RESEARCH PAPER: 2009-4

PREFERENCE VARIABLE IMPACTS IN DIRECT AND INVERSE DIFFERENTIAL DEMAND SYSTEMS

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Abstract

Preference variables are included in the inverse Rotterdam model based on the Tintner-Ichimura-Basmann relationship linking preference effects on quantities demanded to price effects and preference effects on marginal utilities. Restrictions are made on the effects of the preference variables on the marginal utilities, resulting in reductions in the parameter space for the preference variables in both direct and inverse demand systems. The model is used to analyze impacts of product quality on fresh citrus demand.

Key Words: demand, inverse Rotterdam model, preference variables.

JEL Classifications: C51, D12, Q11

Preference Variable Impacts in Direct and Inverse Differential Demand Systems

Non-price, non-income variables such as advertising and quality measures are sometimes added to the consumer utility function and associated direct demand system to measure preference shifts (e.g., Basmann; Phlips; Deaton and Muellbauer, 1980b). In general, the only restrictions on the demand impacts of preference-variables are for adding up. Demand increases for some goods due to a preference variable change must be offset by demand decreases for other goods to satisfy the budget constraint. This implies that for n goods and n product-specific advertising variables, there are n x n-1 advertising impacts in the demand system to estimate. Since the number of such impacts can be quite large and difficult to estimate, additional restrictions on the impacts are sometimes considered. One source of restrictions has been the Tintner-Ichimura-Basmann relationship which shows how impacts of preference variables on utility carry over to the direct demand system. Various restrictions based on this relationship have been explored in order to reduce the preference-variable parameter space to a tractable level in direct demand systems (e.g., Theil, 1980b; Duffy; Brown and Lee, 1997, 2002). What has not been explored, and the subject of this paper, is the use of the Tintner-Ichimura-Basmann relationship as a source of restrictions for the corresponding inverse demand parameter space.

The Tintner-Ichimura-Basmann relationship links preference (variable) effects on quantities demanded to price effects and preference effects on marginal utilities.

Restrictions have been made on the preference effects on marginal utilities, resulting in reductions in the preference-variable parameter space in direct demand systems (e.g., Theil, 1980b; Duffy). The present paper shows the associated implications for the inverse demand parameters. Preference effects in the direct demand system are translated into corresponding effects in the inverse demand system, and an inverse-demand system with a preference-variable parameterization, that can be straightforwardly used to explore preference restrictions as in the direct demand system, is developed.

The results of this paper show that given the direct demand elasticities with respect to n product-specific preference variables can be written as $-\varepsilon^{*}\gamma$ where ε^{*} is a matrix of compensated price elasticities and γ is a matrix of marginal utility elasticities with respect to the preference variables, the inverse demand elasticities are $(I - \iota w')\gamma$, where I is the identity matrix, ι is a unit vector and w is a vector of budget shares.

An empirical analysis of quality impacts in an inverse demand system for U.S. fresh citrus is also discussed. The focus is on how prices for different varieties of citrus are impacted by variety specific quality variables.

The paper consists of a review of direct and inverse demand systems, development of the relationships between preference effects in the two alternative demand systems, discussion of the empirical study and conclusions.

Review of Direct and Inverse Demand Systems in Context of the Rotterdam Model

Consider the utility maximization problem confronting consumers---how to allocate income over available goods. Formally, the problem can be written as maximization of

$u = u(q', z')$ subject to $p'q = x$, where u is utility; $p' = (p_1, \ldots, p_n)$ and $q' = (q_1, \ldots, q_n)$ are price and quantity vectors with p_i and q_i being the price and quantity of good i, respectively; x is total expenditures or income; and $z' = (z_1, \ldots, z_n)$ is a vector of product-specific preferences variables such as advertising. The first-order conditions for this problem are $\partial u/\partial q = \lambda p$ and $p'q = x$, where λ is the Lagrange multiplier which is equal to $\partial u/\partial x$. For direct demand, the solution of the first-order conditions yields $q = q(p, x, z)$, and the Lagrange multiplier equation $\lambda = \lambda(p, x, z)$. Alternatively, for indirect or inverse demand, the solution is $v = v(q, z)$, where $v' = (p_1/x, \ldots, p_n/x)$ or income-normalized prices. The quantities and prices for these two solutions are of course exactly the same. Below, the relationships between the two demand systems are reviewed with a focus on the effects of the preference variables. The Rotterdam demand model[1] is used for this purpose as the relationship between direct and inverse demand with respect to preference variable impacts can be straightforwardly shown for this demand specification. The Rotterdam model is based on the total differential of the first-order conditions, $\partial u/\partial q = \lambda p$ and $p'q = x$, making it convenient to examine preference variable impacts through the Tintner-Ichimura-Basmann relationship which is based on an extension of this total differential.[2]

Following Theil (1975, 1976, 1980a,b), the direct Rotterdam model can be written as

(1) $$w_i\, d(\log q_i) = \theta_i\, d(\log Q) + \sum_j \pi_{ij}\, d(\log p_j) + \sum_j \beta_{ij}\, d(\log z_j) \quad i=1, \ldots, n,$$

where $w_i = p_iq_i /x$ is the budget share for good i; $\theta_i = p_i (\partial q_i/\partial x)$ is the marginal propensity to consume (MPC) for good i; $d(\log Q) = \sum w_i \, d(\log q_i)$ is the Divisia volume index, a measure of the change in real income or utility ($d(\log Q) \approx d(\log x) - \sum w_i \, d(\log p_i)$) (Theil, 1971); $\pi_{ij} = (p_i \, p_j /x) \, s_{ij}$ is the Slutsky coefficient, with $s_{ij} = (\partial q_i/\partial p_j + q_j \, \partial q_i/\partial x)$ being the i,jth element of the substitution matrix S; and $\beta_{ij} = w_i (\partial \log q_i/\partial \log z_j)$. The elasticity of the demand with respect to the jth preference variable is $(\partial \log q_i/\partial \log z_j)$, and, thus, the preference variable coefficient β_{ij} is the budget share times this elasticity. The MPC also equals the budget share times the income elasticity $\eta_i = (\partial \log q_i/\partial \log x)$, i.e., $\theta_i = w_i \, \eta_i$; and the Slutsky coefficient equals the budget share times the compensated price elasticity $\varepsilon^*_{ij} = (\partial \log q_i/\partial \log p_j)|_{u \text{ contant}}$, i.e., $\pi_{ij} = w_i \, \varepsilon^*_{ij}$. The uncompensated price elasticity is $\varepsilon_{ij} = \varepsilon^*_{ij} - \eta_i w_j$. Overall, the Rotterdam model is thus a Hicksian or compensated demand system with the Divisia volume index indicating changes in real income and the Slutsky coefficients indicating compensated effects.

Based on the Tintner-Ichimura-Basmann relationship, the preference-variable coefficients in equation (1) can be written as

(2) $\beta_{ik} = -\sum_j \pi_{ij}\gamma_{jk}$,

where $\gamma_{jk} = \partial \log(\partial u/\partial q_j)/\partial \log(z_k)$, i.e., γ_{jk} is the elasticity of the marginal utility of good j with respect to the preference variable z_k. Equation (2) is the source of preference-variable parameter restrictions in this paper.

Substituting equation (2) into model (1) results in

(3) $w_i \, d(\log q_i) = \theta_i \, d(\log Q) + \sum_j \pi_{ij} \, (d(\log p_j) - \sum_k \gamma_{jk} \, d(\log z_k)) \quad i=1, \ldots, n,$

where the term $d(\log p_j) - \sum_k \gamma_{jk}\, d(\log z_k)$ can be viewed as a preference, adjusted price.

The corresponding inverse Rotterdam model can be written as (e.g., Barten and Bettendorf; Brown, Lee and Seale)

(4) $w_i\, d(\log v_i) = g_i\, d(\log Q) + \sum_j h_{ij}\, d(\log q_j) + \sum_j \alpha_{ij}\, d(\log z_j)$ $i=1, \ldots, n$,

where again $v_i = p_i/x_i$; g_i is the scale coefficient defined as $g_i = w_i(\partial \log v_i/\partial \log k)$, with k being a scalar that can proportionally change some reference bundle ($q=kq^*$ with q^* being the reference bundle)---similar as in the direct Rotterdam parameterization, the scale coefficient is the budget share times the scale elasticity; $h_{ij} = w_i(\partial \log v_i/\partial \log q_j)|_{u\ constant}$ or the budget share times the compensated quantity elasticity or flexibility (the h_{ij}'s are referred to as Antonelli coefficients and are the counterpart of the Slutsky coefficients); and $\alpha_{ij} = w_i\, (\partial \log v_i/\partial \log z_j)$ or the budget share times the inverse-demand, preference variable elasticity. The scale elasticity, compensated flexibility and inverse preference variable elasticity are denoted as $\mu_i = (\partial \log v_i/\partial \log k)$; $\delta^*_{ij} = (\partial \log v_i/\partial \log q_j)\, |_{u\ constant}$; $\rho_{ij} = (\partial \log v_i/\partial \log z_j)$, respectively. The uncompensated flexibility is $\delta_{ij} = \delta^*_{ij} + \mu_i w_j$.

To show the relationship between the direct and inverse demand systems, the models are formulated below in term of matrices and elasticities, i.e.

(5a) $\hat{w}Dq = \theta DQ + \pi(Dp - \gamma Dz)$ (direct demand coefficients),

or,

(5b) $\hat{w}Dq = \hat{w}\eta DQ + \hat{w}\varepsilon^*(Dp - \gamma Dz)$ (direct demand elastictities),

and

(6a) $\hat{w}Dv = gDQ + hDq + \alpha Dz$ (inverse demand coefficients),

or

(6b) $\hat{w}Dv = \hat{w}\mu DQ + \hat{w}\delta^{*}Dq + \hat{w}\rho Dz$ (inverse demand elastictities),

where $\hat{w}$ is a diagonal matrix with the diagonal elements being the budget shares; $Dq = [d \log q_i]$, $Dp = [d \log p_i]$, $Dz = [d \log z_i]$, and $Dv = [d \log v_i]$ are all n x 1 vectors; $DQ = d \log Q = w'Dq \approx Dx - w'Dp$, where $Dx = d\log x$ and w' is a 1 x n vector of budget shares (Theil, 1971); $\theta = [\theta_i]$ is an n x 1 vector of MPCs; $\pi = [\pi_{ij}]$ is an n x n matrix of Slutsky coefficients; $\gamma = [\gamma_{ij}]$ is an n x n matrix of elasticities of marginal utilities with respect to the preference variables; $\eta = [\eta_i]$ is an n x 1 vector of income elasticities; $\varepsilon^{*} = [\varepsilon^{*}_{ij}]$ is an n x n matrix of compensated price elasticities; g is an n x 1 vector of scale coefficients; h is an n x n matrix of quantity or Antonelli coefficients; α is an n x n matrix of preference coefficients; μ is an n x 1 matrix of scale elasticities; δ^{*} is an n x n matrix of compensated quantity elasticities; and ρ is an n x n matrix of advertising elasticities.

The general restrictions on the direct Rotterdam model are (e.g., Theil 1975, 1976, 1980a,b)

(7a) adding up: $\iota'\theta = 1$ $\iota'\pi = 0$

(7b) homogeneity: $\pi\iota = 0$

(7c) symmetry: $\pi = \pi'$,

where ι' is an n x 1 vector of ones. Note that restriction (7a) requires that the preference effects also obey adding up, i.e., given the advertising coefficient matrix β equals $-\pi\gamma$, $\iota'\beta = -\iota'\pi\gamma = 0$, since $\iota'\pi = 0$. Restrictions (7a) through (7c) are for the usual Rotterdam parameterization (5a). The corresponding restrictions on the elasticities in specification

(5b) are a) $w'\eta = 1$ (Engle aggregation) and $w'\varepsilon^* = 0$ or $w'(\varepsilon + \eta w') = w'\varepsilon + w' = 0$ (Cournot aggregation), b) $\varepsilon^*\iota = 0$ or $(\varepsilon + \eta w')\iota = \varepsilon\iota + \eta = 0$ (homogeneity), and c) $\varepsilon^* = \hat{w}^{-1}\varepsilon^{*\prime}\hat{w}$ (symmetry).

The general restrictions on the inverse Rotterdam model are (e.g., Barten and Bettendorf; Brown, Lee and Seale)

(8a)	adding up:	$\iota' g = -1$	$\iota' h = 0$	$\iota'\alpha = 0$,
(8b)	homogeneity:	$h\iota = 0$		
(8c)	symmetry:	$h = h'$.		

The corresponding restrictions on the elasticities in specification (6b) are a) $w'\mu = -1$ and $w'\delta^* = 0$ or $w'(\delta - \mu w') = w'\delta + w' = 0$, b) $\delta^*\iota = 0$ or $(\delta - \mu w')\iota = \delta\iota - \mu = 0$, and c) $\delta^* = \hat{w}^{-1}\delta^{*\prime}\hat{w}$.

Relationship Between Preference Effects in the Direct and Inverse Demand Systems

Anderson has shown the relationship between the direct and inverse demand systems with respect to price, quantity, income and scale effects. Below, these relationships are extended to the preference variable effects. The objective is to transform the direct Rotterdam model to the inverse Rotterdam model to reveal the structure of the inverse demand preference-variable coefficients. The first step is to pre-multiply equation (5b) by $\hat{w}^{-1}$ to find

(9) $$Dq = \eta DQ + \varepsilon^*(Dp - \gamma Dz).$$

If ε^* were nonsingular, we could simply multiply both sides of equation (9) by the inverse of ε^* and rearrange, but this is not the case given (7b). The problem is that price effects of equation (9) are compensated. An inversion, however, can be made by transforming the compensated elasticities of equation (9) to uncompensated ones. To accomplish this, replace DQ in equation (9) by its equivalent Dx – w'Dp, and rearrange the result, i.e.,

(10a) $Dq = \eta(Dx - w'DP) + \varepsilon^{*}(Dp - \gamma Dz)$,

or

(10b) $Dq = \eta Dx + (\varepsilon^{*} - \eta\, w')Dp - \varepsilon^{*}\gamma Dz$,

or

(10c) $Dq = \eta Dx + \varepsilon Dp - \varepsilon^{*}\gamma Dz$,

where again $\varepsilon = \varepsilon^{*} - \eta\, w'$, the uncompensated price elasticities. The homogeneity condition requires $\varepsilon\iota = -\eta$ since $\varepsilon\iota = \varepsilon^{*}\iota - \eta\, w'\iota$ or $\varepsilon\iota = -\eta$, given $\varepsilon^{*}\iota = \hat{w}\pi\, \iota = 0$, by restriction (7b). The corresponding inverse relationship is $\delta\iota = \mu$ since $\delta\iota = \delta^{*}\, \iota + \mu\, w'\, \iota = \mu$, given restrictions (8b).

Based on the inverse function theorem, the uncompensated elasticity matrix ε will be nonsingular, in general, so that multiplying equation (10c) through by its inverse ε^{-1}, denoted by δ, and rearranging yields

(11a) $Dp = \delta Dq - \delta\eta Dx + \delta\varepsilon^{*}\gamma Dz$,

or, further rearranging and simplifying,

(11b) $Dv = \mu\, DQ + \delta^{*}Dq + (I - \iota\, w')\gamma Dz$,

where again $Dv = Dp - \iota Dx$, $\delta^{*} = \delta - \mu w'$, $DQ = w'Dq$, and I is the n x n identity matrix. In equation (11a), the term $-\delta\eta$ equals the n x 1 vector of unit elements, since $\delta\varepsilon = I$, and thus $\delta\varepsilon\, \iota = \iota$ or $-\delta\eta = \iota$ given $\varepsilon\, \iota = -\eta$; this relationship follows from the homogeneity condition noted after equation (7), which for convenience is repeated in the present context as $\delta(\varepsilon^* - \eta\, w')\iota = I\iota = \iota$, or $\delta\varepsilon^*\iota - \delta\eta\, w'\iota = \iota$ or $-\delta\eta = \iota$, since $\varepsilon^*\iota = 0$ based on condition (7b) and $w'\iota = 1$. The term $\delta\varepsilon^*$ in equation (11a) equals $I - \iota\, w'$, since $\delta(\varepsilon^* - \eta\, w') = I$, and thus $\delta\varepsilon^* = I + \delta\eta\, w'$, and given the previous result that $-\delta\eta = \iota$, we have $\delta\varepsilon^* = I - \iota\, w'$. (Similarly, $\varepsilon\delta\iota = \varepsilon\mu = \iota$ and $\varepsilon\delta^* = I - \iota w'$. Thus, $\varepsilon\delta^* = \delta\varepsilon^*$.)

Finally, multiplying (11b) through by $\hat{w}$ yields

(12) $\hat{w}Dv = \hat{w}\mu\, DQ + \hat{w}\delta^* Dq + (\hat{w} - w\, w')\gamma Dz,$

which is the same as equation (6a) or (6b) except for the specification of the coefficients on the preference variables (Dz). In equations (6a) and (6b), these coefficients are $\alpha = \hat{w}\rho$ while in equation (12) they are $(\hat{w} - w\, w')\gamma$. Thus, we conclude

(13) $\alpha = \hat{w}\rho = (\hat{w} - w\, w')\gamma.$

Preference variable coefficient specification (13) can also be obtained using the Hotelling-Wold identity which states

(14) $v_i = \partial u/\partial q_i \,/\, \sum_k q_k\, \partial u/\partial q_k.$

Taking the log of equation (14) results in

(15) $\log v_i = \log(\partial u/\partial q_i) - \log(\sum_k q_k\, \partial u/\partial q_k),$

and differentiating this equation with respect to $\log z_j$ yields

(16) $\partial \log v_i / \partial \log z_j = \partial \log(\partial u/\partial q_i)/ \partial \log z_j$

$- (1/(\sum_k q_k\, \partial u/\partial q_k))\, (\sum_k q_k \partial(\partial u/\partial q_k)/\partial \log z_j + \sum_k (\partial u/\partial q_k)\, \partial q_k/\partial \log z_j).$

Based on a) the first order condition $\partial u/\partial q_j = \lambda p_j$, b) $\sum_j q_j \partial u/\partial q_j = \lambda \sum_j p_j q_j = \lambda x$ and c) given q_k are fixed so that $\partial q_k/\partial \log z = 0$, the last term on the right-hand side of equation (16) can be written as

$-(1/(\sum_k q_k\, \partial u/\partial q_k))\, (\sum_k q_k(\lambda p_k/\partial u/\partial q_k)\, \partial(\partial u/\partial q_k)/\partial \log z_j)$

or

$-(\lambda/(\sum_k q_k\, \partial u/\partial q_k))\, (\sum_k p_k q_k\, \partial \log(\partial u/\partial q_k)/\partial \log z_j)$

or

$-\sum_k w_k\, \partial \log(\partial u/\partial q_k)/\partial \log z_j$.

Hence, equation (16) can be written as

(17a) $\quad \partial \log v_i / \partial \log z_j = \partial \log(\partial u/\partial q_i)/\partial \log(z_j) - \sum_k w_k\, \partial \log(\partial u/\partial q_k)/\partial \log z_j$,

or, multiplying through by w_i,

(17b) $\quad w_i\, \partial \log v_i / \partial \log z_j = w_i\, \gamma_{ij} - w_i \sum_k w_k\, \gamma_{kj}$,

where $\gamma_{ij} = \partial \log(\partial u/\partial q_i)/ \partial \log(z_j)$. Equation (17b) is the non-matrix version of equation (13).

In the above direct and inverse demand systems (5a) and (12), the matrix γ is potentially a source of restrictions on the preference variable impacts; that is, restrictions on the preference variable impacts can be made through restrictions on the effects of the preference variables on marginal utilities. For example, in the direct demand system, Theil (1980b), assumed γ was a scalar times the Identity matrix, while, Duffy assumed it was a diagonal matrix ($\hat{a}$). The Theil (1980b) and Duffy specifications are based on the assumption that the preference variable for good i only effects the marginal utility of that good. For a group of uniform substitutes, Brown and Lee (2002) showed that the same result can be obtained based on the weaker assumption that $\gamma = \hat{a} + \iota b'$ where $b' = (b_1, b_2, \ldots, b_n)$ with b_i being a scalar; this specification allows the preference variable for good i to effect the marginal utility of other goods, uniformly across the goods in the group. In the direct and inverse models, (5a) and (12), the preference variable effects are $-\pi\gamma$ and $(\hat{w} - w\,w')\gamma$, respectively, which for the above structure for γ become $-\pi(\hat{a} + \iota b') = -\pi\hat{a}$ and $(\hat{w} - w\,w')(\hat{a} + \iota b') = (\hat{w} - w\,w')\hat{a}$ with the terms related to b' disappearing given $-\pi\iota = 0$ based on restriction (7b) and $(\hat{w} - w\,w')\iota = 0$. Although assuming b is zero yields the same result, such an assumption may not realistic for a group of closely related goods such as uniform substitutes. Thus, we see that the Duffy assumption can be extended to cases where preference variables have uniform effects across the marginal utilities of the goods in the group.

In estimating the inverse model, there is, however, an endogeneity problem with the budget shares embedded in the term $(\hat{w} - w\,w')\gamma$. This problem might be handled by using mean budget shares in this term, instrumental variables, or perhaps lagged budget shares, as suggested to deal with a similar endogeneity problem in the Almost Ideal Demand System involving budget shares embedded in the Stone price index (Eales and Unnevehr).[3]

Application

Impacts of fresh citrus quality on prices were examined using U.S. retail data on fresh citrus sales (Freshlook Marketing Group). Retail prices, quantities and the percentage of volume sales that are random weight were examined for three varieties of citrus: grapefruit, oranges and tangerines. Fresh citrus is generally sold in two forms: 1) individual pieces, referred to as random-weight (RW) fruit, and 2) bags/cases, referred to as fixed-weight (FW) fruit. The two types of citrus are usually displayed side by side in produce sections in retail stores but priced differently. The quality of RW and FW fruit may differ with respect to size, variety and external look. The RW percentage is treated as a measure of quality but may also reflect merchandising and packaging tactics. A summary of the data are provided in Table 1. The data are weekly from week ending 1/8/2006 through 2/15/2009.

Fresh citrus are seasonal with their availability changing substantially over the course of a year (volumes are greatest during late fall, winter and early spring). When volumes are high, prices tend to be low and vice versa. Given this situation, a conditional demand version of inverse Rotterdam model (4), with the preference-variable coefficients specified as on the right hand side of equation (13), was used to estimate how fresh citrus prices are impacted by scale (overall availability of oranges, grapefruit and tangerines), relative product quantities (Antonelli substitution), and quality/packaging as measured by the RW percentages. Following the preceding section, it is assumed that the quality variable for a variety of citrus (i) has a specific impact (a_i) on the marginal utility of that variety of citrus and a uniform substitute or generic impact (b_i) on the marginal utilities of all varieties (including the variety in question). In this case, $\hat{a}$ is a diagonal matrix with the diagonal elements being (a_1, a_2, a_3) for the three citrus varieties studied, b' = (b_1, b_2, b_3), and the matrix indicating the impacts of quality on the marginal utilities in equation (13) is $\gamma = \hat{a} + \iota b'$. The term $(\hat{w} - w\, w')\, \gamma Dz$ in the model then becomes $(\hat{w} - w\, w')\, \hat{a}\, Dz$, since $(\hat{w} - w\, w')\, \iota b' = 0$. Thus, the impact of preference variable j in the equation for good i can be written as $(w_i\, \Delta_{ij} - w_i w_j)\, a_j\, Dz_j$, where again Δ_{ij} is the Kronecker delta. In estimating this term lagged budget shares were used to avoid the endogeneity problem mentioned earlier.

Homogeneity and symmetry, conditions (8b) and (8c), were imposed as part of the maintained hypothesis in estimating the model. The adding-up condition (8a) holds as the data add up by construction. The infinitely small changes in the logarithms of prices and quantities in the differential model were measured by discrete differences (Theil 1975, 1976). The quality variables, which are percentages of volumes that are RW, were not transformed to log values, and the levels of these variables were similarly differenced. To account for seasonality in demand, the variables were 52[nd] differenced (for the 52 weeks in a year)-- $d(\log p_{it}) = \log p_{it} - \log p_{it-52}$, $d(\log q_{it}) = \log q_{it} - \log q_{it-52}$ and $dz_{kt} = z_{kt} - z_{kt-52}$ (Duffy, Brown and Lee 1997). Average budget share values underlying the differencing were used in constructing the model variables---$w_{i,t}$ was replaced by $(w_{i,t} + w_{i,t-52})/2$.

The demand specifications studied are conditional on expenditure or income allocated to the three citrus varieties. Income allocated to the citrus group is measured by the conditional Divisia volume index for this group which was treated as independent of the error term added to each fresh citrus inverse demand equation for estimation, based on the theory of rational random behavior (Theil 1980a; Brown, Behr and Lee).[4] As the data add up by construction---the sum of the left-hand-side variables in the inverse

Rotterdam model equal the negative of the conditional Divisia volume index--the error covariance matrix was singular and an arbitrary equation was excluded (the model estimates are invariant to the equation deleted as shown by Barten, 1969). The parameters of the excluded equation can be obtained from the adding-up conditions or by re-estimating the model omitting a different equation. The equation error terms were assumed to be contemporaneously correlated and the full information maximum likelihood procedure (TSP) was used to estimate the system of equations.

In estimating the (conditional) inverse Rotterdam model for fresh citrus, first-order autocorrelation was found to exist, which required estimating an additional parameter ρ (Berndt and Savin). Model estimates are shown in Table 2 (equations (4) with α specified in equation (13)). To measure the fit of the system of equations, a system R^2, based on the Wald test and dependent on the equation omitted in estimating the model, was calculated (McElroy, Bewley). The system R^2 ranged from .87 to .99 depending on the equation deleted. Although not appropriate for measuring goodness of fit for a system of equations, single equation R^2 values for grapefruit, orange, and tangerine prices were .86, .99 and .69, respectively.

All coefficient estimates, except that for the tangerine quantity effect on the grapefruit price, are significantly different from zero to the extent their values are twice or greater than their estimated standard errors. The scale coefficients are all negative, indicating that as the overall volume of citrus in the market increases, prices for these varieties decline. All own-quantity or Antonelli coefficients are negative, consistent with the law of demand. The cross-quantity coefficients were positive indicating (net) complementary relationships between these varieties at the compensated demand level, except the insignificant tangerine quantity effect on the grapefruit price indicating a neutral relationship; at the uncompensated level, all cross effects were negative, indicating (gross) substitution, as noted below. The net complementary relationship suggests that some households may be purchasing combinations of these citrus varieties. All coefficients on the quality measures were positive, suggesting the quality of random weight fruit is higher than that for fixed weight fruit. The RW coefficients may also be reflecting a preference for less restricted packaging or the impact of other in-store differences in merchandising of RW and FW fruit.

The uncompensated elasticities (flexibilities) for the inverse citrus demand system are provided in Table 3. The scale elasticities for grapefruit, oranges and tangerines are -.93, -.99 and -1.06, respectively, indicating the if all three quantities increased proportionately, say by 10% , the price of tangerines would decrease the greatest by10.6%, while the price of grapefruit would decrease the least by 9.3%. The own-quantity elasticities for grapefruit, oranges and tangerines are -.38, -.73, and -.39, respectively, indicating the price of oranges is more sensitive to own-quantity than the other two varieties. The cross-quantity elasticities are all negative, reflecting substitution at the uncompensated level.

The quality estimates indicate that if the RW shares of tangerines, grapefruit and oranges are increased say by 10 percentage points, their prices would increase by 7.6%, 2.7% and .8%, respectively, excluding cross effects. The cross-RW estimates indicate negative impacts on competing varietal prices, and if, for example, the RW share of tangerines is increased by 10 percentage points, the prices of grapefruit and oranges would each decrease by about 2.1%. To the extent the fruit in each varietal category is ranked by quality and the highest quality fruit is sold as RW, the results suggest that the retail price and hence revenue might be enhanced by some additional sorting of FW fruit by quality and selling more as RW. Other factors, however, may offset such possible benefits. For example, RW fruit may be subject to a higher spoilage rate as a result of consumer handling of the fruit, and the bags and other containers in which FW fruit is sold may hide external fruit blemishes to some degree, although internal quality of the fruit may be relatively good.

In the empirical application here, the uniform substitute restrictions or essentially equivalent restrictions suggested by Duffy reduce the preference-variable parameter space by a factor of n-1 where again n is the number of goods. Three RW coefficients were estimated for the six RW impacts in the model--three RW variables per equation times two equations with the impacts for the third equation determined from the adding-up condition. More generally, for system of n equations, n coefficients would need to be estimated for n x n-1 preference effects. Thus, to the extent the uniform substitute or Duffy assumptions are acceptable, the reduction in the parameter space could be quite large.

The results of this study may also be of interest for demand analyses where it is useful to have corresponding estimates of direct and inverse demand elasticities or impacts with respect to some preference variables. Based on equation (11b), the inverse demand elasticities with respect to the preference variables are $(I - \iota\, w')\gamma$, while based on equation (10b), the direct demand elasticities with respect to the preference variables are $-\varepsilon^{*}\gamma$. It was also found that $\delta\varepsilon^{*} = I - \iota w'$ where δ is the matrix of uncompensated quantity elasticities for the inverse demand equations. Thus, multiplying $-\delta^{-1}$, which equals $-\varepsilon$ or the negative of the uncompensated price elasticities for the direct demand equations, times the inverse preference variable elasticities $(I - \iota w')\gamma$ yields the direct demand, preference variable elasticities $-\varepsilon^{*}\gamma$. In the present analysis, the direct demand, own-RW impacts ($\partial(\log q_i)/\partial z_i$) at mean budget shares are 1.01 (.27) for grapefruit, .31 (.08) for oranges and 3.91 (.76), with the corresponding inverse demand, own-RW impacts ($\partial(\log p_i)/\partial z_i$) from Table 3 in parentheses. The relatively large direct demand impact for tangerines is a result of a relatively high (direct demand) own-price elasticity, corresponding to the relatively low (inverse demand) own-quantity elasticity, and a relatively high impact (γ) of the RW variable on the marginal utility for tangerines.

Conclusions

This paper extends the Tintner-Ichimura-Basmann relationship to specifying preference variable shifts in inverse demand systems. The Tintner-Ichimura-Basmann relationship indicates how effects of preference variables on quantities demanded are related to price effects and effects of the preference variables on the marginal utilities. This relationship has been a source of restrictions on preference coefficients in direct demand systems. The extension here is based on the relationship between the direct and indirect demand systems and the corresponding preference variable impacts in each system. In both systems, a change in a preference variable has the same basic impacts on the marginal utilities, but in the direct demand system, the impacts on the marginal utilities results in demand impacts through the price effects, while in the inverse demand system the marginal utility impacts result in impacts on prices through the budget shares.

The addition of a set of preference variables like product specific advertising levels results in a relatively large increase in the parameter space of direct and inverse demand systems, which may make estimation of the preference variable impacts difficult. For such demand models, theoretically based restrictions on the preference variable impacts may be of interest. This paper shows that restrictions on the impacts of preference variables on marginal utilities offer an approach to estimating the effects of preference variables in not only direct demand systems but also inverse demand systems. An empirical study of the demand for fresh citrus illustrates the modeling approach. Varietal specific, quality variables are assumed to impact prices through the marginal utilities similarly as has been suggested for direct demand systems. The preference variable specifications of this study may not only be of interest for estimation but may also be useful for converting direct demand system impacts to inverse demand system impacts and vice versa, to help in understanding market behavior.

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Table 1. Summary Statistics: Weekly U.S. Retail Orange Grapefruit and Tangerine Sales, Week Ending 1/8/2006 through 2/15/2009.

Variable	Variety	Unit	Mean	Std Dev
Volume	Grapefruit	mil. lbs	4.242	2.146
	Oranges	mil. lbs	16.797	8.057
	Tangerines	mil. lbs	6.032	6.337
Price	Grapefruit	$/lb	1.006	0.146
	Oranges	$/lb	1.134	0.254
	Tangerines	$/lb	1.405	0.229
Expenditure		mil. $	28.689	11.786
Budget Share	Grapefruit	%	14.4%	2.7%
	Oranges	%	63.7%	12.5%
	Tangerines	%	21.9%	13.8%
RW Share[a]	Grapefruit	%	59.6%	5.1%
	Oranges	%	60.0%	8.9%
	Tangerines	%	15.6%	12.0%

[a] Percentage of total pounds sold that is random weight.

Table 2. Inverse Rotterdam Model Estimates, Equations 4 and 13.

Equation	Explanatory Var.	Coeff. Est.	Std Error	T-Statistic	P-Value
Grapefruit P.	Scale (g_1)	-0.1360	0.0085	-16.0066	[.000]
	Grapefruit Q. (h_{11})	-0.0352	0.0045	-7.7330	[.000]
	Orange Q. (h_{12})[a]	0.0327	0.0042	7.7916	[.000]
	Tangerine Q. (h_{13})[b]	0.0024	0.0019	1.2779	[.201]
	Gft. RW % (a_1)[c,d]	0.3200	0.1069	2.9945	[.003]
Orange P.	Scale (g_2)	-0.6300	0.0227	-27.7246	[.000]
	Orange Q. (h_{22})	-0.0641	0.0070	-9.1997	[.000]
	Tangerine Q. (h_{23})[b]	0.0314	0.0063	4.9517	[.000]
	Oran. RW % (a_2)[c,d]	0.2219	0.0918	2.4165	[.016]
Tangerine P.	Scale (g_3)	-0.2340	0.0233	-10.0211	[.000]
	Tangerine Q. (h_{33})	-0.0338	0.0069	-4.9231	[.000]
	Tan. RW % (a_3)[c,d]	0.9746	0.3676	2.6515	[.008]
Autocorrelation Coeff. (ρ)		0.9008	0.0297	30.3505	[.000]

[a] Parameter h_{12} shared by equation (2) by symmetry.

[b] Parameters h_{13} and h_{23} shared by equation (3) by symmetry.

[c] Percentage of total pounds sold that is random weight.

[d] Parameters a_1, a_2 and a_3 shared by all equations.

Table 3. Inverse Rotterdam Model Elasticities.[a]

Equation	Explanatory Var.	Estimate	Std Error
Grapefruit P.	Scale	-0.931	0.058
	Grapefruit Q.	-0.377	0.037
	Orange Q.	-0.365	0.030
	Tangerine Q.	-0.189	0.017
	Gft. RW %[b]	0.273	0.091
	Oran. RW %[b]	-0.141	0.058
	Tang. RW %[b]	-0.215	0.081
Orange P.	Scale	-0.995	0.036
	Grapefruit Q.	-0.094	0.009
	Orange Q.	-0.731	0.023
	Tangerine Q.	-0.170	0.013
	Gft. RW %[b]	-0.047	0.016
	Oran. RW %[b]	0.081	0.034
	Tang. RW %[b]	-0.215	0.081
Tangerine P.	Scale	-1.061	0.106
	Grapefruit Q.	-0.144	0.018
	Orange Q.	-0.529	0.068
	Tangerine Q.	-0.387	0.042
	Gft. RW %[b]	-0.047	0.016
	Oran. RW %[b]	-0.140	0.058
	Tang. RW %[b]	0.759	0.286

[a] At sample budget share means.

[b] Percentage of total pounds sold that is random weight (z); estimates for RW variables are $\partial(\log p_i)/\partial z_j$.

[1] Barnett, Byron, and Mountain show that the Rotterdam model is a flexible specification comparable to other popular functional forms such as the Almost Ideal Demand System or AIDS (Deaton and Muellbauer, 1980a,b).

[2] Alternative popular demand models based on the cost or expenditure function such as, for example, the recent nested PIGLOG model (Piggott) which embeds the AIDS and related models were not used as their relationship to the Tintner-Ichimura-Basmann relationship is less direct. The AIDS cost function, for example, does not have an associated closed form direct utility function, making the linkage between its demand equations, first-order conditions and the Tintner-Ichimura-Basmann relationship less straightforward.

[3] The preference variable results for the Rotterdam model can also be extended to AIDS-like models. The AIDS model's dependent variable is the budget share $w_i = p_i q_i / x$. Taking the log of this budget share results in (i) $\log w_i = \log p_i + \log q_i - \log x$, and its total differential is (ii) $d(\log w_i) = d(\log p_i) + d(\log q_i) - d(\log x)$, or noting $d(\log w_i) = dw_i / w_i$, (iii) $dw_i = w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log x)$. The latter equality implies (iv) $dw_i / d(\log z_j) = w_i d(\log q_i) / d(\log z_j)$, and since $w_i d(\log q_i)$ is the dependent variable of the direct Rotterdam demand system, this result implies that preference variable effects are the same in each model. That is, the Rotterdam preference variable term, $\sum_j \beta_{ij}\, d(\log z_j)$, in equation (1), is also applicable for AIDS-like models. An approximation of the

differential AIDS model is (v) $dw_i = \sum_j c_{ij}\, Dp_j + b_i\, DQ + \sum_j \beta_{ij}\, Dz_j$ (Deaton and Muellbauer, 1980a; Barten, 1993). Substituting equation (v) into (iii) and rearranging results in (vi) $w_i Dq_i = (b_i + w_i)DQ + \sum_j (c_{ij} - w_i\, \Delta_{ij} + w_i\, w_j)Dp_j + \sum_j \beta_{ij}\, Dz_j$, where Δ_{ij} is the Kronecker delta ($\Delta_{ij} = 1$ if $i = j$, otherwise $\Delta_{ij} = 0$). From equation (vi) we conclude that (vii) $\pi_{ij} = (c_{ij} - w_i\, \Delta_{ij} + w_i\, w_j)$. Thus, in equation (v), the preference variable coefficient can be specified as (vii) $\beta_{ik} = -\sum_j (c_{ij} - w_i\, \Delta_{ij} + w_i\, w_j)\, \gamma_{jk}$, based on equation (2). In estimating AIDS-like model (v) with β_{ij} defined by (vii), budget shares used as explanatory variables might be replaced by their lagged values to avoid endogeneity. If direct estimates of the coefficients of an AIDS-like model (b_i, c_{ij}, and γ_{ij}) are available, the corresponding inverse demand relationship could be found for some set of budget shares as shown in this paper with $\theta_i = (b_i + w_i)$, $\pi_{ij} = (c_{ij} - w_i\, \Delta_{ij} + w_i\, w_j)$, and $\beta_{ik} = -\sum_j (c_{ij} - w_i\, \Delta_{ij} + w_i\, w_j)\, \gamma_{jk}$.

[4] Adding an n x 1 vector of error terms e to the direct demand equations (5a), the error terms in the inverse demand equations (12) are $\hat{w}\delta\hat{w}^{-1}e$. Given e is independent of the Divisia volume index DQ in the direct demand equations (rational random behavior), $\hat{w}\delta\hat{w}^{-1}e$ is independent of DQ in the inverse demand equations.