A Comparison of Timber Models for Use in Public Policy Analysis

Brent L. Sohngen
Roger Sedjo

Discussion Paper 96-12

March 1996

Resources for the Future
1616 P Street, NW
Washington, DC 20036
Telephone 202-328-5000
Fax 202-939-3460

A Comparison of Timber Models for Use in Public Policy Analysis

Brent L. Sohngen and Roger Sedjo

Abstract

In this paper, we compare and contrast two types of timber models that have been used for public policy analysis. These models have been variously used to predict price, inventory and market welfare impacts under different exogenous forces that impact timber markets. The framework and theory for each model type is presented and discussed. We then thoroughly test the two model types across six potential exogenous shocks to timber markets, ranging from instantaneous demand shocks to gradual supply adjustments. Our comparison indicates that these models predict potentially important differences in timber market behavior. These differences are important to consider for those who do public policy analysis.

Keywords: timber markets, models, dynamic adjustment, optimization

JEL Classification Nos.: C62, Q21, Q23

Table of Contents

List of Tables and Figures

A Comparison of Timber Models for Use in Public Policy Analysis

Brent L. Sohngen and Roger Sedjo[1]

1. INTRODUCTION

Today more than ever, forested ecosystems must provide a host of different outputs, ranging from market goods, such as timber, water, or mushrooms, to non-marketed goods, including recreational and existence values. All over the world, traditional timber values, which arise from converting the standing stock of timber into end-products such as lumber, plywood, or paper, have been pitted against the so-called "environmentalist" agenda of reserving some land from timber production. As competition for land becomes more keen, the economist's ability to measure value accurately in different markets, especially over time, will become more and more central to policy debates.

A long list of environmental issues may affect timber markets in the future. Many areas of "old growth," for example, have been removed from timber production for varying reasons, including administrative fiat, high costs, or low value of the timber type. Other supply side issues include acid precipitation and climate change. These two examples present interesting policy considerations because they result from forces that are largely exogenous to timber markets. Nevertheless, they possibly may cause large scale changes in the productive capacity of existing forests (Haynes and Kaiser, 1991; Melillo et al., 1993; and Neilson and Marks, 1994). Land use change may also adjust the future availability of timber supply from tropical

---

[1] Brent Sohngen is a Gilbert White Postdoctoral Fellow at Resources for the Future; Roger Sedjo is a Senior Fellow at Resources for the Future.

and temperate regions. Some researchers have focused on forest health issues, where bug infestations, disease, fire, or other natural disturbances that are related to past management practices may have broad and far-reaching consequences for forests (Sampson and Adams, 1994). On the demand side, many forest industries have been facing consumer challenges to reduce the use of virgin fiber in paper, while some countries have imposed restrictions on wood imports from countries or regions that have "un-sustainable" timber harvest practices.

In any case, it is clear that both timber markets and policy makers face important challenges in the future. Potential environmental effects on forests are broad in both a spatial and a temporal context. They are likely to affect many different species across many parts of the world, and in some cases, such as with climate change, they may involve instantaneous dieback events over large areas, or they may involve long-lasting and gradual adjustments of forests. In order to understand both the consequences of environmental impacts on forests, and timber markets in particular, forest sector models have often been employed to provide analysis (see for example, Haynes and Kaiser, 1991; Winnett et al., 1993; and Sohngen, 1995).

Over the years, many different forest sector models have been utilized for policy purposes. Early on, modelers relied on "gap" analysis, which attempted to determine likely demands and likely supplies. Inevitably, gaps would exist between demand and supply, which would require somehow increasing supplies in the future. Indeed, this type of analysis was one of the factors that led Gifford Pinchot to argue for the establishment of forest reserves in the United States during the late part of the 19th century. Recently, models have been more closely tied to economic theory. Some examples include TAMM (Adams and Haynes, 1980), the CINTRAFOR Global Trade Model (CGTM; described in Kallio et al., 1987), and the global Timber Supply Model (TSM; Sedjo and Lyon, 1990).

These models are similar in some ways, but they differ fundamentally in two respects. First, TAMM and CGTM attempt to capture the complexity of inter-regional trade by using econometrically estimated demand and supply equations for each region and current trade flows to initialize the model. TSM, on the other hand, is strictly a timber supply model, which does not attempt to measure all trade flows simultaneously. Second, the models differ in how they treat timber supply. TAMM and CGTM use econometrically derived estimates of timber supply based on price and total timber inventory. Future supply levels are simulated by the model as price changes and inventory adjusts through growth and harvest. The TSM, however, determines timber supply based on the results of a dynamic social optimization which incorporates the idea of rational expectations. Yearly timber supply is a function of price and the amount of timber in harvestable age classes.

Both model types represent distinct approaches based in economic theory. The first difference, however, can be reconciled if the dynamic optimization models incorporate information on trade flows as well, although this would increase the already burdensome computational demands of the dynamic models. The second difference, however, apparently cannot be reconciled because dramatically different models of supply are utilized. Given the policy ramifications of projections provided by these models, it is instructive at least to understand differences in how supply is modeled.

This difference between the models is important in at least two different situations. Harvesting forests as they transition from old growth to steady state, Faustmann type forests is one. The social optimization model provides a framework for handling this transition, as evident in our discussion above and as shown in Lyon (1981). An equally important situation, however, arises when Faustmann forests are exposed to exogenous changes, such as the types of demand and supply shifts discussed above. These types of supply side changes are unique because they change the entire inventory structure, either instantly or, quite possibly, slowly over time. Because the differences between the econometric simulation and social optimization models revolve mainly around the treatment of inventory, and how it enters into the timber harvesting decision, this may lead to substantial differences between the way the models handle market adjustments to these exogenous effects.

In this paper, we compare and contrast the two types of timber supply utilized by these large scale models. We begin by analyzing the underlying theoretical models, and then we present simple empirical examples. For this analysis, we ignore potential differences that may arise due to trade in order to concentrate on the supply side issues. Under a restrictive set of steady state circumstances, it is likely that these models will produce the same results. When demand or supply side shocks are introduced into the system, however, these models will produce very different behavior in timber markets. The shocks range from instantaneous changes (for example, a reduction in National Forest timber harvest) to more slowly occurring events (acid precipitation and climate change).

In the next section, we review the literature on forest sector modeling, and we present the theoretical underpinnings of two types of models. We then focus on one of the main differences between these models, which is how they assume timber is supplied. In the third section, we present empirical results from comparisons between these models under different demand and supply conditions.

2. FOREST SECTOR MODELS

Forest sector models have been used for many years to project the future of demand and supply in timber markets. For many years in the U.S., "gap" analysis was used to project future levels of consumption in timber markets and future levels of supply of timber stumpage. Because population and demand were growing quickly, and the area of forestland in the U.S. was dwindling, "gap" models inevitably projected that consumption would outpace supply, and that a timber famine loomed on the horizon. The relationship between price, harvest, and regeneration was never clearly understood. While more recent forest sector models have considered market linkages directly, they have developed along two lines of thinking, explained below.

2.1. Econometric Timber Market Models

The econometric timber market models have their roots in the literature on spatial market structures pioneered by Samuelson (1952). One of the first of these, TAMM (Adams and Haynes, 1980), models the spatial structure of markets in the United States by considering separate demand and supply regions. That model recognizes the importance of the transportation costs necessary to move products from manufacturing facilities to demand centers. Because the most productive forests are located remotely from the urban centers where most wood is demanded, transportation costs are an important component in the overall value of wood.

A spatial market model attempts to maximize consumer's plus producer's surplus minus the costs of transporting products to other markets. In any period, then, the objective is to maximize

$$\sum_i \int_0^{Q_i^D} D(Q_1 \ldots Q_I)\,dq_i - \sum_j \int_0^{Q_j^S} S(Q_1 \ldots Q_J)\,dq_j - \sum_i \sum_j C_{i,j} Q_{i,j} \,, \tag{1}$$

subject to

$$Q_i^D \leq \sum_j Q_{i,j} \tag{2}$$

$$Q_j^S \geq \sum_i Q_{i,j} \,. \tag{3}$$

$Q^D{}_i$ is the quantity demanded, $Q^S{}_j$ is the quantity supplied, *D(·)* is the demand function, *S(·)* is the supply function, *i* is the demand region, *j* is the supply region, and $C_{i,j}$ is the cost of transporting from region *i* to *j*. As suggested by Adams and Haynes (1987), this type of model does not need to have a social welfare interpretation, although it implicitly maximized the yearly value of net market welfare, minus transportation costs.

Although this spatial representation concentrates solely on the end-product markets, the TAMM model recognized that both end-product and timber markets must simultaneously be in equilibrium. Derived demand curves for timber can be developed from the production function. Supply can be estimated based on prices and timber inventories. The following specification for the timber market is usually given:

$$Q^D(t) = f\left(P_s(t), P_z(t), k(t),\right) \tag{4}$$

$$Q^S(t) = g(P_s(t), Inv(t)) \ , \tag{5}$$

where $P_s(t)$ is the price of stumpage, $P_z(t)$ price of final products, $k(t)$ is the capacity to produce lumber and plywood, and $Inv(t)$ is the total timber inventory. The timber inventory is completely exogenous to the system of equations, so that it can be used to help identify the demand equations. Because markets clear for end-products, exogenous variables in demand and supply equations at that level are useful for identifying both the supply and demand functions at the timber market level, as well as the equilibrium price and quantity. Although studying the end-product markets is interesting, we consider only the timber market level, represented by equations (4) and (5), in the rest of this paper.

2.2. Dynamic Timber Market Models

The second strain of timber market models is rooted in the theory of renewable and non-renewable resources (Hotelling 1931; Solow 1974) Rather than focusing on end product markets, these models analyze the resource base by using dynamic models to show how benevolent social planners utilize stocks of natural resources over time. If timber markets operate efficiently, the social planner's solution will be the same as that achieved in a competitive market. Berck (1979, 1981), Lyon (1981), and Lyon and Sedjo (1983) showed how these dynamic models could be tied directly to timber markets. Historically, timber had been derived from old-growth stocks, which were considered to be non-renewable resources. Timber prices thus would adjust over time in a way that maximizes the net present value of the net consumer surplus in the market, thereby reflecting the scarcity of the remaining old growth stock.

In dynamic optimization models, a benevolent social planner attempts to maximize the net present value of net consumer surplus. Berck (1981) has shown that the actions of many independent market players will approximate this solution over time. Net consumer surplus is defined as the difference between total consumer surplus (the area underneath the demand curve), and the costs of regenerating timberland and the costs of holding land in timber. Per acre replanting costs, *b*, remains constant over time in this model. *R(t)* represents land rent, or the capital cost of maintaining land in timber. The social planner's dynamic problem is then:

$$\underset{H(t)}{Max}\int_0^\infty e^{-rt}\left\{\int_0^{Q^*(t)}\left\{D\big(Q(H(t),V(a))\big)\right\}dQ - bG(t) - R(t)X(t)\right\}dt\ , \tag{6}$$

where *H(t)* is the number of acres harvested, *V(a)* is the yield function, *a* is the age of the timber, *Q(t)* is the total quantity harvested, G*(t)* is the number of acres replanted, and *X(t)* is the total number of acres in the forest. While *X(t)* represents the total size of the forest, it also contains information on the age structure of the forest, which will be important for deciding how much timber to harvest each year.

The age structure is very important for this type of model, because managers harvest the oldest stock first. The yearly flow of timber depends directly on the amount of timber available in merchantable (economically mature) age classes. The size of the forest, therefore, will vary over time according to:

$$\dot{X} = -H(t) + G(t). \tag{7}$$

Although replanting decisions for the new rotation are made at the same time as are harvesting decisions, they are separate because of the long time lags involved before the next harvest. Assuming that the social planner uses rational expectations, he or she will replant when the present value of future timber rotations on a piece of land are greater than the opportunity cost of doing something else with the land, $LV_{alt}(t)$. This occurs when

$$\sum_{n=1}^{\infty}\left\{P(t_n)V_i(t_n - t_{n-1})e^{-r(t_n - t_{n-1})} - b\right\}e^{-r(t_{n-1}-t_0)} \geq LV_{alt}(t)\,. \tag{8}$$

$LV_{alt}(t)$ is the value of land in the next best alternative to forestry, n is the rotation number, and the difference t_n - t_{n-1}, is the length of the rotation in question. If land is most profitable in forestry, it will remain in forestry, if it is more profitable elsewhere, it will convert to something else. By equation (8), timberland managers have the option to keep land that has been harvested in timber by replanting it or to change it to another use.

Equations (7) and (8) are constraints to the maximization given in (6). Two additional pieces of information must be used to solve the problem First, we must be given a yield function that is concave, initial values for $X(0)$, $P(0)$, b, and r, and an initial age distribution over $X(0)$. Second, we must assume that $H(t)$, $G(t)$, $P(t)$ are greater than 0 in every period. With this, the problem can be defined in terms of a current value Hamiltonian (Kamien and Schwartz, 1981), shown in (9) below.

letting $$W\big(H(t),V(a)\big)=\int_0^{Q(t)}\big\{D\big(Q(H(t),V(a))\big)\big\}dQ$$

(9)

$$\mathrm{H} = W\big(H(t),V(a)\big) - bG(t) - R(t)X(t) + \mu(t)\big[-H(t)+G(t)\big]\,.$$

Utilizing the maximum principle (Pontryagin et al., 1962), the following first order conditions are derived:

$$W_H(t) - \mu(t) = 0, \qquad (10)$$

$$\dot{\mu} - r\mu(t) = -W_X(t), \qquad (11)$$

$$\lim_{T\to\infty} \mu(t)H(t) = 0, \qquad (12)$$

where the final equation is the transversality condition. Equations (10) and (11) can be combined and rewritten to obtain

$$\frac{\dot{W}_H}{W_H(t)} = r + \frac{R(t)}{W_H(t)}. \qquad (13)$$

The social planner will harvest so that the marginal welfare benefit of harvesting rises faster than the rate of interest. This is due to the final term, which is a stock effect. Because there is an opportunity cost involved with holding land in timber, the welfare returns to timber harvesting must increase faster than the rate of interest. The marginal welfare benefits depend on the yield of timber and demand.

This condition is found to be the same as that derived by Lyon and Sedjo (1983) and Brazee and Mendelsohn (1990). To see this, the following welfare function, which satisfies the criteria laid out above, is used:

$$W\big(H(t),V(a)\big) = k + \alpha(H(t)V(a)) - \beta(H(t)V(a))^2 \, . \tag{14}$$

Differentiating (14) with respect to *H(t)* determines

$$W_H = \alpha V(a) - 2\beta H(t)V(a)^2 = \big(\alpha - 2\beta H(t)V(a)\big)V(a) \quad . \tag{15}$$

Equation (15) describes the marginal value of an acre of forest land. Differentiating (15) with respect to *V(a)* gives the marginal value of a unit of timber. In a competitive market place, the marginal value of timber is equal to the stumpage price:

$$P(t) = \alpha - 2\beta H(t)V(a) \, . \tag{16}$$

Substituting (16) into equation (15), and then placing the result in (13), we find the following condition:

$$\dot{P}V(a) + P(t)\dot{V} = rP(t)V(a) + R(t) \, . \tag{17}$$

Equation (17) must be satisfied over all time if the social maximization is to be achieved. This equation has several interesting properties. First, when both the demand function and the stock of land are constant (i.e. steady state), this resolves to the same rotation as if timberland managers were all acting like Faustmann entrepreneurs given price. Assuming prices and the age of the marginal tree harvested stabilize at $\overline{P}$ and $\overline{a}$, the steady state is expressed in terms of:

$$\frac{\dot{\overline{V}}}{\overline{V}} = r + \frac{\overline{R}}{\overline{P}\overline{V}}. \tag{18}$$

Trees are thus harvested when they are growing at the rate of interest plus the stock effect. The stock term results from the assumption that land will be replanted to forestry, and accounts for the individual landowner's decision whether or not to replant. For timber to remain in forestry, *R* must be greater than the next best alternative. Land rent forces owners to harvest sooner than if there was no competitive market for land.

In transition, equation (17) can be used to model the harvesting of old growth, as well as the transition around shocks to a steady state system. The old growth condition is met when tree stands no longer are accumulating harvestable timber (Oliver and Larson, 1990). Some trees will continue to grow, others will stop growing, and still others may die altogether. Mathematically, this occurs when the net growth of timber on each acre approaches 0, or when $\dot{V} \approx 0$. Equation (17), then, can be rewritten as:

$$\frac{\dot{P}}{P(t)} = r + \frac{R(t)}{P(t)V(a)}. \tag{19}$$

Prices will rise faster than the rate of interest if there is land rent. If, as was the case when the settlers first arrived on the North American continent, there is no land rent, $R(t) = 0$, and prices rise exactly at the rate of interest. Over time, the stock of timber will decline and competition for land will increase, thereby increasing land rent. This signals landowners to replant. The depletion of the old growth stock will continue as we harvest successively younger trees. $\dot{V}$ then becomes greater than 0, and ultimately, prices will begin to follow,

$$\frac{\dot{P}}{P(t)} = r + \frac{R(t)}{P(t)V(a)} - \frac{\dot{V}}{V(a)} . \tag{20}$$

Assuming demand is constant, prices will rise, but at a slower and slower rate until they have achieved the steady state. Thus, the old growth case is really just a special case of equation (17). Depending on the particular constraints on the system, the old growth transition of equation (19) will occur, or the regular transition of equation (20) will occur, or the steady state of equation (18) will occur.

Several important points must be made about dynamic models. First, they link aggregate market behavior directly to the well accepted Faustmann formula (Brazee and Mendelsohn, 1990). Timberland managers individually behaving according to Faustmann will anticipate future market conditions by harvesting timber as it achieves the profit maximizing rotation age. Forests are managed just like other renewable natural resource stocks.

Second, behavior in the dynamic market models depends directly on the quantity and age distribution of the initial timber inventory. For example, in the United States, there are substantial old growth stocks remaining on National Forest lands in the west. Including these lands in the social optimization would produce very different results on price and harvest behavior than if they were left out (in reality, most have been removed from harvestable timber stocks by administrative rule).

Finally, the optimization model can account for natural regeneration processes, but has no mechanism for harvesting multi-cohort stands. If prices are too low in the future, there is no incentive to spend money on regeneration, but forestland is likely to replace itself naturally anyhow. Stocking levels, of course, will be lower, a factor that must be captured in a natural land yield function. Multi-cohort stands that result from certain disturbance patterns, certain harvesting patterns, or long periods of natural regeneration processes, are more difficult to incorporate into the optimization model. This remains one of the most difficult aspects of utilizing optimization models.

2.3. Similarities and Differences Between the Models

It is clear from the discussion above that there are large differences between the two types of timber market models we have presented. The most apparent difference between the models is that econometric models have been developed with multiple market layers, that is, they describe the vertical market for forest products, and simultaneously solve for equilibrium between supply and demand at each market level. Although Lyon et al. (1987) developed a dynamic model that solved both market levels simultaneously (by solving endogenously for both capital investment and timber harvest), most other modelers have considered only the timber stumpage or log markets. The reason for this usually lies in the computational burden of solving all time periods simultaneously.

An equally important difference is how the models treat timber supply. The econometric models rely on a timber supply specification that adjusts according to the total timber inventory. The optimization models, on the other hand, adjust according to the amount of timber in economically mature age classes. Furthermore, in the optimization models, prices and harvests are determined in a forward looking manner according to a rational expectation's derived time path. In the dynamic problem described above, a represents the age of the oldest timber. Harvest begins with the oldest stock and continues until the marginal opportunity costs of waiting an additional moment to harvest an additional acre just equal the marginal benefits. Not only is the age of the timber important, but the price path matters as well.

A similar harvest rule does not result from econometric models. Instead, econometric models determine the total timber quantity harvested, without reference to which age classes are harvested. The modeler is then free to choose a harvesting rule, which may include only the oldest trees, or which may include a host of trees. Depending on the particular harvest rule chosen, the econometric models may produce a very different adjustment over time.

Suppose, for example, that an exogenous supply shock kills younger timber inventories. This impact will affect the early phase of the transition in the econometric model as the supply curve shifts inward. Because plenty of timber exists in merchantable age classes immediately following the supply shock, in the optimization model, consumers and producers will react at the moment of the shock, but their reaction will appear muted compared to the econometric model.

One potential weakness in the optimization models is that they capture high intensive timber management fairly well, but at the expense of modeling lower intensity management. By nature, for all timberland within the initial inventory, optimization models assume that the only ownership objective is to harvest timber. Unfortunately, this may not capture other types of landowners who manage for many alternative objectives, which may or may not include timber. For example, many non-industrial private forest land (NIPF) owners do not harvest timber at the optimal Faustmann rotation age (indeed, by looking at aggregate inventories, Sohngen, 1995, has found that both timber industry and NIPF owners manage land for multiple purposes). Many of these NIPF stands may contain multiple age classes, or the owners may prefer selection cuts to clear cuts. Either way, these landowners do not follow Faustmann rules exactly, so they should not be incorporated into the inventory described above.

The econometric models need to make fewer assumptions about alternative management patterns. They do not distinguish between different management strategies because the overall quantity of timber harvested is sensitive only to the total timber inventory and price. Because most supply and demand curves are econometrically estimated based on past data, they implicitly include information on harvest patterns from different management intensities. The economic modeler then, is free to choose a harvest rule that satisfies the market clearing conditions in equations (5) and (6) above, but that also satisfies the distribution of ownership types and objectives observable.

3. A COMPARISON OF BEHAVIOR

Because these two types of models may be used for policy analysis, we will compare transitional price and inventory pathways determined made by these two types of models for six external forces. These models should produce different transition pathways, given their structure. Although the final steady state, resting point of the system may vary, we assume that these models begin at the same initial steady state conditions. Differences in the modeling structure will determine how the systems respond to the various exogenous forces, and how the systems approach their new steady state conditions.

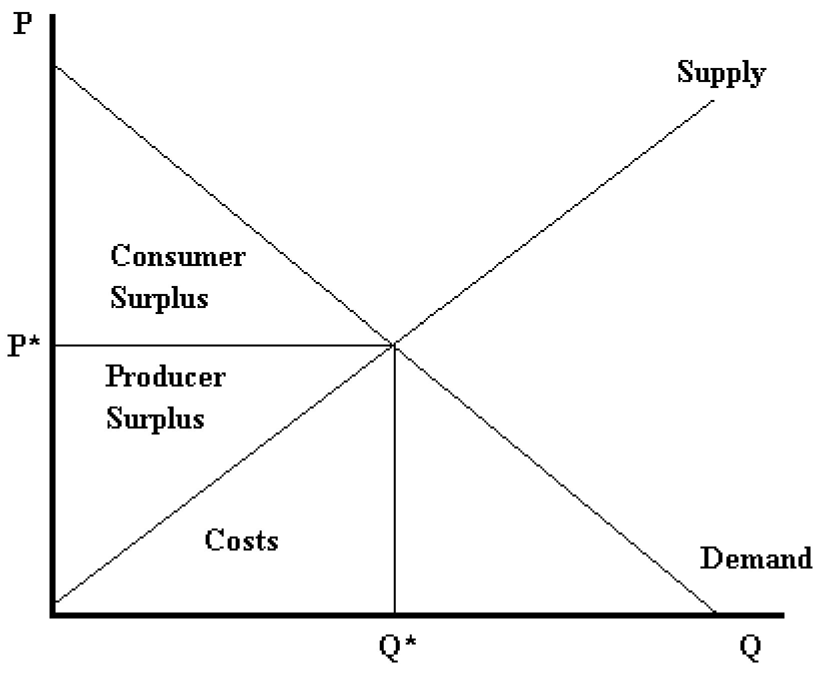
The results of six different exogenous forces under the two models are compared in this section. We analyze three instantaneous shocks and three gradual adjustments, as shown in Table 1. These six examples provide us with a mixture of possible forces that are affecting timber markets now or will affect them in the future. Our interest lies mainly in the transitional behavior, so we assume that initial steady state harvest and price levels are the same for each. We begin our comparison of the econometric and optimization models first by describing these steady state conditions, and then by discussing the transition predicted by each of these models in response to the given shocks.

Table 1. Six exogenous shocks over which econometric and optimization models are compared. The numbers in parenthesis next to the specific scenarios correspond to the appropriate section of the paper where the scenario is discussed.

Exogenous Shocks	
(I) Instantaneous Adjustment	(II) Continuous Adjustment
General Attributes • Each impact occurs in the initial period. • Land supply is inelastic.	*General Attributes* • Forcing factors gradually change over time. • Forcing factors stabilize at new steady state. • Land supply is inelastic
Specific Scenarios (3.2.) **Instantaneous Demand Shock** • Demand increases 20% in the first year and stabilizes. (3.3.) **Fire Damage** • Fire destroys 20% of timber aged 10-20 years in the first year. • All fire damaged acres are replanted instantly to the same species. (3.4.) **Old Timber Disturbance** • Disturbance destroys all timber 27 years and older. •All fire damaged acres are replanted instantly to the same species.	*Specific Scenarios* (3.5.) **Slow Demand Increase** • Demand increases .5% each year for 40 years and then stabilizes. (3.6.) **Slow Timber Growth Increase** • Timber growth rates increase .5% each year for 50 years and then stabilize. (3.7.) **Slow Area Increase** • Timber acreage increase by 50,000 each year for 50 years and then stabilizes. • Timber acres are added to first age class.

Three different comparisons between the models are made for each shock: price, inventory, and market welfare. A future path of price results directly from each model's predicted transition path after or during the exogenous shock. The inventory adjustment occurs as timber is harvested from the oldest age classes (recall that we must assume this for the econometric model), and growth increases the size of the younger age classes. We compare the total volume of timber inventory for each model. Volume is closely related to the stock of carbon in the forest, which bears policy relevance for climate change research.

Figure 1. Diagram of welfare measures for a timber market. The value of the market in any particular period is measured as the sum of consumer's plus producer's surplus.



The value of the market in any particular period is measured as net consumer surplus. This is shown graphically in Figure 1, as the total area under the demand curve less the total costs of producing the quantity Q^*. The exact shape of the supply function depends on the model used. In the econometric models, equation (1) provides an exact Marshallian measure of value for each period. In the optimization model, value is determined from equation (6) as total consumer surplus minus the costs of regenerating and maintaining land in timber. By considering a stumpage market, harvesting and transportation costs to the mill are incorporated.

Over the entire transition, the value of the market is measured by summing the discounted value of all future net consumer surplus (*NCS(t)*). This leads to an aggregate value of the market, *MW*:

$$MW = \int_0^\infty e^{-rt}\left(NCS(t)\right)dt \tag{21}$$

If we assume that our baseline case (the case that would occur without one of the exogenous shocks), is the initial steady state, then we can measure the value of the exogenous shocks by comparing market welfare in a "shocked" case, MW^1 , with market welfare if the initial steady state conditions lasted forever, MW^0. This is:

$$\Delta MW = MW^1 - MW^0 . \tag{22}$$

In the cases we consider below, MW^0 will vary between the econometric model and the optimization model because slightly different calculations are used (see equations 1 and 6 above). For either model type, however, MW^0 depends only on the initial steady state conditions, so it remains constant regardless of the shock considered. For policy purposes one would be interested in comparing market welfare in the case where the market must adjust to some exogenous shock (MW^1), to the case where the market does not need to adjust (MW^0). In this comparison of the two models, we are most interested in comparing the relative value of ΔMW for each example of a shock. Although both absolute market welfare changes and percentage changes are reported, the most important results revolve around the percentage changes.

We also will disaggregate this total change in market welfare effects into individual producer and consumer surplus effects. ΔNPV(Producer Surplus) is therefore the difference in the net present value of producer surplus between the initial steady state case and the adjustment case and ΔNPV(Consumer Surplus) is the same measure for consumer surplus. When summed, these last two measures equal ΔMW.

3.1. Initial Steady State

The models are calibrated initially under steady state conditions. The steady state is described by a stable demand function (i.e. a demand function that is not a function of time), completely inelastic land supply, a 4% rate of interest, and 16 million acres of land divided into approximately 32 age classes of 500,000 acres each. The demand function is:

$$Q^D(t) = 34242 - 171.21 * P(t)\,. \tag{23}$$

The initial demand elasticity in this model is 1.0. The yield function is derived from a typical stand of southern pine, with the following functional form:

$$\ln\big(V(a)\big) = 12.54 - (52.9 / a)\,, \tag{24}$$

where *a* is the age of the timber.

Supply in the econometric model assumes that harvests occur similarly to the optimization model in that they begin with the oldest timber first, and continue until the demand and supply functions are equilibrated. This will maximize the area under the demand curve and above the supply curve in any given period. The supply function for the econometric model is defined by:

$$Q^S(t) = -6441.2 + 31.7 * P(t) + .113 * I(t). \tag{25}$$

Supply is a function of price, *P(t)*, and total timber inventory (I_t). The initial price elasticity of supply is .186, while the inventory elasticity is 1.18. The actual slope terms in equation (24) were determined by approximating elasticity values found in the literature (Adams and Haynes, 1980, and Newman, 1987). Under these conditions, prices equilibrate at $100/MBF, and yearly harvests are 17,121 MMBF. Timber is harvested at 32.20 years of age, and there are approximately 32 equal age classes of timber, with 500,000 acres in each age class. Assuming that all of the land harvested is replanted each year, a Faustmann forest will result in the econometric model.

In the optimization model, the same demand function is utilized, but no explicit functional form is defined for supply. Supply is determined endogenously within the dynamic optimization procedure. In steady state, the model solution predicts that harvests will stabilize when the age of timber is approximately 32.21 years. At this age, 17,114 MMBF are harvested each year, on 497,234 acres, and prices are approximately $99.60/MBF. A

Faustmann forest also arises here, as timber is replanted into the same timber type as soon as the land is harvested.

Under these initial steady state conditions, timber harvests and prices for the two types of models are nearly identical. The harvest rule for the econometric model was chosen so that the same timber is harvested in it as in the optimization model. Following the notion discussed above, these models will behave similarly under conditions of stability in the market. We now show the three different cases where market behavior adjusts, beginning with a 20% increase in demand.

3.2. Instantaneous Demand Shock

In the first case, we assume that demand instantly increases 20%. This increase is implemented by adjusting the constant in the demand function outwards to 40,912 in the initial period, and holding it constant at this level throughout the model runs. No attempt was made to adjust any of the other parameters in either model. Harvests consequently adjust to their new steady state levels. We define the adjustment period as the length of time it takes for price and harvest levels to stabilize at a new steady state level.

Figure (2a) shows the price schedule for both models as they adjust harvests and inventories to account for the change in the level of demand. Recall that prices initially are \$100/MBF, and in both cases they jump upwards at the time of the shock, t^*. After that, prices increase slowly over time in the econometric model, while they jump instantly to their new steady state level in the optimization model. Very small price and harvest adjustments

Figure 2. Comparison of price paths for both models across the six examples of potential exogenous impacts on timber markets.

Figure 2a: Instantaneous 20 % demand increase

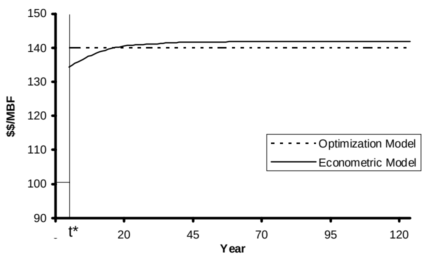


Figure 2d: Slow (.5 % annual) increase in demand

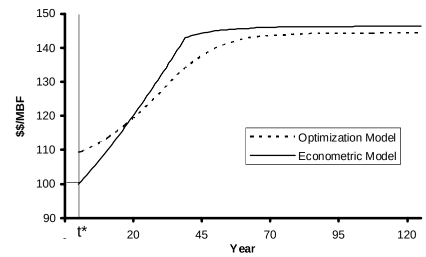


Figure 2b: Fire damage to younger age classes

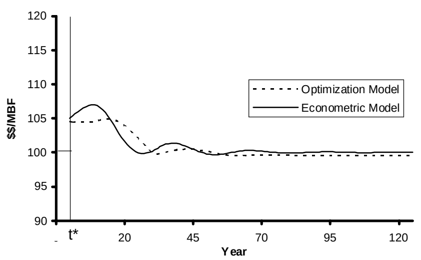


Figure 2e: Slow increase in timber yield

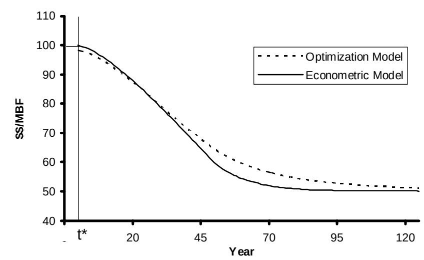


Figure 2c: Disturbance in older aged timber

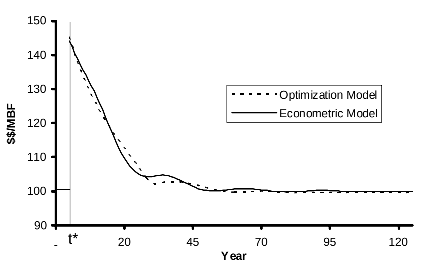
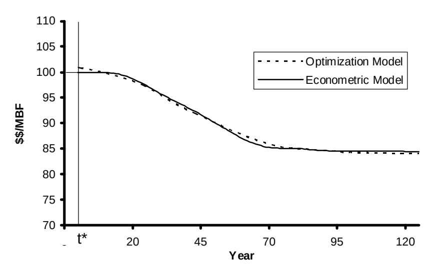


Figure 2f: Slow increase in timber acres



occur in the optimization model, but it is initially so close to the final steady state that these changes are barely perceptible.

In the long run, prices in the optimization model are $1.82 lower than in the econometric model. This contrasts to the initial steady state where they were the same. Timberland owners in both models respond by adjusting harvests to new conditions, but the econometric model evokes a much larger reduction in harvest age (down to 30.86 years of age), because it does not capture the marginal trade-offs that timberland owners make across time-periods. At a steady state price of $141.83/MBF, the Faustmann formula predicts that the harvest age should be 31.95 years, rather than the 30.86 years predicted by the econometric model. Although prices are higher, forestland values decline because producers are harvesting their forests too soon. The optimization model provides a direct mechanism for harvests to be re-established in steady state Faustmann rotations.

Inventories have a similarly diverging time path (Figure 3a). The final steady state inventory in the econometric model is approximately 7% lower than the optimization model. This result mirrors the fact that forests are harvested sooner in the econometric model, and the forest has been converted to one which has younger timber on average, and therefore, less biomass. Timber inventories in the optimization model remain stable, with only a few minor adjustments.

Our final comparison is across the change in market welfare, shown in Table 2. In the econometric model, ΔMW is negative, indicating that market welfare declines relative to the initial steady state. In the optimization model, market welfare increases. The differences are explained by considering the individual changes in the net present value of net consumer and

Figure 3: Comparison of total inventory for both models across the six examples of potential exogenous impacts on timber markets.

Figure 3a: Instantaneous 20 % demand increase

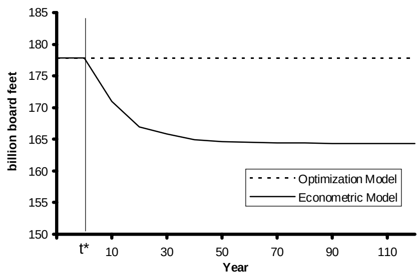


Figure 3b: Fire damage to younger age classes

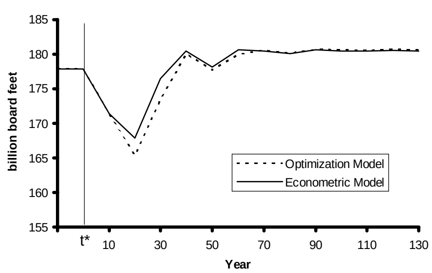


Figure 3c: Disturbance in older aged timber

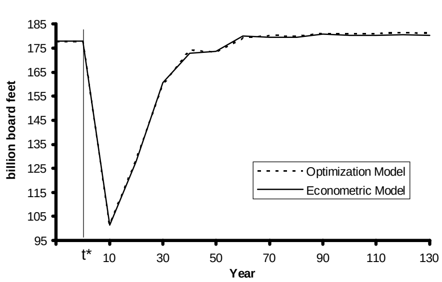


Figure 3d: Slow (.5% annual) increase in demand

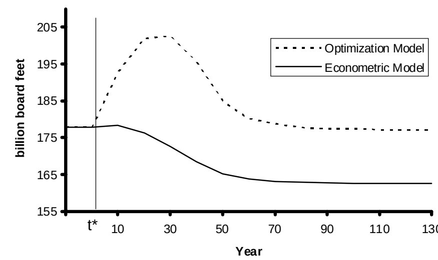


Figure 3e: Slow increase in timber yield

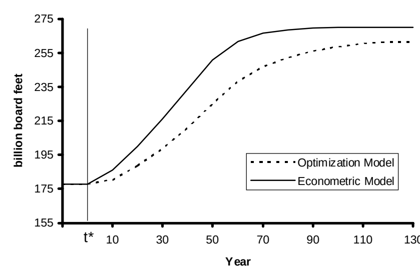
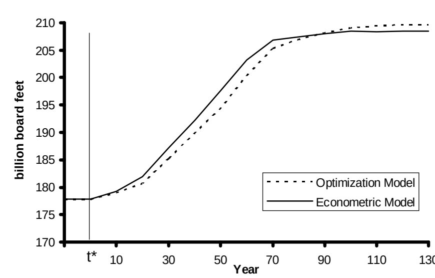


Figure 3f: Slow increase in timber acres



producer surplus. Although consumer surplus increases over time in the econometric model, it is outweighed by a larger decline in producer surplus. This decline results from harvesting timber that is below economic maturation. The supply function adjusts inward much more than Faustmann would suggest is necessary. Although the exact size of the reduction in supply will depend on the inventory elasticity, over a range of different values for this elasticity, the same behavior was observed.

Table 2. Comparison of Δ*MW*, ΔNPV(Consumer Surplus), ΔNPV(Producer Surplus) across the two models and the six exogenous impacts.

	Δ*MW*		ΔNPV(Consumer Surplus)		ΔNPV(Producer Surplus)	
	absolute (billions $$)	percent	absolute (billions $$)	percent	absolute (billions $$)	percent
Optimization						
Demand Change	$9.93	21.16 %	$0.02	0.09 %	$9.71	37.84 %
Old Death	(8.90)	(18.96)	(6.31)	(29.67)	(2.59)	(10.09)
Fire Damage	(1.39)	(2.96)	(1.15)	(5.41)	(0.24)	(0.94)
Slow Dem. Inc.	2.24	4.77	0.07	0.33	2.18	8.50
Slow Yield Inc.	7.49	15.96	7.62	35.83	(0.13)	(0.51)
Slow Acre Inc.	3.02	6.44	1.57	7.38	1.45	5.65
Econometric						
Demand Change	(5.28)	(1.88)	0.18	0.93	(6.01)	(2.08)
Old Death	(59.27)	(19.18)	(6.25)	(32.23)	(39.88)	(13.77)
Fire Damage	(10.96)	(3.55)	(1.16)	(5.98)	(9.80)	(3.38)
Slow Dem. Inc.	(0.76)	(0.25)	0.37	1.91	(1.12)	(0.39)
Slow Yield Inc.	158.62	51.33	5.66	29.19	152.96	52.81
Slow Acre Inc.	31.81	10.29	0.97	5.00	30.84	10.65

3.3. Fire Damage to Younger Age Classes

Here, a discrete shock in the supply of timber occurs in the form of fires over 20% of the timberland in age classes 10-20. We assume that this is a one-time, unexpected event; that all timber on the land that burns dies back; and that there is no salvage associated with the dieback. All land that burns is replanted instantly into the same timber type. Similar to the demand increase, the fire occurs in the first year and stocks subsequently must adjust to new steady state levels.

Figure 2b shows the price paths for both models. Although both models experience an instantaneous jump in prices, the initial price adjustments and transition paths are different. The supply function immediately begins to shift inward in the econometric model, reflecting the inventory loss. Due to this shift, prices in the econometric model increase and peak approximately 10 years after the fire shock. After peaking, prices decline rapidly as the land that was regenerated after the fire enters older age classes. Despite the fact that this timber is still well below the merchantable age, it has a large effect on timber prices during years 10 to 30.

In the optimization model, marginal adjustments move timber stocks from lower valued periods to higher valued periods. Prices jump up initially because consumers and producers are aware of the coming shortage. They correctly foresee the shortage of timber in certain future age classes, as well as the large slug of timber coming through the system after year 30.

Producers, for example, know that prices will be higher after 10 years when timber becomes relatively scarce, so they begin harvesting less in the first few periods. This pushes timber inventories into future, more highly valued periods. This activity smoothes the transition across the shortfall in inventories that occurs when fire affected age classes are mature. This benefits consumers, because they do not have to endure a price "spike" of the size encountered in the econometric model. When the shortfall has been crossed, supply increases and prices decline. As the fire damaged acres approach maturity, producers begin to harvest more heavily to take advantage of prices that are higher than prices in the new steady state.

Over time, producers arbitrage between current and future periods using the interest rate to assess the marginal value of timber stocks in each period. At the same time, consumers arbitrage. They are willing to accept slightly higher prices initially in order not to have to endure a price spike when inventories are reduced. These marginal adjustments will maximize the present value of net consumer surplus (market welfare).

Under the constraint that land supply is completely inelastic, the models take quite a few years to achieve their new optimal, long-run equilibrium harvest levels. Small adjustments continue many years into the future, as stocks are returned to Faustmann rotations. The optimization model appears to return the forest to the Faustmann conditions much quicker than the econometric model, an observation similar to one for the demand increase shown above. Unlike the demand case above, however, the models both return the forest to the same steady state conditions after the shock.

Despite these differences in the price pathways, the inventory adjustment does not vary significantly between these two models. Inventories in the optimization model remain slightly below the econometric model throughout the middle part of the transition because the optimization model harvests more heavily out of the "slug" of timber that is available around year 30. Market welfare decreases in each case, although the decline is greater in the econometric model.

3.4. Old Timber Dieback

In the third case, we consider an exogenous environmental damage that kills the oldest five age classes of timber. We assume that all timber in age classes affected dies back, landowners do not salvage, no one expects this event, and landowners replant the land immediately to the same timber type. Prices jump in both models (Figure 2c), and the initial jump is about the same in each because the oldest trees die. Prices then decline in both models, although they do so more quickly during the first 15 years in the optimization model, because timber in mature age classes increases fairly rapidly in that model. Prices stabilize in both models along the same long run path.

In the econometric model, the price jump results from the immediate loss in timber inventory (recall that the oldest age classes are destroyed, and these just happen to have the greatest volume per acre). Prices remain higher than in the optimization model for the first few years of the adjustment. During this time period, the land that died back has been returned to younger age classes, but the timber is too young to make a difference in the overall inventory. As this timber matures, it becomes a larger factor in the inventory, and prices begin to decline more rapidly. Prices take longer to bottom out in the econometric model, as they do not hit their minimum level until 50 years.

As before, producers and consumers arbitrage between current and future periods in the optimization model. Prices decline rapidly at first as the age of the oldest timber increases, and the stock grows. Timberland owners also know that a "slug" of stock will enter the market when the timber that previously died back becomes economically mature. They are willing to sell immature timber stocks for the first 30 years because prices are relatively high.

Over time, prices are driven down. They bottom out when the stock that died back becomes merchantable.

Inventory adjusts along a similar pathway for each model. When the oldest timber dies back, as in this case, econometric models will behave similarly to optimization models. Market welfare decreases for both shocks, and again, the percentage change is greater in the econometric model than in the optimization model.

3.5. Slow Demand Increase

The slow demand increase scenario is the first of our comparisons over gradual exogenous forces that may affect timber markets. In most policy discussions, some baseline change in demand will be included to account for gradual changes in underlying economic conditions, such as population and income growth. For this reason, we analyze a case where demand increases 0.5% per year for 40 years and then stabilizes at a total of 22% above the initial steady state demand level.

An interesting difference in the price paths occurs for these two models (Figure 2d). The econometric model suggests a slight initial increase in prices, and then a gradual increase until about year 40, when prices begin to stabilize. The optimization model, on the other hand, predicts a larger initial increase in prices, but a more gradual adjustment after that. Prices ultimately stabilize at a level lower than the econometric model (similar to the observation above for the instantaneous increase).

The reason for the larger increase in initial prices in the optimization model stems from equation (20) above. Producers hold on to their timber for a little longer, because they know that prices will be higher in the next period. Timber rotations during the transition when demand is increasing will be higher than in steady state (this result has also been found by Berck, 1981 and Newman et al., 1985). From the initial rotation period, harvests will decline, and prices will increase. Older inventories, however, ultimately will increase the yearly supply of timber from any particular acre, so that prices will end up lower than in the econometric model.

By comparing these models around the year 40, when demand stabilizes, one can also see the effect of perfect foresight on the optimization model. Consumers and producers do not recognize in advance that demand will stabilize in the econometric model, so they are caught by surprise and a big shift in price growth occurs in year 40. In the optimization model, however, future demand conditions are considered in today's harvesting decisions, so that the transition to stable prices is smoother and quicker.

Inventory initially increases in the optimization model example (Figure 3d), and ends up slightly higher than initially, whereas it decreases throughout the model run for the econometric model. The final steady state follows from our intuition in the instantaneous demand increase. The "hump" in inventories projected by the optimization model follows from the discussion above about the path of price and harvest. Recall that we suggested that timber is harvested efficiently at older ages during a period of demand growth. As the stock adjusts to this older age distribution, the stock of timber increases. After demand stops growing, the inventory declines back to its new steady state level.

Market welfare increases under the optimization model whereas it decreases under the econometric model. As in the first demand case, this stems mainly from decreases in producer surplus in the econometric model. The change in market welfare in both models turns out to be smaller in this case than the first case, reflecting the relatively smaller increases in demand during the earlier, more highly valued periods (due to discounting).

3.6. Slow Yield Increase

The slow yield increase involves a gradual, 1.0% annual increase in the growth rate of timber. This change is implemented as an adjustment to the current yearly increment to growth, such that the adjusted yearly growth of timber at *t* years after the shock is

$$\dot{V}(a)_{adjusted} = (1{+}.01t)\dot{V}(a)_{initial} \quad . \tag{25}$$

This adjustment continues for 50 years and then the yearly growth increment stabilizes at a level 50% greater than initially. Such a gradual adjustment may occur from ecosystem effects that arise from global warming. On the other side, reductions in growth like this may occur from acid precipitation, or other long term, gradual environmental stimulus.

Figure 2e present the price paths for the econometric and optimization models. In the econometric models, prices decline slowly at first from their original level of $100/MBF, but then more rapidly after about 30 years. Prices continue to fall until about year 80, well after the gradual increment in growth has stopped (year 50). Recall that only trees planted after year 50 will have the full change in yield associated with the adjustment. Any tree planted before year 50 will obtain only some fraction of the 50% total increase in timber growth that occur.

For the optimization model, prices jump downward at the start of the transition period. Perfect foresight convinces consumers and producers to harvest more heavily in early periods for two reasons. First, consumers and producers realize that the new steady state rotation age should be a little younger than before. They must therefore harvest somewhat more heavily in early periods. Second, they realize that additional timber will be available in future periods because the yearly growth is increasing. They move future stocks into current periods by harvesting heavily in order to take advantage of this.

Over the long term, prices decline steadily, but at a slower rate than in the econometric model. In the econometric model, recall that enhanced timber volume in younger age classes, for example those younger than 25, shifts the supply function outward during those early periods. This, in turn, reduces prices more quickly. In the optimization model, on the other hand, the model realizes that increased growth rates will increase later timber stocks, but it limits those benefits in the first few periods. During later periods, harvests are lower than in the econometric model.

Figure 3e presents the inventory adjustment for this slow increase in yield. Inventories increase significantly more in the econometric model because that model holds the rotation age above the optimal Faustmann age. The optimization model, on the other hand, adjusts by decreasing the optimal age at which the timber is harvested. Market welfare increases in both model types for this experiment, although the change in the econometric model is relatively larger than the optimization model. The supply function in the econometric model shifts outward relatively quickly during early periods as growth increases, because younger stocks benefit significantly. The optimization model does not allow the benefits of that additional growth nearly until it is time to harvest those acres.

3.7. Slow Increase in Timber Area

In this final example, we consider potential increases in the area of forests due to replanting or natural regeneration. Similar changes occurred throughout the United States in the last century as land in the northeastern and southeastern U.S. converted from agriculture back to forestland (Powell et al., 1993). These trends continue today in some regions of the country. It is important to realize, however, that these types of area changes are important in a global sense as well, as plantation forests continue to arise in different regions. We implement this scenario by allowing 50,000 additional acres to regenerate for each of the first 50 years of the simulation. No additional land enters after that. This increases the total land base by 2.5 million acres.

Figure 2f shows that prices in the optimization model instantaneously increase. This odd results stems from the long term changes in timber inventories that must occur in this example. Looking to the long term, the optimal rotation period will be extended. The market begins to adjust instantly by reducing harvests in the initial period, and increasing prices. The number of acres replanted during the first 50 years is high due to the exogenous factors, however. This story in the early periods contrasts with the econometric model, where prices begin at about the same level, and they do not begin changing until the extra acres planted begin to have significant amounts of timber growing on them (10 - 15 years after the start of the model run).

Prices in the optimization model later decline rather rapidly as the model harvests more heavily into younger timber age classes. This behavior occurs because the model anticipates the year when it will have additional timber in harvestable age classes. Rather than wait until the moment those age classes become merchantable, the model moves some of those acres into present periods for harvesting purposes. Price declines level out after year 80, but the forest continues to adjust until well after that.

In the econometric model, prices begin to decline rapidly after about year 25, but they do not decline as quickly as in the optimization model. The reason for this is that the only way the model can "anticipate" future conditions, is by incorporating younger aged timber into the calculation of total timber inventory. Because these age classes are at a lower point on the yield function, they do not enhance supply as significantly.

The inventory adjustment is shown in Figure 3f. The adjustment in both models is actually quite similar, although slight differences occur. The optimization model ends up with more timber because rotation ages are slightly extended. During the early part of the transition, however, the econometric model actually has a greater inventory because rotation ages are higher during that period. This occurs when the optimization model harvests heavily out of timber stocks in order to begin the process of equalizing them and achieving the new steady state. Market welfare increases in both models. For similar reasons as in the previous case, even when timber is well below its harvest date, it can have a significant impact on supply through inventory in the econometric model. In the optimization model, the benefits of this additional land are not realized until later.

4. DISCUSSION AND CONCLUSION

In this study we have compared two basic types of timber supply models used to predict future timber market behavior. One model results from econometrically estimated demand and supply equations, which equilibrate in each period. This maximizes the single period value of consumer's plus producer's surplus. The other model optimizes harvests so that the net present value of net consumer surplus is maximized. The models were compared under six different examples to show how differences might affect market projections for policy analysis.

Fairly substantial differences are found to occur between the two types of models. When timber demand adjusts, for example, the Faustmann formula predicts that prices rise and rotation ages decline. While both models incorporate this behavior, the size of the reduction in rotation ages is very different for each model. The econometric model predicts a larger reduction than does the optimization model. Because the optimization model accounts for future periods, it penalizes deep harvests into current stocks because the marginal benefits of waiting an extra period to harvest that timber are great.

The econometric models are more sensitive to shocks that affect younger age classes, because supply relies on inventory that is measured over the entire distribution of timber ages. Although similar behavior occurs within the optimization model, the price effect is less severe, and the mechanism for the response differs drastically. The forward looking nature of the model smoothes out the transition from the initial conditions to the final steady state. A smaller difference occurs in the old timber dieback case. Similarly, the differences are not large in the gradual adjustment cases when the changes occur over time.

The welfare analysis provides an interesting comparison. In the demand adjustment cases, the models predict exactly opposite effects. The loss in the econometric models is driven by the large decrease in producer surplus due to higher prices. In the other cases, the optimization model predicts smaller relative changes in market welfare. Because the optimization model solves each period simultaneously, it has the luxury of maximizing the our measure of market welfare. By maximizing over both the baseline case and the adjustment cases, the optimization model essentially minimizes the change in welfare between the two cases.

These differences have some interesting implications for timber modeling. First, these models predict qualitatively different welfare results in the demand cases, and quantitatively different results in the others. Given that most policy applications will consist of a shifting demand function, analysts should understand why these differences arise. Drastically different welfare results and policy implications will result from each model.

Policy makers need to be careful about the implications of changes in inventory. Recent reductions in Forest Service harvests in the Pacific Northwestern U.S., for example, have increased the demand for stumpage in the South. This may be like the first case we considered, an instantaneous and permanent increase in demand. The optimization model suggests that prices will jump upwards, but that they will quickly stabilize. With no uncertainty about the future supply of timber in the Pacific Northwest, there would be a limit to how deeply Southern landowners will cut into their timber. The econometric model, however, may lead to the conclusion that landowners will continue to harvest more and more deeply into their timber and prices will continue to rise for some time. Under the optimization model, policy makers are led to believe that timber markets adjust relatively quickly to large changes such as occurred in the early part of this decade. Under the econometric model, however, policy makers may believe that it takes many more years for timber markets to adjust to these changes, that prices will be higher, and that timber stocks will be reduced (absolutely, as well as relatively to the optimization model).

Finally, the differences in inventory adjustments are largest for the demand changes. This can have significant policy consequences, particularly for climate change research, where policy makers are interested in utilizing forests to enhance short term storage of carbon. Given that differences are particularly notable in the short term in all of the panels in Figure 3, policy makers should use alternative models to get a sense for the range of potential additional storage for programs before they spend money on expensive programs.

We have not attempted to show that either model is clearly superior to the other. Each will have its place in modeling efforts. It is clear, however, that for policy analysis on timber market behavior, analysts should have a clear understanding of the differences between the models. Furthermore, modelers themselves must be clear about the benefits and limitations of the approaches that they have chosen.

REFERENCES

Adams, D. M., and R. W. Haynes. 1980. "The 1980 Timber Asset Market Model: Structure, Projections, and Policy Simulations," *Forest Science*, vol. 26, no. 3, Monograph 22.

Adams, D. M., and R. W. Haynes. 1987. "Interregional Modeling," in M. Kallio, D. P. Dykstra, and C. S. Binkley, eds., *The Global Forest Sector: An Analytical Perspective* (New York, N.Y., John Wiley and Sons).

Berck, P. 1979. "Economics of Timber: A Renewable Resource in the Long Run," *Bell Journal of Economics*, 10, pp. 447-462.

Berck, P. 1981. "Optimal Management of Renewable Resources with Growing Demand and Stock Externalities," *Journal of Environmental Economics and Management*, 8, pp. 105-117.

Brazee, R., and R. Mendelsohn. 1990. "A Dynamic Model of Timber Markets," *Forest Science*, 36, pp. 255-264.

Haynes, R. W., and H. F. Kaiser. 1991. "Forests: Methods for Valuing Acidic Deposition and Air Pollution Effects," in P. M. Irving, ed., *Acidic Deposition: State of Science and Technology, Vol. IV. Control Technologies, Future Emissions, and Effects Valuation* (Washington, D.C., U. S. Government Printing Office).

Hotelling, H. 1931. "The Economics of Exhaustible Resources," *Journal of Political Economy*, 39, pp. 137-175.

Kallio, Markku, Dennis P. Dykstra, and Clark S. Binkley. 1987. *The Global Forest Sector: An Analytical Perspective* (New York, N.Y., John Wiley and Sons).

Kamien, M. J., and N. L. Schwartz. 1981. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management* (New York, N.Y., Elsevier North Holland).

Lyon, K. S. 1981. "Mining of the Forest and the Time Path of the Price of Timber," *Journal of Environmental Economics and Management*, vol. 8, no. 4, pp. 330-345.

Lyon, K. S., and R. A. Sedjo. 1983. "An Optimal Control Theory Model to Estimate the Regional Long Term Supply of Timber," *Forest Science*, vol. 29, no. 4, pp. 798-812.

Lyon, K. S., R. A. Sedjo, and A. P. Bambang. 1987. "An Optimal Control Model for Analysis of Timber Resource Utilization in Southeast Asia," *Natural Resource Modeling*, vol. 2, no. 1, pp. 55-80.

Melillo, J. M., A. D. McGuire, D. W. Kicklighter, B. Moore, III, C. J. Vorosmarty, and A. L. Schloss. 1993. "Global Climate Change and Terrestrial Net Primary Production," *Nature*, 363, pp. 234-240.

Neilson, R. P., and D. Marks. 1994. "A Global Perspective of Regional Vegetation an Hydrologic Sensitivities from Climatic Change," *Journal of Vegetation Science*, 5, pp. 715-730.

Newman, D. H. 1987. "An Econometric Analysis of the Southern Softwood Stumpage Market: 1950-1980," *Forest Science*, vol. 33, no. 4, pp. 932-945.

Newman, D. H., C. E. Gilbert, and W. F. Hyde. 1985. "The Optimal Forest Rotation with Evolving Prices," *Land Economics*, vol. 61, no. 4, pp. 357-394.

Oliver, C. D., and B. C. Larson. 1990. *Forest Stand Dynamics* (New York, N.Y., McGraw-Hill, Inc.).

Pontryagin, L. S., V. S. Boltyanskii, R. V. Gamkrelidze, and E. F. Mischenko. 1962. *The Mathematical Theory of Optimal Processes* (New York, N.Y., Wiley).

Powell, D. S., J. L. Faulkner, D. R. Darr, Z. Zhu, and D. W. MacCleery. 1993. *Forest Resources of the United States, 1992*, General Technical Report RM-234, US Department of Agriculture, Forest Service, Rocky Mountain Forest and Range Experiment Station, Fort Collins.

Sampson, R. N., and D. L. Adams. 1994. *Assessing Forest Ecosystem Health in the Inland West* (New York, N.Y., Food Products Press).

Samuelson, Paul A. 1952. "Spatial Price Equilibrium and Linear Programming," *American Economic Review*, 42, pp. 283-303.

Sedjo, R. A., and K. S. Lyon. 1990. *The Long Term Adequacy of the World Timber Supply* (Washington, D.C., Resources For the Future).

Sohngen, Brent L. 1995. "Integrating Ecology and Economics: The Economic Impacts of Climate Change on Timber Markets in The United States." Unpublished dissertation, Yale School of Forestry and Environmental Studies, New Haven, Conn.

Solow, R. M. 1974. "The Economics of Resources or the Resources of Economics," *American Economic Review*, 64, pp. 1-14.

Winnett, S. M., R. W. Haynes, and W. G. Hohenstein. 1993. "Economic Impacts of Individual Climate Change Mitigation Options in the U.S. Forest Sector," *Climate Research*, 3, pp. 121-128.