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# **Economic Value of Information: Wheat Protein Measurement**

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# **Economic Value of Information: Wheat Protein Measurement**

**(Preliminary Draft - Please do not Quote without Permission)**

## **Abstract**

In this paper we study U.S. wheat farmers' willingness to pay for near infrared (NIR) sensor that can segregates wheat grains according to their protein concentration. We first develop a microeconomic optimization model of wheat farmers' segregating and commingling decisions. Then we use U.S. wheat prices and stocks to estimate a wheat protein stock demand system. This allows us to establish the effects of changes in the protein profile of wheat stocks on protein premiums. The paper's simulation section combines the results from the microeconomic optimization model and from the econometric estimations to simulate wheat farmers' WTP for the sorting technology. Preliminary findings from the simulation show that a typical hard red winter (hard red spring) wheat farmer's WTP for the sorting technology is 5.6 (4.8) cents per bushel.

**Key words:** information, economic value, wheat, protein, market structure.

**JEL classification:** Q12, Q16, D81.

# 1 Introduction

Wheat is the world's second largest crop by average annual (1990-2009) production. The average annual wheat production over 1990 to 2009 is 593.45 million metric tons. The U.S. is the third largest wheat producer which produces about 10% of world production. The top two wheat producers in the world are China and India, which respectively produce about 17% and 11% of world wheat production. The top twenty countries produce about 86% of total world wheat.<sup>1</sup>

Protein concentration is one of the major factors that affect prices of wheat and barley. For wheat that is used to produce bread or pasta, higher protein concentration is preferred due to the favorable end-use properties added by the higher protein level, and hence higher protein wheat often receives protein premiums. For example, U.S. Wheat Associates (<http://www.uswheat.org>) and Agricultural Marketing Service (AMS) at USDA (<http://www.ams.usda.gov/AMSV1.0/>) report wheat prices based on protein concentrations. Like U.S. wheat markets, wheat markets in other major wheat countries pay protein premiums as well. In China and France, protein levels directly determine the grading of wheat (Tab. 37 and Tab. 58 in Popper, Schäfer, and Freund, 2007). In Canada, No. 1 Canada Western Red Spring (CWRS) wheat and No. 2 CWRS wheat are often sold at different protein levels (p50, Popper, Schäfer, and Freund, 2007). Australian Wheat Board (AWB) has maximum or minimum requirements for wheat protein levels for its six main wheat grades (Tab. 28 in Popper, Schäfer, and Freund, 2007). India also has such requirements for its five classes of wheat (p86, Popper, Schäfer, and Freund, 2007). In Argentina, wheat experts proposed to further divide wheat classes by protein levels (p94, Popper, Schäfer, and Freund, 2007).

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<sup>1</sup>Data source of this paragraph: Food and Agricultural Organizations of the United Nations (<http://faostat.fao.org/site/567/default.aspx#ancor>), accessed on October 19th, 2010.

A new technology named near infrared (NIR) sensor makes sorting wheat grains according to their protein concentrations possible (Long, Engel, and Siemens, 2008). If the sorting technology is widely adopted by wheat farmers, then one should expect that the supply of wheat with favorable protein concentration levels will increase. Given that the demand is unchanged, the protein premium will be negatively affected by the sorting technology. The purpose of this article is to study wheat farmers' willingness to pay (WTP) for the sorting technology. To fulfill the purpose we first develop a microeconomic optimization model of wheat farmers' segregating and commingling decisions. Then we use U.S. wheat prices and stocks to estimate a wheat protein stock demand system. This allows us to establish the effects of changes in the protein profile of wheat stocks on protein premiums. The paper's simulation section combines the results from the microeconomic optimization model and from the econometric estimations to simulate wheat farmers' WTP for the sorting technology.

Our analysis focuses on Hard Red Winter (HRW) wheat and Hard Red Spring (HRS) wheat, which account for about 64% wheat production in the U.S. Initial wheat stocks are important to determine WTP since they affect wheat price schedules. We simulated a normalized WTP (i.e., WTP for a sorting service which sorts wheat production from 100 acres every year and for 10 years). For HRW wheat, results show that if we take sets of 10 year continuous historical data and use them as wheat stocks facing HRW wheat farmers, then the normalized WTP ranges from from 1,821 to 2,273, depending on which 10-year data we excerpt from the data set.

The article proceeds as follows. In Section 2 we develop a conceptual model of a typical wheat farmer's optimal segregating and commingling decisions facing various protein premium schedules. Section 3 estimates a wheat protein stock demand system. Section 4 simulates the WTP of wheat farmers' for the sorting technology. Section 5

concludes.

## 2 Conceptual Model

In this section we develop a microeconomic optimization model of a wheat farmer's segregating and commingling decisions according to wheat protein concentrations. The goal of the wheat farmer is to maximize the profit from selling her wheat by optimally segregating and commingling her wheat given the protein premium schedule and the distribution of protein concentration of her wheat. At this moment we assume that segregating and commingling costs are zero. Therefore, the profit maximizing goal is equal to maximizing the revenue from selling wheat.

Processing wheat with various protein levels is different from processing wheat with various dockage rate. Since protein is part of wheat kernels, protein within one load of wheat does not have linear separability that dockage has. For example, 1,000 bushels of grain with average 10% protein level cannot be segregated into 900 bushels of zero percent protein wheat and 100 bushels of 100% protein wheat. This means that the segregating result will be constrained by the distribution of protein concentration in one load of wheat. For instance, if 1,000 bushel of 12% protein wheat is a mix of 500 bushels of pure 10% protein wheat (imagine that each grain of this 500 bushels contains 10% protein) and 500 bushels of 14% protein wheat (imagine that each grain of this 500 bushels contains 14% protein). Suppose the farmer segregate this 1,000-bushel load into 1,000 one-bushel loads. Then the protein level of any load among the 1,000 one-bushel loads must lie in the interval  $[10\%, 14\%]$ . Such constraint does not apply when processing wheat with various dockage rate (Hennessy and Wahl 1997). For example, 1,000 bushels of grain with 1% dockage for unacceptable materials can be decomposed into 990 bushels of zero

percent dockage grain and 10 bushels of cleanings, or 500 bushels of 1.1% dockage and 500 bushels of 0.9% dockage. Therefore, regarding processing wheat at protein dimension, like Hennessy and Wahl (1997), perfect commingling is an available action to the wheat farmer; unlike Hennessy and Wahl (1997), however, perfect segregating is not an available action to the wheat farmer. For one load of wheat, the extent of segregation depends on this load's distribution of protein concentration. This future non-linear separability makes the analysis of this article different from the analysis in Hennessy and Wahl (1997), in which both perfect segregating and perfect commingling are available as choices to maximize revenue.

## **2.1 Model Setup**

Unlike Hennessy and Wahl (1997) that studied optimal segregating or commingling decision at elevator level, we consider the optimal decision at farm level. This is because the near infrared (NIR) technology can be readily applied in field and farmers have the incentive to adopt the technology. If wheat farmers and elevators are facing the same protein premium schedule, then once farmers adopt the technology to explore the arbitrage opportunity, there will be very little benefit for elevators to process wheat according to protein concentration levels. We assume that before the NIR technology is adopted, the wheat farmer sell her wheat in one load. That is, without the NIR technology, the farmer cannot segregate high protein wheat from his harvest to obtain the protein premium. After adopting the NIR technology, then she has the freedom to sorting her wheat according to protein concentration, and then optimize her revenue either by segregating or commingling. The method developed in this article can be readily applied to analyzing optimal arrangements of multiple wheat loads.

To better present the results in this paper, a series of definitions about the wheat

farmer's activities (i.e., sorting, segregating, commingling, and processing) dealing the load of wheat are necessary.

**Definition 1.** *To sort one load of wheat is to read the protein concentration of each smallest unit (e.g., one grain or one bushel of grains) of the load, and then label units with same protein concentrations as a group but label units with different protein concentrations as different groups.*

Here the verb “label” is used only for conceptual convenient. As we will see in this article that during the optimal processing, the action of “label” is not necessary because a unit of wheat will be placed into a corresponding sub-load once its protein concentration is read.

**Definition 2.** *Commingling two loads (or two groups of a load as sorting results in Definition 1) is to mix the them so that any sample of the mix has the same protein concentration.*

**Definition 3.** *Segregating one load of wheat is to separate the load into two or more sub-loads with different protein concentrations. Complete segregating one load means that wheat in this load is separate into as many as possible sub-loads such that each sub-load only contains wheat with the same protein concentration and that protein concentration of each sub-load differs.*

**Definition 4.** *Processing one load of wheat means to sort the load of wheat and then do commingling or segregating.*

From Definitions 1 to 4 one can see that sorting is necessary for both commingling and segregating. Without sorting the farmer cannot know the protein concentrations of any part in the load, and hence she cannot conduct segregating or commingling. Therefore, in the rest of this article if we say that “no sorting is needed to a load of wheat,”

then it implies that neither segregating nor commingling is needed, which also means no processing is needed to this load of wheat.

Suppose a wheat farmer has one load of wheat with mean protein level  $\mu$ . The mass of this load of wheat is normalized to one. The protein concentration distribution of this load of wheat is  $F(l)$  with density function  $f(l)$  and support  $[0, \bar{L}]$ . Here  $\bar{L} \leq 1$  is the upper bound of protein concentration of one unit of wheat. We assume there is no atoms on the protein concentration distribution. For simplicity we assume the farmer knows the protein distribution before she adopts the NIR technology. If the farmer does not know the protein distribution until she utilizes the NIR technology, then the estimation of willingness to pay for the NIR technology would require the farmer's belief about protein distributions of her harvest. In the situation that farmers only have a belief about the protein concentration distribution, our analysis in optimal processing decisions is still essential. This is because for any given protein concentration distribution under a belief our analysis can be used to obtain the optimal processing decisions.

In our model the wheat farmer is assumed as a price taker. This is reasonable considering the large number of wheat farmers in the United States. Let the non-decreasing wheat price function facing the farmer be  $p(l)$ , where  $l$  is protein concentration of one unit of wheat. Protein premium is imbedded in the price schedule because high protein wheat receives high price. In the following subsections we study the wheat farmer's optimal segregating and commingling decisions under four price schedules: 1) uniformly curved schedules (i.e., concave or convex), 2) non-uniformly curved schedules (i.e., concave at low protein levels and convex at high protein levels, or the reverse), and 3) three-step schedules.

## 2.2 Uniformly Curved Schedules

Incentives to segregate and commingle grain with different dockage when the price quality schedule is uniformly curved (i.e., concave or convex) have been studied in Hennessy and Wahl (1997). Regarding wheat with different protein concentrations, incentives to segregate and commingle is similar as what is in Hennessy and Wahl (1997). But we still demonstrate the results here because some of them will be utilized repeatedly in obtaining optimal processing decisions when price schedules are not uniformly curved.

**Proposition 1.** *If the price schedule is concave, then no sorting is needed to the load. That is, this load of wheat will be sold as it is. If the price schedule is convex, then the load should be sorted and completely segregated.*

*Proof.* Suppose that the price schedule,  $p(l)$ , is concave and that the processing outcome is segregating the load into  $n \geq 2$  sub-loads, namely sub-loads  $1, \dots, n$ . If we show that this processing outcome is not optimal under the concave price schedule, then we prove the first half of the proposition. Let sub-load 1 have weight  $W_1$  and protein concentration  $l_1$  per unit weight, and let sub-load 2 have weight  $W_2$  and protein concentration  $l_2$  per unit weight. The total revenue from sub-load 1 and from sub-load 2 is  $W_1p(l_1) + W_2p(l_2)$ . If the farmer commingles sub-load 1 and sub-load 2, then the protein concentration of the mix is  $(W_1l_1 + W_2l_2)/(W_1 + W_2)$ . Therefore, the revenue from the mix is  $(W_1 + W_2)p((W_1l_1 + W_2l_2)/(W_1 + W_2))$ . By Jensen's inequality, we have  $(W_1 + W_2)p((W_1l_1 + W_2l_2)/(W_1 + W_2)) > W_1p(l_1) + W_2p(l_2)$  when  $p(l)$  is concave. The second part of the proposition can be proved by a complete reversal of the above procedure. □

From Proposition 1 we have the following corollary.

**Corollary 1.** *Commingling any two loads increases (decreases) a wheat farmer's revenue when the price schedule is concave (convex). Segregating one load increases (decreases) a wheat farmer's revenue when the price schedule is convex (concave).*

Next proposition studies the effects of protein concentration distribution on revenue when price schedule is concave and convex.

**Proposition 2.** *Suppose wheat load A and wheat load B have protein concentration distribution  $F(l)$  and  $G(l)$ , respectively, where  $G(l)$  is a mean-preserving spread of  $F(l)$ . And suppose loads A and B have the same weight. If price schedule is convex, then a wheat farmer can receive higher maximum revenue from selling load B than load A. If price schedule is concave, then the maximum revenue from selling loads A and B is the same.*

*Proof.* If the price schedule is convex, then by Proposition 1 both load A and load B will be completely segregated. Therefore, the maximum revenue from selling loads A and B is  $\int_0^1 p(l)dF(l)$  and  $\int_0^1 p(l)dG(l)$ , respectively. By Proposition 6.D.2 on page 199 of Mas-Colell, Whinston, and Green (1995) we can obtain  $\int_0^1 p(l)dF(l) < \int_0^1 p(l)dG(l)$ . If the price schedule is concave, then by Proposition 1 neither load A nor load B is sorted. Because loads A and B have the same weight and mean protein concentration, they bring the same amount revenue to the farmer. □

### 2.3 Nonuniformly Curved Schedules

Instead of being uniform curves, price schedules may have nonuniform curves. Figures 1a and 1b present two possibilities of these schedules. Figure 1a shows a price schedule that is concave at low protein concentration levels and convex at high protein concentration levels. Figure 1b shows a price schedule that is convex at low protein concentration levels

and concave at high protein concentration levels. Following Hennessy and Wahl (1997) we call schedules with the curvature of Figure 1a as shape type I and schedules with the curvature of Figure 1b as shape type II. Without loss of generality it is assumed that  $p(0) = 0$ , so the schedules pass through the origin. In Figure 1a and Figure 1b, points  $O$  and  $O'$  are two ends of price schedule curve. Point  $B$  is the inflexion point where the schedule changes from being concave (convex) to being convex (concave) in Figure 1a (Figure 1b).

In Figure 1a, there are two tangent lines of price curve  $OO'$  that are of critical interest. Line  $O'A$  is the tangent of the price curve with tangency point at  $A$ . If there is no tangency point, then set point  $A$  as origin  $O$ . Line  $CD$  is defined as in Definition 5. Let the coordinates of points  $A$ ,  $B$ ,  $C$ , and  $D$  are  $[l_A, p(l_A)]$ ,  $[l_B, p(l_B)]$ ,  $[l_C, p(l_C)]$ , and  $[l_D, p(l_D)]$ , respectively.

**Definition 5.** *Line  $CD$  in Figure 1a is defined as: (1) Line  $CD$  is a tangent of curve  $OB$  with tangency point at  $C$ . (2) Line  $CD$  intersects curve  $O'B$  at point  $D$ . (3) The  $l$ -coordinates of points  $C$  and  $D$  is such that  $l_A < l_C < l_B$  and  $\int_0^{l_D} f(l)dl / \int_0^{l_D} f(l)dl = l_C$ .*

Definition 5 indicates that the commingle of wheat with protein concentration no higher than  $l_D$  has mean protein concentration  $l_C$ . Let  $\mu$  be the mean protein concentration of the initial load of wheat. Therefore, if  $\mu \leq l_A$ , line  $CD$  does not exist.

In Figure 1b, there are two critical tangent lines as well. Line  $OA$  is the tangent of the price curve with tangency point at  $A$ . If there is no tangency point, then set point  $A$  as  $O'$ . Line  $CD$  is defined as in Definition 6.

**Definition 6.** *Line  $CD$  in Figure 1b is defined as: (1) Line  $CD$  is a tangent of curve  $O'B$  with tangency point at  $C$ . (2) Line  $CD$  intersects curve  $OB$  at point  $D$ . (3) The  $l$ -coordinates of points  $C$  and  $D$  are such that  $l_B < l_C < l_A$  and  $\int_{l_D}^{\bar{l}} f(l)dl / \int_{l_D}^{\bar{l}} f(l)dl = l_C$ .*

Definition 6 indicates that the commingle of wheat with protein concentration no less than  $l_D$  has mean protein concentration  $l_C$ . Therefore, if  $\mu \geq l_A$ , line  $CD$  does not exist.

For schedules with shape type I and schedules with shape type II, the optimal processing arrangements are presented in next proposition.

**Proposition 3.** *For shape type I schedules, (i) when  $\mu \leq l_A$ , then no sorting is needed in the optimal arrangements; and (ii) when  $\mu > l_A$ , then in the optimal arrangements wheat in this load with protein concentration higher than  $l_D$  should be completely segregated and the remaining wheat should be completely commingled. Here  $l_D$  is the  $l$ -coordinate of point  $D$  defined in Definition 5.*

*For shape type II schedules, (i) when  $\mu \geq l_A$ , then no sorting is needed in the optimal arrangements; and (ii) when  $\mu < l_A$ , then in the optimal arrangements wheat in this load with protein level higher than  $l_D$  should be completely commingled and the remaining wheat should be completely segregated. Here  $l_D$  is the  $l$ -coordinate of point  $D$  defined in Definition 6.*

We first prove a lemma that will be used repeatedly in the proof of the proposition. Suppose one unit of wheat with protein concentration  $\alpha$  is a commingle of wheat with purely  $\alpha_1$  protein concentration and wheat with purely  $\alpha_2$  protein concentration. And suppose this unit of wheat is segregated into two sub-loads, namely  $A$  and  $B$ , with mean protein concentration  $l_A$  and  $l_B$ , respectively. Without loss of generality, we assume that  $\alpha_1 < \alpha_2$  and  $l_A < l_B$ . Then the following two items are true:

**Lemma 1.** *(i)  $\alpha_1 \leq l_A < l_B \leq \alpha_2$ . (ii)  $l_A$  (or  $l_B$ ) can be any value on the interval of  $[\alpha_1, \alpha)$  (or  $(\alpha, \alpha_2]$ ).*

*Proof.* The proof of item (i) is trivial because the unit of wheat dose not contain any wheat that with protein concentration less than  $\alpha_1$  or higher than  $\alpha_2$ .

Now let us prove item (ii) is true. Let  $l_A$  equal  $\lambda_A$  and  $l_B$  equal  $\lambda_B$ , where  $\lambda_A$  (or  $\lambda_B$ ) is an arbitrary value on the interval of  $[\alpha_1, \alpha]$  (or  $[\alpha, \alpha_2]$ ). To show item (ii) is to show that the unit of wheat can be segregated into sub-loads  $A$  and  $B$  such that  $l_A = \lambda_A$  and  $l_B = \lambda_B$ . Assume the weights of sub-load  $A$  and  $B$  are  $W_A$  and  $W_B$ , respectively. Then we have

$$(1) \quad \begin{cases} W_A + W_B = 1, \\ W_A \lambda_A + W_B \lambda_B = \alpha. \end{cases}$$

Suppose the weights of  $\alpha_1$  wheat and  $\alpha_2$  wheat are  $W_1$  and  $W_2$ , respectively. Then  $W_1$  and  $W_2$  can be uniquely determined by

$$(2) \quad \begin{cases} W_1 + W_2 = 1, \\ W_1 \alpha_1 + W_2 \alpha_2 = \alpha. \end{cases}$$

Suppose we commingle  $x$  units of  $\alpha_1$  wheat and  $y$  units of  $\alpha_2$  wheat to obtain sub-load  $A$ . Then we must have

$$(3) \quad \begin{cases} x + y = W_A, \\ x \alpha_1 + y \alpha_2 = W_A \lambda_A. \end{cases}$$

Solving equation system (3) generates

$$(4) \quad \begin{cases} x = \frac{\alpha_2 - \lambda_A}{\alpha_2 - \alpha_1} W_A, \\ y = \frac{\lambda_A - \alpha_1}{\alpha_2 - \alpha_1} W_A. \end{cases}$$

If we show that the remaining wheat (i.e., wheat not in sub-load  $A$ ) has weight  $W_B$  and mean protein concentration  $\lambda_B$ , then the proof is completed. Clearly, the weight of remaining wheat is  $(W_1 - x) + (W_2 - y) = (W_1 + W_2) - (x + y) = 1 - W_A = W_B$ . And the

protein of the remaining wheat is  $(W_1 - x)\alpha_1 + (W_2 - y)\alpha_2 = (W_1\alpha_1 + W_2\alpha_2) - (x\alpha_1 + y\alpha_2) = \alpha - W_A\lambda_A = W_B\lambda_B$ . Therefore, the mean protein concentration is  $\lambda_B$ .  $\square$

If we use vector  $(l_A, l_B)$  to denote the protein concentrations of sub-loads  $A$  and  $B$  segregated from this one unit of wheat and if we relax the constraint  $l_A < l_B$ , then Lemma 1 can be rewritten as

**Lemma 2.** *Suppose one unit of wheat with protein concentration  $\alpha$  is a commingle of wheat with purely  $\alpha_1$  protein concentration and wheat with purely  $\alpha_2$  protein concentration. If this unit of wheat is segregated into two sub-loads, then it can and only can be segregated into two sub-loads such that  $(l_A, l_B)$  is majorized by  $(\alpha_1, \alpha_2)$ , where  $l_A$  and  $l_B$  are the protein concentrations of these two sub-loads, respectively.*

Based on Lemma 1 we can prove Proposition 3. The proof is presented in Appendix A.

## 2.4 Three-Step Schedules

In wheat markets the price schedules are often of the step function forms. In this subsection we study a wheat farmer's optimal processing decisions when price schedules are three-step function forms. When price schedule in wheat market is an  $N$ -step schedule ( $N > 3$ ), unfortunately we cannot obtain an elegant uniformly concave or convex effective price schedule by eliminating dominated discontinuous points on the step price schedule like Hennessy and Wahl (1997) did. This is because of the non-linear separability in the protein dimension. However, we still can use price schedule convexity and the results of optimal decisions facing three-step schedules to simplify the effective price schedule once we know the protein distribution of one load.

Suppose the three-step price schedule is

$$(5) \quad p(l) = \begin{cases} p_1 & \text{if } 0 \leq l < l_1 \\ p_2 & \text{if } l_1 \leq l < l_2 \\ p_3 & \text{if } l_2 \leq l \leq \bar{L}, \end{cases}$$

where  $l \in [0, \bar{L}]$  is protein concentration of one load of wheat;  $p_3 > p_2 > p_1 > 0$  are prices; and  $l_1$  and  $l_2$  are constants such that  $0 \leq l_1 < l_2 \leq \bar{L}$ . Figure 2 depicts this three-step price schedule. In this subsection we further assume that the mean protein concentration of one load of wheat,  $\mu$ , is such that  $\mu \in (0, l_2)$ . If  $\mu \geq l_2$ , then it will receive the highest price and hence its owner has no incentive to further process it.

The farmer's problem is to maximize her revenue by optimally processing her wheat. Because the price schedule has a three-step function form, the farmer's problem is equal to optimally segregating his wheat into three sub-loads, namely  $S_1$ ,  $S_2$ , and  $S_3$ , to get the maximized revenue. Let  $\mu_i$ ,  $p_i$ , and  $q_i$  be the mean protein concentration, the price received, and the weight of sub-load  $S_i$ ,  $i = 1, \dots, 3$ , respectively. By construction we have  $\mu_1 \in [0, l_1)$ ,  $\mu_2 \in [l_1, l_2)$ ,  $\mu_3 \in [l_2, \bar{L}]$ , and  $\sum_{i=1}^3 q_i \mu_i = \mu$  (please recall that the total quantity is normalized to 1). Then the farmer's problem is to maximize  $\sum_{i=1}^3 p_i q_i$  subject to certain conditions.

For the next result, several definitions are necessary.

**Definition 7.** We define  $c_1$  and  $c_2$  as two minimum non-negative constants that are such that  $\frac{\int_{c_1}^{\bar{L}} f(l) dl}{\int_{c_1}^{\bar{L}} f(l) dl} \geq l_1$  and  $\frac{\int_{c_2}^{\bar{L}} f(l) dl}{\int_{c_2}^{\bar{L}} f(l) dl} = l_2$ , respectively.

Definition 7 says that the average protein level of the mix of all wheat with protein level higher than  $c_1$  is greater than or equal to  $l_1$ ; and the average protein level of the mix of all wheat with protein level higher than  $c_2$  is  $l_2$ . Clearly we have  $c_2 < l_2$  and  $c_1 < l_1$ .

**Definition 8.** When  $c_2 > l_1$ , then  $\hat{l}_1$  is defined as the minimum non-negative constant that satisfies  $\frac{\int_{\hat{l}_1}^{c_2} f(l)dl}{\int_{\hat{l}_1}^{c_2} f(l)dl} \geq l_1$ .

Definition 8 says that  $\hat{l}_1$  is the minimum non-negative constant such that the average protein level of wheat distributed on  $[\hat{l}_1, c_2]$  is no less than  $l_1$ .

We also need to present two non-linear programming problems.

$$(6) \quad \begin{aligned} & \max_{q_i} \sum_{i=1}^3 p_i q_i \\ & s.t. \\ & q_i \geq 0, \\ & \sum_{i=1}^3 q_i = 1, \\ & \int_0^{F^{-1}(q_1)} f(l)dl + q_2 l_1 + q_3 l_2 = \mu, \\ & F(c_1) \leq q_1 \leq F(c_2). \end{aligned}$$

$$(7) \quad \begin{aligned} & \max_{q_i} \sum_{i=1}^3 p_i q_i \\ & s.t. \\ & q_i \geq 0, \\ & \sum_{i=1}^3 q_i = 1, \\ & \int_0^{F^{-1}(q_1)} f(l)dl + q_2 l_1 + q_3 l_2 = \mu, \\ & F(c_1) \leq q_1 \leq F(\hat{l}_1). \end{aligned}$$

For the optimal processing outcomes under three-step price schedules, we have the

following proposition.

**Proposition 4.** *Suppose the mean protein concentration of one load of wheat,  $\mu$ , is such that  $\mu \in (0, l_2)$ . The optimal processing outcomes are (i) the solutions of problem (6) if  $c_2 \leq l_1$ ; (ii) the solutions of problem (7) if  $c_2 > l_1$  and if  $\hat{l}_1 > 0$ ; and (iii)  $q_1^* = 0$ ,  $q_2^* = F(c_2)$ , and  $q_3^* = 1 - F(c_2)$  if  $c_2 > l_1$  and if  $\hat{l}_1 = 0$ .*

Visual presentations of the three items in Proposition 4 are depicted in Figure 3, Figure 4, and Figure 5, respectively. To prove the proposition, several lemmas are necessary.

**Lemma 3.** *The maximized amount of wheat with mean protein concentration at  $l_2$  ( $l_1$ ) that can be segregated out from the initial load is  $1 - F(c_2)$  ( $1 - F(c_1)$ ).*

*Proof.* The proof is trivial. Suppose now all wheat with protein concentration that is no less than  $c_2$  is segregated into sub-load  $S_3$ . By the definition of  $c_2$  we know that the mean protein concentration of sub-load  $S_3$  is  $l_2$ . In order to increase the weight of sub-load  $S_3$ , one must add some of the remaining wheat into sub-load  $S_3$ . However, the wheat in the remaining now has protein concentration lower than  $c_2$ , which is lower than  $l_2$ . Adding such wheat into sub-load  $S_3$  will make the mean protein concentration in the sub-load lower than  $l_2$ . The same argument applies when proving the other part of this lemma.  $\square$

**Lemma 4.** *In the optimal arrangements, (i) if  $q_1^* > 0$ , then  $\mu_2 = l_1$  and  $\mu_3 = l_2$ ; (ii) if  $q_2^* > 0$  or if  $q_3^* > 0$ , then  $\mu_3 = l_2$ .*

*Proof.* The proof is completed by simple arbitrage arguments. For item (i), if  $q_1^* > 0$  but  $\mu_2 > l_1$ , then the farmer can always increase her revenue by commingling some wheat from sub-load  $S_1$  to sub-load  $S_2$  as long as  $\mu_2 \geq l_1$ . This is because the wheat that is moved from sub-load  $S_1$  to sub-load  $S_2$  now is sold at price  $p_2$  instead of price  $p_1$  and the price of wheat initially in sub-load  $S_2$  is not affected. The same argument applies for

$\mu_3 = l_2$  of item (i) and for the first part of item (ii). If  $q_3^* > 0$ , then  $q_2^* > 0$  or  $q_1^* > 0$ , or both. This is because  $\mu < l_2$ . By item (i) and the first part of item (ii) we know that  $\mu_3 = l_2$ .  $\square$

Let  $\bar{l}_{S_1}$  denote the protein concentration of wheat that has the highest protein in  $S_1$ . Let  $\underline{l}_{S_2}$  (or  $\underline{l}_{S_3}$ ) denote the protein concentration of wheat that has the lowest protein in sub-load  $S_2$  (or  $S_3$ ). The next lemma can be stated as

**Lemma 5.** *In the optimal arrangements, we have (i)  $\bar{l}_{S_1} \leq \min[\underline{l}_{S_2}, \underline{l}_{S_3}]$  and (ii) any wheat with protein concentration no higher than  $\bar{l}_{S_1}$  is in sub-load  $S_1$ .*

*Proof.* Suppose in the optimal arrangement we have  $\bar{l}_{S_1} > \min[\underline{l}_{S_2}, \underline{l}_{S_3}]$ . That is, protein concentration of some wheat in sub-load  $S_1$  is higher than protein concentration of some wheat in sub-load  $S_2$  or in sub-load  $S_3$ . Without loss of generality we assume that  $\min[\underline{l}_{S_2}, \underline{l}_{S_3}] = \underline{l}_{S_2}$ . Then the farmer can increase her revenue by doing step (1) exchanging 1 unit of  $\bar{l}_{S_1}$  wheat from sub-load  $S_1$  with 1 unit wheat with protein concentration lower than  $\bar{l}_{S_1}$  from sub-load  $S_2$ ; and step (2) moving  $\delta$  amount of wheat with protein concentration lower than  $l_1$  from sub-load  $S_1$  to sub-load  $S_2$  as long as  $\mu_2$  is no less than  $l_1$ . By doing step (1),  $\mu_2$  is increased and but the revenue is not affected; by doing step (2),  $q_2$  is increased by  $\delta$  and  $q_1$  is decreased by  $\delta$ . So is the revenue is increased by  $\delta(p_2 - p_1)$ . In sum, we must have  $\bar{l}_{S_1} \leq \min[\underline{l}_{S_2}, \underline{l}_{S_3}]$  in the optimal arrangement. Item (ii) follows naturally.  $\square$

**Lemma 6.** *In the optimal arrangements, (i) if  $c_2 \leq l_1$ , then  $\bar{l}_{S_1} \in [c_1, c_2]$ ; (ii) if  $c_2 > l_1$ , then  $\bar{l}_{S_1} \in [c_1, \hat{l}_1]$ .*

*Proof.* First, we show that  $\bar{l}_{S_1} \geq c_1$ . If  $\bar{l}_{S_1} < c_1$ , then the mean protein concentration of the commingle of wheat in sub-load  $S_1$  and wheat in  $S_3$  will be lower than  $l_1$ , which contradicts that  $\mu_2 \in [l_1, l_2)$  and  $\mu_3 \geq l_2$ .

Second, we show that if  $c_2 \leq l_1$  then  $\bar{l}_{S_1} \leq c_2$ . Suppose when  $c_2 \leq l_1$  we have  $\bar{l}_{S_1} > c_2$ . So the mean protein concentration of the commingle of wheat in sub-load  $S_2$  and wheat in  $S_3$  will be higher than  $l_2$ , which contradicts that in the optimal arrangements  $\mu_2 \in [l_1, l_2)$  and  $\mu_3 = l_2$  (Lemma 4).

Third, we show that if  $c_2 > l_1$  then  $\bar{l}_{S_1} \leq \hat{l}_1$ . Please note that  $\hat{l}_1$  has definition only if  $c_2 > l_1$ . Suppose  $\bar{l}_{S_1} > \hat{l}_1$  when  $c_2 > l_1$ . Therefore we have  $q_1^* = F(\bar{l}_{S_1}) > 0$ . Taking  $q_1^*$  as fixed, to maximize the revenue is equal to maximize  $q_3$  under the constraint of  $\mu_2 \geq l_1$ . The maximized  $q_3$  is  $1 - F(c_2)$ . Since  $c_2 > l_1$  and  $\bar{l}_{S_1} > \hat{l}_1$ , we must have  $\mu_2 > l_1$ , which is not optimal (Lemma 4).  $\square$

**Lemma 7.** *In the optimal arrangements, (i) if  $c_2 \leq l_1$ , then  $\mu_2 = l_1$ ; (ii) if  $c_2 > l_1$  and if  $\hat{l}_1 > 0$ , then  $\mu_2 = l_1$ .*

*Proof.* If  $q_1^* > 0$ , then items (i) and (ii) are true according to Lemma 4. Now we prove that items (i) and (ii) are true when  $q_1^* = 0$ .

If  $q_1^* = 0$ , then we must have  $q_2^* > 0$  and  $q_3^* \geq 0$ . This implies that  $\mu \geq l_1$ . If  $c_2 \leq l_1$ , then the initial load of wheat can be seen as a commingle of wheat with  $l_2$  protein concentration and wheat with  $l_0$  protein concentration, where  $l_0 \equiv \int_0^{c_2} f(l)l dl / \int_0^{c_2} f(l) dl < l_1$ . According to Lemma 1, the initial load of wheat can be segregated into two sub-loads with one sub-load having protein concentration at  $l_1$  and the other sub-load having protein concentration at  $l_2$ . Given  $q_1^* = 0$ , this segregation is optimal. It is because that if  $q_1^* = 0$ , then the optimal segregation should be to maximize  $q_3$  while keeping  $\mu_2 \geq l_1$ . Some algebra can show that  $q_3$  is not maximized when  $\mu_2 > l_1$ .

If  $q_1^* = 0$ , then  $q_2^*$  and  $q_3^*$  should be such that

$$(8) \quad \begin{cases} q_2^* + q_3^* = 1 \\ q_2^* \mu_2 + q_3^* l_2 = \mu. \end{cases}$$

Solving (8) we obtain  $q_3^* = \frac{\mu - \mu_2}{l_2 - \mu_2}$ . Then we have

$$(9) \quad \frac{dq_3^*}{d\mu_2} = \frac{\mu - l_2}{(l_2 - \mu_2)^2} < 0.$$

Therefore, in the optimal arrangement  $\mu_2$  must be equal to  $l_1$  if  $c_2 \leq l_1$ . The same procedure follows when proving item (ii).  $\square$

The proof of Proposition 4 is presented in Appendix B. One may intend to think that the programming problems (6) and (7) without the fourth constraint will generate the same optimal solutions. Her argument could be the programming problem naturally prefers a smaller  $q_1$  over a bigger  $q_1$ ; therefore, the programming problem without the constraint  $F(c_1) \leq q_1 \leq F(\hat{l}_1)$  will automatically drive  $q_1$ s small enough so that  $F(c_1) \leq q_1 \leq F(\hat{l}_1)$  is met.

But this is not necessarily true. Decreasing one unit of  $q_1$  from  $q_1^*$  means that the farmer will gain  $p_2 - p_1$ . But to keep  $\mu_2 = l_1$ , the farmer has to move  $x$  amount of wheat from sub-load  $S_3$  (with averagely  $l_2$  protein level) to sub-load  $S_2$ ; otherwise wheat in sub-load  $S_2$  will have average protein level lower than  $l_1$ . Therefore, the revenue loss is  $x(p_3 - p_2)$ . If  $x(p_3 - p_2) > p_2 - p_1$ , then decreasing  $q_1^*$  is not profitable.

Suppose we delete the constraint  $F(c_1) \leq q_1 \leq F(\hat{l}_1)$  in problem (7). And suppose  $F(c_2) > q_1 > F(\hat{l}_1)$ . By construction of  $\hat{l}_1$  we have  $\frac{\int_{\hat{l}_1}^{c_2} f(l)dl}{\int_{\hat{l}_1}^{c_2} f(l)dl} > l_1$ . In this case the achievable  $q_2$  and  $q_3$  given  $q_1$  should be  $q_2 = \int_{\hat{l}_1}^{c_2} f(l)dl$  and  $1 - F(c_2)$ , respectively. However, since we delete the constraint  $F(c_1) \leq q_1 \leq \hat{l}_1$ , the programming problem will obtain  $q_2$  and  $q_3$  based on  $q_1$  by its two equality constrains. And always the  $q_2$  (or  $q_3$ ) obtained from the two equality constraints is smaller (or greater) than the true value. Therefore, the programming problem without constraint  $F(c_1) \leq q_1 \leq F(\hat{l}_1)$  biases the revenue to a bigger value.

Sivaraman *et al.* (2002) claim that their method applies to step premium schedules (page 157, Case 4). However, their claim is not correct. They assume that in the optimal outcomes the protein levels in one bin are continuous, (i.e.,  $D_i = [d_{i-1}, d_i]$  in the last paragraph on page 156). But this may not be true. Here is an example. Suppose the protein concentrations of one load of wheat is uniformly distributed on  $[11.4\%, 13.6\%]$ . Then the average protein level of this load is 12.5%. The price schedule is wheat with protein level higher than or equal to 13% receives 13% protein price; wheat with protein level lower than 12% receives 11% protein price; the rest of wheat receives 12% price. Suppose wheat prices encourage commingling and the optimal solution is that  $q_1^* = 0$ ,  $q_2^* > 0$ , and  $q_3^* > 0$ , here  $q_1^*$ ,  $q_2^*$ , and  $q_3^*$  are quantities of wheat that receive 11%, 12%, and 13% protein price, respectively. Then  $q_2^* = 1/2$  and  $q_3^* = 1/2$  can be solved by

$$(10) \quad \begin{cases} q_2^* + q_3^* = 1 \\ 0.12q_2^* + 0.13q_3^* = 0.125. \end{cases}$$

Based on the uniform distribution, how to achieve  $q_2^* = 1/2$  and  $q_3^* = 1/2$ ? Is it possible to find  $d \in [11.4\%, 13.6\%]$  such that  $(d - 11.4\%)/(13.6\% - 11.4\%) = 1/2$  and  $(d + 11.4\%)/2 = 12\%$ ? The answer is no. One procedure that can make  $q_2^* = 1/2$  and  $q_3^* = 1/2$  is as follows: Step 1. Put wheat with protein level between 12.4% and 13.6% into one bin, say bin A, and mix them completely; so the average protein level in bin A is 13%; Step 2. Put wheat with protein level between 11.4% and 12.4% into another bin, say bin B; so the average protein level in bin B is 11.9%. Step 3. Move some wheat (with average protein level 13%) from bin A to bin B until the average protein level in bin B reaches 12%. Clearly protein levels of wheat in bin B is not continuous. For example, Bin B could includes wheat with protein levels between 11.4% and 12.4% and wheat with protein levels at 13%.

Since in reality wheat protein premium often has step-form schedules, the result in Proposition 4 will be utilized as the basis in the empirical part of this article. In next section we estimate the wheat protein stock demand system that will be utilized when simulating the WTP of the sorting technology.

### 3 Wheat Protein Stock Demand System

Since price differences, not price levels, matter for calculating the WTP, in our econometric model we focus on price differences instead of price levels. That is, the dependent variables in the econometric model are price differences. We expect that the protein premiums are mainly affected by the wheat stocks and seasonality based on the standard supply-demand analysis. Specifically, the econometric models are

$$(11) \quad p_{3t} - p_{2t} = \alpha X + e_{1t}$$

$$(12) \quad p_{2t} - p_{1t} = \beta X + e_{2t},$$

where  $p_{it}$  ( $i=1,2,3$ ) is wheat price,  $p_i$ , in period  $t$ ; and  $X = (1, sh_{2t}, sh_{3t}, ts_t, sea_{1t}, sea_{2t}, sea_{3t})$  is the independent variable vector in which  $sh_{2t}$  and  $sh_{3t}$  are the shares of wheat stocks that receive  $p_{2t}$  and  $p_{3t}$ , respectively;  $ts_t$  is the total stocks in period  $t$ ;  $sea_{jt}$ ,  $j = 1, 2, 3$  is seasonal dummy of season  $j$  in period  $t$ ;  $e_{1t}$  and  $e_{2t}$  are error terms.

Regressions in equation (11) can alleviate the endogeneity problem existing in the model. The reason is that there are omitting variables that affect wheat prices and are correlated with wheat stocks, such as weather, foreign exchange rate, and prices of live cattle or hogs. By differencing prices, these variables will likely be canceled out so that regressions in equation (11) can fit the classical linear model assumptions for ordinary least square (OLS) estimator to be best linear unbiased estimator (BLUE). We use the

feasible generalized least squares (FGLS) method to correct autocorrelation problem in the model.

### **3.1 Data**

We use daily cash price data of HRW wheat and HRS wheat from Montana Wheat and Barley Committee (<http://wbc.agr.mt.gov/>). For HRW wheat, the prices are broken down to prices for 11% protein wheat, 12% protein wheat, and 13% protein wheat, respectively. For HRS wheat, the prices are broken down to prices for 13% protein wheat, 14% protein wheat, and 15% protein wheat, respectively. The time range for the price data is from 1980 to 2009. To make time series continuous, we use monthly averages of daily cash price data. For some years price data are missing for one or two months. We use cubic spline interpolation to fill the missing monthly average data. Therefore we get 360 observations, in which 8 observations are data filled by cubic spline interpolation. Figures 6 and 7 provide visual presentations of monthly HRW wheat prices and HRS wheat prices, respectively.

The monthly stocks of HRW wheat with 11%, 12%, and 13% protein level are calculated using the following procedure. Step 1), all wheat quarterly stocks (1980-2009) are obtained from NASS of USDA;<sup>2</sup> Step 2), calculate the percentage of HRW wheat production in all wheat production using data of wheat production by class from Crop Quality Reports published by U.S. Wheat Associates; Step 3), quarterly all wheat stocks in step 1) are multiplied by the percentage in step 2) to get the quarterly HRW wheat stock; Step 4), the percentages of HRW wheat with different protein levels in every year from 1980 to 2009 are obtained from annual Crop Quality Report published by U.S. Wheat Asso-

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<sup>2</sup>The website address is: <http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1079> (accessed on October 28, 2010)

ciates. Step 5), using data from Step 4), calculate the percentages of the following three categories of HRW wheat in total HRW wheat production: HRW wheat with protein level less than 12%, HRW wheat with protein level higher than or equal to 12% but less than 13%, HRW wheat with protein level higher than or equal to 13%; Step 6), quarterly HRW wheat stocks in step 3) were multiplied by percentages in step 5) to get the quarterly stocks of HRW wheat at 11%, 12%, and 13% protein level, respectively; Step 7), using cubic spline interpolation to quarterly stocks obtained in step 6) to get monthly HRW wheat stocks. The monthly stocks of HRW wheat at the three protein levels are presented in Figure 8. By the similar steps we can obtain the monthly stocks of HRS wheat at protein levels of 13%, 14%, and 15%, respectively. The result is presented in Figure 9.

### 3.2 Regression Results

The results of regression in equation (11) for HRW wheat and HRS wheat are reported in Table 1 and Table 2, respectively. From Table 1 we can see that the coefficients of  $sh_{2t}$  are significant at 5% level in regression (12). The coefficients of  $sh_{3t}$  are significant at 5% level in regressions (11) and (12). For example, in the case of HRW wheat, if the share of 13% protein wheat is increased by one percentage point, then the price difference between 13% protein wheat and 12% protein wheat will decrease by 0.42 cent. The coefficient of  $ts_t$  are not significantly different from zero, which means total stocks do not affect wheat protein premium. Seasonal dummies do not affect price difference between 13% protein wheat and 12% protein wheat, but they do affect price difference between 12% protein wheat and 11% protein wheat and price difference between 13% protein wheat and 11% protein wheat.

From Table 2 we can see that the coefficients of  $sh_{3t}$  are significant at 5% level in each regression. If the share of 15% protein wheat is increased by one percentage point,

then the price difference between  $p_3$  and  $p_2$  will decrease by 0.53 cent. The coefficient of  $sh_2$  is significantly different from 0 in regression (12) at 10% level but is not significant in regression (11). The coefficient of  $ts_t$  is not significantly different from zero in each regression, which means total stocks do not affect wheat protein premium. The price differences in the third season (July, August, and September), the harvest season of HRS wheat, are not significantly different from price differences in the fourth season (October, November, and December). But the price differences in the second season (April, May, and June) are significantly lower than the price differences in the fourth season. This indicates that the protein premium reaches the highest value during the harvest season and decreases as wheat stocks shrink.

### **3.3 Field-Level Protein Concentration Distribution Model and Data**

In order to estimate how much wheat with different protein levels can be sorted out from a load of wheat produced by a farmer, we need to know the protein concentration distribution of this load of wheat. Denote  $\Phi(\cdot)$  as the distribution function of the protein concentration. Our task in this subsection is to estimate  $\Phi(\cdot)$ . Washington State University Extension Cereal Variety Testing Program (<http://variety.wsu.edu/>) provides wheat and barley variety testing data and cultural data that can be traced back to 1997. One variety is usually planted in several different locations. The data report the variety's yield, test weight and protein concentration in each location and each year. In this subsection we focus on HRW wheat. The method we develop here can be easily applied to HRS wheat variety testing results.

From 1997 to 2009, the Program tested 194 HRW varieties in 15 locations across the State of Washington. Even though there were so many varieties tested, only a few of them were widely planted by wheat farmers. According to data from National Agricul-

tural Statistics Service (NASS) of USDA, in each crop year, the top ten varieties usually accounted for more than 90 percent of planted area for the same class of wheat in the State of Washington.<sup>3</sup> Therefore, in our analysis we only focused on the top 10 varieties in a crop year. According to this standard, 16 HRW wheat varieties were chosen from 1997 to 2009. The names of varieties and locations are listed in Table 3.

Top varieties varied from year to year. One variety may be popular in some years but disappeared from the list of top varieties in another year. In addition, not every variety was tested in every location every year. In one year, some locations may have more varieties than other locations. It was also possible that one of our 16 varieties in some years did not get tested at all. We collect the observation of one variety's performance into our dataset if: a) it is in the list of top varieties; and b) it was tested at least one time between 1997 and 2009. Using this screen we collected 538 observations. For each observation, we know the variety's name, yield, test weight, protein level, trial location where it was tested, and year when it was tested. We also know detailed cultural information about the trial, such as the type of soil, fertilizer usage, precipitation, latitude and longitude, etc.

Table 4 presents summary statistics of the 538 observations of HRW wheat protein concentrations. Its sample average is 11.87%. Its maximum and minimum values are 16.6% and 7.4%, respectively. The sample standard deviation is 0.017. However, this is not a satisfying estimator of the variability of HRW wheat protein concentration. The reason is that it is at the farm level for a given variety and in a given year that protein segregation occurs. But these 538 observations include 16 varieties planted on 15 locations within 13 years. If we accept this standard deviation as the estimator of protein variability, then we will over-estimate protein variability. To get a better estimation, we need to control for the effect of varieties, locations and years.

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<sup>3</sup>Data source: [http://www.nass.usda.gov/Statistics.by\\_State/Washington/Historic\\_Data/smallgrains/whtvar.pdf](http://www.nass.usda.gov/Statistics.by_State/Washington/Historic_Data/smallgrains/whtvar.pdf)

We applied regression analysis to estimate the protein variability. We assume that protein concentration has normal distribution conditional on variety, location, and year. That is,

$$(13) \quad pr\_conc|x \sim Normal(c + \sum_{i=2}^{13} \alpha_{year_i} + \sum_{i=2}^{16} \beta_j variety_j + \sum_{i=2}^{15} \gamma_k location_k, \sigma^2),$$

where  $x$  is a shorthand for the vector of a constant and dummy control variables:  $(constant, year_2, \dots, year_{13}, variety_2, \dots, variety_{16}, location_1, \dots, location_{15})$ . Here  $i$ ,  $j$ , and  $k$  start from 2 because we set  $year_1$ ,  $variety_1$ , and  $location_1$  as bases. Since in the dataset there is only one observation of protein concentration for a variety at one location in one year, it is difficult to test the normality of protein concentration conditional on variety, location, and year. However, we can test the normality of protein concentration conditional on variety or location, or both. If we cannot reject the normality in these tests, then we will have good reason to assume that protein concentration conditional on variety, location, and year has a normal distribution as well.<sup>4</sup>

The linear regression model can be written as

$$(14) \quad protein = c + \sum_{i=2}^{13} \alpha_{year_i} + \sum_{i=2}^{16} \beta_j variety_j + \sum_{i=2}^{15} \gamma_k location_k + u,$$

where  $u \sim Normal(0, \sigma^2)$  is the error term. An unbiased estimator of  $\sigma^2$  is  $s^2 = e'e / (n - K - 1)$  where  $e$  is least squares residuals,  $n$  is the number of observations, and  $K$  is the number of independent variables.

The results of regression (14) are listed in Table 5. The value of  $s^2$  is 0.0001. Combining the coefficients in Table 5, we will know exactly the distribution of protein concen-

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<sup>4</sup>Results of Lilliefors' test (Lilliefors, 1967) on the normality of protein concentration show that in most cases we cannot reject the hypothesis that our sample of protein concentration (conditional on one or two variables of year, variety, and location) comes from a normal distribution at the 0.01 significance level.

tration for a given variety at one location in one year. For example, the estimated protein distribution of variety Boundary at Connell in 2009 is  $Normal(0.126, 0.0001)$ . We are confident that the distribution  $Normal(0.126, 0.0001)$  is a satisfactory approximation of protein distribution of variety Boundary at Connell, Washington in 2009 at farm level. The reason is that the trials of one location are very closed to each other. For example, in Connell Washington from year 2005 to 2009, the shortest distance between two trials is only 200 feet; and the longest distance is 1.4 miles. Therefore, we can see these trails as a reasonable sample from an individual farm.

Regarding HRS wheat, the data set includes variety testing results of 13 years, 15 varieties, and 28 locations in the State of Washington. The regression for HRS wheat is

$$(15) \quad protein = c + \sum_{i=2}^{13} \alpha_{year_i} + \sum_{i=2}^{15} \beta_j variety_j + \sum_{i=2}^{28} \gamma_k location_k + e,$$

whose results are presented in Table 6.

Now we have protein distribution conditional on variety, location, and year. Together with the protein-price relations, we can calculate the revenue difference for a farmer between segregating and not segregating. We now have all the elements we need to estimate the WTP of wheat farmers for the sorting technology. In the next section we show how to utilize these elements in the simulation, and we report the simulation results.

## 4 WTP Simulation

In this section we focus on the WTP of HRW wheat farmers. Simulations for WTP of HRS wheat farmers follow the same methods. We take sets of continuous stock data from the data set as wheat stocks facing farmers. For example, we may take wheat stocks from crop year 1980/1981 to crop year 1989/1990 from the data set as wheat stocks facing

farmers. Then the WTP based on wheat stocks from 1980/1981 to 1989/1990 is the WTP for the technology if it had been launched in that period of time. Depending on which period of time we take, there are multiple possibilities of wheat stocks facing farmers.

The general idea under the simulation procedure is as follows. Given initial HRW wheat stocks, we find out what the wheat stocks will be after the sorting technology is adopted by farmers who can benefit from the technology considering the technology's market equilibrium effect. When we find out the wheat stocks after sorting, then we calculate the new price differences. Based on the new price differences, we then calculate the WTP of wheat farmers for the sorting technology. We calculate the WTP on 100-acre and 10-year basis. We name it as normalized WTP. Here "10-year" is the lifespan of a sorter and "100-acre" is the area of field whose production is sorted by the sorter. In other words, the normalized WTP stands for the net present value of a farmer's willingness to pay for sorting wheat from 100 acres every year and for 10 years. The interest rate is set at  $r=0.05$ . Once we have the normalized WTP, it is easy to calculate a farmer's WTP for a sorter if the sorter can last  $M$  years and can sort production from  $N$  acres per year. We use the trend yield in this report.<sup>5</sup> The yield trend is described as

$$(16) \quad yield_t = const + b(t - 1932) + e_t,$$

where  $t = 1933, 1934, \dots, 2009$ . The statistics for regression (16) are summarized in Table 7.

We take sets of 10-year continuous stock data directly from the data set and use them as the wheat stock facing wheat farmers. For example, we may use historical wheat stock data from crop year 1999-2000 to crop year 2008-2009. There are 30 calendar years (1980-2009) and 29 complete crop years (1980-81, , 2008-09) in the data set. Hence

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<sup>5</sup>Data source: NASS, all wheat yield, 1933-2009, website: <http://www.nass.usda.gov/QuickStats/>

we have 20 different sets of wheat stocks to take. They are: wheat stocks of crop years 1980-81 to 1989-90, wheat stocks of crop years 1981-82 to 1990-91, . . . , wheat stocks of crop years 1999-00 to 2008-09. We go through simulation for each of the 20 wheat stock sets. For HRW wheat, the mean WTP from the 20 stock sets is \$2,028 (about 5.6 cents per bushel). For HRS wheat, the mean WTP from the 20 stock sets is \$1,910 (about 4.8 cents per bushel). Figure 10 depicts the WTP of HRW wheat farmers. Figure 11 depicts the WTP of HRS wheat farmers. In Figures 10 and 11, a crop year on the x-axis means that the ten-year continuous stock data starts in that year.

## **5 Conclusions and Future Work**

Two important and related trends in food markets are a) growth in demand for differentiated products, and b) capacity to distinguish between quality attributes at the commercial level. U.S. planted wheat acres are declining in the face of stiff international competition in premium product markets and demand for crop acres from biofuels. A sorting technology could allow wheat growers to better identify grain that can be directed to premium markets while also increasing consumer surplus. Our work provides a coherent methodology for evaluating the benefits for a farm-level information technology. A microeconomic optimization model of wheat farmers' segregating and commingling decisions is developed. Wheat farmers' WTP for the sorting technology is simulated using U.S. HRW and HRS wheat prices and stocks based on an estimation of a wheat protein stock demand system. Our preliminary findings from the simulation show that a typical HRW (HRS) wheat farmer's WTP for the sorting technology is about 5.6 (4.8) cents per bushel. Future work such as studying the sorting technology's impacts on wheat protein premiums or studying the sorting technology's market prospects may be done based on the analysis in

this paper.

## Appendixes

### Appendix A

In this appendix we prove Proposition 3.

*Proof.* Here we only prove the results for shape type I schedules. The same procedure applies when proving results related with shape type II schedules.

**Part A.** In this part we prove that under shape type I schedules item (i) is true. Suppose in the optimal arrangement the load is segregated into  $n \geq 2$  sub-loads with different protein concentrations. Then there must be at least one sub-load, say sub-load  $i$ , with protein concentration less than  $l_A$ . If not, then  $\mu$  would be greater than  $l_A$ , which contradicts  $\mu \leq l_A$  in item (i). Next we are going to show that the wheat farmer can increase her revenue by commingling sub-load  $i$  with any other sub-load  $j \neq i$ .

Let  $l_i$  and  $l_j$  be the protein concentration of sub-loads  $i$  and  $j$ , respectively. If  $l_j \leq l_A$ , then sub-load  $i$  and sub-load  $j$  are under the segment of the price schedule that is uniformly concave, therefore, according to Corollary 1 the farmer can always increase her revenue by commingling sub-load  $i$  and sub-load  $j$ . If  $l_j > l_A$ , then from Figure A1 we see that points  $[l_i, p(l_i)]$ ,  $A$ , and  $[l_j, p(l_j)]$  form a concave price schedule. It is easy to show that for any two points  $E$  and  $F$  on the price curve, if point  $E$  ( $F$ ) is on the left (right) of point  $A$ , then points  $E$ ,  $A$ , and  $F$  form a concave price schedule. Similarly, by Corollary 1 the farmer can always increase her revenue by commingling sub-load  $i$  and sub-load  $j$ . In sum, for shape type I schedules, when  $\mu \leq l_A$ , then no sorting is needed in the optimal arrangements.

**Part B.** Now let us show that for shape type I schedules item (ii) is true. In Step 1 we show that when  $\mu > l_A$ , then the tangent line  $CD$  defined in Definition 5 uniquely exists. Step 2 shows that in the optimal arrangement there is one and only one sub-load that has protein concentration lower than  $l_B$ . Let  $Z$  be the name of this sub-load. Step 3 shows that sub-load  $Z$  has protein concentration  $l_C$ . Step 4 shows that in sub-load  $Z$  there is no wheat with protein concentration higher than  $l_D$ . Step 5 concludes the proof.

*Step 1.* In this step we show that when  $\mu > l_A$ , then line  $CD$  defined in Definition 5 uniquely exists. Let us start from the tangent line  $AO'$ . Imagine that line  $AO'$  is rotated in a clockwise direction while the tangent point between the line and the curve  $OB$  moves rightward from point  $A$ . Let  $[l_j, p(l_j)]$  denote the coordinates of the tangent point,  $J$ . And let  $[l_k, p(l_k)]$  denote the intersection point,  $K$ . During the rotation the value of  $\int_0^{l_k} f(l)dl / \int_0^{l_k} f(l)dl$  (i.e., the mean protein concentration of the commingle of wheat with protein concentration no higher than  $l_k$ ) is decreasing and the value of  $l_j$  is increasing. At point  $B$  we have  $l_j = l_k$  and  $\int_0^{l_k} f(l)dl / \int_0^{l_k} f(l)dl < l_j$ .

When the coordinate of the tangency point is  $[l_j, p(l_j)]$ , then the slope of the tangent is  $p'(l_j)$ . Hence the equation of the tangent is  $p = p'(l_j)l + [p(l_j) - p'(l_j)l_j]$ . Then given  $l_j$ , the l-coordinate of the intersection point,  $l_k > l_j$ , can be determined by an equation system as follows

$$(A-1) \quad \begin{cases} p = p(l_k), \\ p = p'(l_j)l_k + [p(l_j) - p'(l_j)l_j], \end{cases}$$

where  $l_k > l_j$ . From equation system (A-1) we can obtain that the relationship between  $l_j$  and  $l_k$  is determined implicitly by

$$(A-2) \quad H(l_k; l_j) \equiv p(l_k) - p'(l_j)l_k - [p(l_j) - p'(l_j)l_j] = 0.$$

By implicit function theorem we have

$$\begin{aligned}
\text{(A-3)} \quad \frac{dl_k}{dl_j} &= -\frac{\partial H/\partial l_j}{\partial H/\partial l_k} \\
&= -\frac{-p''(l_j)l_k - p'(l_j) + p'(l_j) + p''(l_j)l_j}{p'(l_k) - p'(l_j)} \\
&= -\frac{-p''(l_j)l_k + p''(l_j)l_j}{p'(l_k) - p'(l_j)} \\
&= \frac{p''(l_j)(l_k - l_j)}{p'(l_k) - p'(l_j)}
\end{aligned}$$

Since curve  $OB$  is concave, we have  $p''(l_j) < 0$ . Together with  $l_k > l_j$  we have  $p''(l_j)(l_k - l_j) < 0$  in equation (A-3). Because at point  $[l_k, p(l_k)]$  the slope of curve  $O'B$  is greater than the slope of the line  $JK$ , it is true that  $p'(l_k) - p'(l_j) > 0$ . Therefore, we show  $dl_k/dl_j < 0$ .

When  $l_j = l_B$ , which means the tangency point is at point  $B$ , then we have  $l_k = l_B$  as well because point  $B$  is the inflection point. This implies that when  $l_j = l_B$  then the tangency point and the interception point coincide with point  $B$ .

Let us construct a function

$$\text{(A-4)} \quad M(l_j) = \int_0^{l_k(l_j)} f(l)dl / \int_0^{l_k(l_j)} f(l)dl - l_j,$$

where  $l_k(\cdot)$  is a function of  $l_j$  implicitly determined in equation (A-2). When  $l_j = l_A$ , then  $M(l_j) > 0$ , which is because  $\mu = \int_0^{O'} f(l)dl / \int_0^{O'} f(l)dl > l_A$ . When  $l_j = l_B$ , then  $M(l_j) < 0$ , which is because  $\int_0^{l_B} f(l)dl / \int_0^{l_B} f(l)dl < l_B$ . Therefore, according to the intermediate value theorem, there must be an  $l_C \in (l_A, l_B)$  such that  $M(l_C) = 0$ . That is  $\int_0^{l_k(l_C)} f(l)dl / \int_0^{l_k(l_C)} f(l)dl = l_C$ . This shows that when  $\mu > l_A$ , then line  $CD$  defined in Definition 5 exists.

Now we show that the line defined in Definition 5 is unique. The uniqueness will be

proved if we show  $dM(l_j)/dl_j < 0$ .

$$\begin{aligned}
\text{(A-5)} \quad \frac{dM(l_j)}{dl_j} &= \frac{f(l_k)l_k l'_k(l_j) \int_0^{l_k(l_j)} f(l)dl - f(l_k)l'_k(l_j) \int_0^{l_k(l_j)} f(l)ldl}{\left(\int_0^{l_k(l_j)} f(l)dl\right)^2} - 1 \\
&= \frac{f(l_k)l_k l'_k(l_j) \left(\int_0^{l_k(l_j)} f(l)dl\right) \left(1 - \frac{\int_0^{l_k(l_j)} f(l)ldl}{l_k \int_0^{l_k(l_j)} f(l)dl}\right)}{\left(\int_0^{l_k(l_j)} f(l)dl\right)^2} - 1 \\
&< 0.
\end{aligned}$$

The inequality in expression (A-5) holds because  $f(l_k)l_k l'_k(l_j) \left(\int_0^{l_k(l_j)} f(l)dl\right) > 0$  and  $\frac{\int_0^{l_k(l_j)} f(l)ldl}{l_k \int_0^{l_k(l_j)} f(l)dl} < 1$ .

*Step 2.* In this step we show that in the optimal arrangement there is one and only one sub-load that has mean protein concentration less than  $l_B$ . We denote this unique sub-load as  $Z$ . Suppose there are two or more sub-loads that have protein concentration less than  $l_B$ . Since curve  $OB$  is concave, according to Corollary 1 the farmer can increase her revenue by commingling these sub-loads. Therefore, having more than one sub-loads that are with protein concentration less than  $l_B$  is not optimal. If there is not any sub-load that has protein concentration lower than  $l_B$ , then there must be one sub-load, namely sub-load  $J$ , with protein concentration  $l_j \geq l_B$  that is a commingle of wheat with protein concentration  $l_i < l_B$  and wheat with protein concentration  $l_k > l_B$ . If  $l_j > l_B$ , then since curve  $O'B$  is convex, by Proposition 1 sub-load  $J$  should be completely segregated. If  $l_j = l_B$ , then for any point, say point  $E$ , with l-coordinate  $l_E$  such that  $l_B < l_E < l_k$ , we can always find a point, say point  $F$  with l-coordinate  $l_F$  such that  $l_i < l_F < l_B$ , so that points  $E$ ,  $B$ , and  $F$  form a convex price schedule (Figure A2). By Lemma 1, the sub-load with protein concentration  $l_j$  can be segregated into two smaller sub-loads. One is with protein concentration at  $l_E$  and the other one with protein concentration  $l_F$ . Again, by Proposition 1 sub-load  $J$  should be segregated. Therefore, having no sub-load whose

protein concentration is less than  $l_B$  is not optimal either.

*Step 3.* In this step we show that the protein concentration of sub-load  $Z$ ,  $l_Z$ , is equal to  $l_C$ . Here  $l_C$  is the  $l$ -coordinate of point  $C$  defined in Definition 5. Suppose in the optimal arrangement we have  $l_Z > l_C$ . Then there are two types of configuration of this unique sub-load  $Z$ . The first one is that there is some wheat with protein concentration  $l_j > l_D$  in sub-load  $Z$ ; the second one is that some wheat with protein concentration lower than  $l_Z$  is not included in sub-load  $Z$ . These two types of configuration exist because  $\int_0^{l_D} f(l)l dl / \int_0^{l_D} f(l) dl = l_C$ . Intuitively, since the mean protein concentration of wheat with protein concentration lower than  $l_D$  is  $l_C$ , then to form a sub-load with protein concentration higher than  $l_C$  one needs either to include some wheat with protein concentration higher than  $l_D$  in the sub-load or to exclude some wheat with protein concentration lower than  $l_Z$ , or both. The two types of configuration are not mutual exclusive.

Now we show the first configuration is not optimal. Suppose sub-load  $Z$  has some wheat with protein concentration  $l_j > l_D$ . Since the the mean protein concentration of this sub-load is equal to  $l_Z$ , this sub-load must have some wheat with protein concentration lower than  $l_Z$ . Draw a line that connects points  $Z$  and  $D$  (see Figure A3). Then we can always find a point, say point  $E$ , that is very close to point  $Z$  from the left side so that  $l_E > l_i$ , here  $l_E$  is the  $l$ -coordinate of point  $E$ . Points  $E$ ,  $Z$ , and  $D$  form a convex price schedule. By Lemma 1, sub-load  $Z$  can be segregated into two smaller loads, one is with protein concentration at  $l_D$ ; and the other one is with protein concentration  $l_E$ . By Corollary 1, segregating sub-load  $Z$  increases the farmer's revenue. Therefore, the first type of configuration is not optimal.

Now we show the second one is not optimal either. If some wheat with protein concentration lower than  $l_Z$  is not included in sub-load  $Z$ , then this wheat must be commingled with some wheat with protein concentration higher than  $l_B$  to form a sub-load with mean

protein concentration no less than  $l_B$ . If not, then there are at least two sub-loads that have protein concentration lower than  $l_B$ , which has been shown not optimal in Step 2. Suppose it is that wheat with protein concentration equal to  $l_i < l_Z$  is commingled with wheat with protein concentration equal to  $l_j > l_B$  to form a sub-load  $K$  with protein concentration equal to  $l_k \geq l_B$ . If  $l_k > l_B$ , then it is always beneficial to segregate sub-load  $K$  because curve  $O'B$  is convex. If  $l_k = l_B$ , then (with the same argument we made in Step 2) we can always find a point, say  $E$ , which is very close to point  $B$  from the left side, so that points  $E$ ,  $B$ , and  $[l_j, p(l_j)]$  form a convex price schedule (See Figure A4). According to Corollary 1, however, segregating this sub-load is beneficial. Therefore, the second type is not optimal either.

Now we show the unique sub-load  $Z$  cannot have  $l_Z < l_C$ . If  $l_Z < l_C$ , then there must be some wheat with protein concentration at  $l_j$  such that  $l_Z \leq l_j \leq l_D$  that is not in sub-load  $Z$ . Otherwise the mean protein concentration of sub-load  $Z$  will be  $l_C$  or higher. However, the three points,  $Z$ ,  $C$ , and  $[l_j, p(l_j)]$ , form a concave price schedule (See Figure A5). According to Corollary 1 the farmer can increase her revenue by commingling wheat in sub-load  $Z$  with wheat that has protein concentration  $l_j$ .

*Step 4.* This step shows that in sub-load  $Z$  there is no wheat with protein concentration higher than  $l_D$ . Here  $l_D$  is the 1-coordinate of point  $D$  defined in Definition 5. Suppose this is not true, then sub-load  $Z$  contains some wheat with protein concentration  $l_k > l_D$ . Therefore, there must be some wheat with protein concentration  $l_j$  such that  $l_C \leq l_j \leq l_D$  that is not in sub-load  $Z$ . This is because if all wheat with protein concentration between  $l_C$  and  $l_D$  is in sub-load  $l_Z$ , then together with some wheat with protein concentration higher than  $l_D$  being in sub-load  $l_Z$  as well, the mean protein concentration of sub-load  $Z$  must be higher than  $l_C$ . Please recall that the mean protein concentration of wheat with protein concentration less than  $l_D$  is  $l_C$ . We name the sub-load that contains wheat

with protein concentration  $l_j$  as sub-load  $J$ . By the result in Step 2 we know the mean protein concentration of sub-load  $J$  is no less than  $l_B$ . We claim that sub-load  $J$  only contains wheat with protein concentration at  $l_j$ . If sub-load  $J$  is a commingle of wheat with different protein concentrations and if its mean protein concentration is higher than  $l_B$ , then according to Corollary 1 it is profitable to segregate sub-load  $J$ . If sub-load  $J$  is a commingle of wheat with different protein concentrations and if its mean protein concentration is equal to  $l_B$ , then on the price curve we can always find two points, say  $E$  and  $F$ , such that (1)  $E$  is on the left of point  $B$  and  $F$  is on the right of point  $B$ ; and (2) points  $E$ ,  $B$ , and  $F$  form a convex price shape. According to Corollary 1, under this situation segregating sub-load  $J$  is profitable.

From sub-load  $Z$  we can separate out one unit of wheat with mean protein level  $l_j$  that is a mix of wheat with protein concentration  $l_k$  and some wheat with mean protein concentration  $l_C$ . Exchanging this unit of mix separated from sub-load  $Z$  with one unit wheat from sub-load  $J$  does not affect the mean protein concentrations of both sub-load  $Z$  and sub-load  $J$ . Therefore, the total revenue is not affected by this exchange. However, the farmer can increase her revenue by segregating the unit of mix originally from sub-load  $Z$  but now in sub-load  $J$ . One way of the segregation is to segregate the mix into two groups, one group has mean protein concentration at  $l_C$ ; the other group has mean protein concentration at  $l_D$ . The three points,  $C$ ,  $J$ , and  $D$ , form a convex price schedule (See Figure A6). Therefore, according to Corollary 1 the farmer can increase her revenue by segregating the unit of mix.

*Step 5.* We have shown in Step 1 that line  $CD$  defined in Definition 5 exists when  $\mu > l_A$ . We also have shown that there is one and only one sub-load, namely sub-load  $Z$ , that has mean protein concentration less than  $l_B$  but equal to  $l_C$  in Step 2 and Step 3. In Step 4 we showed that there is no wheat with protein concentration higher than

$l_D$  in sub-load  $Z$ , which implied that sub-load  $Z$  is a commingle of wheat with protein concentration no higher than  $l_D$ . Because wheat with protein concentration higher than  $l_D$  is under a convex price schedule and there is no commingling opportunity for such wheat, these wheat will be completely segregated according to Proposition 1. In sum, for the type I price schedules, when  $\mu > l_A$ , then in the optimal arrangements wheat with protein concentration higher than  $l_D$  should be completely segregated and the remaining wheat should be completely commingled.

□

## Appendix B

In this appendix we prove Proposition 4.

*Proof.* To optimally process one load of wheat is to explore the benefit of commingling and segregating based on the information from measuring protein concentration. Once the benefit of commingling and segregating is completely obtained, the processing reaches its optimal results. Since the farmer's goal is to find out the optimal  $q_1$ ,  $q_2$ , and  $q_3$  to maximize her revenue, the objective function  $\max_{q_i} \sum_{i=1}^3 p_i q_i$  in problem (6) and problem (7) is correct. The major work of specifying an appropriate form of programming problem for the farmer is to correctly dealing with the non-linear segregating property imposed by protein concentration distribution. In this proof we show that the constraints specified in problem (6) and problem (7) achieve this goal.

The first three constraints in problem (6) (or problem (7)) are necessary for clear reasons. The first constraint says that the weight of each sub-load cannot be negative. The second constraint says the total weight of three sub-loads sums up to one. The third constraint indicates that the protein concentration of wheat in sub-loads  $S_2$  and  $S_3$  are

$l_1$  and  $l_2$ , respectively (Lemma 4 and Lemma 7). It also says the total protein in three sub-loads is equal to  $\mu$ , the aggregate protein in the initial load.

The last constraint in problem (6) and problem (7) is the key. When  $c_2 \leq l_1$ , then by Lemma 5 and Lemma 6, we have  $F(c_1) \leq q_1 \leq F(c_2)$ . Therefore, the wheat not in sub-load  $S_1$  can be seen as a commingle of wheat with protein concentration  $l_2$  and wheat with protein concentration  $l_0$ , here  $l_0 \equiv \int_{l_{S_1}}^{c_2} f(l)dl / \int_{l_{S_1}}^{c_2} f(l)dl < l_1$ . Then by Lemma 1 we know this commingle can be segregated into two sub-loads, one with protein concentration at  $l_1$  and the other with protein concentration  $l_2$ . Given  $q_1$ , by Lemma 7 we know that this segregation is optimal. For each  $q_1 \in [F(c_1), F(c_2)]$ , the second and the third constraints in problem (6) uniquely determine the optimal  $q_2$  and  $q_3$  (i.e., optimal conditional on  $q_1$ ). Therefore, the non-linear programming problem (6) will search out the optimal  $q_1, q_2$ , and  $q_3$ . This shows that item (i) is true. The same procedure follows when proving item (ii).

When  $c_2 > l_1$  and  $\hat{l}_1 = 0$ , then according to Lemma 6 we have  $q_1^* = 0$ . Then in the optimal arrangements  $q_3$  must be maximized constrained by  $\mu_2 \geq l_1$ . By the definition of  $\hat{l}_1$  we have  $\int_{\hat{l}_1}^{c_2} f(l)dl / \int_{\hat{l}_1}^{c_2} f(l)dl \geq l_1$ . By Lemma 3 we know that  $q_3^* = 1 - F(c_2)$ . Therefore,  $q_2^* = F(c_2)$ . This concludes the proof.  $\square$

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**Table 1. Results of Regressions (11) to (12) of HRW Wheat**

variables	Regression (11)		Regression (12)	
	coefficients	t value	coefficients	t value
const.	41.78	6.17	34.43	7.21
sh <sub>2</sub>	-0.31	-1.66	-0.44	-3.21
sh <sub>3</sub>	-0.42	-5.56	-0.21	-3.84
ts	-0.19	-1.31	-0.16	-1.67
sea <sub>1</sub>	-0.75	-0.71	-1.99	-2.64
sea <sub>2</sub>	-2.08	-1.71	-2.31	-2.62
sea <sub>3</sub>	0.63	0.59	-1.48	-1.98
	F-statistic: 6.78		F-statistic: 6.15	

**Table 2. Results of Regressions (11) to (12) of HRS Wheat**

variables	Regression (11)		Regression (12)	
	coefficients	t value	coefficients	t value
const.	47.60	3.45	116.27	3.21
sh <sub>2</sub>	-0.32	-1.29	-1.38	-1.98
sh <sub>3</sub>	-0.53	-3.28	-1.54	-3.72
ts	0.24	0.53	1.48	0.84
sea <sub>1</sub>	-3.88	-2.81	-5.28	-1.19
sea <sub>2</sub>	-3.28	-2.06	-9.98	-1.99
sea <sub>3</sub>	-0.99	-0.73	3.04	0.68
	F-statistic: 4.03		F-statistic: 4.34	

**Table 3. Trial Locations and Names of Top Varieties  
of HRW Wheat in the State of Washington**

<b>Locations</b>	(1) Almira	(6) Dusty	(11) Pullman
	(2) Bickleton	(7) Finley	(12) Reardan
	(3) Connell	(8) Horse heaven	(13) Ritzville
	(4) Coulee city	(9) Lamont	(14) St andrews
	(5) Dayton	(10) Lind	(15) Walla walla
<b>Varieties</b>	(1) Agripro paladin	(7) Eddy	(12) Residence
	(2) Bauermeister (1)	(8) Estica	(13) Semper
	(3) Boundary	(9) Finley	(14) Symphony
	(4) Buchanan	(10) Hatton	(15) Wanser
	(5) Columbia – 1	(11) Quantum hybrid	(16) Weston
	(6) Declo	542	

**Table 4. Summary Statistics of HRW Wheat Testing Results**  
(Observations: 538)

	<b>Mean</b>	<b>Variance</b>	<b>Maximum</b>	<b>Minimum</b>
<b>Protein</b>	11.87%	$2.88 \times 10^{-4}$	16.6%	7.40%
<b>Yield (bu/acre)</b>	63.25	$1.06 \times 10^3$	165.90	9.90
<b>Test Weight (lb/bu)</b>	60.48	6.16	64.50	47.10

**Table 5. Results of Regression (14)**

<b>variable</b>	<b>coefficient</b>	<b>t value</b>	<b>variable</b>	<b>coefficient</b>	<b>t value</b>
<b>constant</b>	0.1153	22.6973	<b>variety10</b>	-0.0084	-1.8672
<b>year2</b>	-0.0165	-6.5310	<b>variety11</b>	-0.0033	-0.7456
<b>year3</b>	0.0038	1.3690	<b>variety12</b>	-0.0102	-1.9260
<b>year4</b>	-0.0042	-1.3678	<b>variety13</b>	-0.0082	-1.5482
<b>year5</b>	0.0137	5.4812	<b>variety14</b>	-0.0011	-0.2564
<b>year6</b>	0.0028	1.0606	<b>variety15</b>	-0.0058	-1.1062
<b>year7</b>	-0.0034	-1.2555	<b>variety16</b>	0.0016	0.3564
<b>year8</b>	0.0080	2.9421	<b>location2</b>	-0.0184	-6.3658
<b>year9</b>	-0.0007	-0.2265	<b>location3</b>	0.0088	3.5184
<b>year10</b>	0.0052	1.7054	<b>location4</b>	-0.0006	-0.1847
<b>year11</b>	0.0127	3.9615	<b>location5</b>	0.0004	0.1355
<b>year12</b>	0.0145	4.6577	<b>location6</b>	0.0067	1.2968
<b>year13</b>	0.0099	3.1529	<b>location7</b>	0.0119	3.0240
<b>variety2</b>	-0.0095	-2.2997	<b>location8</b>	0.0209	8.8342
<b>variety3</b>	-0.0079	-1.9563	<b>location9</b>	-0.0080	-2.4570
<b>variety4</b>	-0.0141	-3.2322	<b>location10</b>	0.0150	6.3996
<b>variety5</b>	0.0002	0.0295	<b>location11</b>	0.0015	0.6535
<b>variety6</b>	0.0007	0.1719	<b>location12</b>	0.0059	1.8062
<b>variety7</b>	-0.0033	-0.8035	<b>location13</b>	0.0035	1.3373
<b>variety8</b>	-0.0089	-1.9462	<b>location14</b>	-0.0105	-4.4670
<b>variety9</b>	-0.0067	-1.6678	<b>location15</b>	0.0056	2.0292

**Table 6. Coefficients of Regression (15)**

<b>variable</b>	<b>coefficient</b>	<b>t value</b>	<b>variable</b>	<b>coefficient</b>	<b>t value</b>
<b>constant</b>	0.1185	43.57	<b>location2</b>	0.0067	1.82
<b>year2</b>	0.0051	2.77	<b>location3</b>	0.0055	1.11
<b>year3</b>	0.0148	7.72	<b>location4</b>	0.0039	1.79
<b>year4</b>	0.0233	11.44	<b>location5</b>	0.0202	8.54
<b>year5</b>	0.0318	15.41	<b>location6</b>	0.0031	1.48
<b>year6</b>	0.0357	17.19	<b>location7</b>	0.0073	3.13
<b>year7</b>	0.0321	15.43	<b>location8</b>	0.0084	2.67
<b>year8</b>	0.0350	17.72	<b>location9</b>	-0.0135	-5.83
<b>year9</b>	0.0400	19.79	<b>location10</b>	0.0006	0.27
<b>year10</b>	0.0332	16.53	<b>location11</b>	0.0130	6.27
<b>year11</b>	0.0344	16.34	<b>location12</b>	0.0033	1.59
<b>year12</b>	0.0356	17.07	<b>location13</b>	0.0165	5.27
<b>year13</b>	0.0381	18.37	<b>location14</b>	0.0234	7.25
<b>variety2</b>	-0.0060	-2.72	<b>location15</b>	0.0090	3.34
<b>variety3</b>	-0.0100	-2.93	<b>location16</b>	0.0259	9.24
<b>variety4</b>	-0.0105	-3.86	<b>location17</b>	-0.0007	-0.32
<b>variety5</b>	-0.0089	-5.28	<b>location18</b>	0.0083	3.19
<b>variety6</b>	-0.0035	-1.97	<b>location19</b>	0.0107	3.56
<b>variety7</b>	-0.0062	-2.39	<b>location20</b>	-0.0008	-0.37
<b>variety8</b>	-0.0096	-5.79	<b>location21</b>	0.0058	1.21
<b>variety9</b>	0.0040	1.54	<b>location22</b>	0.0061	2.93
<b>variety10</b>	-0.0005	-0.13	<b>location23</b>	-0.0029	-1.31
<b>variety11</b>	-0.0114	-6.85	<b>location24</b>	0.0158	6.13
<b>variety12</b>	-0.0097	-4.09	<b>location25</b>	0.0144	5.04
<b>variety13</b>	-0.0080	-4.44	<b>location26</b>	0.0010	0.50
<b>variety14</b>	-0.0043	-2.48	<b>location27</b>	0.0059	1.54
<b>variety15</b>	-0.0106	-2.77	<b>location28</b>	0.0030	1.36

<b>Table 7. Results of Regression (16)</b>		
<b>variables</b>	<b>coefficient</b>	<b>t value</b>
<i>const</i>	<b>11.58</b>	<b>20.68</b>
<i>b</i>	<b>0.43</b>	<b>36.08</b>
<b>R_square: 0.94</b>		
<b>F test: 1203.5</b>		

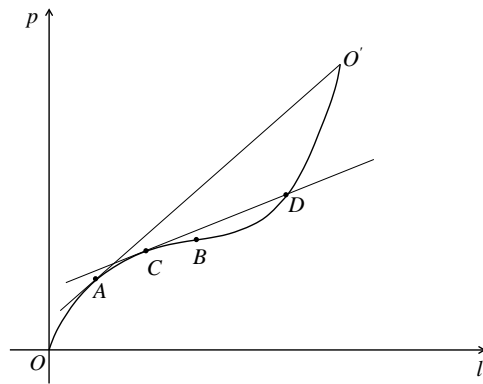


Figure 1a. Price Schedule which Turns from being Concave to Convex

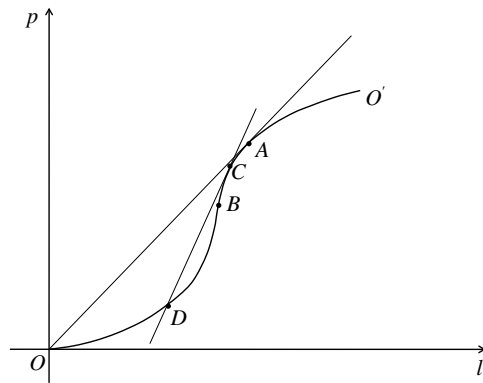


Figure 1b. Price Schedule which Turns from being Convex to Concave

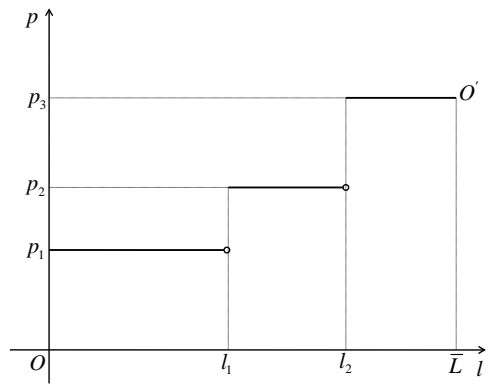


Figure 2. A Three-Step Price Schedule

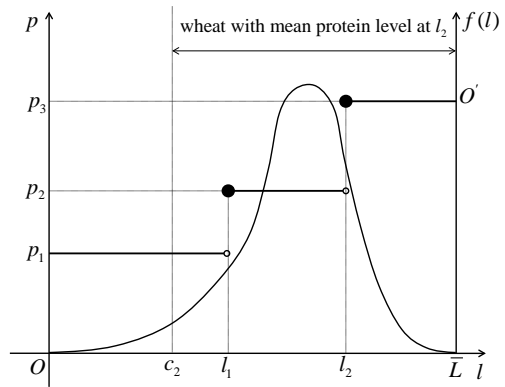


Figure 3.  $c_2 \leq l_1$

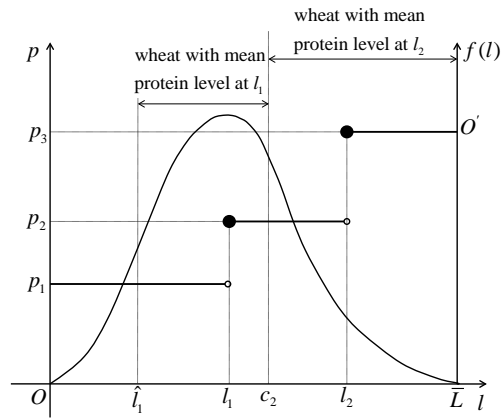


Figure 4.  $c_2 > l_1$  and  $\hat{l}_1 > 0$

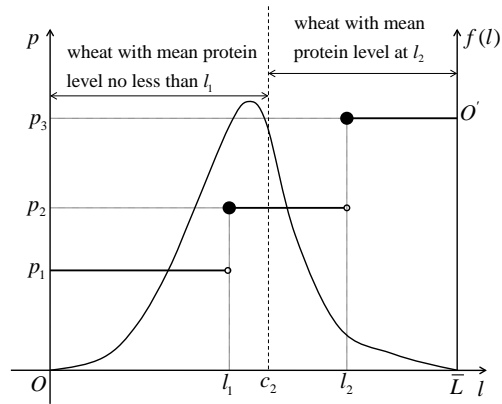


Figure 5.  $c_2 > l_1$  and  $\hat{l}_1 = 0$

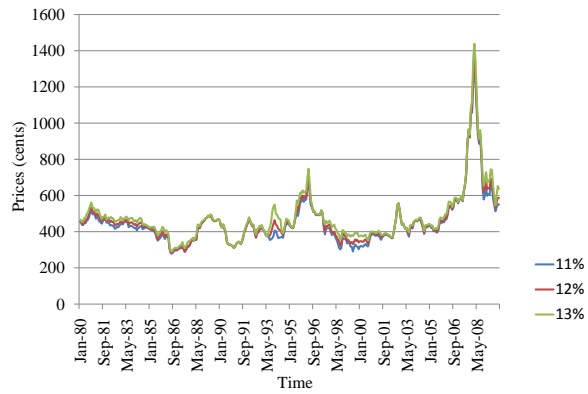


Figure 6. Prices of HRW Wheat of Protein Level 11%, 12%, and 13%

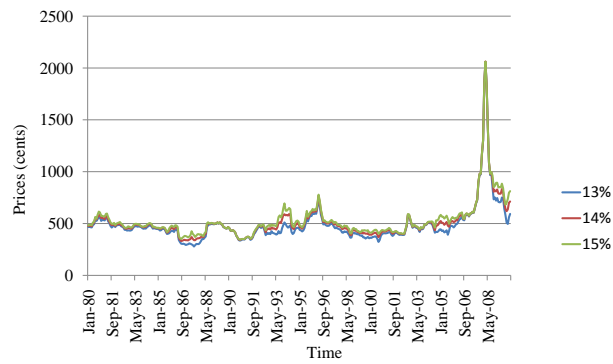


Figure 7. Prices of HRS Wheat at Protein Level 13%, 14%,

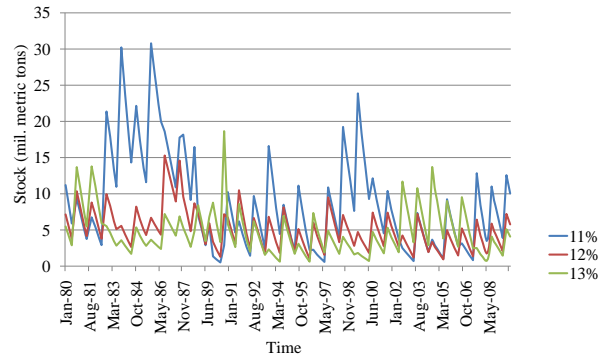


Figure 8. Monthly HRW Wheat Stocks with 11%, 12%, and 13% Protein Level

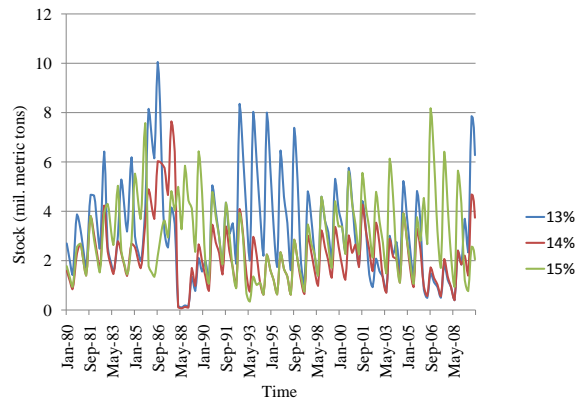


Figure 9. Monthly HRS Wheat Stocks with 13%, 14%, and 15% Protein Level

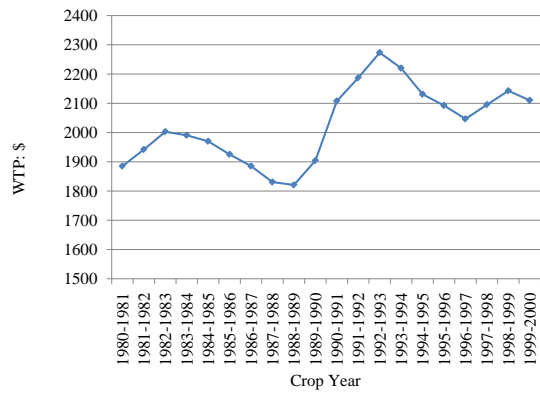


Figure 10. WTPs of HRW Wheat Farmers if the Sorting Technology Had Been Launched in a Historical Crop Year

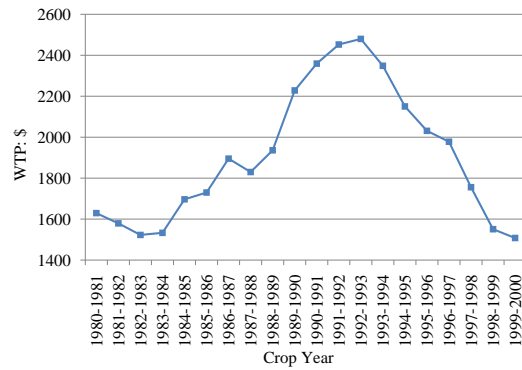


Figure 11. WTPs of HRS Wheat Farmers if the Sorting Technology Had Been Launched in a Historical Crop Year

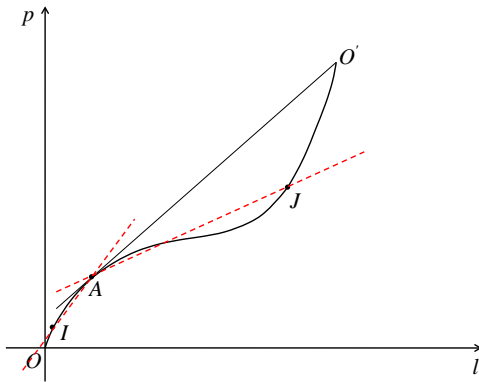


Figure A1. Points  $I$ ,  $A$ , and  $J$  Form a Concave Price Schedule

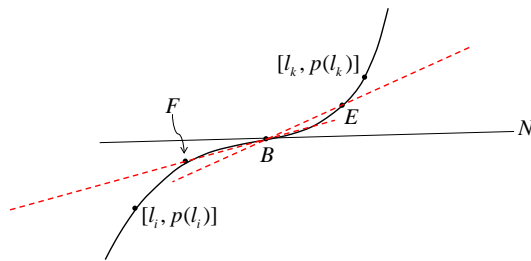


Figure A2. Points  $E$ ,  $B$ , and  $F$  Form a Convex Price Schedule

*Note:* The slope of line  $BN$  is the same as the slope of price schedule curve at point  $B$ .

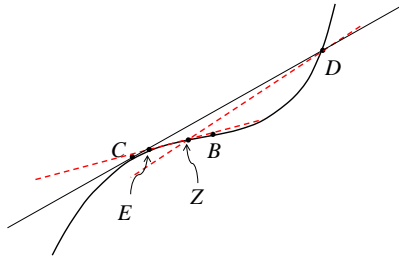


Figure A3. Points E, Z, and D Form a Convex Price Schedule

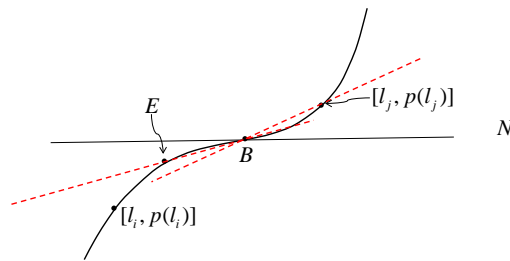


Figure A4. Points E, B, and  $[l_j, p(l_j)]$  Form a Convex Price Schedule

Note: The slope of line  $BN$  is the same as the slope of price schedule curve at point B.

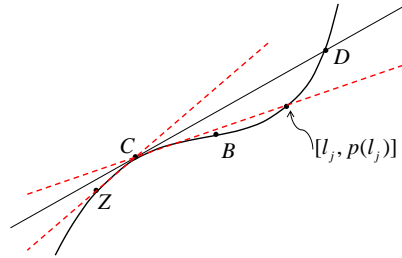


Figure A5. Points Z, C, and  $[l_j, p(l_j)]$  Form a Concave Price Schedule

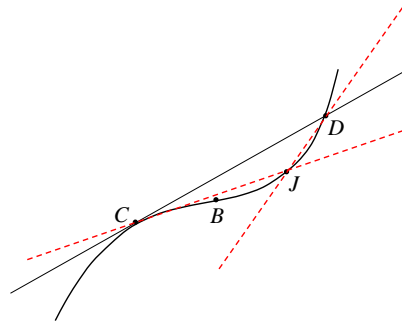


Figure A6. Points C, J, and D Form a Convex Price Schedule