Instrument Selection Through Bayesian Model Average and Directed Acyclic Graph Approaches :

Case Study In Childhood Obesity and Parental Time Allocation

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Introduction

Childhood obesity has been an active empirical research area. Although reduced form analysis can explain demographic and profile differences in childhood obesity causes, structural econometric modeling is still much needed in order to explain how endogenous variables (i.e., behavioral choices, intervention program participation, parental involvement) evolve according to fundamental processes (i.e., taste shocks, policy changes).

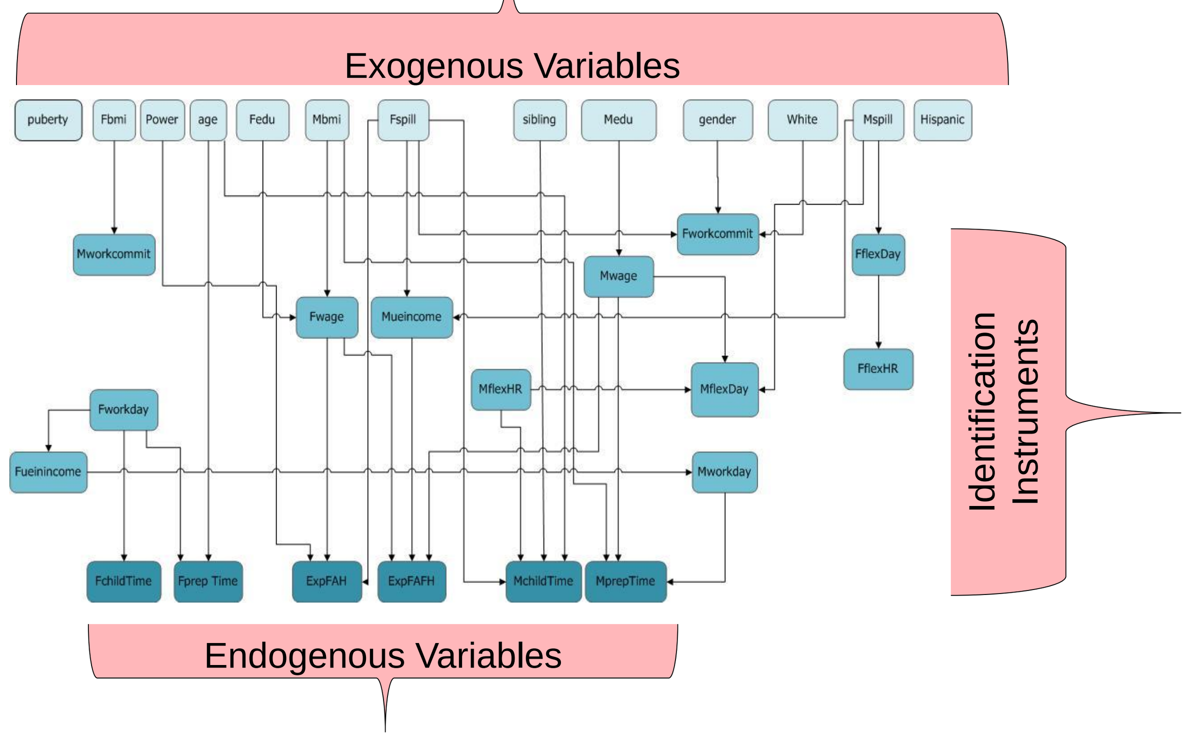
- Endogeneity bias invalidates least squares estimation.
- Instrumental variables estimation is a popular method used.
  - ✓ Theoretical framework is still the backbone (e.g., Deaton 2010)
  - ✓ Within a well-defined problem of inference, instrumental variables can be a solution.
- Challenge: weak instruments problem (e.g., Donald and Newey 2001)
  - ✓ Finite sample properties of estimators are sensitive to the choice of valid instruments used
- Two sources of uncertainty in 2SLS:
  - ✓ Model uncertainty common in all empirical analysis.
  - ✓ Instruments uncertainty in handling endogeneity while facing many weak but valid instruments.

Objectives and Methodology

- We face the challenges of model uncertainty, instruments uncertainty and weak instruments challenges through adapting two existing procedures which have been extended to the endogeneity problems:
  - ✓ Directed Acyclic Graph (DAG) (e.g., Wang and Bessler 2006, Stockton, Capps, and Bessler 2008)
  - ✓ Bayesian Model Averaging (BMA) (e.g., Moral-Benito, 2010; Durlauf et al., 2008; Eicher et al., 2009)
- Furthermore, concurring with Deaton (2010), this study roots the instrumental variable estimations in theoretical framework:
  - ✓ The empirical case study is based on the unique theoretical model developed by You and Davis (2010).
  - ✓ The model depicts the interaction between parents and the child in order to guide empirical analysis of childhood weight production process.
  - ✓ Specifically, the model identifies a pool of valid instruments for parental inputs that are of policy interests (e.g., parental time allocations).

DAG: Visual Presentation

- DAG is a graphical model that shows causal flows among variables which can provide helpful insights to the instrument selection stage.



(Based on the model and data used in You and Davis, 2010)

BMA: IVBMA Method

- IVBMA: Eicher et al. (2009) extended BMA to account for the instruments selection uncertainty through a two-step procedure using Bayesian Information Criteria weights.

Let the model be: $Y=\beta'\begin{pmatrix}W\\X\end{pmatrix}+\eta,\quad W=\theta'_Z Z+\theta'_X X+\varepsilon$ where $\begin{pmatrix}\eta\\ \varepsilon\end{pmatrix}\sim N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\sigma^2_\eta & \sigma_{\eta\varepsilon}\\ \sigma_{\varepsilon\eta} & \sigma^2_\varepsilon\end{pmatrix}\right)$

First Stage: Nesting BMA into the 1st stage estimation of the 2SLS process

By Bayesian rule, the posterior probability is: $\pi_i = pr(M_i\mid D)=\dfrac{pr(D\mid M_i)pr(M_i)}{\sum_i pr(D\mid M_i)pr(M_i)}$

where $pr(D\mid M_i)=\int pr(D\mid\theta^i,M_i)pr(\theta^i\mid M_i)d\theta^i$

✓ The posterior mean of the 1st stage parameter θ is the weighted average of all possible models estimates (weighted by posterior probability):

$$\hat\theta=\sum_{i=1}^I \pi_i\hat\theta^i$$

✓ The posterior variance of the 1st stage parameter θ has two components:
- The weighted average of the variances of all possible models;
- The weighted sum of the coefficient deviances for each possible models.

$$\hat\sigma^{BMA}(\theta)=\sum_{i=1}^I\pi_i\hat\sigma_i^2+\sum_{i=1}^I\pi_i(\hat\theta^i-\hat\theta^{BMA})^2$$

Second Stage of the 2SLS process nested with BMA:

Given all the possible models in the 1st stage $M=\{M_1,M_2,\cdots M_I\}$, we can either use least squares or nest BMA again in the 2nd stage.

Let $\hat b(\tilde w_i)$ be the coefficient of the 2nd stage given the 1st fitted values of the endogenous variables $\tilde w_i$, then

✓ The posterior mean of 2nd stage parameter:

$$\hat b^{IVBMA}=\sum_{i=1}^I p_i\hat b(\tilde w_i)$$

✓ The posterior variance of 2nd stage parameter:

$$s^2_{IVBMA}=\sum_{i=1}^I p_i\,\mathrm{var}(\hat b\mid M_i)+\sum_{i=1}^I p_i(\hat b(\tilde w_i)-\hat b^{IVBMA})$$

Note:

- DAG can inform the setting of priors (i.e., $pr(M_i)$ and $pr(\theta^i\mid M_i)$) in the 1st stage.
- IVBMA provides extra information about instruments selected through posterior possibilities.

Simulation: Compare IVBMA, 2SLS, OLS

- OLS: suffers endogeneity bias
- 2SLS: finite sample properties (i.e., biasedness and inefficiency) suffer when using too many weak instruments
- IVBMA: mitigates the many instrument bias providing that instrument candidates are valid.

Scenario 1: Small numbers of weak instruments

	100 observations			500 observations		
	BMA	2SLS	OLS	BMA	2SLS	OLS
Bias	0.00734	0.03318	0.29916	0.00011	0.00589	0.29906
MSE	0.00514	0.00568	0.09202	0.00099	0.001	0.08996
Inter quartile	0.09929	0.09371	0.06708	0.00097	0.00097	0.03177
Median bias	0.01155	0.03674	0.29772	0.00169	0.00805	0.2981
Abs deviance	0.07406	0.06916	0.20132	0.02997	0.03001	0.02311

Scenario 2: Many weak instruments

	10 IVs, b_1=1 ,rest 0.01			15 IVs, b_1=0.5 ,rest 0.05		
	BMA	2SLS	OLS	BMA	2SLS	OLS
bias	0.0663	0.074	0.0816	0.0844	0.1869	0.5899
MSE	1.1649	0.0098	0.0019	0.0184	0.0462	0.3592
inter quartile	0.1645	0.1268	0.0589	0.1711	0.1332	0.0776
median bias	0.0796	0.078	0.0812	0.0983	0.1946	0.5807
abs deviance	0.1229	0.0938	0.0436	0.0113	0.0997	0.0579

Case Study: Parental Choice, Childhood Obesity

- To further demonstrate the application of DAG and IVBMA, we estimate the children's weight production function from You and Davis (2010) by 2SLS and IVBMA.
- The instrument pool is based on the theoretical framework developed in You and Davis (2010).
- We use the same data as You and Davis (2010) which provides information on children's weight (measured by Body Mass Index (BMI)), parental time allocation, household food expenditures, and other important identification instruments (e.g., parental work enviornment, unearned income, wage rate etc.

The policy relevant equation:

$$\log(kidbmi)=b_1 ExpFAH+b_2 ExpFAFH+b_3 \Pr epTime+b_4 ChildTime+X'q+e$$

- The variables of interests are the household food-at-home expenditure (ExpFAH), food-away-from-home expenditure (ExpFAFH), parental time spent in food preparation (PrepTime) and parental time spent with the child (ChildTime).
- These policy relevant variables are endogenous since they are all choices made by the parents which are most likely to be influenced by uncontrolled factors that also affect the child's weight production.

1st Stage Estimation: IVBMA vs. OLS

	ExpFAH			ExpFAFH			PrepTime			ChildTime		
	post prob	Mean coef	OLS coef.	post prob	Mean Coef	OLS coef.	post prob	Mean Coef	OLS coef.	post prob	mean coef	OLS coef.
FUEin	0	0	0.0004	100	0.0006*	0.0007*	0	0	0.0014	18.9	0.0021	0.0099
MUEin	0	0	-0.0007	100	0.0053***	0.0056***	0	0	-0.002	0	0	0.0191
Fwage	100	0.0012*	0.0011	100	0.0023***	0.0019***	0	0	-0.0044	0	0	-0.0017
Mwage	0	0	-0.0005	100	0.0019***	0.0020***	100	-0.0108***	-0.0081	0	0	-0.0084
FflexHR	100	0.0413**	0.033	100	-0.0321**	-0.0418**	0	0	-0.0525	0	0	-0.0899
MflexHR	8.6	-0.0009	-0.0036	0	0	-0.0125	0	0	-0.1242	0	0	0.0912
FflexDay	0	0	0.0155	55.4	0.0106	0.0221	3	0.0038	0.1143	0	0	-0.0593
MflexDay	0	0	-0.0051	8	0.0006	0.0211	2.4	0.0025	0.2002	100	0.6054***	0.5158
FWorkCom	0	0	-0.002	52.5	-0.0065	-0.0191*	31.9	-0.0443	-0.1877**	0	0	0.1004
MworkCom	0	0	0.0144	100	0.0203*	0.0243*	0	0	0.0576	0	0	-0.027
FworkDay	0	0	0.009	100	0.0564**	0.0522	14.6	-0.0503	-0.3043	100	-2.024***	-2.03***
Mworkday	100	-0.0476	-0.0305	7.5	0.001	0.0211	100	-0.7546***	-0.7011***	0	0	0.1919

- ✓ We include all the exogenous variables in the first stage IVBMA estimation. The prior for these exogenous variables are informed by a pre-estimation of BMA on the structural weight production equation.
- ✓ The priors for the extra identification instruments can be informed by DAG as shown in the DAG section.

2nd Stage Estimation: IVBMA vs. 2SLS

		ExpFAH	ExpFAFH	PrepT	ChildT	Power	fspillOver	mspillover	age	gender	puberty	sibling	mombmi	dadbmi
2sls	beta	0.6331	0.0937	0.0216	-0.0133	-0.053	-0.0431	0.0392	0.0183	0.0423	0.08	0.0236	0.0112	0.0125
	sd	0.5137	0.267	0.0554	0.0262	0.0342	0.0286	0.0275	0.0115	0.0424	0.0687	0.0286	0.0042	0.007
IVBMA	wbeta	0.0961	-0.0215	0.1717	0.0401	-0.0269	-0.0301	0.038	0.0219	0.0231	0.0878	0.0336	0.0105	0.0087
	wsd	0.5111	0.0249	0.2763	0.0558	0.023	0.025	0.0257	0.0108	0.0364	0.0627	0.0228	0.004	0.006

Conclusion

- Guided by theoretical framework, we can identify valid instruments pool.
- Data availability and measurement difficulty usually leave us with weak instruments.
- Weak instruments can cause biasedness and inefficiency.
- We demonstrate that combining DAG and BMA in the instrumental variable estimation process (2SLS) have the potential to gain efficiency and mitigate weak instrument bias.
- DAG not only can provide visual revelation of the causal flow among variables but also can inform the prior assumptions in BMA procedure.
- BMA applied to the 1st stage of the 2SLS can contribute in reducing the numbers of potential instruments used and model averaging will provide a way to combine the strength of different instruments available.