation of these damages often involve holding long-term emissions to a level below what is believed to be "dangerous." This is the approach taken, for example, in Article 2 of the United Nations Framework Convention on Climate Change. Sometimes "dangerous" is interpreted to be a loading within the capacity of the environment to neutralize or otherwise assimilate damaging effects over the long term.

Rather than regulating damaging emissions, policymakers could simply ban (immediately or after an adjustment period) the production and use of the offending substances, if there are substitutes. This was the approach taken for ozone-depleting chlorofluorocarbons. It is also implicit in, for example, policies to "virtually eliminate" persistent pollutants in the Great Lakes (International Joint Commission (1989); see also Foran (1991)). Alternative goods or technologies also could be subsidized to induce a changeover to their use. Clearly there are relationships among these various strategies, e.g., high enough taxes on damaging substances will drive them out of the market and induce substitution to more benign alternatives. Policies which focus on "green investment" to bring forth substitutes in a finite period of time are attracting policy interest as possible ways to limit environmental damage as well as rejuvenating the economic productivity of capital stocks.

There is a substantial academic literature concerned with optimal control of accumulative pollutants.[1] This paper builds upon previous studies by addressing the optimal long-term management of an accumulative but assimilatable pollutant through policies that restrict more damaging production processes and thereby induce more benign alternatives. In this respect we follow the simple general equilibrium approach taken by Keeler, Spence, and Zeckhauser (1971) in their "Model II." Unlike these authors, however, we consider the possibility that the assimilative capacity of the environment is diminished and eventually exhausted by pollution accumulation, following in particular the partial equilibrium analysis of Tahvonen and Withagen (1996). In this case there is a nonconvexity in the problem that gives rise to multiple potential optima, complicating the characterization of the optimal path and the determination of decentralized policies that can support an optimal outcome.[2] In particular, environmental quality may be preserved or completely degraded in the long term. This makes the question of whether polluting processes or products should be banned more complicated and more interesting.

In Section 2 of the paper we give the basic structure and assumptions of the model. In Section 3 we develop our basic results about the characterization of optimal paths for the two different classes of pollution degradation function. In Section 4 we consider an interesting special case of the model with discrete technology options. In Section 5 we consider how the analysis informs the design of practical policies for dealing with accumulative pollutants. In Section 6 we offer some brief concluding remarks.

2. THE MODEL

We use a fairly simple model of social surplus maximization in which households' well-being depends on environmental quality as well as consumption. Pollution occurs as a byproduct of input utilization by competitive firms producing the consumption good. In the general case input proportions and thus the pollution intensity of output are variable; however, restricting the use of the polluting input raises the opportunity cost of producing the consumption good. Environmental quality is related to the concentration or "stock" of pollution. In the absence of government policy to internalize environmental externality,

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[1] See, for example, Keeler, Spence, and Zeckhauser (1971); Plourde (1972); d'Arge and Kogiku (1973); Cropper (1976); Asako (1980); Heal (1982); Becker (1982); Pethig (1991); van der Ploeg and Withagen (1991); Conrad and Olson (1992); Falk and Mendelsohn (1993); Hoel (1993); Tahvonen and Kuuluvainen (1993); Tahvonen (1995, 1996); Tahvonen and Withagen (1996); Tahvonen and Salo (1996); Weyant et al. (1996); and Toman and Palmer (1997). Here we use the term "optimal" in the conventional sense to refer to intertemporal programs that maximize the present value of social surplus. A few studies have been concerned with other social criteria, e.g., maximum long-term sustainable surplus with a zero rate of time preference (e.g., Forster (1973); see also Toman, Pezzey and Krautkraemer (1995) for further discussion and references). In this paper we address only present-value-maximizing programs.

[2] Tahvonen and Withagen do not discuss policies for supporting an optimal path, and their formulation does not incorporate the possibility of production with a pollution-free technology, which is one of the characteristics of the Keeler et al. framework.

production of the consumption good takes place with polluting means and both pollution and environmental damage accumulate.[3]

We consider several forms of the pollution assimilation function. One has the quantity of pollution stock decay or assimilation as an increasing function of the stock of pollution. This is a reasonable specification if one is thinking about the decay or dispersion of chemical substances in the environment, like radioactive materials or greenhouse gas concentrations in the environment. The second specification assumes that degradation or assimilation monotonically decreases as the pollution stock grows until a saturation level is reached, at which point assimilative capacity is irreversibly lost. This is a reasonable assumption for pollutants whose degradation depends on biological action which can be poisoned by pollution accumulation.[4] This threshold introduces a nonconvexity, as we discuss below.[5] Finally, the most general case allows for an inverted-U shape for the assimilation function, with capacity initially rising then falling.

To formalize the presentation, we assume the economy has the capacity to produce a homogeneous consumer commodity (q) according to a technology that employs labor (l) and some other input (y), for example energy from fossil fuels, that causes pollution. By appropriate scaling we define the pollution flow also by y. The production technology is described by a function F with the following properties:

A1) $F(l, y)$ is concave and continuously differentiable for all $(l, y) > 0$ with nonnegative partial derivatives. Moreover $F(0, y) = 0$ and $F_{ly} \geq 0$. We treat the cases $F(l,0) = 0$ and $F(l,0) > 0$ separately.

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[3] For simplicity we do not address pollution occurring directly from product consumption or the prospects for environmental protection through end-use measures such as recycling.

[4] Hediger (1991) distinguishes between pollution assimilation and pollution degradation as follows: if a pollutant is nondegradable then its accumulation monotonically decreases the assimilative capacity of the environment, whereas a degradable pollutant may have increasing then decreasing assimilative capacity as a function of the pollution stock (i.e., an inverted U shape). (See also Fiedler (1992) and Pezzey (1996) for a more detailed characterization of alternative decay functions and their implications.) We assume that our pollutant is degradable and can be assimilated, but we distinguish between the physical properties of pollution degradation and the bioeconomic aspects of pollution assimilation. We assume in particular that the degradation rate either increases or decreases with the pollution stock, while the assimilation rate monotonically decreases with the pollution stock (this is reflected in our specification of the damage function). We do not directly address threshold effects in assimilation that would appear as kinks or discontinuities in the damage function (e.g., the possibility of environmental "catastrophe" addressed in Cropper (1976)). However, our results can be extended to the case of irreversible environmental damage, even when the pollutants do chemically decompose. To see this, let x denote the pollutant stock, let $\bar{x}$ denote the saturation level of x, and define y by $y = x$ if $x \leq \bar{x}$ and $y = \bar{x}$ if $x > \bar{x}$. If we represent environmental damage as a function of y rather than x, we can capture the idea that damage can get stuck at a high level of environmental degradation, even with pollutant decay.

[5] Nonconvexity issues also are addressed in Tahvonen and Salo (1996). They are formally dual to the problem of optimal fisheries management with a nonconcave regeneration function (Cropper, Lee and Pannu 1979).

The stock of pollutants is denoted by x. It accumulates according to the rate of pollution y but there is also decay of pollution which is described by the function A, depending on the existing stock. This gives the following differential equation for the stock of pollutants:

$$\dot{x} = y - A(x), \quad x(0) = x_0 > 0 \text{ given.} \tag{2.1}$$

With respect to the decay function we make three alternative sets of assumptions:

A2.i) $A : R^+ \to R^+$ is continuously differentiable and concave. Furthermore, $A(0) \geq 0, A'(x) \geq 0$ for all $x \geq 0$ and there exists $y \geq 0$ such that $A(y) > 0$.

A2.ii) $A : R^+ \to R^+$ is continuous, $A(0) > 0$, and there exists $\bar{x} > 0$ such that $A(x) = 0$ for all $x \geq \bar{x}$. A is continuously differentiable and concave for $x \geq \bar{x}$, with $A'(x) \leq 0$. Finally, $x_0 < \bar{x}$.

A2.iii) There exists $z > 0$ such that A satisfies the conditions of A2.i for $0 \leq x \leq z$ and the conditions of A2.ii for $z \leq x \leq \bar{x}$.

A typical representation of the decay function is given in Figure 2.1.

Instantaneous social welfare depends on consumption of the consumer commodity, leisure and the stock of pollution. The instantaneous welfare function is strongly separable in these entities. The utility attached to the consumer commodity is given by a function U satisfying

A3. $U : R^+ \to R^+$ is continuously differentiable on R^{++} and strictly concave, with $U'(0) = \infty$ and $U'(\infty) = 0$.

Second there is a disutility D associated with the stock of pollutants.

A4. $D : R^+ \to R^+$ is continuously differentiable and strictly convex, with $D(0) = D'(0) = 0$ and $D'(\infty) = \infty$.

We assume that leisure, which equals $\bar{l} - l$ where $\bar{l}$ is the total amount of time available, enters linearly into the social welfare function. By appropriate scaling we can attribute a utility weight of unity to leisure.[6] We further assume the usual discounted

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[6] Note that the formulation of the social welfare function can be obtained in another way as well. It is not necessary to restrict attention to leisure as an argument in the welfare function. It could as well have been assumed that instantaneous welfare is obtained from a large set of other commodities besides the consumption commodity considered here along with pollution, and that utility is quasi-linear in other consumption goods.

Figure 2.1
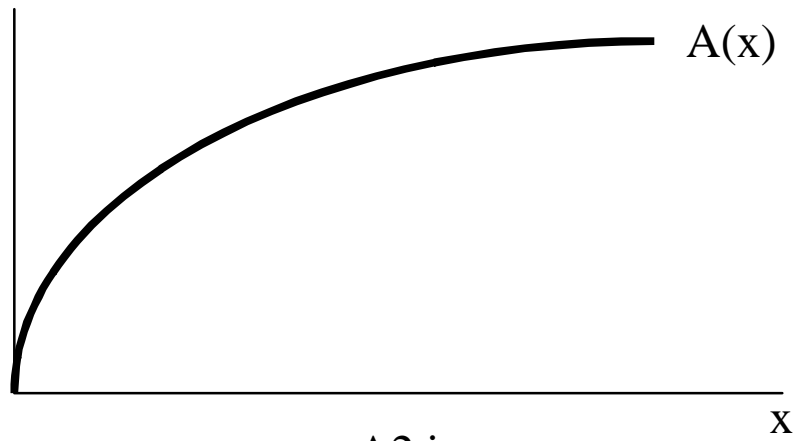
A2.i
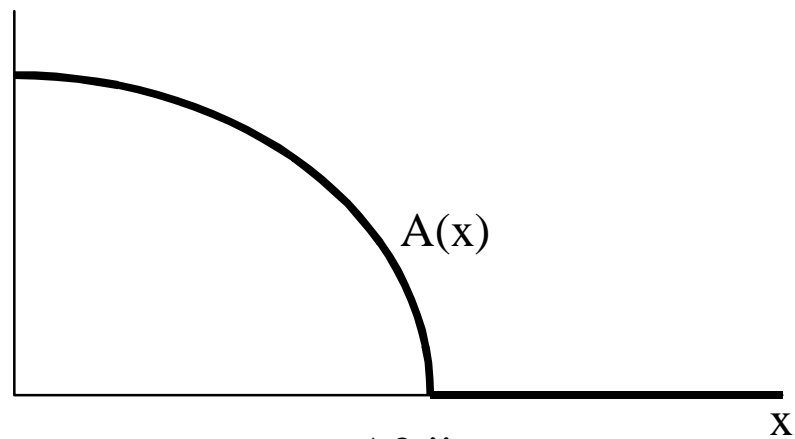
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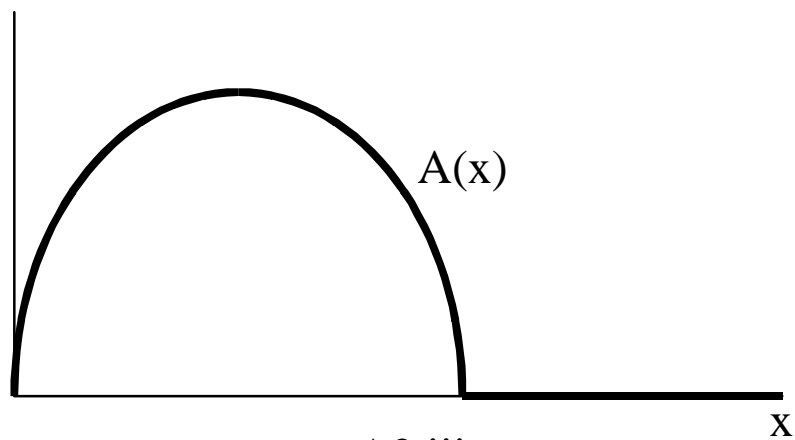
A2.iii

utilitarian criterion for analyzing a social optimum, with a positive and constant rate of time preference r. The intertemporal social welfare function then can be written as

$$\int_0^\infty e^{-rt}[U(q)-(\bar{l}-l)-D(x)]dt \tag{2.2}$$

This integral has to be maximized with respect to the labor input and the use of the polluting input/flow of emissions, taking into account the evolution over time of the stock of pollution (2.1) and the pure state constraint $x \geq 0$. This is a standard optimal control problem, except for the pure state constraint, which makes the set of necessary conditions for optimality a little complicated. We return to this and other technical points below.

Finally, we make one other assumption to guarantee an interior solution for labor and the pollutant stock.

A5. Suppose $D(x) \equiv 0.$ Then the problem posed above has a solution, denoted by (l^c, y^c) such that $l^c < \bar{l}$ and $y^c > A(0)$.

The first part of the assumption says that in the absence of pollution damage not all labor is allocated to production. This necessarily will also hold when pollution is damaging. The second part of the assumption says that, in the absence of pollution damage, there exists an optimal level of the flow of pollution which causes a positive stock of pollution eventually.

The current value Hamiltonian of the problem is

$$H(l, y, x, m) = U(F(l, y)) + (\bar{l} - l) - D(x) + m[y - A(x)]$$

Here, m is the shadow price of the pollution stock. Intuitively it can be seen that this shadow price of damage should be negative; we discuss this point formally in the Appendix. To simplify the notation, define $t = -m$. Given a positive pollution stock along an optimal path, assumptions A1 and A3 (in particular necessity of labor input and infinite marginal utility at zero consumption) imply that necessary conditions for an optimum are

$$\partial H / \partial l = 0: \ U'(q)F_l(y,l) = 1 \tag{2.3}$$

$$\partial H / \partial y \leq 0, \ y \partial H / \partial y = 0: \ U'(q)F_y(y,l) \leq \tau, \ = \tau \ if \ y > 0 \tag{2.4}$$

$$\partial H / \partial x = \dot{\tau} - \rho\tau: \ \dot{\tau} = -D'(x) + [\rho + A'(x)]\tau \tag{2.5}$$

The interpretation of the above conditions is straightforward if t is viewed as a pollution tax. Condition (2.3) says that the marginal benefits from labor should equal the

value of leisure. Condition (2.4) requires that the marginal benefits from using the polluting input should equal the marginal damage, which equals the tax rate, except in a corner solution with $y=0$. Equation (2.5) indicates that the co-state is the current value shadow cost of the pollution stock. To see this define the present value tax $\bar{t}$ and integrate to obtain

$$\bar{t}(t) = \int_{t}^{\infty} e^{-rs} D'(x(s)) ds - \int_{t}^{\infty} \bar{t}(s) A'(x(s)) ds \tag{2.6}$$

The first term on the right hand side is the present value of future damage costs from a current increase in the pollution stock. The second term is the present value of a future change in the state dynamics, valued at the co-state $\bar{t}$. Our t has the same interpretation except for the change in focal date from zero to date t.

An important property of the optimal path is given by Lemma 2.1, which is proved in the Appendix.

Lemma 2.1. Along an optimal path, $x(t)$ is monotonic and converges.

This property of the optimal solution is useful in constructing graphical representations of the optimal pollution path and tax rate.

The development to this point has ignored the possibility of the constraint $x \geq 0$ being binding. Lemma 2.2, whose proof also is given in the Appendix, provides a sufficient condition for treating this constraint as nonbinding.

Lemma 2.2. $x(t) > 0$ for all t if and only if $r + A'(0) > 0$.

We assume throughout the rest of the paper that the condition $r + A'(0) > 0$ holds, since according to Lemma 2.2 the problem of long-term environmental management is not very significant in the opposite case (in that case, there is some t_1 such that $x(t) = 0$ for all $t \geq t_1$ along an optimal path).

3. PROPERTIES OF OPTIMAL PATHS

Since the model contains only one state variable it is in principle possible to perform a phase diagram analysis. We define for $x \geq 0$ and $r + A'(x) \neq 0$ the function

$$t_1(x) = \frac{D'(x)}{r + A'(x)} \tag{3.1}$$

In view of (2.5), this gives the locus of points where the optimal tax rate is constant. Due to the concavity of the decay function and the convexity of the damage function, the locus is upward sloping wherever it is defined. Moreover, under Assumption A4 we have $t_1(0) = 0$.

If there exists $x_r > 0$ such that $r + A'(x_r) = 0$ then this x_r is an asymptote. Given (2.5), if $t > t_1(x)$ for some $x < x_r$ then $\dot{t} < 0$, and conversely (intuitively, with a lower tax than the critical level $t_1(x)$, usage of the dirty input is encouraged, the pollution stock grows, marginal damages rise, and the tax needed to internalize damages grows).

   The set of points for which the stock of pollution is constant is described by (2.3), (2.4), and $y = A(x)$ from (2.1). We denote this locus by $t_2(x)$. To derive its properties, assume first that $y = A(x) > 0$. We can solve (2.3) for l as a function of y, which we denote by $l = y(y)$. Straightforward calculations (assuming differentiability) yield

$$\frac{dy(y)}{dy} = -\frac{U''F_l F_y + U'F_{ly}}{U''F_l F_l + U'F_{ll}}$$

Then some straightforward but extremely messy algebra exploiting the concavity of the production function F shows that

$$\text{sgn}(dt_2/dx) = -\text{sgn}(A'), \tag{3.2}$$

Thus, the t_2 locus is increasing if and only if A' is decreasing (see the Appendix for further discussion). Finally, from the construction of t_2 it follows that if $t < t_2(x)$ for some x then $\dot{x} > 0$, and conversely (intuitively, if the tax rate is lower than the critical level then use of the dirty input is encouraged and the pollution stock grows).

   The analysis in the previous paragraph requires modification in two respects to deal with corner solutions. If Assumptions A2.ii or A2.iii hold for the pollution assimilation function A, so that $A \to 0$ as $x \to \bar{x}$, then t_2 introduced above is well-defined only for $x < \bar{x}$. For $x \geq \bar{x}$ there is no assimilation capacity and the pollution stock can be constant only if $y = 0$, i.e., only if "clean" production is possible.

   Now suppose that $F(l,0) > 0$, i.e., completely "clean" production is possible. Define $\hat{l} > 0$ and $\hat{t} > 0$ by

$$U'(F(\hat{l},0))F_l(\hat{l},0) = 1, \ U'(F(\hat{l},0))F_y(\hat{l},0) = \hat{t} \tag{3.3}$$

Then the locus $t_2(x)$ described above is well-defined only for $t_2(x) \leq \hat{t}$. Intuitively, the point is that for a pollution tax larger than $\hat{t}$, $y = 0$ and clean production is pursued with $l = \hat{l}$.

   Many possibilities arise with respect to the phase plane and optimal paths, depending on the various assumptions invoked. In the balance of this section we illustrate some of the possibilities, drawing upon related analyses in Tahvonen and Withagen (1996).

3.1 Nondecreasing Pollutant Decay

If $A' \geq 0$ as in Assumption A2.i, then we have the picture shown in Figure 3.1. The top panel of this picture shows a unique saddle point equilibrium (x^*, t^*) with the optimal steady-state tax satisfying $t^* < \hat{t}$. This outcome will obtain in particular if $F(l,0) = 0$, meaning y is essential and completely clean production is impossible. The bottom panel shows an intersection of the t_1 and t_2 loci where $\hat{t} < t^*$. In this case the long-term steady-state features clean production ($y=0$) and a pollution stock $\hat{x}$ given by $t_1(x) = \hat{t}$.

3.2 Assimilation Capacity that Decreases and Can Disappear

The analysis is much more complex if Assumption A2.ii holds for the assimilation function. As already noted, $t_1 \rightarrow \infty$ as $x \rightarrow x_r$ where $r + A'(x_r) = 0$. We also have that $t_2' > 0$, and Assumption A5 implies that $t_2(0) > 0$ (since if $t = 0$ the optimal choice of y is $y^c > A(0)$).

One possible set of outcomes is shown in Figure 3.2, in which we assume that clean production is possible. In this figure, $x_r < \bar{x}$ and t_1 and t_2 intersect exactly once at a point (x^*, t^*) with $t^* < \hat{t}$. For any initial point $x_0 < \bar{x}$ there is an extremal path, indicated by a in the diagram, which converges monotonically to (x^*, t^*). Given the possibility of clean production, part of the convergence from above on this path involves a period of clean production, but as the extremal path nears its limit use of the dirty input on this path resumes.

There is, however, another possibility. In Figure 3.2 we also assume that $\bar{x} < \hat{x}$, where $t_1(\hat{x}) = \hat{t}$, the tax that triggers use of the clean technology. Suppose that the initial pollution stock satisfies $\bar{x} \leq x_0 < \hat{x}$. With this environmentally unfavorable starting point, pollution further accumulates and the optimal pollution tax increases monotonically until the shadow price of pollution is high enough to trigger the elimination of dirty input use and the stock of pollutants ceases to grow but remains at a level beyond any assimilative capacity. This path is shown as b in Figure 3.2.

We can now exploit the fact that solutions of the differential equations for $(\dot{x}, t)$ along the extremal path vary continuously with the initial conditions to argue that path b can be extended somewhat to the left of $\bar{x}$ to some pollutant stock level $\tilde{x}$. Behind this mathematical construct is the economic observation that the disappearance of assimilative capacity creates a nonconvexity in the social planner's optimization problem. With this nonconvexity, the first-order conditions cannot be counted on exclusively to determine the optimum. In Figure 3.2, if the initial environmental state is in the critical range $\underline{x} \leq x_0 < \bar{x}$, then both the a path of environmental restoration to the steady-state x^* and the b path to the
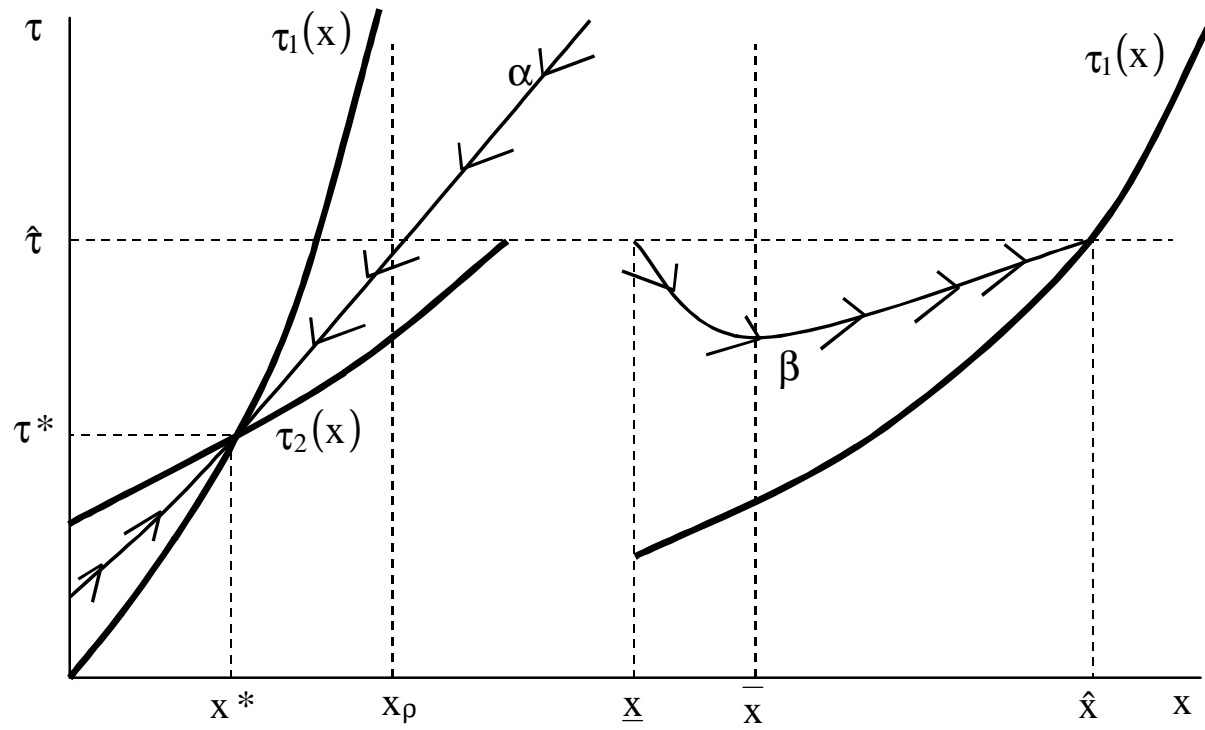
Figure 3.1
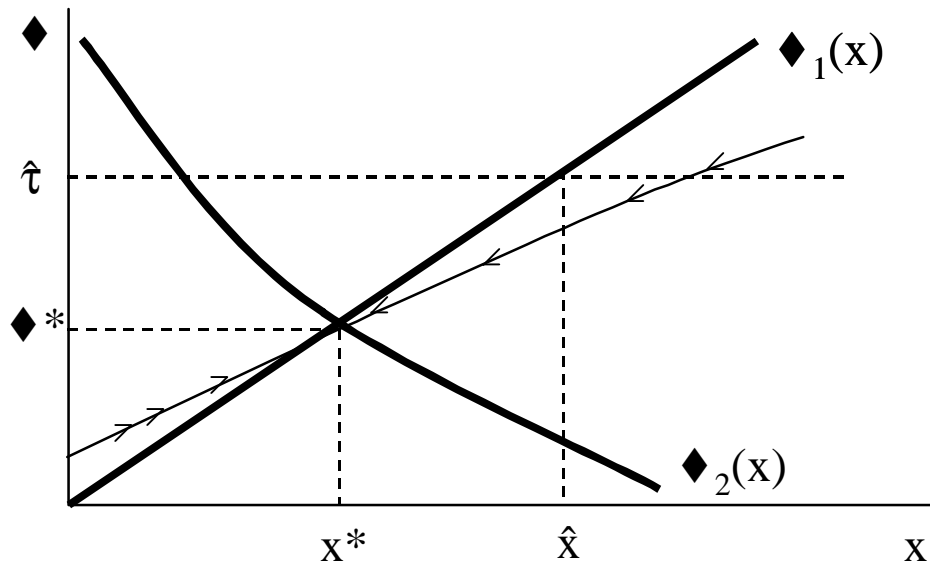
(a)
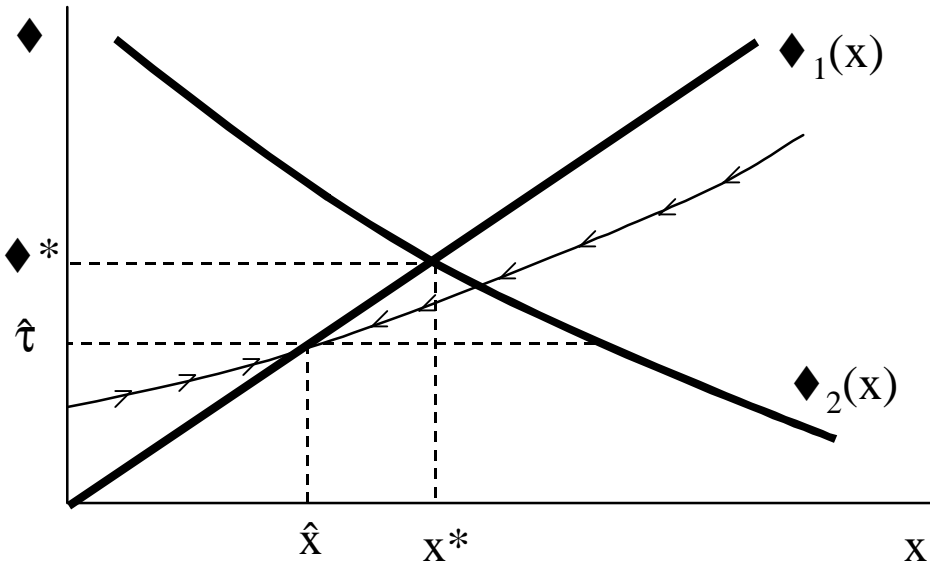
(b)

Figure 3.2

polluted steady-state $\hat{x}$ are possibilities, and only a direct comparison of the two paths (through an integral test) can indicate which has higher discounted social surplus.[7]

As noted, the situation pictured in Figure 3.2 is only one of many possibilities. Another, somewhat more optimistic situation is shown in Figure 3.3, Here we assume that the clean technology is more affordable that in Figure 3.2, so that the critical tax rate $\hat{t}$ is lower and $\hat{x} < \bar{x}$. In this case there is no prospect of a path starting at $x_0 < \bar{x}$ going beyond the limit of assimilative capacity like the b path in Figure 3.2, since with these initial conditions in Figure3.3 the optimal pollution tax will be high enough to trigger immediate use of the clean technology and the associated environmental recovery.[8]

On the other hand, if clean technology does not exist (y is an essential input), then in Figure 3.2 we have the possibility of the pollutant stock exhausting assimilative capacity. Still other possibilities arise if $t_2 > t_1$ for all x, in which case both the pollution stock and tax increase and are bounded only by the existence of a clean technology (if at all), or if there are multiple intersections of these two loci (with stable and unstable potential equilibria but still the potential for a dirty as well as a cleaner steady-state).

Clearly the most general assumption on the assimilation function, A2.iii, combines the results of the two cases considered above. Since there is no need to develop these details, we turn instead to a somewhat closer look at the issue of clean technology choice in a special case.

4. DISCRETE TECHNOLOGY CHOICE

In this section we consider a special case where the economy has the capacity to produce the consumer commodity in exactly two discrete ways. The first way employs a "clean" technology, while the second way employs a "dirty" but cheaper technology. We assume that both technologies are linear in labor:

$$l_a = c_a q_a, \;\; l_b = c_b q_b, \;\; c_a > c_b \tag{4.1}$$

where l, c and q denote labor input, labor requirement and output respectively and a and b refer to the clean and the dirty technology respectively. Pollution is a joint product of technology b, and we assume without loss of generality that the pollution flow also can be represented by q_b. Therefore we have

$$\dot{x} = q_b - A(x) \tag{4.2}$$

---

[7] A similar outcome occurs if there is no root of the equation $A'(x) + r = 0$; in this case it can be shown that, in effect, $x_r = \bar{x}$ and the analysis is virtually identical to that given in the text.

[8] The same kind of outcome would occur if the clean technology were so cheap that it obviated the steady state given by the intersection of the two t loci, as shown in the bottom panel of Figure 3.1.

Figure 3.3
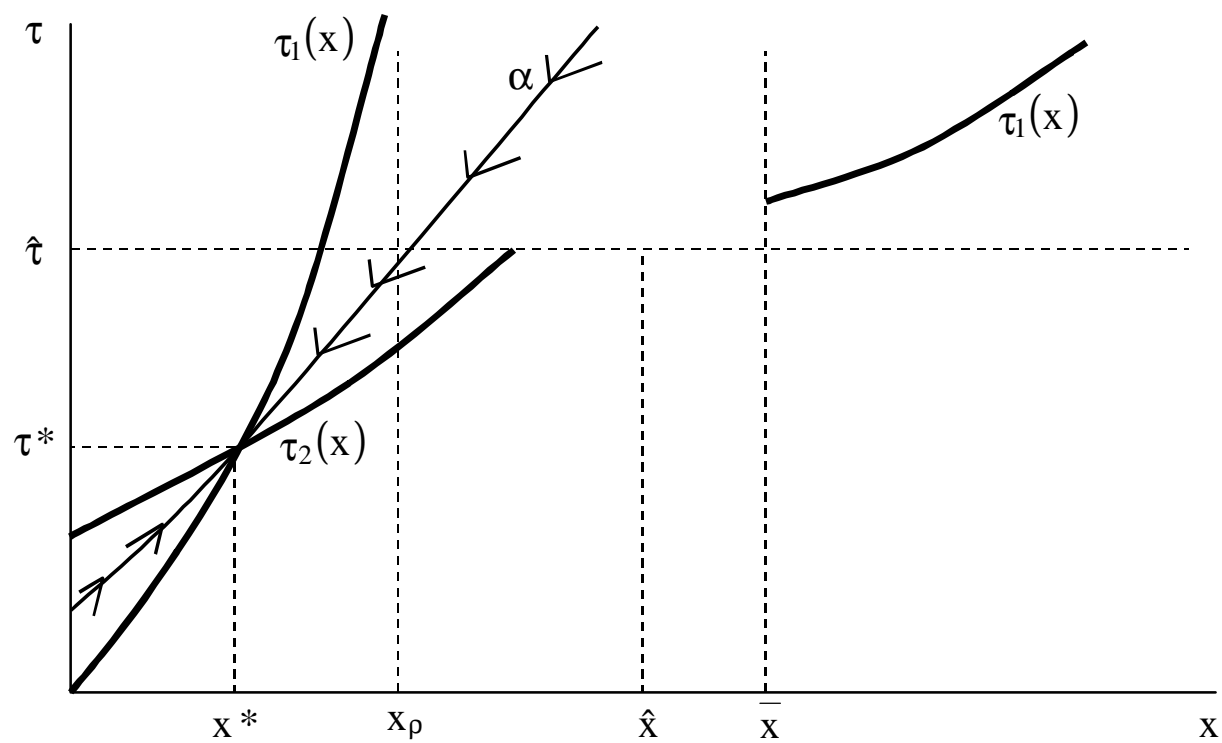
The current value Hamiltonian now is

$$H(q_a, q_b, x, t) = U(q_a + q_b) + \bar{l} - c_a q_a - c_b q_b - D(x) - t[q_b - A(x)] \tag{4.3}$$

The necessary conditions include the dynamic equation (2.5) for the pollution tax, and

$$U'(q_a + q_b) \leq c_a (= c_a \text{ if } q_a > 0) \tag{4.4}$$

$$U'(q_a + q_b) \leq c_b + t (= c_b + t \text{ if } q_b > 0) \tag{4.5}$$

Condition (4.4) says that the marginal instantaneous utility of the non-toxic commodity should not exceed the marginal cost of producing it. This is also the meaning of (4.5), where marginal cost of the toxic commodity consists of the sum of marginal cost of production and the marginal environmental cost or pollution tax.

There exists a strong similarity between the model outlined above and the literature on renewable natural resources. One could look upon x as the reciprocal of a renewable resource, which is depleted by extracting the commodity q_b, but which is subject to a regeneration process given by A as a function of the stock, say $y = 1/x$. The resource stock has amenity value $-D(1/x)$ and the rate of extraction yields a profit $U(q_b) - c_b q_b$. The other commodity is produced according to a backstop technology which, by definition, does not need the natural resource (see also Levhari and Withagen 1992).

Now define $\hat{t}$ by

$$\hat{t} = c_a - c_b \tag{4.6}$$

If the actual tax is larger than $\hat{t}$ it is too costly to produce the toxic commodity because its production cost plus tax exceeds the production cost of the clean commodity. Therefore, τ will not converge to a value larger than $\hat{t}$. It might be that τ converges to $\hat{t}$, at which tax rate producers are indifferent as to what production process they will employ. Alternatively, $t < \hat{t}$ eventually so that producers will only produce the dirty commodity.

Analyzing these cases involves phase plane analysis that is essentially identical to that presented in the previous section. We can define the locus $t = t_1(x)$ for $\dot{t} = 0$, as before, and the locus $t = t_2(x)$ for $\dot{x} = 0$ in this case is given by (4.5) and

$$t_2(x) = U'(A(x)) - c_b \tag{4.7}$$

provided $A(x) > 0$ and $t < \hat{t}$. These loci have the same properties as described above for the general model, and thus the solutions also are similar – including the challenges of multiple extremal paths and the potential for dirty and clean steady states when there is nonconvexity under Assumption A2.ii. In particular, we have use of the clean technology if and only if

$t > \hat{t}$. Thus, in outcomes corresponding to the top panel of Figure 3.1, the clean technology is used only if the pollutant stock is high; as the environment recovers, the economy reverts to the dirty technology. In the bottom half of Figure 3.1, in contrast, an economy with a relatively clean environment ($x_0 < \hat{x}$) uses the dirty technology, while an economy with a dirtier environment ($x_0 > \hat{x}$) relies exclusively on the clean technology. The pattern of technology use along the b path in Figure 3.2 is even more striking, in that the economy following this path would avoid using the clean technology until environmental degradation has gone beyond the limit of assimilative capacity, but would then deploy the clean technology to prevent the now-unavoidable ongoing flow of pollution damages from growing too large.[9]

We emphasize these different patterns of technology choice in the discrete technology model for two reasons. The first is simply to illustrate the range of possibilities and their dependence on the underlying features of the problem, including both the behavior of the assimilation function and the relative cost of the clean technology. These differences have some implications for policy, which we discuss in the next section. The other reason to emphasize the patterns is a critical one. In not just the special case we consider here but also in the general case (and in similar models, e.g. by Keeler et al 1971), it is possible to move between the clean technology and dirtier technologies at will. There are in particular no fixed set-up costs governing the deployment or re-deployment of the clean technology. Set-up costs can make the choice of technique "stickier," and incorporating them into this analysis is a topic of ongoing research.

5. **LESSONS FOR POLICY DESIGN**

In this section we combine the analysis from previous sections with reasoning about optimal policy design related to both pollution control and the stimulation of new technologies. The first question we address is the desirability of banning dirty products or processes. As noted in the Introduction, this policy seems to be growing in popularity. The analysis in Sections 3 and 4 suggests that there is at best a limited rationale for a banning policy in the case that $A' \geq 0$. This case presents no fundamental environmental irreversibilities, and a well-designed environmental policy instrument (see below), perhaps combined with policies to help ameliorate any failures in the R&D market, will induce

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[9] George Tolley has pointed out to us that our assumptions about the damage function exclude the plausible and interesting case in which once assimilative capacity is exceeded, damages are unavoidable but eventually attenuate over time. We could model this in our setup, for example, by modifying the damage function such that if $x = \bar{x}$ at some time s, damages from that time onward (for any $x \geq \bar{x}$) are given by $\overline{D}(x) = D(\bar{x})e^{-l\,(t-s)}$ and l is the rate of damage decay. Our results would be largely unchanged by this modification, except that with this permanent but attenuating damage the prospects for the b path being optimal in Figure 3.2 would be enhanced, and there would be no human-induced brake on the accumulation of pollution (no reason to deploy the clean technology to limit growth in x).

substitution to the clean technology when this is warranted (taking into account sunk costs in actual practice as well).

Even with environmental irreversibilities the rationale for a pollution banning policy remains unclear. In Figure 3.2, suppose that the optimal path for the economy is the clean path a, rather than b. If the environment starts out degraded but not destroyed $(x^* < x_0 < \bar{x})$, and pollution control policy creates a high shadow price consistent with path a, then the economy will in fact find it attractive to switch to the clean technology or product without a banning policy.[10] Banning is even less warranted if the situation is as shown in Figure 3.3. Banning thus would be useful in our context only if there is concern that market forces supplemented by appropriate pollution control policies will not lead the market to the right technology choices over the longer term (or that this process somehow will take "too long" in relation to some potential environmental catastrophe). And if this is the case, an important question which lies largely beyond the scope of this paper is whether banning the offending products or processes is superior to other policies that correct market failures in the development and diffusion of new technology, such as R&D support and information/ demonstration campaigns.[11]

The second question we turn to is the sensitivity of optimal outcomes to key parameters and the implications of these comparative dynamics for policy. The phase diagrams highlight the sensitivity of outcomes to the relative costliness of the clean technology (this is seen most starkly in the discrete choice case, as indicated by equation (4.6), but it is true for the more general case as well). In particular, Figures 3.2 and 3.3 show that a fall in the relative cost of the clean technology not only is likely to improve the environment in general but may also help to avoid the conundrum of dirty versus clean long-term futures with declining assimilative capacity. The obvious message here again is that if there are R&D or other market failures that impede progress in the development or diffusion of cleaner technology, there may be a high social value in reducing these impediments.

It is also intuitively obvious that the optimal outcomes are sensitive to the discount rate employed in the social cost-benefit analysis. It can be shown that an increase in r rotates the t_1 locus in equation (3.1) to the right and increases the asymptote x_r. This has the effect of reducing the prospects for occurrence of the clean steady state in Figure 3.2 versus the dirty steady state, a result consistent for example with Krautkraemer (1985), among others. Aside from raising broader issues beyond our scope concerning the nature of social decision criteria for long-term environmental problems (see, e.g., Azar 1998), and concerns for the sources of high discounting in practice (e.g., poverty and misfunctioning capital markets), the sensitivity of the outcome to the discount rate in our analysis also underscores the potential importance

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[10] Indeed, the question in this case becomes the longer-term viability of the clean technology when the environment improves. But since our model includes neither sunk costs nor learning-by-doing, it is not really that capable of addressing this question.

[11] A banning policy is a blunt instrument in terms of the timing of switchover; but other policies may encounter the problem of trying to "pick winners."

of clean technology development: greater availability of such technology may be able to increase the prospects for a socially preferred clean outcome.

The third point we address concerns the robustness of various pollution policy designs. In practice, achieving anything like a dynamically optimal pollution tax may be too much to hope for. The most that may be possible in practice with tax policy is the setting of a pollution charge that must be maintained for a long period of time. Consider the limiting case of this hypothesized institutional rigidity: a fixed pollution charge must be set for all time. Suppose one knows the desired steady state $x^* < \hat{x}$, or one simply has specified a priori some other long-term target for pollution stock $x^\# < \hat{x}$. If $A' \geq 0$ then a fixed tax can be set in Figure 3.1 to support either long-term environmental quality goal. For example, in the discrete technology case one can determine the desired tax rate $t^\#$ by $U'(q_b^\#) = c_b + t^\#$ and $A(x^\#) = q_b^\#$. In particular, if $x^\# = x^*$ this policy will amount to setting the optimal steady-state tax from the start and having inefficient convergence to the steady-state environmental quality.

In this case, a small variation in whatever long-term tax is set will trigger only a small variation in the long-term environmental quality that is induced. This is not the case if $A'(x) < 0$ for $x < \bar{x}$ and $A(x) = 0$ for $x \geq \bar{x}$. To see this, consider the discrete technology case and suppose the fixed tax rate $t^\#$ is set at a high level (though below $\hat{t}$), leading to a low level of dirty commodity output, $q_b^\#$, relative to initial neutralization capacity $A(x_0)$. Then x will be decreasing, and since $A' < 0$, there will be an ever growing decrease in x in the future until x approaches 0. Similarly, if the tax $t^\#$ is set too low, x will increase to the limit of the environment's neutralizing capacity and beyond. In short, a constant tax policy in this case does not yield a stable solution in terms of the allocation of natural absorptive capacity.

An alternative approach for addressing long-term management of an accumulative pollutant would be an intertemporal emissions trading policy based on the long-term capacity of the environment to neutralize pollutants. This approach has received significant recent attention in connection with the regulation of greenhouse gases (see, e.g., Kosobud et al. 1994). In this approach, one starts again with some specification (optimal or otherwise) of a long-term environmental target $x^\#$ and an estimate of the associated pollution neutralization capacity $A(x^\#)$. This long-term allowable pollution flow is allocated annually through the economy in some fashion through use-or-lose emissions permits that can be traded among sources.[12] In addition, assuming that $x_0 < x^\#$, the remaining unexploited capacity $x^\# - x_0$ can

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[12] This approach would not necessarily require the capacity to monitor actual pollution releases. For example, if pollution releases are a readily predictable function of outputs or inputs, as is the case for example with fossil fuel combustion, then allowances could be assigned based on these observable quantities. The monitoring problem is more challenging if pollution can be contained or recycled.

be allocated as a bankable stock of single-use emissions permits that emitters can trade or use as they see fit.[13]

This approach would provide greater dynamic efficiency in the adjustment toward a long-term environmental target than a rigid tax, precisely because the permits market would be able to induce more dynamically efficient adjustments in the shadow price of the pollutant over time.[14] In particular, the approach would provide reasonable long-term market signals for the development and diffusion of clean technology. Moreover, the quantity-based approach would be more robust in the case of decreasing assimilative capacity. The Weitzman (1974) issue of price versus quantity instruments could argue in favor of a tax-based approach with uncertain costs (see Pizer 1997 for a discussion of this in the context of greenhouse gases), though this depends on the slope of the marginal damage function. Moreover, issues of time consistency (Kydland and Prescott 1977) arise in attempting to implement an intertemporal emissions trading program, but the same issues arise in connection with *any* dynamic pollution control program that involves a rising shadow price of emissions. We therefore conclude that especially in the case of declining and uncertain assimilative capacity, a dynamic pollution trading program warrants serious consideration as a control strategy.

6. CONCLUDING REMARKS

We have shown in this paper that the introduction of cleaner substitution possibilities considerably complicates the dynamics of optimal accumulative pollutant control, especially with limited assimilative capacity. The analysis does not provide support for easy prescriptions such as a permanent ban on polluting goods when cleaner ones are available, or a subsidy for the cleaner good designed to achieve the same end. The analysis instead underscores the virtues of adapting incentive-based economic instruments to the situation under consideration, in particular the use of dynamic quantity-based policies.

As we have noted already, an important extension of the current analysis involves addressing optimal investment in the capacity to produce using cleaner methods. A better understanding of how product substitution interacts with optimal environmental remediation policies (extending the work of Levhari and Withagen (1992)) also would be useful. Beyond these points, an obvious but longer-term objective is the treatment of the implications of ecological and economic uncertainties.

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[13] In practice the problem is not so simple since the timing of emissions will affect interim pollutant concentration; this approach does not ensure the achievement of a particular environmental goal by a fixed time.

[14] These cost savings may be substantial, as illustrated by the analysis in Wigley, Richels and Edmonds (1996).

APPENDIX

In this appendix we prove Lemmas 2.1 and 2.2. We also derive some comparative statics results that are useful in drawing the phase diagrams of section 3.

The current value Hamiltonian of the problem of section 2 reads

$$H(l, y, x, m) = U(F(l, y)) + (\bar{l} - l) - D(x) + m[y - A(x)]$$

1. $\mu(t) \leq 0$ *virtually everywhere*.

A necessary condition for optimality is that at each instant of time the Hamiltonian is maximized with respect to y subject to the condition that $y \geq 0$. Given the fact that the production function is non-decreasing there exists no maximum if the co-state would ever be positive.

2. *Proof of lemma 2.1*.

Suppose on the contrary that x displays a bulge as depicted in Figure A1. Here it is assumed without loss of generality that x starts increasing at time zero and returns to the old level at time T.

Given the continuity of x there exist $0 < t_1 < t_2 < T$ such that $x(t_1) = x(t_2) > 0, \dot{x}(t_1) > 0$ and $\dot{x}(t_2) < 0$. In view of the concavity of F and A and, in particular, the strict concavity of U and D, the path x is the unique optimal trajectory. But, this being so, the optimal path from t_2 on should be the same as from t_1 on, which is not the case by construction. This proves the monotonicity of the optimal stock of pollutants. Convergence of the stock follows from the fact that it is bounded from below (by zero) and from above, due to the fact that $D(\infty) = \infty$.

3. *Proof of lemma 2.2*.

First we consider the case where $A(0) > 0$.

Fix some t. Without loss of generality we take it equal to 0 in order to save on notation. Consider two adjacent intervals of time with length h starting at 0. The optimal stocks of pollutants at the beginning of the intervals are denoted by $x(0), x(h)$ and $x(2h)$. Suppose it is optimal to have $x(0) = x(h) = x(2h) = 0$. Then the optimal rates of pollution are $y(0) = y(h) = y(2h) = A(0)$. These are supposed to be constant along the intervals. The optimal labor input is constant as well and will be suppressed henceforth. Now we consider an alternative feasible trajectory, denoted by hats, such that $\bar{y}(0) = A(0) + e$, with e a positive constant still to be determined. The labor input is unaltered. We take care that

Figure A1
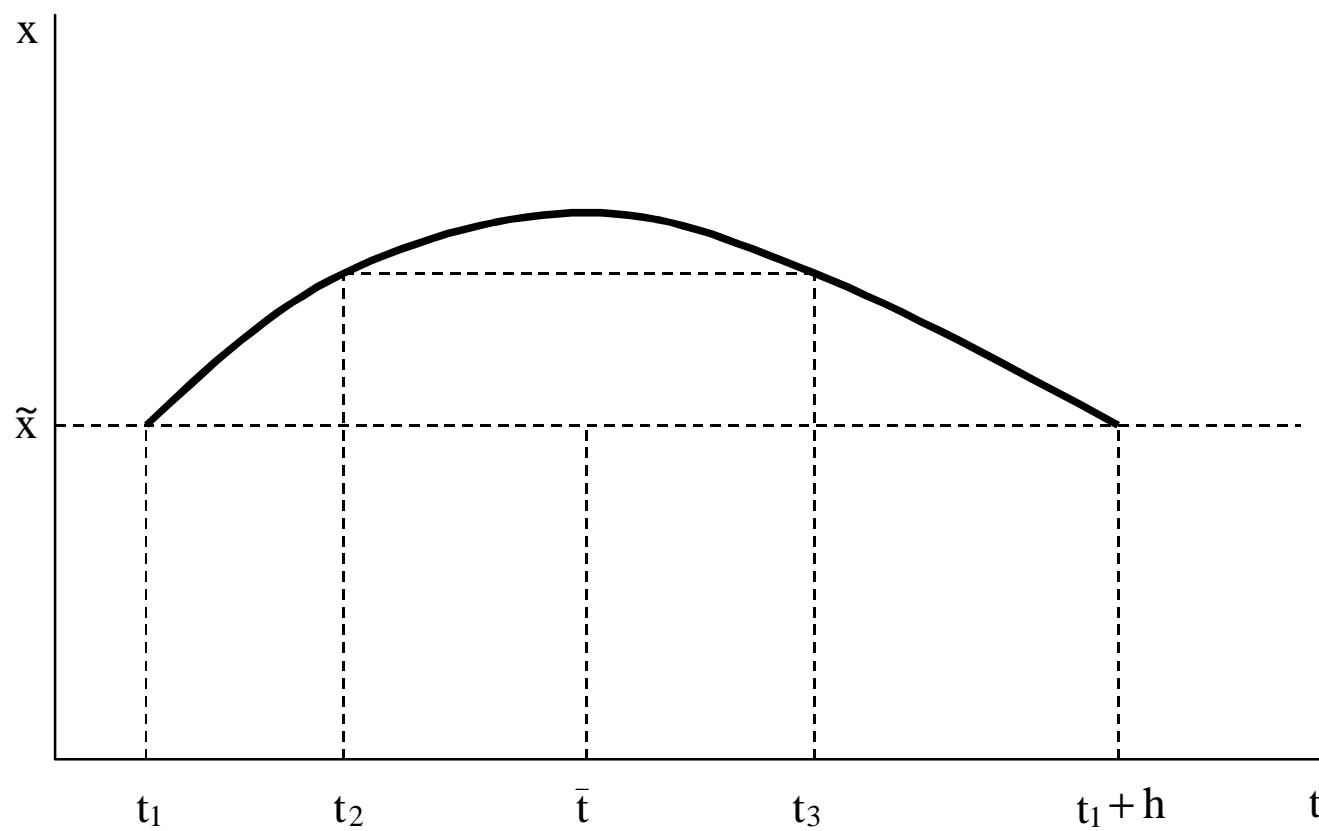
$\bar{x}(0) = x(0) = \bar{x}(2h) = x(2h)$. Hence $\bar{y}(h) = A(0 + h\mathrm{e}) - \mathrm{e}$. The difference between total welfare along the new and the old paths over the two periods equals:

$$\Delta W = - h[U(F(A(0)) - D(0)] - \frac{1}{1 + \mathrm{r}\,h} h[U(F(A(0)) - D(0)] +$$

$$h[U(F(A(0) + \mathrm{e})) - D(0)] + \frac{1}{1 + \mathrm{r}\,h} h[U(F(A(0) + A(0 + h\mathrm{e}) - \mathrm{e} - A(0)) - D(h\mathrm{e})]$$

This expression can be rewritten as follows:

$$\Delta W = h\mathrm{e}\frac{U(F(A(0) + \mathrm{e}) - U(F(A(0)))}{\mathrm{e}} - h\frac{1}{1 + \mathrm{r}\,h} h\mathrm{e}\frac{D(h\mathrm{e}) - D(0)}{h\mathrm{e}}$$

$$+ h\mathrm{e}\frac{1}{1 + \mathrm{r}\,h}[h\frac{A(0 + h\mathrm{e}) - A(0)}{h\mathrm{e}} - 1]\frac{U(F(A(0) + A(0 + h\mathrm{e}) - A(0) - \mathrm{e}) - U(F(A(0)))}{A(0 + h\mathrm{e}) - A(0) - \mathrm{e}}$$

$$= \frac{h\mathrm{e}}{1 + \mathrm{r}\,h}\left[\begin{array}{l}(1 + \mathrm{r}\,h)\dfrac{U(F(A(0 + \mathrm{e}) - U(F(A(0)))}{\mathrm{e}} + \\[4pt] h[\dfrac{A(0 + h\mathrm{e}) - A(0)}{h\mathrm{e}}]\dfrac{U(F(A(0) + A(0 + h\mathrm{e}) - A(0) - \mathrm{e}) - U(F(A(0)))}{A(0 + h\mathrm{e}) - A(0) - \mathrm{e}}\end{array}\right]$$

$$- \frac{h\mathrm{e}}{1 + \mathrm{r}\,h} h\frac{D(h\mathrm{e}) - D(0)}{h\mathrm{e}}$$

The expression between brackets converges to $hU' F_y[\mathrm{r} + A'(0)]$ as e converges to zero. Therefore if $\mathrm{r} + A'(0) > 0$ the optimal path is overtaken, a contradiction.

This procedure cannot be used if $A(0) = 0$, because then the flow of pollution would have to be negative in the second period. But an alternative construction is as follows.

Fix some t. Without loss of generality we take it equal to 0. Suppose $x(t) = 0$ for all $t \geq 0$. For this to be optimal it is necessary that $F(l,0) > 0$. Consider an alternative feasible path, denoted by upper bars, constructed as follows. Fix a small positive e. Choose $\bar{l}(t) = l(t)$, $\bar{y}(t) = y(t) + A(\mathrm{e}) = A(\mathrm{e})$. And let $\bar{x}(t)$ be the solution of $\dot{x}(t) = A(\mathrm{e}) - A(x)$, $x(0) = 0$. Then $\bar{x} < \mathrm{e}$ and

$$U(F(l,\bar{y}) - D(\bar{x}) - U(F(l,0) + D(0) \geq A(e)\frac{U(F(l,A(e)) - U(F(l,0))}{A(e)} - e\frac{D(e) - D(0)}{e}$$

which expression is positive for e small enough, contradicting the optimality of x being zero.

4. Comparative statics results.

If the stock of pollutants is constant it follows from the necessary conditions 2.1 and 2.3

$$y = A(x)$$

$$U'(F(l,y))F_l(l,y) = 1$$

that

$$dy = A'(x)dx$$

and

$$[U''F_yF_l + U'F_{ly}]dy + [U''F_lF_l + U'F_{ll}]dl = 0$$

Total differentiation of 2.4

$$U'(F(l,y))F_y(l,y) = t$$

and some algebraic manipulation yield

$$\frac{dt_2}{dx} = \frac{A'}{U''F_lF_l + U'F_l}\{U'U'[F_{yy}F_{ll} - F_{ly}] + U'U''[F_y^2F_{ll} - 2F_yF_lF_{ly} + F_l^2F_{yy}]\}$$

In view of the assumptions made, in particular the concavity of U and F and $U'>0, U''<0, F_l >0, F_y >0, \; F_{ly} >0$, it follows that

$$\text{sgn}\frac{dt_2}{dx} = -\text{sgn}\, A'(x)$$

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