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## **Externalities, Decreasing Returns, and Common Ownership**

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# **Externalities, Decreasing Returns, and Common Ownership**

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## **Abstract**

Placing production units under common ownership is often suggested as a solution to the problem of externalities. This will not always be true when there are decreasing returns to scale. An atomistic industry could be more efficient than a monopoly in some instances. Even when the “optimal” industry configuration would involve a finite number of producers, no two may have appropriate incentives to combine. An omniscient and benign regulator can always assure a more efficient outcome than would result from the combination of private producers. Whether real-world regulators should be called upon, however, is less clear.

**Key Words:** Externalities, Mergers, Returns to Scale, Incentives

**JEL Classification Numbers:** L23, Q24

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# Externalities, Decreasing Returns, and Common Ownership

R. David Simpson

## Introduction

Externalities can be corrected in several ways. A regulator may impose direct restrictions. The regulator may impose a tax on the externality or the activities generating it. Alternatively, the regulator can specify a total ceiling on the amount of the externality to be generated among several sources, and allow individuals to trade permits for generating the externality. A private solution could, in theory, be implemented by contracts among parties providing payments either to reduce or to allow the generation of externalities.

Another private solution would be to combine all parties generating and receiving externalities into a single entity. The owner of the resulting combination would have an incentive to restrict generation of the externality to the efficient level. Such a combination could have ambiguous welfare effects. On one hand, common ownership internalizes externalities. On the other hand, a monopolist with market power may have an incentive to restrict output beyond the point justified by the internalization of marginal social costs (Buchanan 1969).<sup>1</sup>

Interest in common ownership as a solution to environmental problems has risen in recent years. A school of free-market environmentalists asserts that many environmental problems can be solved by private markets, such as those for properties affected by externalities (see, e.g., Anderson and Leal 2001). Concern with the ecosystem services provided by natural landscapes (see, e.g., Daily 1997) has also led some researchers to investigate in greater detail the conditions under which uniting a landscape under single ownership can result in socially optimal performance. Numerous examples have been cited in which property developers, for example, have followed “smart growth” strategies. By preserving<sup>2</sup> certain areas of a development in a

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<sup>1</sup> The standard employed for measuring social welfare throughout this paper will be net monetary benefits. The concentration of ownership will in general have distributional as well as efficiency effects, but we will not consider the former.

<sup>2</sup> Such preservation may also involve contracts with purchasers requiring them to preserve the natural areas in perpetuity as well.

natural state, for example, a land developer may be maximizing her total earnings from sales in the development. Geoffrey Heal has both cited examples of such phenomena (2000) and provided rigorous conditions under which common ownership is optimal (2001). In essence, the conditions are first, that the owner of the property be able to fully internalize all benefits and costs from its environmental features, and second, that the owner perfectly price-discriminate in her sales of products whose attributes depend on those environmental features.

This paper demonstrates that common ownership does not, in general, result in the efficient generation of externalities unless we further require constant returns to scale in production. The reasoning is straightforward. One might ascribe the existence of multiple production units to decreasing returns to scale. If these production units have not been combined yet, it may be because doing so would not prove efficient.<sup>3</sup> Decreasing returns to scale are due to the presence of factors that cannot be replicated, such as proximity to immobile natural resources.

A common owner could instruct each externality-generating production unit to act so as to maximize the profit of the entire concern.<sup>4</sup> This would, in theory, match the objective of a benign regulator and result in the efficient outcome. Realistically, however, ownership oversight must itself be regarded as a limited resource. To suppose otherwise would be to deny the relevance of vast literatures on corporate governance, principal-agent problems, and the design of managerial incentives (for one recent survey, see Prendergast 1999).

The conclusions of these literatures can be summarized as follows. It is costly, and in some instances impossible, for the owner of an enterprise to observe exactly what its managers are doing. Leverage over risk-averse managers can be achieved by tying their compensation to the performance of the units they manage. Such incentives have two limitations, however. If compensation is tied to *aggregate* performance, one manager will have an incentive to free-ride

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<sup>3</sup> Another possible reason such production units have not been combined is that doing so would violate antitrust laws. The present paper abstracts entirely from market-power issues in order to concentrate on the implications of returns to scale.

<sup>4</sup> I will refer to profit here, although externalities affecting consumers as well as producers might not be reflected in market transactions.

on another's efforts. If compensation is tied to *individual unit* performance, managers may be tempted to impose externalities on other production units. The limitations of the owner's oversight might prevent her from inferring the source of such externalities. In short, owner oversight is a scarce resource whose limitations may prevent the replication of production units under constant returns to scale.

Decreasing returns to scale can have several implications for the ability of private actors to achieve mutually beneficial combinations. Three results are demonstrated. First, optimal industry structure in the presence of externalities may involve more than one producer and may, in some instances, be atomistic. Second, benign regulation can result in a more efficient outcome than even the most efficient configuration of independent producers. Third, individual producers generally do not face appropriate incentives to enter into combinations that would enhance the efficiency of the industry in which they operate.

The third result has a precedent in work by Salant et al. (1983) on industrial mergers. Those authors found that merging Cournot oligopolists typically do not profit. The concentration of market power such combinations facilitate is analogous to the internalization of externalities in this analysis (which does not consider market power issues in order to concentrate on externalities). In each instance, combining entities may provide greater benefits to the producers who are not merging than they receive themselves. A somewhat broader range of outcomes is possible in the present model than in that of Salant et al. (1983), who find that mergers among duopolists, and only such mergers, are profitable. In this paper there are examples in which the only two producers would not wish to combine, and also examples in which no two producers among any number would choose to combine, even though industry profits would be maximized with a finite number of producers.

The results speak to issues in environmental regulation. The analysis could be applied to many examples but will take an example from agriculture. Production relies upon ecosystem services generated by land that is not cultivated. Such services are notoriously difficult to value. How shall we assure that they are provided, then? Geoffrey Heal (2000, 125) has written, "Valuation is neither necessary nor sufficient for conservation. We conserve much on which we do not place economic value, and we do not conserve much that we value economically." The wisdom of these observations is unexceptionable, but the question remains how we might best motivate conservation of the assets that provide ecosystem services. Heal suggests that

organizing transactions in such assets and services can mitigate the problem (Heal 2000, 134–41). This paper questions the mitigation thus achieved.<sup>5</sup>

The analysis proceeds by constructing an example in which production both generates and depends upon an externality, and individual producers experience decreasing returns to scale. A concrete example is presented in terms of agricultural production. Because comparisons must be made between levels of total profit rather than marginal conditions, it is necessary to employ a specific functional form. Individual farmers have a Cobb-Douglas production function distinguished by three properties. First, an individual farmer's output depends positively on the amount of land he cultivates, and negatively on the total amount of land cultivated in the community. The latter assumption can be motivated by biological considerations: Natural habitat may be essential to populations of beneficial wild animals, and these animals may be far-ranging.<sup>6</sup> The second distinguishing property of the model is that the amount of the externality any given farmer experiences depends not only on the total amount of land remaining outside cultivation but also on the individual farmer's share of land under cultivation. The larger the individual farmer's field (in proportion to nearby fields), the more likely it is that wildlife will use it. The final distinguishing property is that the production function exhibits decreasing returns to scale in land cultivated and the service provided by natural habitats.

We then characterize a noncooperative outcome. In game-theoretic terms, we solve for the Nash equilibrium of a one-shot game in which each farmer decides how much of the land at his disposal to cultivate, taking other farmers' land use choices as given. Having derived individual land use decisions and profits, we can then treat the corresponding aggregate values as functions of the number of landowners. The optimal number of landowners is that number for which aggregate profit is greatest. We can then ask what incentives there are for combining holdings into this optimal number. Following this, we ask whether a benign regulator might produce a more efficient solution by restricting the land use choices of individual farmers.

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<sup>5</sup> The intention is not to portray the author of thoughtful work as a straw man. A reasonable reading of the work cited is that Heal suggests, entirely correctly, that markets can provide *some* appropriate incentives. What to do beyond that remains an open question.

<sup>6</sup> Animals with complex life cycles may be particularly appropriate examples. Insects may, for example, require one type of habitat in their larval stage, but then spread over large distances as adults.



Results are easily understood as consequences of, on one hand, externalities, and on the other, decreasing returns to scale. Competing forces are at work when holdings are combined. Production is enhanced by internalizing externalities, but at the same time it may be stifled by stretching fixed factors too thin. Depending upon which of these effects dominates, and to what degree, the optimal structure of landholding among noncooperative farmers may call for innumerable atomistic landowners, for a finite number, or for all land to be concentrated under a single owner. In contrast, an atomistic structure is always optimal when a regulator can control land use choices directly. Regulatory authority can be used to address externalities. With this problem resolved, there is no reason to sacrifice the efficiency inherent in large numbers of producers.

The policy implications of the analysis may not be as straightforward as they first appear. An omniscient regulator might improve performance by requiring every farmer to cultivate less land. An omniscient regulator is, however, no less a fiction than an indefatigable entrepreneur. Although the paper suggests reasons why private markets might not achieve the *optimum*, it is not the author's intention to dismiss an idea for which he feels considerable philosophical sympathy. It remains an open question whether real-world regulation, with all the limitations it faces, could assure better outcomes than do real-world private decisionmakers.

## I. The Model

Suppose that the production of each farm is a function of two arguments. The first is the flow of ecosystem services the farm appropriates. The other is the land the farmer devotes to production. We will suppose that the production function exhibits decreasing returns to scale.

Notation is as follows. Farms are indexed by an integer  $i = 1, 2, \dots, m$ . Farm  $i$ 's production is  $q_i$ ; its land under cultivation is  $a_i$ ; and its ecosystem services are  $n_i$ . Corresponding uppercase variables denote total production,  $Q$ ; total land under cultivation,  $A$ ; and the total flow of ecosystem services,  $N$ . The total flow,  $N$ , is a function of the amount of potential farmland maintained in natural habitat. Let  $H$  denote total land available. Then  $H - A$  is the amount maintained in natural habitat.

Any number of examples could be given to illustrate this general structure, but one is particularly apposite. Total output on a farm may depend on the land the farmer has under cultivation and the wildlife that serves the land by pollinating crops or eating pests. Such wild animals may require natural habitat for foraging, breeding, refuges to escape predators, or other purposes. Life-cycle considerations may be important. Wildlife may breed when crops are not

available, and hence eggs, larvae, or other life stages may require that natural habitats be maintained. Mobile populations may move to the land of another farmer, and vice versa. Each farmer is, then, providing some externalities for her neighbors when she maintains habitat. Common models of biological growth model wildlife populations as functions of habitat area. If natural habitat constitutes a limiting factor on wildlife populations, it is appropriate to model the services that such populations provide as a function of land maintained in a natural state.

Similar considerations may apply in modeling natural ecosystems that regulate water flow and other examples (see, e.g., Daily 1997 for a list of the services that natural ecosystems provide). Essentially the same analysis would also apply to situations such as pesticide overspray, which may kill wildlife serving not only the fields of the farmer doing the spraying but those of his neighbors as well.

It is useful to have a concrete example to motivate the exposition, so  $N$  will henceforth be the population of pollinators. Let us use the simplest possible form to relate pollinators to natural habitat:

$$N = H - A. (1)$$

Similarly, and in keeping with the example, suppose that the number of pollinators attracted to a given farm is proportional to that farm's area under cultivation as a fraction of the total:

$$n_i = N \frac{a_i}{A}. (2)$$

This seems a reasonable representation of the way pollinating insects might spread over adjoining and homogeneous fields.

Let farm  $i$ 's production function be of the Cobb-Douglas form,

$$q_i = n_i^\alpha a_i^{\beta-\alpha} (3)$$

We will generally assume that  $\beta < 1$ , so the production technology exhibits decreasing returns to scale.

Using the definitions given thus far,

$$q_i = \left( N \frac{a_i}{A} \right)^\alpha a_i^{\beta-\alpha} = \left[ (H - A) \frac{a_i}{A} \right]^\alpha a_i^{\beta-\alpha} = \left( \frac{H - A}{A} \right)^\alpha a_i^\beta. (4)$$

No important generality would be obtained by making the price of output explicit, incorporating a set of costly inputs, or including a price for land. Let us suppose, in order to

concentrate attention on the matters of interest, that the price of output is normalized to one, demand for output is perfectly elastic, and there are no other inputs.

If all  $m$  farmers act independently, each maximizes profit by choosing an area  $a_i$  to cultivate such that

$$\frac{\partial q_i}{\partial a_i} = -\alpha \left( \frac{H-A}{A} \right)^{\alpha-1} \frac{H}{A^2} a_i^\beta + \beta \left( \frac{H-A}{A} \right)^\alpha a_i^{\beta-1} = 0. (5)$$

Rearrange expression (5) as

$$\alpha \frac{H}{A^2} a_i = \beta \left( \frac{H-A}{A} \right). (6)$$

As the outcome should be symmetric,  $A = ma_i$ , so

$$\alpha \frac{H}{m^2 a} = \beta \left( \frac{H-ma}{ma} \right),$$

or

$$a = \frac{\beta m - \alpha}{\beta m^2} H. (7)$$

Total agricultural land use is

$$A = \frac{\beta m - \alpha}{\beta m} H. (8)$$

The results (7) and (8) can be used to solve for  $q_i$  in (4),

$$q_i = \alpha^\alpha \left( \frac{H}{\beta} \right)^\beta \frac{[\beta m - \alpha]^{\beta-\alpha}}{m^{2\beta}}. (9)$$

Multiplying by  $m$  to determine total output, we have

$$Q = \alpha^\alpha \left( \frac{H}{\beta} \right)^\beta \frac{[\beta m - \alpha]^{\beta-\alpha}}{m^{2\beta-1}}. (10)$$

## II. The Optimal Number of Farms

Conventional wisdom has it that externalities can be internalized by placing the entities both generating and receiving the externalities under common ownership. This is not necessarily

true. Consider the problem of maximizing joint production. Use expression (10) now to express total production as a function of the number of separate farms.

Differentiate expression (10) with respect to  $m$ :

$$\frac{dQ}{dm} = \alpha^\alpha \left( \frac{H}{\beta} \right)^\beta (\beta m - \alpha)^{\beta-\alpha-1} \frac{\beta(1 - (\alpha + \beta))m + (2\beta - 1)\alpha}{m^{2\beta}}. \quad (11)$$

If there is to be an interior solution for the optimal number of farms, which we will denote as  $m^*$ , the two terms in the numerator of the fraction must be of opposite sign.<sup>7</sup> An interior solution is obtained when  $\alpha + \beta > 1$ . Under any other circumstances the optimal structure involves atomistic farms. Recall that  $\alpha < \beta$ . The externalities one farm imposes on others are limited when  $\alpha + \beta \leq 1$ , so the decreasing-returns-to-scale consideration dominates, and optimal structure is atomistic.

The second-order condition for a maximum is satisfied when  $\alpha + \beta > 1$  and can be used to rule out other potential solutions. The details are more tedious than enlightening, however, and are relegated to Appendix A.1.

When  $\alpha + \beta > 1$ ,  $m$  can be chosen to set (11) to zero, and we would have

$$m^* = \frac{\alpha}{\beta} \left( \frac{1 - 2\beta}{1 - (\alpha + \beta)} \right). \quad (12)$$

We ought also require that a meaningful solution have  $m^* \geq 1$ . It is obvious on inspection that when  $\beta = 1$ ,  $m^* = 1$ . Recall that  $\beta = 1$  when the production function displays constant returns to scale in the two arguments of land and ecosystem services. Not surprisingly, then, the optimal industry structure under constant returns and externalities is a single farm. There are no disadvantages associated with the concentration of production in a single entity, so the advantages of internalizing the externalities dominate.

A single firm would also be optimal if  $\beta = \alpha$ , but this case can be dismissed as uninteresting, as it would imply that the marginal product of cultivated land is zero. The two extremes we have identified,  $\beta = 1$  and  $\beta = \alpha$ , are the roots of the quadratic equation in  $\beta$  defined by (12) when  $m^* = 1$ . For all  $\alpha < \beta < 1$  and  $\alpha + \beta > 1$ ,  $m^* > 1$ .

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<sup>7</sup> Meaningful solutions will have  $m^*$  a positive integer. Thus  $\beta m^* - \alpha > 0$ .

Results of a number of numerical examples are reported in Table 1. Note the triangular pattern. When  $\alpha + \beta < 1$  the optimal structure is atomistic. Since  $\alpha$  is necessarily less than  $\beta$ , values of  $\beta$  less than  $\frac{1}{2}$  are not reported, and the section of Table 1 for which  $\alpha > \beta$  is obviated. The numerical examples confirm that the approximations afforded by treating  $m$  as a continuous variable are reliable.

**Table 1: Optimal Number of Farms as a Function of  $\alpha$  and  $\beta$**

		$\beta$				
		0.55	0.65	0.75	0.85	0.95
$\alpha$	0.1	$\infty$	$\infty$	$\infty$	$\infty$	2
	0.2	$\infty$	$\infty$	$\infty$	3	1
	0.3	$\infty$	$\infty$	4	2	1
	0.4	$\infty$	4	2	1	1
	0.5	2	2	2	1	1
	0.6	*	1	1	1	1
	0.7	*	*	1	1	1
	0.8	*	*	*	1	1
	0.9	*	*	*	*	1

\* Not meaningful, as  $\alpha < \beta$ .

### III. Incentives for Efficiency-Enhancing Combinations

We have just seen that the most efficient outcome is not necessarily achieved under single ownership. Even if single ownership, or concentration of holdings under a finite number of owners, were most efficient, it is not necessarily easy to achieve such a combination. The problem is that any subset of entities that combine generates positive externalities for all other farmers. Unless enough farmers make payments to compensate those who have agreed to benefit the community by withdrawing land from production, efficient consolidations will not occur.

Let us consider the incentives of two farms to join under common ownership. The production realized by two among  $m$  farms is  $2q(m)$ . If these farms were to merge, they would constitute one production unit among  $m - 1$ , and produce  $q(m - 1)$ . Such a merger would be profitable, then, if

$$q(m-1) > 2q(m). \quad (13)$$

Answers will be more easily derived if we continue to abstract from the integer constraint and suppose

$$q(m) \approx q(m-1) + \frac{dq}{dm} \cdot 1. (14)$$

In order to avoid some awkward notation, expression (14) is treated as though a *divestiture*, as opposed to a merger, were contemplated, but in this model divestitures and combinations are entirely symmetric. Now using (14) in (13), a merger between two farms would be profitable if

$$-q(m-1) > 2 \frac{dq}{dm},$$

or, suppressing arguments and rearranging,

$$\frac{m}{2} < -\frac{dq/q}{dm/m}. (15)$$

The elasticity on the right-hand side is

$$\frac{dq/q}{dm/m} = \frac{\beta}{\beta m - \alpha} [(2-m)\alpha - \beta m]. (16)$$

If  $m \geq 1$ , and recalling that  $\beta > \alpha$ , the fraction  $\beta/(\beta m - \alpha)$  is positive, and the expression in square brackets is negative. The elasticity of individual farm output with respect to the number of farms is declining. The comparison in (15) can be expressed as

$$\frac{\beta(\alpha + \beta)m}{\beta m - \alpha} - \frac{2\alpha}{\beta m - \alpha} > \frac{m}{2}. (17)$$

It can be shown (see Appendix 2) that this inequality can be satisfied only for  $m < 4$ . Numerical examples establish both that the limiting behavior described in the appendix occurs and that the differentiable approximation we have employed follows closely the exact results obtained for integer values of  $m$ .

Although individually remunerative combinations can happen only if  $m < 4$ , there is no assurance that the optimal industry structure can always be achieved by combinations among existing farms. Consider, for example, the situation depicted in Table 2. Although industry

output is maximized when there are four farms, no pairwise combination among a larger number is attractive to the owners who would combine.

**Table 2: An Example in Which Efficient Combinations Are Never Privately Attractive**

<i>Number of farms</i>	<i>Output per farm</i>	<i>Total output</i>
1	0.608	0.608
2	0.329	0.658
3	0.222	0.666
4	0.167	0.667
5	0.133	0.666
6	0.111	0.664
7	0.095	0.662
8	0.082	0.659
9	0.073	0.657
10	0.065	0.655

Parameter values:  $H = 1$ ,  $\alpha = 0.25$ ,  $\beta = 0.80$

#### IV. Optimal Regulation

Implicit in the analysis above is the assumption that firms cannot be compelled by regulatory authority to restrict their land use. Let us now consider a situation in which a regulator could require such restrictions. The regulator's objective would be to maximize output over all firms. We can characterize this objective by summing the individual farmer's objective, (4), over all farms:

$$\max_{a_i} \sum_{j=1}^m q_j = \sum_{j=1}^m \left( \frac{H-A}{A} \right)^\alpha a_j^\beta. \quad (18)$$

The first-order condition for each farm's land use decision would then be

$$\frac{\partial q_i}{\partial a_i} = - \sum_{j=1}^m \alpha \left( \frac{H-A}{A} \right)^{\alpha-1} \frac{H}{A^2} a_j^\beta + \beta \left( \frac{H-A}{A} \right)^\alpha a_i^{\beta-1} = 0. \quad (19)$$

Assuming symmetry, we have, on rearranging,

$$m\alpha \frac{H}{A^2} a_j = \beta \left( \frac{H-A}{A} \right),$$

or

$$A = \left( \frac{\beta - \alpha}{\beta} \right) H. (20)$$

Each farm would cultivate one  $m^{\text{th}}$  of this total, that is,

$$a_j^r = \left( \frac{\beta - \alpha}{\beta m} \right) H. (21)$$

Note that we are using a superscript  $r$  to denote these first-best outcomes.

Expressions (20) and (21) may be used to compute output per farm,

$$q_j^r = \left( \frac{H - A^r}{A^r} \right)^\alpha (a_j^r)^\beta = (\alpha)^\alpha \left( \frac{H}{\beta} \right)^\beta \frac{(\beta - \alpha)^{\beta - \alpha}}{m^\beta}. (22)$$

Total output becomes

$$Q^r = m q_j^r = \alpha^\alpha \left( \frac{H}{\beta} \right)^\beta (\beta - \alpha)^{\beta - \alpha} m^{1 - \beta}. (23)$$

If there are decreasing returns—that is, if  $\beta < 1$ —the regulator would always prefer an atomistic structure. Since the regulator can compel jointly optimal performance, she can take advantage of the economies of atomistic production without having to combine farms in order to provide stronger incentives for internalization.

How does the first-best outcome induced by the regulator compare with that from the output-maximizing combination of farms with no regulatory compulsion? Consider the quotient of expression (10) derived in Section II divided by the expression we have just derived, (23):

$$\frac{Q}{Q^r} = \left[ \frac{\beta m - \alpha}{\beta - \alpha} \right]^{\beta - \alpha} / m^\beta. (24)$$

This ratio is necessarily less than or equal to one, with equality holding only when  $\alpha = 0$  or  $m = 1$ . This limit on the ratio is obvious by construction. The objective of regulation is presumed to be the maximization of output, so the regulated outcome can result in no less production than the unregulated. This conclusion is also demonstrated mathematically in Appendix 3.



## V. Conclusion

Three principal results have emerged. First, combining all farms under common ownership does not necessarily maximize industry profits. Second, farmers will not necessarily have appropriate incentives to combine into an efficient industry structure. Third, a hypothetical regulator could induce superior performance by imposing restrictions on each farmer's use of land.

These results have been demonstrated in a very simple model. In the interest of clarity we have omitted considerations such as elasticity of demand for output, costs of inputs, and dynamic aspects of biological resources. Although a richer model would be more realistic, the three conclusions would still obtain.

Two considerations might be particularly important in interpreting the results of the simple exercises we have conducted. First, land use decisions have been treated as one-time choices among farmers. Individuals interacting with one another over generations may be able to achieve more efficient outcomes by adopting strategies that depend on past actions (the extensive literature on the "folk theorem" in repeated games treats these issues; see, e.g., Mas-Colell et al. 1995, 404). In that context as well, however, efficient outcomes will be more difficult to achieve with larger numbers of actors. Rewards to deviation from collectively efficient strategies typically increase with the number of actors, as do the costs of detecting such deviations in the presence of stochastic factors.

The second important issue that has been sidestepped concerns heterogeneity across farmers and landscapes. Land use choices among each of a group of farmers have an equal effect on all farmers in the group (and, implicitly, on no one outside the group). The reality is probably very different—albeit also very poorly understood. It is often plausible to suppose that farmers' land use choices have the greatest effect on their own production and that of their immediate neighbors, but declining effects as distances increase. Spatial and temporal scales are important in making such determinations, however. To continue with our example of pollinating insects, areas foraged by an existing hive of wild bees are limited, but one farmer provides a positive externality for others if he maintains bee populations that could colonize other areas.

The Cobb-Douglas model with which we have worked is patently unrealistic in one respect. Output is unbounded if production is divided among infinitely many infinitesimal farms.

This unrealistic aspect could be obviated by including a fixed cost of operation in the specification of the individual farmer's and the regulator's objective functions.<sup>8</sup> As there are diminishing returns in the number of farms, adding a linear cost term in the number of farms would necessarily result in an interior solution for both the optimal number of noncooperative farms and the optimal number of farms under regulation.

The result embodied in Equation (24) would continue to hold. For any given number of farms, a regulator could effect a more efficient outcome than would result from even the optimal combination among noncooperative firms. Still more efficient outcomes might result if the regulator could also choose the appropriate number of farms.

The policy prescription emerging from this analysis may be less obvious than it seems. "Impose regulation" may seem good advice, but this begs the question "How much?" The analysis suggests that some small restriction on land use among numerous farmers would always be a step in the right direction. It is less clear how far such steps can be taken before they become excessively restrictive. The point of the paper is that there may be significant impediments to purely private actions resulting in the *optimum optimorum*. To suppose that an omniscient, purely benevolent regulator would achieve a substantially better outcome is no less of an appeal to a *deus ex machina* than is putting blind faith in the efficiency of market outcomes. It is worth bearing in mind as well that regulation has its costs. Even the most benign regulators expend social resources in performing their function. A private market outcome could, in the final analysis, still be the best practical alternative.

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<sup>8</sup> This would be reasonable. The operation of a successful business requires the attention of an entrepreneur, and there are both upper and lower bounds on the attention an individual can devote to the enterprises she controls. We have not incorporated such factors into the exposition because the analytical difficulties introduced would complicate results considerably: Closed-form solutions would no longer obtain.

## Appendixes

### A.1 Conditions under Which Atomistic Farms Are Optimal

In order for the upper bound on the collective output of farms to occur for some positive  $m^*$ , expression (11) must be satisfied as an equality. This can occur if  $\alpha + \beta > 1$ . Expression (11) can also be an equality if  $\beta$  is less than  $\frac{1}{2}$ . Inspection of expression (12) shows that the implied value of  $m^*$  would then be a fraction, however (recall that  $\alpha < \beta$ ). Moreover, such a point is a *minimum* rather than a maximum. To see this, calculate the second derivative of joint output with respect to the number of farms:

$$\frac{d^2Q}{dm^2} = \alpha^\alpha \left(\frac{H}{\beta}\right)^\beta \left[ \frac{\beta(\beta - \alpha - 1)(\beta m - \alpha)^{\beta - \alpha - 2} \frac{\beta(1 - (\alpha + \beta))m + (2\beta - 1)\alpha}{m^{2\beta}} + \frac{(\beta m - \alpha)^{\beta - \alpha - 1}(1 - 2\beta) \frac{\beta(1 - (\alpha + \beta))m + 2\alpha\beta}{m^{2\beta + 1}}}{m^{2\beta + 1}} \right] \quad .(A1)$$

The first term in brackets is zero if the expression is evaluated at  $m^*$  for which  $dQ/dm = 0$ . If  $\beta$  is less than  $\frac{1}{2}$ , the second terms in brackets is positive, and hence the entire second derivative is positive when evaluated at  $m^*$ . Thus the second-order condition for joint output maximization is not satisfied. We conclude, then, that if  $\beta < \frac{1}{2}$ , the optimal industry structure is one with innumerable tiny farms. Note that this second-order condition is satisfied for  $\alpha + \beta > 1$ .

If  $\beta > \frac{1}{2}$  and  $\alpha + \beta \leq 1$ , the first-order condition (11) is always positive, and innumerable atomistic farms are again optimal.

### A.2 A Limit on the Number of Farms for Which a Pairwise Combination Would Be Profitable

Differentiate the left-hand side of inequality (17) with respect to  $\alpha$  for fixed  $\beta$  and  $m$ .

$$\frac{\partial}{\partial \alpha} \left[ \frac{\beta(\alpha + \beta)m}{\beta m - \alpha} - \frac{2\alpha}{\beta m - \alpha} \right] = \frac{2\beta(1 - \beta)m}{(\beta m - \alpha)^2} > 0 \quad .(A2)$$

Now differentiate the same expression with respect to  $\beta$ :

$$\frac{\partial}{\partial \beta} \left[ \frac{\beta(\alpha + \beta)m}{\beta m - \alpha} - \frac{2\alpha}{\beta m - \alpha} \right] = \frac{(\beta^2 - \alpha^2)m + 2\alpha(1 - \beta)m}{(\beta m - \alpha)^2} > 0. \quad (A3)$$

We conclude, then, that the left-hand side of inequality (17) reaches its greatest value in the limit as  $\alpha$  approaches  $\beta$  and  $\beta$  approaches 1.

Making these substitutions in (17), we have

$$2 > \frac{m}{2}, \text{ or } m < 4. \quad (A4)$$

### A.3 The Ratio of Noncooperative to Regulated Output

Expression (24) in the text gives the ratio of industry output under the noncooperative equilibrium to that under optimal regulation:

$$\frac{Q}{Q^r} = \left[ \frac{\beta m - \alpha}{\beta - \alpha} \right]^{\beta - \alpha} m^{-\beta}. \quad (A5)$$

When  $\alpha = 0$  or  $m = 1$ , the expression is one. Differentiating with respect to  $\alpha$  for fixed  $\beta$  and  $m$ , we have

$$\frac{\partial(Q/Q^r)}{\partial \alpha} = \left[ \frac{\beta m - \alpha}{\beta - \alpha} \right]^{\beta - \alpha} m^{-\beta} \left[ \frac{(m-1)\beta}{m\beta - \alpha} - \ln \left( \frac{\beta m - \alpha}{\beta - \alpha} \right) \right]. \quad (A6)$$

A sufficient condition for this entire expression to be negative (maintaining the assumptions  $m \geq 1$  and  $1 > \beta > \alpha > 0$ ) is that the expression in square brackets be negative.

Again, evaluate the expression in square brackets at  $\alpha = 0$  and  $\alpha = \beta$ . At the former,  $(m-1)/m < \ln m$  for all values of  $m$  of 2 or more. When  $\alpha$  approaches  $\beta$ , the logarithm expression tends to infinity while the first term approaches 1. Finally, differentiating the expression in square brackets with respect to  $\alpha$  again demonstrates that it is decreasing in  $\alpha$  for  $\alpha < \beta$ .

We have concluded, then, that the term in square brackets in (A6) is less than zero for all values of  $\alpha$  between zero and  $\beta$ . This means that expression (A5) is declining for all values of  $\alpha$  between zero and  $\beta$ . One must then be its maximum on that interval, and would be achieved only in the limit as  $\alpha$  approaches zero.

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