How Market Power Changes in Monopoly: Using Lau's Hessian Identities

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Abstract

This research examines market power using Lau's Hessian Identity relationships based on the empirical properties of duality theory. We compare the performance of the proposed dual approach using Lau's Hessian Identity relationships with the simple traditional dual approach.

Keywords: Lau's Hessian Identity, Monte Carlo simulation, Market Power, Monopoly

Introduction

Through the "New Empirical Industrial Organization" (NEIO) literature, there are many market power studies in recent years. Following Bresnahan (1982), most NEIO studies estimate monopoly market power exertion from first-order profit maximization conditions using aggregate industry (or country) data. Several studies test for market power such as Ashenfelter and Sullivan (1987), Schroeter (1988), Azzam (1997), Sexton (2000) and Paul (2001).

In contrast, Love and Shumway (1994) suggest a nonparametric approach to test for market power exertion that does not require specifying functional forms for supply or demand. Love and Shumway (1994) extended market power tests from previous studies (Chavas and Cox 1988; Fawson and Shumway 1987; Ashenfelter and Sullivan 1987) for an input market. Love and Shumway (1994) developed a nonparametric deterministic test for monopsony market power using a normalized quadratic restricted cost function with one variable input and one input for which the firm has potential market power. Their nonparametric market power estimates are consistent with actual Lerner index and results indicate that monopsony market power decreases with factor supply elasticity. However there are exceptions where nonparametric market power estimates with technical change and shifting supply are inconsistent with actual Lerner index (Love and Shumway, 1994).

The dual approach assumes price taking behavior for a profit maximizing firm and cost minimizing firm. The unrestricted profit function contains the same economic information as the indirect cost function (Mas-Colell et al., 1995). Lau (1976) developed a general set of Hessian Identities under perfect competition that permits additional valuable information to be derived from the profit function. Lusk et al. (2002) empirically examined the relationship between the parameters of production function, unrestricted profit function and restricted profit function.

Objectives

The objective of this paper is twofold. First, the study proposes to examine market power using Lau's Hessian Identity relationships (Lau 1976). Second, the study assesses the performance of the proposed dual approach using Lau's Hessian Identity relationships comparing with the simple dual approach.

Methods and Procedures

Our approach is completed using two steps: 1) using a production function and a market demand function optimal input and output quantities are estimated under different input price regimes with output choice determining output price under monopoly power, and 2) estimate cost function and profit function using Lau's Hessian Identities, estimate the cost function from the profit function estimates.

Following Lusk et al. (2002), data used to estimate market power are simulated through Monte Carlo simulation techniques. Monte Carlo simulation techniques are used for obtaining data by simulating a statistical model that has all assumed numeric parameters. For the data generation process, we use the firm's profit maximization problem for a single output and four input production function. We assume a quadratic production function of one output-four input function as:

$$Y = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + 0.5[\alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \alpha_{33} x_3^2 + \alpha_{44} x_4^2 + 2\alpha_{12} x_1 x_2 + 2\alpha_{13} x_1 x_3 + 2\alpha_{14} x_1 x_4 + 2\alpha_{23} x_2 x_3 + 2\alpha_{24} x_2 x_4 + 2\alpha_{34} x_3 x_4]$$

where Y is the output quantity and x_i is the *i*th input quantity. We set an intercept to zero so that no output comes without any inputs. Following Lusk et al. (2002), the parameters are chosen so that economic regularity conditions were met.

Since this study's purpose is to examine market power using Lau's Hessian Identity relationships, output price P is not given. We assumed the output price P is an inverse demand function for monopoly case so that we can also simulate prices as an inverse demand function with a quadratic form of output. The inverse demand function that the monopolist faces is assumed to be:

$$P = 250 - .01Y$$

After set up the production function and the inverse demand function, we can set the firm's profit maximization problem as:

$$\max \pi = PY - \sum_{i=1}^{4} w_i x_i$$

After substituting the inverse demand function into the firm's profit maximization and rearranging the profit function is:

$$\max \pi = 250Y - 0.01Y^2 - \sum_{i=1}^{4} w_i x_i$$

where Y is the production function previously defined and w_i is the *i*th input price. The first-order conditions of the profit maximization problem for the four inputs are determined an set to zero:

$$\frac{\partial \pi}{\partial x_1} = 100\{\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + 0.5[\alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \alpha_{33} x_3^2 + \alpha_{44} x_4^2 + 2\alpha_{12} x_1 x_2 + 2\alpha_{13} x_1 x_3 + 2\alpha_{14} x_1 x_4 + 2\alpha_{23} x_2 x_3 + 2\alpha_{24} x_2 x_4 + 2\alpha_{34} x_3 x_4]\}^{-6} * \{\alpha_1 + \alpha_{11} x_1 + \alpha_{12} x_2 + \alpha_{13} x_3 + \alpha_{14} x_4\} - w_1 = 0$$

$$\frac{\partial \pi}{\partial x_2} = 100\{\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + 0.5[\alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \alpha_{33} x_3^2 + \alpha_{44} x_4^2 + 2\alpha_{12} x_1 x_2 + 2\alpha_{13} x_1 x_3 + 2\alpha_{14} x_1 x_4 + 2\alpha_{23} x_2 x_3 + 2\alpha_{24} x_2 x_4 + 2\alpha_{34} x_3 x_4]\}^{-6} * \{\alpha_2 + \alpha_{22} x_2 + \alpha_{12} x_1 + \alpha_{23} x_3 + \alpha_{24} x_4\} - w_2 = 0$$

$$\frac{\partial \pi}{\partial x_3} = 100\{\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + 0.5[\alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \alpha_{33} x_3^2 + \alpha_{44} x_4^2 + 2\alpha_{12} x_1 x_2 + 2\alpha_{13} x_1 x_3 + 2\alpha_{14} x_1 x_4 + 2\alpha_{23} x_2 x_3 + 2\alpha_{24} x_2 x_4 + 2\alpha_{34} x_3 x_4]\}^{-6} * \{\alpha_3 + \alpha_{33} x_3 + \alpha_{13} x_1 + \alpha_{23} x_2 + \alpha_{34} x_4\} - w_3 = 0$$

$$\frac{\partial \pi}{\partial x_4} = 100\{\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + 0.5[\alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \alpha_{33} x_3^2 + \alpha_{44} x_4^2 + 2\alpha_{12} x_1 x_2 + 2\alpha_{13} x_1 x_3 + 2\alpha_{14} x_1 x_4 + 2\alpha_{23} x_2 x_3 + 2\alpha_{24} x_2 x_4 + 2\alpha_{34} x_3 x_4]\}^{-6} * \{\alpha_4 + \alpha_{44} x_4 + \alpha_{14} x_1 + \alpha_{24} x_2 + \alpha_{34} x_3\} - w_4 = 0$$

Given input prices, we use SHAZAM for solving the system of four equations simultaneously. Input prices are randomly generated and firms take input prices as exogenous (a competitive input market). A normal distribution is assumed and input prices were randomly generated in SHAZAM. In this study, the output price and the output were calculated by the input prices and input quantity values that were calculated from the system of first-order conditions.

Lau's Hessian Identities

Lau (1976) provided the Hessian identities to show the equivalence of estimates from the restricted profit, unrestricted profit and production functions. We used the coefficients for the production function that Lusk et al. (2002) assumed for their estimation. Thus the true Hessian matrix is as:

$$\begin{bmatrix} \frac{\partial^2 Y}{\partial x_{1-3}^2} & \frac{\partial^2 Y}{\partial x_{1-3}\partial x_4} \\ \frac{\partial^2 Y}{\partial x_4 \partial x_{1-3}} & \frac{\partial^2 Y}{\partial x_4^2} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{bmatrix}$$

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{12} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{13} & \beta_{23} & \beta_{33} & \beta_{34} \\ \beta_{14} & \beta_{24} & \beta_{34} & \beta_{44} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{bmatrix}^{-1} = \begin{bmatrix} (\Theta_3 + \Theta_1' \Theta_2^{-1} \Theta_1) & (\Theta_2 \Theta_1)' \\ (\Theta_2 \Theta_1) & \Theta_1 \end{bmatrix}$$

where α_{ij} are the second-order derivatives of the production function, β_{ij} are the second-order derivatives of the unrestricted profit function and Θ_i are the matrix identities of the second-order derivatives of the restricted profit function defined by Lau (1976). Above direct matrices relationship between production, unrestricted profit and restricted profit functions are shown by Lau (1976). Therefore, using Lau's Hessian ideantities, "*estimates from any one of the three forms can be used to determine estimates from the other two*" (Lusk et al. 2002). Table 1 shows simulated input and output prices by Monte Carlo simulation techniques. Table 2 lists assumed coefficient values for the production function from Lusk et al. (2002). Table 3 shows given input prices for the profit maximization problem. Table 4, 5 and 6 are the estimated results for restricted, unrestricted profit functions and production function.

If we find output is monopoly, then the estimated cost function derived from the profit function and the cost function will not be equal. A test will then be constructed from that difference. We expect to find different results of market power for two approaches. This result is partly because market power using Lau's Hessian Identity relationships.

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Table 1. Simulated Input and Output Quantities by Monte Carlo Simulation Techniques

Quantity	Value
X1	0.71789
X2	1.5221
X3	0.62113
X4	1.1102
Y	127.16

Reported values are the mean input and output quantities from 5000 repetitions.

Table 2. Assumed Coefficient Values for the Production Function from Lusk et al. (2002)

Coefficient	Value
a1	20
a2	10
a3	30
a4	70
a11	-0.9
a22	-0.7
a33	-0.8
a44	-0.3
a12	0.1
a13	-0.37
a14	0.15
a23	0.2
a24	0.1
a34	0.13

Source Lusk et al. (2002)

Table 3. Input Prices for the Profit Maximization Problem from Lusk et al. (2002)

Input Price	Value
W1	100
W2	45
W3	105
W4	95

Reported values are the mean input and output quantities from 5000 repetitions.

Table 4. Estimated Results for Restricted Profit Function

	Coefficient	Std. Err.	T-Ratio
CONST	119.27	100.8	1.1832
W1	-0.76304	0.49171	-1.5518
W2	-2.1189	0.89623	-2.3643
W3	-0.81101	0.46397	-1.748
X4	-27.858	201.29	-0.1384
W11	-2.08E-03	1.15E-03	-1.812
W22	-4.19E-03	1.51E-03	-2.7745
W33	-3.29E-03	6.58E-04	-5.0088
W44A	-33.358	200.93	-0.16602
W12	-1.20E-03	7.38E-04	-1.6214
W13	2.37E-03	8.81E-04	2.694
W14A	0.21141	0.4868	0.43428
W23	2.80E-03	8.55E-04	3.2814
W24A	-1.1443	0.90192	-1.2688
W34A	0.20858	0.46129	0.45215

Table 5. Estimated Results for Unrestricted Profit Function

	Coefficient	Std. Err.	T-Ratio
CONST	130.98	0.83209	157.42
W1	-1.0089	0.064801	-15.569
W2	-0.9065	0.15375	-5.8959
W3	-1.1139	0.06462	-17.237
W4	-0.89295	0.082147	-10.87
W11	-2.32E-03	4.64E-03	-0.49955
W22	7.04E-03	1.56E-02	0.45211
W33	-7.91E-03	3.21E-03	-2.4671
W44	0.0050808	0.007004	0.72547
W12	-4.89E-03	5.40E-03	-0.9057
W13	4.10E-03	2.64E-03	1.5516
W14	0.0003967	0.003285	0.12075
W23	1.00E-02	5.16E-03	1.942
W24	-0.010429	0.009168	-1.1377
W34	-0.0001998	0.004814	-0.0415

Table 6. Estimated Results for Production Function

	Coefficient	Std. Err.	T-Ratio
X1	-201.38	14.17	-14.22
X2	-70.043	13.63	-5.138
X3	-1.0496	14.69	-7.14E-02
X4	10.592	6.9	1.535
X11	-152.83	19.57	-7.81
X22	-17.83	7.826	-2.278
X33	10.676	8.068	1.323
X44	0.113	0.9013	0.1254
X12	-46.303	9.052	-5.115
X13	-8.7084	7.352	-1.184
X14	6.4669	3.804	1.7
X23	-6.4498	5.23	-1.233
X24	0.51601	1.816	0.2841
X34	3.5169	2.428	1.449