Welfare Measurement and Representative Consumer Theory

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Abstract

This paper generalizes results from Anderson, De Palma, and Thisse [1992] linking individual random utility and aggregate representative individual demand models, to consider a comparable relation for the willingness to pay functions for quality attributes of marketed goods. It also suggests how the logic can be used to describe links between choice occasion and aggregate models (across occasions) for an individual.

Key Words: willingness to pay, RUM, aggregation

JEL Classification No(s).: DG1, Q2

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I. INTRODUCTION

Anderson, de Palma, and Thisse [1987, 1992] transformed the conceptual basis for modeling consumer behavior in selecting among differentiated products by demonstrating that the aggregate demands implied by a discrete choice random utility model (RUM) would be equivalent to those from a representative consumer description for specific combinations of utility functions. This paper uses their results to suggest that the same linkage can be established for the Hicksian willingness to pay (WTP) due to changes in one or more environmental amenities. Moreover, the link also holds for Verboven's [1996] recent extension to the Anderson, et al. results with both the case of nested logit models using linear conditional indirect utility (CIU) functions and with the log linear micro CIU specification giving rise to the aggregate group constant elasticity of substitution functions (GCES).

This relationship is important because there has been extensive use of the RUM framework in evaluating the benefits from improvements in one or more attributes of environmental amenities.[1] As a rule, benefit measures sought for policy purposes are on a

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[1] The relationship describes conditions sufficient to assure compatibility between "top down" and "bottom up" measures for the benefits of environmental improvements often associated with benefit transfers. This is a common concern of policy studies that seek to transfer benefit estimates developed from one context to another policy application. See Freeman [1984], Desvousges, Naugton and Parsons [1992], and Smith [1992].

seasonal basis and those constructed from a RUM are estimated for a choice occasion. Multiplying the WTP per choice occasion by the number of occasions has been treated as an arbitrary (and usually incorrect) approximation (Morey [1994]). Our results provide several examples where this practice would be consistent with a framework that links a RUM choice occasion model to a seasonal demand with downward sloping demand function. This is established by treating the Anderson et al. aggregation as involving a pre-defined set of independent choice occasions for each individual.

II. REPRESENTATIVE VERSUS INDIVIDUAL WTP

Assume there are N statistically independent and identical consumers. As with Anderson, et al. [1992], each consumer is allowed to purchase a variable amount of differentiated goods. In a specific choice occasion they select only one variety. The consumer's conditional direct utility, U_j, function for variant j is assumed to be given by equation (1):

$$U_j = ln(a(q_j)\cdot x_j) + \alpha ln x_0 + \varepsilon_j \qquad j = 1,2,\ldots,n \tag{1}$$

where x_j is the amount of variant j (n in total)

q_j is the non-marketed environmental amenity ($a(\cdot)$ is monotonic function that can differ with j, but not across individuals).[2]

x_0 is the numeraire good

$\in_j$ is a random error ($E(\in_j)=0$)

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[2] The source of the augmentation form for describing how quality attributes are associated with the measurement of the amount of constant quality units of a commodity is often attributed to Griliches [1964]. See Triplett [1990] for discussion.

Following Anderson, et al. [1992] the conditional indirect utility function, given, $x_0 = y - p_j x_j$, and $m = y/(1+\alpha)$ (the amount of income spent on one of these products), is given in equation (2):

$$V_j = (1+\alpha)ln(y) - lnp_j + ln(a(q_j)) + B + \varepsilon_j \quad j = 1,2,...n \quad \text{where } B = \alpha ln\alpha - (1+\alpha)ln(1+\alpha) \tag{2}$$

Establishing the aggregate (over N consumers or over N choice occasions) for a single consumer demand for each variety involves assuming $\in_j$ follow independent Gumbel distributions. In this case the choice probabilities for variant j would be given by equation (3):

$$\pi_j = \frac{exp[(-lnp_j + ln(a(q_j)) + (1+\alpha)ln\,y + B)/\mu]}{\sum_{k=1}^{n} exp[(-lnp_k + ln(a(q_k)) + (1+\alpha)ln\,y + B)/\mu]} \tag{3}$$

with μ a scale parameter

The expected demand for variant j for all consumers is then given in (4):

$$X_j = N \cdot x_j \cdot \pi_j = N \cdot \left(\frac{m}{p_j}\right)\pi_j \quad j = 1,2,..,n \tag{4}$$

Deriving the ex ante Hicksian WTP for a quality change requires that we consider how that change influences an individual's choice. This requires defining the random variable for the maximum of the conditional indirect utilities for different values of the q_j's. In the case of independent Gumbel's errors, this maximum will also be a Gumbel with location parameters, θ. For our problem it is given in (5) (see Ben-Akiva and Lerman [1985] pp. 105-106):

$$\theta = \mu ln\left(\sum_{k=1}^{n} exp\left[\frac{1}{\mu}(-lnp_k + ln(a(q_k)) + (1+\alpha)ln\, y + B)\right]\right) \tag{5}$$

The individual's WTP is defined from this location parameter. It specifies how the expected value of this maximum of the conditional indirect utilities changes as q_j changes. Thus evaluating θ at each quality level and introducing WTP for the improved state (q_j^1), yields (6) for a change from q_j^0 to q_j^1, j = 1, 2,...,n.[3]

$$WTP = y\left[1 - \left[\frac{\sum_j \left(\frac{a(q_j^0)}{p_j}\right)^{\frac{1}{\mu}}}{\sum_j \left(\frac{a(q_j^1)}{p_j}\right)^{\frac{1}{\mu}}}\right]^{\mu/1+\alpha}\right] \tag{6}$$

The constant elasticity of substitution (CES) preference function for the aggregate (representative) individual yields an aggregate willingness to pay, WTP^A, for a corresponding quality change that is N times the expression for the individual WTP given in equation (6). This expression can be assumed to describe different individuals or difference choice occasions. To establish this result we follow Anderson, et al. [1992]. The aggregate CES is given in equation (7)

$$U = \left(\sum_{j=1}^{n}(a(q_j)\cdot X_j)^{\rho}\right)^{1/\rho} X_0^{\alpha} \tag{7}$$

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[3] The expected value of maximum differs from the expression for the location parameter, θ, by a constant that is invariant with changes in the q_j's so these do not enter the expression for the WTP.

and the aggregate indirect utility in (8)

$$V = (1+\alpha)lnY + \left(\frac{1-\rho}{\rho}\right) ln\left(\sum_{j=1}^{n}\left(\frac{a(q_j)}{p_j}\right)^{\rho/_{1-\rho}}\right) \tag{8}$$

Algebraic manipulation of equation (8) based on the definition for the WTP

$(i.e.\ V(Y - WTP^A, p_1, p_2, ...p_n, q_1^1, q_2^1, ..., q_n^1) = V(Y, p_1, p_2, ..., p_n, q_1^0, q_2^0, ..., q_n^0))$ *yields* $WTP^A = N \cdot WTP$, *with* $\mu = (1-\rho)/\rho$.

These results can be extended to the case described by Verboven [1996] for the direct utility function of the nested logit with the linear and log linear conditional indirect utility functions. The expected value of the maximum of conditional indirect utilities with a generalized extreme value function can be expressed in terms of the logarithm of the generating function plus a term that is invariant to changes in the arguments of the conditional indirect utility functions (McFadden [forthcoming]). This allows nested logit with log linear CIU functions to be solved for the WTP function, as given in (9):

$$WTP = y\left[1 - \left[\frac{\sum_{g=1}^{G}\left(\sum_{j=1}^{J_g}\left(\frac{a(q_{jg}^0)}{p_{jg}}\right)^{\frac{1}{\mu_g}}\right)^{\frac{\mu_g}{\mu}}}{\sum_{g=1}^{G}\left(\sum_{j=1}^{J_g}\left(\frac{a(q_{jg}^1}{p_{jg}}\right)^{\frac{1}{\mu_g}}\right)^{\frac{\mu_g}{\mu}}}\right]^{\frac{\mu}{(1+\alpha)}}\right] \tag{9}$$

The notation in equation (9) adapts the format used in Verboven [1996] (his equation (2)) to fit our notation. Equation (9) can be shown to correspond to the WTP for the GCES utility function for the representative individual (with the adjustment to the income, and with $\mu = (1\text{-}\rho)/\rho$; $\mu_g = (1\text{-}\rho_g)/\rho_g$ for Verboven equation (11))

III. DISCUSSION

Linkages between welfare measures derived from discrete choice and representative consumer models offer opportunities to describe the connections between consumer decisions for each choice occasion and a longer time horizon. This distinction has received considerable attention in linking RUM estimates of Hicksian consumer surplus (HCS) per choice occasion with seasonal measures. Parsons and Kealy [1995], for example, propose interpreting travel cost demand models for the number of trips taken in a season as the result of recreationists maximizing expected utility subject to expected prices, with a RUM framework (for site choice) providing the probabilities to estimate these expectations. Equation (4) illustrates why this intuitively appealing proposal is incorrect for our adaptation of the Anderson, et al. framework. Using their formulation with a log linear conditional indirect utility function we allow for all the features sought in a linked RUM/travel cost demand model: (i) variety in choices, (ii) more than one unit of each variety consumed over a predefined time horizon (i.e. $x_j > 1$); and (iii) a consistent link between the RUM and aggregate model (across choice occasions). However, the resulting demand function is not consistent with what Parson and Kealy's model would imply. Their demand in our two good world would not be able to distinguish trips to different sites. Trip demand is the result of expected prices $\sum_j \pi_j p_j$.

Thus with a given expenditure on recreation trips, m, the implied quantity demanded is: $m / \sum_j \pi_j p_j$. This format would not be consistent with the demands for each variety implied by our adaptation to Anderson et al., (i.e. m / p_j). Even if we sum the demands across choice occasions (which we have argued can be used as an alternative to the Anderson et al. treatment of separate consumers) the aggregate demand is not consistent with the Parsons and Kealy description. This relationship is given in equation (4).[4]

Several caveats should be noted in considering the use of these results. First, errors are allowed to enter explicitly the choice occasion decisions but not the aggregate (over occasions). These stochastic effects are assumed to arise because the analyst has incomplete knowledge. While this interpretation seems appropriate, we do not account for unobserved individual heterogeneity. Moreover, this strategy does not overcome the other issues posed by Morey [1994]. Quality attributes enter the model as the equivalent of price adjustments. Finally, the analytical links between choice occasion and aggregate individual models are limited to a narrow range of functions where such solutions are feasible. Of course, this does not preclude adapting the logic to more general cases with the use of numerical solution methods.

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[4] Parsons and Kealy [1995] acknowledge that their framework is an approximation, noting that: "We develop a site choice model and a trip demand model so that each is utility theoretic at its own level and in such a way that behavior in the two models is consistent. The models are not, however, derived from a single overall utility maximization - problem in which site and trip demand decisions are made simultaneously." (p. 360)

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