A dynamic structural model of household migration decisions and their effects.

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1 Introduction

Studies of the relationship between migration and rural development have been implemented in a variety of places and circumstances, and have led to conflicting findings. The heterogeneity of both findings and settings is suggestive of a conditional relationship, where many different outcomes are possible given the right circumstances. This paper proposes a structural model of time and human capital allocation, where the different circumstances impact both the process of migration decisions by the household and development outcomes in the wake of these decisions. Unique panel data from Mexico is used to estimate the model and to clarify the conditions under which migration out of rural areas leads to investment and economic boom against those under which it leads to desertion and economic stagnation.

An illustration of the conflicts that exist in the literature about migration and development can be drawn by comparing the work of Stark, Yitzhaki and Taylor (1986) to that of Barham and Boucher (1998). Both of these papers use Gini decompositions to look at how the migrant remittance component of income is distributed and how changing that income source impacts the income distribution. Stark et al. study an area in rural Mexico, whereas Barham and Boucher study Bluefields Nicaragua, a port city. These regions have different kinds of production, different kinds of wage work opportunities and so in many ways, we should not be surprised that they also have different relationships with migration and migrant remittances.

The core idea of the current work is that the variations seen in households and across regions may have as much to do with the different migration outcomes seen in the literature as methodological or technical questions raised therein. In this paper, we build a structural model capable of identifying circumstances that differentiate between the two outcomes above. In order to learn from empirical circumstances, the model is estimated using the ENHRUM dataset, which is representative of rural Mexico at the national level. By using this dataset, we can simultaneously incorporate many households and villages in different areas, with different economic and social conditions. This gives us the ability to compare many different circumstances while having congruent data for each location.

2 A basic theoretical model, or the innermost doll of the migration matryoshka

In order to make a more complete and concrete investigation into the connections between migration and rural development, we need to start at a basic level. In the following, I will construct a model that considers the rural household as the manager of a limited resource, human capital. The household maximizes its utility over time by allocating human capital into different activities, and by choosing consumption and investment levels in each period.

Suppose there is a household that consumes a single commodity, c. The household can either produce the commodity itself, if it has the necessary capital, or buy the commodity at a price p in the market. This market price may or may not be subject to transaction costs. The household facing transaction costs for output sales can only earn $p - s$ for their output, where s represents the transaction costs. To simplify the exposition, we can use the term p^* to represent the price net of transaction costs, so it represents $p - s$ or p as relevant for an individual household. Production is governed by a function f, that converts human capital, H, and capital, k, into the commodity, producing x units of the commodity. The household can also utilize its human capital in the labor market to earn wages, ω. Wage earnings are also a function g of human capital invested into the labor market, $\bar{H} - H$, where the household has a total of $\bar{H}$ units of human capital to divide between wage earning and the home production activity. Consumption in the most basic model is the sum of home production and the value (in units of consumption) of wages earned. The basic form of the model is:

$$\max_{c,H} \quad U = u\left(c\right) \tag{1}$$

subject to

$$c = \frac{\omega}{p} + x \tag{2}$$

$$x = f\left(H, k\right) \tag{3}$$

$$\omega = g\left(\bar{H} - H\right) \tag{4}$$

and with appropriate substitution, collapses to:

$$\max_{H} \quad U = u\left(\frac{g\left(\bar{H} - H\right)}{p^{*}} + f\left(H, k\right)\right) \tag{5}$$

and the first order condition for an interior solution (assuming that $u'(c) > 0$ is

$$u'(c)\left[\frac{\partial f}{\partial H}\left(H, k\right) - \frac{g'\left(\bar{H} - H\right)}{p^{*}}\right] = 0 \tag{6}$$

or

$$\frac{\partial f}{\partial H}\left(H, k\right) = \frac{g'\left(\bar{H} - H\right)}{p^{*}} \tag{7}$$

The choice of the household in this basic situation will depend on the nature of the functions f and g. For example, if both are linear, whichever has the greater slope will command the entirety of the household's assets. If either were convex, we would again expect to see a corner solution. There are also many circumstances which would lead us to an interior solution. For example, if one of the functions is concave, and the other is quasi-concave, the household should pursue an interior solution.

Increasing transaction costs in this simple model will make the wage labor market less appealing. This result is, however, very conditional on how the model is set up. In this case, the transaction costs impede the conversion of money into consumption goods, and since there is only one consumption good, and that can be produced at home, the transaction costs only work in that direction. However, if there were multiple consumption goods, and one or more could not be produced at home, there could be a transaction cost that impedes the conversion of the home production into cash to purchase the other consumption goods. In a more nuanced framework like this, the impact of the transaction cost is much less straightforward. Since our goal at the moment is not too directly related to transaction costs, we will put off discussion of such a framework until a later point.

There are other aspects that could lead the household to an interior solution. If the household endowment of human capital is not entirely homogeneous, for example, we could see some interesting results. Suppose that there are two types of people, and each type is relatively more skilled at a different type of work. Since the returns to each type of human capital would differ across activities, the household would have different protocols to allocate each type of human capital. As a basic example, imagine two types of human capital, H_a and H_b, such that:

$$\frac{g'(H_a)}{p^*} > \frac{\partial f}{\partial H_a}$$

and

$$\frac{g'(H_b)}{p^*} < \frac{\partial f}{\partial H_b}$$

when evaluated at the levels of a, b, and k possessed by the household. In this case the household would assign assets of type a to wage work, while those of type b would be dedicated to household production (given appropriate assumptions about capital).

Another potential source of an interior solution would be a dynamic model with investment and a cash constraint. This would closely resemble some of the ideas that have been linked with the New Economics of Labor Migration. If the marginal product of capital in home production is high, the household will want to acquire capital. But if capital can only be acquired with cash up front, then the household will have to find some way to earn that cash before capital acquisition can take place. Depending on model assumptions (or empirical circumstances), it may be possible for the household to sell some of its output from home production to finance a desired increase in capital stock. There are some circumstances under which this might not be the optimal thing to do, for example if the household faces high transaction costs in the output market. If the transaction costs in the labor market are lower, or if they can be overcome in some way (perhaps by stationing a member of the household in a location with easy access to the labor market, or sending a member to school in order to increase returns in the labor market) it may be optimal for the household to forego current production in order to acquire cash to allow for an increased future capital stock.

In order to build this dynamic version of the model, we need a current period cash constraint, a capital market, and at least two periods. Additionally, an imperfect output market can produce the interior solution under conditions in which the marginal productivities of human capital in the two activities would not immediately suggest a movement into the wage labor market to finance capital acquisition. Suppose that the marginal product of human capital (measured in the amount of the consumption good it can either produce or buy with earned wages) is greater in home production than it is in the wage labor market. If, however, we have investment possibilities, a cash constraint, and transaction costs in the output market, it could be that the cash earned from the marginal unit of wage work is greater than that earned from the marginal unit of produce sold in the market. In other words, because of the transaction cost, the comparison between wage work and household production is different when converted into the consumption good than when converted into cash. These circumstances could drive the household into the labor market even when the marginal product of labor in home production is relatively high.

When we allow for investment in the dynamic model, we can also allow for disinvestment. This would occur if the household's optimal investment level is less than zero. There are many reasons that this could be the case. If a household has some capital holdings, but it is highly productive in the labor market, it may not wish to use that capital. It is also possible that a household could start with more capital than it can effectively use with its endowment of human capital. In the simple version of the model we are constructing here, we will allow the household to sell capital in order to increase consumption, although the option to rent out capital would be an important future development. We will also confine capital to be non-negative in all periods.

To show this mathematically, we can add a cash constraint and an equation of motion for capital that includes investment, I (and divestment should $I < 0$ be the case), and constrains capital to remain non-negative. We also include a transaction cost for selling home production in the output market, s in the model, and subscript for each period:

$$px_t + \omega_t \geq I_t + pc_t + s\left(x_t - c_t\right) \tag{8}$$

$$k_{t+1} = k_t + I_t \tag{9}$$

$$k_t \geq 0 \quad \forall t \tag{10}$$

for households that sell some of their produce in the market. For the households who do not sell in the market, the cash constraint is:

$$px_t + \omega_t \geq I_t + pc_t \tag{11}$$

In either case the non-negativity constraint on capital applies as does teh equation of motion, and the objective function is then the sum across periods of the discounted per-period utility:

$$\max_{c_t, H_t, I_t} \quad U = \sum_t e^{-rt} u\left(c_t\right) \tag{12}$$

To try to keep things simple, we can look at a solution for a two period version of this basic model. In this version, I assume that there is nothing left after period two, so that there is only investment in period one. Also, the household starts with an exogenously determined capital endowment $\bar{k}$. With the appropriate substitutions, the household faces a maximization problem with one constraint, although that constraint varies depending on whether the household sells its output in the first period. The objective function is:

$$\max_{c_1, H_1, H_2, I} \quad u(c_1) + e^{-r} u\left(f\left(H_2, \bar{k} + I\right) + \frac{g\left(\bar{H} - H_2\right)}{p^*} \right) \tag{13}$$

subject to the budget constraint:

$$(p^*)\left(f\left(H_1, \bar{k}\right) - c_1\right) + g\left(\bar{H} - H_1\right) - I \geq 0. \tag{14}$$

The solution can be characterized by three conditions derived from the derivatives of the Lagrangian. The optimal level of investment is governed by

$$e^{-r} u' \left(f\left(H_2, \bar{k} + I\right) + \frac{g\left(\bar{H} - H_2\right)}{p^*} \right) \frac{\partial f}{\partial k}\left(H_2, \bar{k} + I\right) = \frac{u'(c_1)}{p^*} \tag{15}$$

or, investment is made to set the discounted marginal utility of additional consumption in the second period times the marginal prdouct of capital equal to the marginal utility in the first period divided by price. The term on the left hand side represents the current value (discounted from the future period) in utility terms of the marginal unit of investment, converted through f into consumption. On the right is the marginal utility that is foregone by converting some consumption into cash for this last unit of investment. So, not surprisingly, we are equating the cost (foregone current period utility) to the benefit (marginal gains in discounted future utility). Assuming concavity in production (which I have done with the method of solving the problem) this also suggests that higher net prices lead to greater levels of investment.

Human capital allocation is simplified here for notational simplicity. The amount H_t is allocated to home production in period t, and the remainder of the endowment $\bar{H} - H_t$ is allocated to the labor market. Essentially, the marginal benefit, in terms of the consumption good, is equated across activities. So in the initial period, the solution is governed by the relationship:

$$\frac{\partial f}{\partial H}\left(H_1, \bar{k}\right) = \frac{g'\left(\bar{H} - H_1\right)}{p^*} \tag{16}$$

and in the subsequent period the solution rule is:

$$\frac{\partial f}{\partial H}\left(H_2, \bar{k} + I\right) = \frac{g'\left(\bar{H} - H_2\right)}{p^*} \tag{17}$$

assuming that $u'(c_t) > 0$ for all t, or that $u(c)$ is monotonically increasing in c.

From these basic conditions, and before using any kind of functional form (other than the assumptions of convexity) we have a few insights. First, the net price of the consumption commodity is an important determinant of investment. The relationship between net price and the initial endowment of capital determines whether or not the household has motivation to invest. Capital rich households will not have as much motivation, obviously, but also those who face high transaction costs (which lowers p^*) or low output prices will not have the motivation to do so.

Another important factor that can be seen here is that the cross partial of the home production function plays an important role in the allocation of human capital and its relationship with investment. If the cross partial is positive, investment will boost the use of human capital in the home production activity. However, if the cross partial is negative (i.e. if capital is a net substitute for human capital) then investment will push human capital into the wage labor market.

2.1 Some possible cases

In order to start thinking about some inter-household comparisons, we can impose a little bit of structure onto the model. As a starting point, we will assume that H enters both the home production function and the wage function linearly, and that f is concave in k. This leads to three possibilities, or three 'types' of households, holding k constant:

$$\frac{\partial f}{\partial H} > \frac{g'}{p^*} \quad \text{for all } H \tag{18}$$

$$\frac{\partial f}{\partial H} < \frac{g'}{p^*} \quad \text{for all } H \tag{19}$$

$$\frac{\partial f}{\partial H} = \frac{g'}{p^*} \quad \text{for all } H \tag{20}$$

These criteria divide the households into different types, where one type specializes in the home production activity, another type in wage work, and the third type splits its time between the two activities. For a more explicit example, we can assume a functional form for the production functions. Following the form used by Weitzman (1974) we can set up a function that allows for the coefficients on each factor to vary across households. Whereas Weitzman's variations were stochastic, ours will instead be generated by factors that differentiate households from each other. The function f can be written as

$$f = \delta + f_{Hi}H + f_{Hki}kH + f_{ki}k + \frac{1}{2}f_{kki}k^2 \quad \text{if} \quad H, k > 0 \tag{21}$$

$$= 0 \quad \text{otherwise} \tag{22}$$

where i indexes the relevant households. The idea behind this form is that the coefficient for each household is not necessarily the same as for other households. More directly from Weitzman, we can imagine that each of these subscripted terms consists of two parts, an average value and a term that is household specific. For example, we could have

$$f_{Hi} = f_H + \eta \tag{23}$$

$$\eta \sim (0, \sigma_\eta) \tag{24}$$

where η represents the household's skill level in or inclination towards labor in the home production activity, and can be described as a function of household characteristics.

The wage labor function, g, can be written

$$g = \omega + g_{Hi}\left(\bar{H} - H\right) \quad \text{if} \quad H < \bar{H} \tag{25}$$

$$= 0 \quad \text{otherwise} \tag{26}$$

where g_{Hi} is a term like f_{Hi} above that relates to the household's inclination towards or ability in wage work and can take different values for different households. From this we can derive a set of conditions which govern the choice between interior and corner solutions.

$$f_{Hi} + f_{Hki}k > \frac{g_{Hi}}{p^*} \quad \rightarrow H_t = \bar{H} \tag{27}$$

$$f_{Hi} + f_{Hki}k < \frac{g_{Hi}}{p^*} \quad \rightarrow H_t = 0 \tag{28}$$

$$f_{Hi} + f_{Hki}k = \frac{g_{Hi}}{p^*} \quad \rightarrow 0 \leq H_t \leq \bar{H} \tag{29}$$

In the first case, the marginal product of human capital in home production is high enough that the household will not benefit by switching any of its time into the labor market. The second case is one where the opposite occurs, where the labor market is the best option for the entirety of the household's time allotment. Notice that the household's capital allocation plays a role here. All else equal, low capital households will be more likely to participate in the wage labor market if f_{Hki} is positive. Assuming a linear utility function for simplicity, we can solve for the optimal level of investment from (15) above

$$I = \frac{\frac{e^r}{p^*} - f_{ki} - f_{Hki}H_2}{f_{kki}} - \bar{k}. \tag{30}$$

Assuming that the denominator is negative, we get essentially what should be expected; increasing either the net price, the expected marginal product of capital or the households ability to use capital effectively in home production will increase the optimal level of investment. Higher discount rates lead to lower levels of optimal investment. Additionally, we can see that the human capital allotment plays a role in determining the optimal level of investment. So we can, to an extent, relate investment levels to household type.

One thing to note while starting this discussion is that there are terms that determine type that impact investment only indirectly through activity choice. That is to say, two of the production function parameters that appear in the activity choice solutions do not appear in the optimal investment equation. In particular there could be two households which have identical values for all the parameters in the investment equation except for H. One household could have a higher value of f_{Hi} (relative to g_{Hi}) than the other, causing the first household to specialize in the home production activity and the second to split into both types of work. In this case, the optimal level of investment for the specialized household will be higher than for the diversified household, assuming that activity choice carries over into the second period.

On that note, we can see that it is in fact possible for a household to change their activity choice between periods. For the situation in the above paragraph, we are implicitly assuming that the level of investment for the diversified household times that household's value of f_{Hki} is not large enough to overcome the difference between the household's values of f_{Hi} and g_{Hi}. But we should note that the investment equation above is directly a function of activity choice in the second period. Activity choice in the first period is not a direct determinant of optimal investment, but it does impact investment through earnings and production, and consumption decisions. To talk about investment, then, we need to think about the pair of household activity choices. This also means that observing household activity choice in one period does not give us much information about investment.

To illustrate this, lets think of two identical households, with the exception being that one has a very small allocation of capital ($\bar{k}$) in the beginning. If we move $\bar{k}$ to the left side of the investment equation, we can define an optima level of capital in the second period

$$k^* = \frac{\frac{e^r}{p^*} - f_{ki} - f_{Hki} H_2}{f_{kki}} \tag{31}$$

which does not depend on the initial capital stock. So, with these two households that differ only in their initial capital allotment, if the f_{Hki} term or the difference in $\bar{k}$ is large enough we can have the household that originally works in the wage labor market making a larger investment than the home production household to end up at identical levels of capital in the second period, both then being specialized in the home production activity. As a counter example, we can imagine another household that is identical to the first household in our above example (the one that specializes in home production) in every way except for the f_{Hki} term. It could even be the case that this household has, in the first period, identical values for marginal product of human capital in the two activities to the values that the second household above has. So again, we have two households, one specialized in the home production activity, the other in the wage labor market, but since the source of differentiation in this case is related to the optimal capital level, the differentiation can persist over time. If the second household's value of f_{Hki} is low enough, they will continue to specialize in wage labor in the second period, and will not invest.

Relating this all to the real world, we know optimal investment is partly determined by the allocation of human capital. This allocation decision is in turn a function of the household's inclination towards the different types of work. We can see in this way that the household characteristics which determine household 'type' will affect investment. So we have to think seriously here about what characteristics of the household will affect each of these 'type' parameters. Experience in either wage work or home production would be an important determinant of ability in each of these roles, and even experience of the parents of the household head could play into this. Network effects could also be important here. For example, connections in the wage labor market may make that activity more profitable, whereas access to other people engaged in home production could help to develop skills or disseminate new knowledge about production techniques.

The demographic makeup of the household as a whole could also be a factor that helps assign a household into a type. For instance, if wage work tends to be male dominated, it might be harder for a household composed primarily of women to earn high returns in that field. Alternatively, there are probably some types of wage work that are traditionally female dominated, and so the opposite could be true. This also introduces a potential interaction between the demographic characteristics of the household with some characteristics of the regional labor market.

The first step is to define the different 'types' of households that can exist, and examine the paramter relationships that make them. To classify these different cases we will use the activity choices in each period. The model includes many variables, and so making statements on how these all relate to each other in each case can be somewhat cloudy. To facilitate this, we will discuss the levels of initial and optimal capital in each case and how these capital levels relate to the other model parameters, as well as how they relate to each other, or, what investment is like in each case. Each of these cases represents a 'type' of household, or a set of activity choices across the two periods.

2.1.1 Households that specialize in home production in both periods

Here we have a couple of possibilities. The first we can call the 'high capital' case. It involves the household having an initial endowment of capital large enough that the capital-human capital interaction term can overwhelm the households wage work abilities and push the household into the home production activity. The other would be a situation involving the relative magnitudes of human capital productivity in either activity. In the linear model as written at this point, the household will never have a motivation to switch out of home production specialization. The possible exception to this is that if capital is strictly a substitute for labor ($f_{Hki} < 0$), then the acquisition of capital may lower the productivity of human capital in home production and make a move into the wage labor market beneficial. However, we will consider this to be an extreme case.

The question of investment is somewhat ambiguous in this case. Levels of investment depend on initial capital stock as well as on the parameters of the production function, prices and discount rates. However, since the f_{Hi} and g_{Hi} terms do not appear in the investment equation, the level of investment is not directly related to initial period activity choice. We can also note the fact that households that specialize in the wage labor market in the second period will have no incentive to invest. Another fact to be noted: for two households that fall into this category and that differ only in their capital endowment, the household with the smaller capital endowment will have a larger investment. Both households will move to the same amount of capital in the second period.

Mathematically, for any household that specializes in home production in period t, the following must be true:

$$k_t \geq \frac{1}{f_{Hki}} \left(\frac{g_{Hi}}{p^*} - f_{Hi} \right). \tag{32}$$

So in this case we know that both $\bar{k}$ and k^* satisfy the above equation. As discussed above, a particularly low value for g_{Hi}, $\left(\frac{g_{Hi}}{p^*} \leq f_{Hi} \quad \text{for example} \right)$ can guarantee this regardless of capital allotment. We can also see that the size of the capital-human capital interaction term adjusts the magnitude of the capital stock necessary to keep the household away from wage work.

2.2 Households that specialize in wage work in both periods

In this situation we can imagine two possibilities again. One is that the household has some sort of skill set that makes productivity in the wage labor market particularly high. The other is that capital in the home production activity is not very productive. What either of these really mean in terms of the model is that $\frac{g'(H)}{p^*}$ is greater than $f_{Hi} + f_{Hki}k$, and that adding additional k, even up to k^*, is not enough to change this relationship. The difference between the two cases, mathematically, would be that in one case g_{Hi} is particularly high, and in the other f_{Hki} is particularly low.

This is the case in which we know that the household does not have investment incentives. Mathematically, this means that no amount of capital raises the productivity of human capital in the home production activity above the level of productivity that human capital exhibits in the wage labor market. We can write this as:

$$f_{Hi} + f_{Hki}k^* < \frac{g_{Hi}}{p^*} \tag{33}$$

or

$$k^* = \frac{\frac{e^r}{p^*} - f_{ki} - f_{Hki}\bar{H}}{f_{kki}} < \frac{1}{f_{Hki}}\left(\frac{g_{Hi}}{p^*} - f_{Hi}\right). \tag{34}$$

Investment in this case will be less than zero. Since the household will not use their capital, it will be advantageous to sell it off in order to increase consumption. So we can expect that

$$I = -\bar{k}. \tag{35}$$

2.2.1 Households that don't specialize in either period

This case will only occur in the linear model if the household starts with a capital allotment equal to k^*, and that allotment equalizes human capital productivity in the home production activity to that in the wage labor market. If the household starts with a capital allotment smaller than this, it will be beneficial to increase investment, which will increase the marginal product of labor in the home production activity, and cause the household, in this linear model, to specialize in home production. In a model where the marginal product of labor in either activity is dependent on the amount of labor applied this case will be more interesting. However in the linear case we have built here, this pair of choices results from optimal investment being zero and optimal human capital allotment being the same in each period.

2.2.2 Households that move from labor market specialization into mixed activities

This basically would involve a household acquiring enough capital through investment to raise the marginal product of labor in the home production activity to be equal to that in the wage labor market. It also implies that the peak of human capital productivity (when the household invests to reach k^*) in the home production activity is equal human capital productivity in the wage labor market. Households in this category start with either low capital levels, or else relatively low skill levels in the home production activity (when compared to the same households skill level in the wage labor market).

Investment in this case may be high for those households that suffer from low capital endowments in the beginning. However, some households may only need small investments to move to their optimal level of capital, depending on both their original allotment and on the parameters of the households home production function. The parameters related to capital are particularly important in determining optimal investment levels in all cases, and this is no different. In terms relative to the other possible activity choice combinations, however, those that end in mixed activities are related with lower parameter values (f_{ki}, f_{kki} and f_{Hki}) for the home production function when compared to scenarios that end in home production specialization, and therefore lower optimal investment levels (all else equal).

As with most of the cases, we can assign some conditions to capital levels in each period here. Specifically, for this case to occur we have a condition on initial capital levels that leads to the labor market specialization:

$$\bar{k} < \frac{1}{f_{Hki}} \left(\frac{g_{Hi}}{p^*} - f_{Hi} \right) \tag{36}$$

and another condition on the optimal level that is very similar:

$$k^* > \frac{1}{f_{Hki}} \left(\frac{g_{Hi}}{p^*} - f_{Hi} \right). \tag{37}$$

These conditions illustrate the lack of definitiveness that we have to talk about investment levels in this case. The difference between capital in the two periods need only be marginally greater than zero, but can be quite large. The level of welfare improvement that the household sees from the switch will of course be related to investment, at least to the degree that the improvement must be worth the utility foregone to make the investment.

2.2.3 Households that move from Mixed activities into home production specialization

Through investment, these households are able to raise the marginal product of labor in the home production activity above the level of that in the wage labor market. This is associated with any level of investment, really, as the household starts with an equal marginal product of human capital in the two markets. Increasing the productivity of human capital in the home production activity even marginally will make transferring all human capital into that activity optimal in this linear model.

2.2.4 Households that move from the labor market to home production specialization

Moving from one specialization to another requires that investment has been made to a high enough degree to make home production in the second period more advantageous than wage work, which dominates in the first period with only the initial capital endowment available for home production. We can show these conditions mathematically as follows:

$$\bar{k} \leq \frac{1}{f_{Hki}} \left(\frac{g_{Hi}}{p^*} - f_{Hi} \right) \tag{38}$$

$$k^* = \frac{\frac{e^r}{p^*} - f_{ki} - f_{Hki}\bar{H}}{f_{kki}} \geq \frac{1}{f_{Hki}} \left(\frac{g_{Hi}}{p^*} - f_{Hi} \right). \tag{39}$$

The original dedication to the wage labor market market could come from a particularly high level of skill in that market (a high value for g_{Hi}) or from a particularly low level of capital in the first period. Either one of these could lead to high investment levels. If the household exhibits a high level of skill in the wage labor market, movement into the home production activity would only come after overwhelming that skill level. This would require either a lot of investment or very high values for the capital related parameters of the home production function, which at the least imply a very high optimal capital level.

2.3 General Discussion

One change that we would not see with this model is a household that moves from home production specialization to anything other than home production specialization. Basically, this is a consequence of some of the model assumptions, but mostly of the fact that we have not installed any method to increase productivity in the wage labor market. So if home production is advantageous in the first period, there is no way that it would be less advantageous in the second.

It is hard to reach clear conclusions from the discussion above. One thing that we can see is that in this model, each household must be examined individually. Average values of any of the parameters across a village or community will not be at all informative, as these averages values do not inform us of the relative magnitude of individual parameters in any single household. The model above is built on the assumption that the differences between individual households are important, and so use of the model for empirical analysis will depend on data that includes this information. An interesting next step that might follow the analysis of individual households by their unique characteristics is to look at how the agglomeration of different types of households into a village results in different sets of decisions and effects regarding migration.

One thing that we can say from the model is that there is no group of households which, when observed in the first period and classified according to activity choice, can be said for sure to have no investment incentive. Households exhibiting any of the activity choices may have some investment incentive. Only if the household already holds the optimal level of capital, or if the households ability in the wage labor market is higher than the households ability in home production with the optimal level of capital will the household have no reason to invest.

2.4 Expansion

This is a good place to start thinking about the limitations of the model that we have built so far. By doing this we can start to address the possibilities for expanding the simple model presented above into one that can apply to many real world situations. So far we have assumed that all households start off with capital, or that those that start with zero capital have no activity choices and can only access the wage labor market. In order to address this shortfall, as well as to get into the heart of the problem that we have set in front of us here we could add a third activity, migration. To do so, we would essentially set up another wage labor market, with different parameters and a cost to entry that could vary by households.

We have mentioned above the possibility of a capital rental market, which would be an important addition. Another change to the nature of capital could involve adding multiple types. These types could be separated by how they interact with human capital. So a household could invest in capital that is either a complement or a substitute to its application of labor.

2.5 Adding concavity in f with respect to human capital

One of the most basic and probably important ways to expand the model is to modify the home production function to be concave with respect to human capital. We commonly see situations where households participate in both home production and wage labor, and so it is desirable to have a model that provides ways for this to happen. Mathematically, we need only add an additional squared human capital term to f to make this happen:

$$f = \delta + f_{Hi}H + \frac{1}{2}f_{HHi}H^2 + f_{Hki}kH + f_{ki}k + \frac{1}{2}f_{kki}k^2 \quad \text{if} \quad H, k > 0 \tag{40}$$

$$= 0 \quad \text{otherwise} \tag{41}$$

This changes the partial derivative with respect to human capital as well:

$$\frac{\partial f}{\partial H} = f_{Hi} + f_{HHi}H + f_{Hki}k \tag{42}$$

Since the partial derivative now depends on H, there are multiple points for many households that lead to an interior condition. Of course the corner solutions are still possibilities, but they are more limited now. The corner solution where the household specializes in wage labor is actually identical, but the corner solution where the household specializes in the home produciton activity is more restricted. Specifically, for wage labor specialization it must be true that

$$f_{Hi} + f_{Hki}k < \frac{g_{Hi}}{p^*} \tag{43}$$

where we have evaluated the partial of f with respect to H when H is equal to zero. Given the nature of the function, this is the highest value for $\frac{\partial f}{\partial H}$, so if it is lower than the right hand side at that point, it will also be lower for all other points. The corner solution in the direction is a little more complicated. Since we have a declining marginal product of human capital in the home production activity, if it exhibits values above the mp of human capital in the wage labor market at low levels of H, it must eventually, as H increases, fall to a point where these two are equal. However, since we have a limited endowment of human capital, it is possible that the value of human capital that would set the two marginal productivities equal is greater than the level of human capital that the household has access to. We can call the point at which the the human capital productivity is equal in the two activities $\tilde{H}$ and solve for it, both with the inital capital endowment (fairly straightforward) and at the optimized, post investment capital level. In the first period, we find:

$$\tilde{H}_1 = \frac{\frac{g_{Hi}}{p^*} - f_{Hi} - f_{Hki}\bar{k}}{f_{HHi}} \tag{44}$$

We can substitute our optimal k^* in for $\bar{k}$ and solve for the value in the second period

$$\tilde{H}_2 = \frac{\left(\frac{g_{Hi}}{p^*} - f_{Hi}\right) f_{kki} - f_{Hki}\left(\frac{e^r}{p^*} - f_{ki}\right)}{f_{kki}f_{HHi} - f_{Hki}^2} \tag{45}$$

To put this together a little bit more clearly, we can turn to the types of households that we might see in this new formulation and the conditions that each must exhibit.

2.5.1 Households that specialize in home production in both periods

In this new situation we have a few things that can be said about this category of households. Basically, the household's allotment of human capital is not large enough to drive down human capital productivity in the home production activity to the level of human capital productivity in the wage market. So the household starts with

$$f_{Hi} + f_{Hki}\bar{k} > \frac{g_{Hi}}{p^*} \tag{46}$$

and

$$\bar{H} < \frac{\left(\frac{g_{Hi}}{p^*} - f_{Hi}\right) f_{kki} - f_{Hki}\left(\frac{e^r}{p^*} - f_{ki}\right)}{f_{kki}f_{HHi} - f_{Hki}^2} \tag{47}$$

Under these conditions the household will never find it profitable to switch into the wage labor market. Investment in this case depends as above on the initial level of capital. The optimal level of capital (k^*) is identical to the similar case above.

2.5.2 Households that specialize in wage labor in both periods

As stated above, the conditions that must hold for households in this category are the same as they were in the linear case. Investment will be negative, as the household sells off their capital to increase consumption.

2.5.3 Households that do not specialize in either period

In the linear model this case borders on a novelty, but in the new formulation there is a lot more to be said. Since the marginal product of human capital in home production now depends on the level of human capital used in home production this case can occur under a broader set of circumstances. The household must exhibit a condition that pushes them away from labor market specialization, basically that the mp of human capital starts higher in the home production activity than in wage labor. Additionally, the households allotment of human capital must be large enough to move some of that resource into the wage labor market, or:

$$\bar{H} > \frac{\left(\frac{g_{Hi}}{p^*} - f_{Hi}\right) f_{kki} - f_{Hki}\left(\frac{e^r}{p^*} - f_{ki}\right)}{f_{kki} f_{HHi} - f_{Hki}^2} \tag{48}$$

We can also calculate the optimal level of capital for a mixed activity household, where $H_2 = \tilde{H}_2$:

$$k^* = \frac{f_{HHi}\left(\frac{e^r}{p^*} - f_{ki}\right) - f_{Hki}\left(\frac{g_{Hi}}{p^*} - f_{ki}\right)}{f_{kki} f_{HHi} - f_{Hki}^2} \tag{49}$$

At this point we can think about how $\tilde{H}_1$ and $\tilde{H}_2$ compare for different households. While the two expressions written above may seem difficult to compare, we should remember that $\tilde{H}_2$ is essentially the same as $\tilde{H}_1$ but with the optimal capital level k^* substituted in for $\bar{k}$. So the relationship between labor use in the home production activity across periods is directly derived from the relationship between levels of capital used in the home production activity across periods. If the physical capital-human capital interaction term (f_{Hki}) is positive, situations where the household increases the capital level in the second period through investment will also lead to an increase in the human capital applied to the home production activity in the second period compared to the first.

The biggest increases in $\tilde{H}$ across periods will be associated with low values for $\bar{k}$ and high values for f_{Hki}. They will also make the biggest difference to larger households, since smaller households may switch from diversified activities to specialization in the home production activity as $\tilde{H}$ becomes greater than $\bar{H}$ after investment. Both of these parameters are likely to exhibit high levels of regional correlation. In particular, $fHki$ will be dependent on the character of the local home production process. For example, if this is an agricultural process (as in our empirical example) then the local crop mix can make a big difference in how physical and human capital interact in the process. Other local characteristics that could impact this include geographic factors and climate conditions. Another point of interest is that the impact of net price (and therefore transaction costs) is an empirical matter, and is enters $\tilde{H}_2$ in two places with opposite signs.

2.6 Including migration as an explicit activity

At this point, it is appropriate to include a third activity, to specifically represent migration in the model. This activity will be similar to the wage labor market, but will include an up front cost that the household must bear to enter the migrant labor market. The f and g functions remain the same, but we introduce a new function, $m(H)$ that will represent the earnings of the migrants. We also introduce the costs of migration, ν, and modify the price of consumption to reflect the fact that some members of the household are in a different, presumably more expensive location. This new price reflects the fact that the household budget is spent on consumption goods in both the home village and in the migrant destination. The final new term that we introduce is $\hat{H}$, which will be the portion of $\bar{H}$ that is left in the village, so that if the household sends no migrants, $\hat{H} = \bar{H}$. Costs of migration, ν_i, will be a function of several things, including characteristics of the destination, of the household, the number of persons migrating, the number of trips they take and possibly other macro factors. We can lump the factors other than human capital together into a vector called z, and write $\nu_i = \nu\left(z, \bar{H} - \hat{H}\right)$. Our new price of consumption, p^m, is a weighted average of the price in the village and the price in the migrant destination:

$$p^m = \frac{(p^*)\,\hat{H} + p'\left(\bar{H} - \hat{H}\right)}{\bar{H}} \tag{50}$$

where p^* is the price of consumption in the village and p' is the price of consumption in the migrant destination. Our function for migrant earnings is a function of the portion of the household that migrates, and other wise resembles the g function:

$$m = \mu + m_{Hi}\left(\bar{H} - \hat{H}\right) \tag{51}$$

In this initial, simple version, we will consider migration to be a one-time, one way decision. When migrants leave, they will be excluded from the future labor pool for local activities. The two period model will have the following budget constraints:

$$\nu\left(z, \bar{H} - \hat{H}\right) + p^m c_1 \leq p^* f\left(H_1, \bar{k}\right) + g\left(\hat{H} - H_1\right) + m\left(\bar{H} - \hat{H}\right) \tag{52}$$

$$p^m c_2 \leq p^* f\left(H_2, \bar{k} + I\right) + g\left(\hat{H} - H_2\right) + m\left(\bar{H} - \hat{H}\right) \tag{53}$$

Assuming that the budget constraint is satisfied with equality, we can derive terms for c_1 and c_2 and substitute these into the objective function:

$$\begin{aligned}\max_{H_1, H_2, \hat{H}, I} \; & u\left(\frac{p^*}{p^m} f\left(H_1, \bar{k}\right) + \frac{1}{p^m}\left(g\left(\hat{H} - H_1\right) + m\left(\bar{H} - \hat{H}\right) - I - \nu\left(z, \bar{H} - \hat{H}\right)\right)\right) \\ & + e^{-r} u\left(\frac{p^*}{p^m} f\left(H_2, \bar{k} + I\right) + \frac{1}{p^m}\left(g\left(\hat{H} - H_2\right) + m\left(\bar{H} - \hat{H}\right)\right)\right)\end{aligned} \tag{54}$$

2.6.1 Interior Solutions

Many of the first order conditions (namely, those with respect to H_1, H_2, and I) for this new problem are actually identical to those from the previous set up. However the first order condition with respect to our new choice variable, $\hat{H}$, shows us something interesting. Since this can be illustrated without functional form, we will do it that way for simplicity. The basic condition is:

$$\begin{aligned} u'(c_1)\left(\frac{p'-p^*}{(p^m)^2\bar{H}}\left(\frac{\partial g}{\partial H}\left(\hat{H}-H_1\right)-\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right)+\frac{\partial \nu}{\partial \hat{H}}\right)\right) \\ +e^{-r}u'(c_2)\left(\frac{p'-p^*}{(p^m)^2\bar{H}}\left(\frac{\partial g}{\partial H}\left(\hat{H}-H_2\right)-\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right)\right)\right)=0 \end{aligned} \tag{55}$$

which is a bit cluttered. If we continue to assume that marginal utility is always positive, and that the price of consumption changes with migration, we can derive two possible conditions from this. The first is a set of conditions that must be true

$$\begin{aligned} \frac{\partial g}{\partial H}\left(\hat{H}-H_1\right)&=\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right)-\frac{\partial \nu}{\partial \hat{H}} \\ \frac{\partial g}{\partial H}\left(\hat{H}-H_2\right)&=\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) \end{aligned} \tag{56}$$

is not actually feasible in the model that we have formulated as it would require a non-linear form for either g, m, or both, or else a zero value for the partial derivative of ν. With our functional forms above, it would require

$$g_{Hi}=m_{Hi}-\frac{\partial \nu}{\partial \hat{H}}$$

and

$$g_{Hi}=m_{Hi} \tag{57}$$

$$\tag{58}$$

which cannot both be true. However, if the household does not pursue a strict interior solution, this condition can actually change slightly so that the partial of f replaces that of g in the second period for households that abandon the wage market after the initial period. Holding off on that for a moment, we have an alternate condition:

$$\frac{u'(c_1)}{e^{-r}u'(c_2)} = \frac{\frac{\partial m}{\partial H}\left(\bar{H} - \hat{H}\right) - \frac{\partial g}{\partial H}\left(\hat{H} - H_2\right)}{\frac{\partial g}{\partial H}\left(\hat{H} - H_1\right) - \frac{\partial m}{\partial H}\left(\bar{H} - \hat{H}\right) + \frac{\partial \nu}{\partial \hat{H}}} \tag{59}$$

or, with our functional form:

$$\frac{u'(c_1)}{e^{-r}u'(c_2)} = \frac{m_{Hi} - g_{Hi}}{g_{Hi} - m_{Hi} + \frac{\partial \nu}{\partial \hat{H}}} \tag{60}$$

which essentially sets the marginal rate of (intertemporal) substitution equal to the price ratio. It may not be apparent at first glance that the right hand side of this expression is a price ratio, but if we break it apart we can see that it has both the cost of migrating in the first period and the cost of not having migrated in the second period. The individual that migrates in the first period gives up the wages that would be earned in the local labor market (which are also equal to the value of marginal product of labor in the home production activity) plus the cost of migration, minus the wages earned in the migrant labor market:

$$\frac{\partial g}{\partial H}\left(\hat{H} - H_1\right) - \frac{\partial m}{\partial H}\left(\bar{H} - \hat{H}\right) + \frac{\partial \nu}{\partial \hat{H}} \tag{61}$$

and the individual that does not migrate gives up the second period benefits, or the additional wages that would be earned in the migrant labor market over those in the home wage market, which again has a very specific relationship with the marginal product of labor in home production:

$$\frac{\partial m}{\partial H}\left(\bar{H} - \hat{H}\right) - \frac{\partial g}{\partial H}\left(\hat{H} - H_2\right) \tag{62}$$

So what the right hand side of our transformed first order condition shows us is the relative size of the gains from migration, for situations where the gains accumulate in the second period. Households that send out a migrant incur a cost in the first period but not in the subsequent one. This condition describes a possible relationship to us that will allow us to see a strict interior solution to the problem in both periods. If the relative size of the second period gains from migration, compared to the first period costs, is equal to the marginal rate of intertemporal substitution, then this strict interior situation is possible. If there is no point where this relationship holds, then we do not expect to see a household participate in both wage labor and migration for both periods.

If we add in the functional forms that we have specified we can get another picture of these conditions. Our first set of conditions translates to:

$$g_{Hi} = m_{Hi} \tag{63}$$

$$g_{Hi} = m_{Hi} - \frac{\partial \nu}{\partial \hat{H}} \tag{64}$$

which can only both be true if the partial derivative of the ν function is zero, or if migration costs do not change with the number of people that are migrating. The more interesting condition for our model is:

$$g_{Hi} = m_{Hi} - \frac{\partial \nu}{\partial \hat{H}} \frac{u'(c_1)}{e^{-r} u'(c_2) + u'(c_1)} \tag{65}$$

This tells us that in order for a household to participate in both of these activities (migration and the local wage labor market) the difference between the gains made in either of the activities must be equal to the marginal cost of migration for one more unit of human capital times a ratio that represents the relative size of the first period level of marginal utility compared to that in the second period.

2.6.2 Corner solutions and partial corner solutions

What we have written above deals with a strict interior solution. There are several other possibilities, including corner solutions where the household only participates in one activity, and partial corner solutions where the household participates in more than one but not in all activities in all periods. A subset of these is the set of solutions in which the household does not choose to migrate. The household in this case will set $\tilde{H}$ equal to $\bar{H}$ and the above discussion of the model without an explicit migration activity will apply. That is to say, all solutions of the earlier model are a special subset of this version of the model. The other possibilities follow.

2.6.3 Households that switch from all activities in the first period to only home production and migration in the second

Households that land in the strict interior solution in the first period do not necessarily stay there. In the model as built so far, they may have an optimal level of investment that increases the marginal productivity of human capital in the home production activity above the level of marginal productivity in the wage labor market. Assuming that marginal utility is strictly positive, we have the familiar condition determining the split between home production and wage labor in the first period:

$$\frac{\partial f}{\partial H}\left(H_1, \bar{k}\right) = \frac{1}{p^m}\frac{\partial g}{\partial H}\left(\bar{H} - H_1\right) \tag{66}$$

or

$$f_{Hi} + f_{HHi}H_1 + f_{Hki}\bar{k} = \frac{1}{p^m}g_{Hi} \tag{67}$$

The household will abandon the wage labor market if the following is true:

$$\frac{\partial f}{\partial H}\left(\hat{H}, \bar{k} + I\right) > \frac{1}{p^m}\frac{\partial g}{\partial H}\left(H'\right) \qquad \forall H' \in \left[0, \hat{H}\right] \tag{68}$$

or, using our functional forms,

$$f_{Hi} + f_{HHi}\hat{H} + f_{Hki}k^* > \frac{1}{p^m} g_{Hi} \tag{69}$$

The final condition we have here is the one that determines migration. In this case, we can set H_2 equal to $\hat{H}$, and then take the first order condition with respect to $\hat{H}$. One possibility gives us a relationship between marginal productivity tradeoffs and marginal utilities across the two periods, which gives us:

$$\frac{u'(c_1)}{e^{-r}u'(c_2)} = \frac{\frac{1}{p^m}\frac{\partial m}{\partial H} - \frac{\partial f}{\partial H}\left(\hat{H}, \bar{k} + I\right)}{\frac{1}{p^m}\left[\frac{\partial g}{\partial H}\left(\hat{H} - H_1\right) - \frac{\partial m}{\partial H}\left(\bar{H} - \hat{H}\right) + \frac{\partial \nu}{\partial H}\left(\bar{H} - \hat{H}\right)\right]} \tag{70}$$

Using the functional forms adopted above gives us

$$\frac{u'(c_1)}{e^{-r}u'(c_2)} = \frac{\frac{1}{p^m}m_{Hi} - \left(f_{Hi} + f_{HHi}\hat{H} + f_{Hki}k^*\right)}{\frac{1}{p^m}\left[g_{Hi} - m_{Hi} + \frac{\partial \nu}{\partial H}\right]} \tag{71}$$

This is basically the same as our interior solution condition, with the exception that in the $\hat{H}$ condition, for the second period we have substituted the marginal productivity in the home production activity for that in the wage labor market. The other possibility gives us a relationship between the marginal productivities of the relevant activities in each period, in which case both of the following must be true:

$$\frac{\partial g}{\partial H}\left(\hat{H} - H_1\right) = \frac{\partial m}{\partial H}\left(\bar{H} - \hat{H}\right) - \frac{\partial \nu}{\partial H}\left(\bar{H} - \hat{H}\right)$$

and

$$\frac{\partial f}{\partial H}\left(\hat{H}, \bar{k} + I\right) = \frac{1}{p^m}\frac{\partial m}{\partial H}\left(\bar{H} - \hat{H}\right) \tag{72}$$

This possibility is much different in this case than in the strict interior solution, as we do not require the marginal productivities in either migration or the wage labor market to be variable. This could work with the set of functional forms proposed above, and in that case looks like

$$\begin{aligned} g_{Hi} &= m_{Hi} - \frac{\partial \nu}{\partial H} \\ f_{Hi} + f_{HHi}\hat{H} + f_{Hki}k^* &= \frac{1}{p^m} m_{Hi} \end{aligned} \tag{73}$$

2.6.4 Households that participate in the wage labor market in the first period, home production in the second, and migration in both

This is not much different than the case right above, except that in the first period the household sees bigger returns from using all human capital in the wage market. Investment still drives that labor into home production in the second period. Mechanically, we can say a few things about these households. In order to keep all non-migrant human capital in the wage labor market, the following must be true:

$$\begin{gathered} \frac{1}{p^m}\frac{\partial g}{\partial H}\left(\hat{H}\right) > \frac{\partial f}{\partial H}\left(H', \bar{k}\right) \\ \frac{\partial f}{\partial H}\left(\hat{H}, \bar{k} + I\right) \geq \frac{1}{p^m}\frac{\partial g}{\partial H}\left(H'\right) \\ \forall H' \in [0, \hat{H}] \end{gathered} \tag{74}$$

which says that, given the initial level of physical capital, human capital is always more productive in the wage labor market. After investment, however, human capital is always more productive in the home production activity. The equivalent with the functional forms we are proposing is:

$$\begin{aligned} \frac{1}{p^m} g_{Hi} &\geq f_{Hi} + f_{HHi}H' + f_{Hki}\bar{k} \\ f_{Hi} + f_{HHi}\hat{H} + f_{Hki}k^* &\geq \frac{1}{p^m} g_{Hi} \end{aligned} \tag{75}$$

Our migration conditions are almost identical, except that the partial derivative of the g function is evaluated at a different point. Again, we have two possibilities for the migration condition, the first being:

$$\frac{u'(c_1)}{e^{-r}u'(c_2)} = \frac{\frac{1}{p^m}\frac{\partial m}{\partial H} - \frac{\partial f}{\partial H}\left(\hat{H}, \bar{k}+I\right)}{\frac{1}{p^m}\left[\frac{\partial g}{\partial H}\left(\hat{H}\right) - \frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) + \frac{\partial \nu}{\partial H}\left(\bar{H}-\hat{H}\right)\right]} \tag{76}$$

Migration occurs at either a point where that condition is true, or else where the following two both hold:

$$\frac{\partial g}{\partial H}\left(\hat{H}\right) = \frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) - \frac{\partial \nu}{\partial H}\left(\bar{H}-\hat{H}\right) \tag{77}$$

and

$$\frac{\partial f}{\partial H}\left(\hat{H}, \bar{k}+I\right) = \frac{1}{p^m}\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) \tag{78}$$

2.6.5 Households that participate in the wage labor market in both periods and migration in both periods

It could be the case that a household sees returns in the wage labor market that are high enough so that no level of investment in the home production activity can draw human capital away form the wage labor market. The question of migration is a fairly limited one here. If marginal returns to human capital in the wage labor market are constant, one of the possible migration conditions is eliminated. In order for the household to avoid home production, we must have:

$$\frac{1}{p^m}\frac{\partial g}{\partial H}\left(\hat{H}\right) > \frac{\partial f}{\partial H}\left(H', \bar{k}\right) \tag{79}$$

$$\frac{1}{p^m}\frac{\partial g}{\partial H}\left(\hat{H}\right) > \frac{\partial f}{\partial H}\left(H', k^*\right) \tag{80}$$

$$\forall H' \in \left[0, \hat{H}\right] \tag{81}$$

Determination of the amount of human capital devoted to migration will be grounded in the following rule:

$$\frac{u'(c_1)}{e^{-r}u'(c_2)} = \frac{\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) - \frac{\partial g}{\partial H}\left(\hat{H}-H_2\right)}{\frac{\partial g}{\partial H}\left(\hat{H}-H_1\right) - \frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) + \frac{\partial \nu}{\partial \hat{H}}} \tag{82}$$

which is the same as the rule that we see in the full interior solution, although in this case the marginal product of human capital in the wage labor market does not hold the same relationship to home production as it does in the interior solution.What we do see here is that only in the case where this relationship between migration and wage labor market tradeoffs and consumption over time holds can we expect to see the household participate in both of these activities. If this relationship fails to hold at any point of the household problem, we would expect the household to instead specialize in either the wage labor market or migration, whichever it finds to be more profitable. Alternately, we could say that households who participate in this combination of activities in both periods will have a marginal rate of intertemporal substitution that matches this particular relationship between migration and wage labor market tradeoffs.

2.6.6 Households that participate in home production in both periods and in migration in both periods

For some households, potential earnings in both home production and migration could outweigh those from the wage labor market. Mathematically, this means that the marginal product of human capital in the wage market is never higher than the marginal product of human capital in the home production activity, or:

$$\frac{\partial f}{\partial H}(H', k') > \frac{1}{p^m}\frac{\partial g}{\partial H}(H'') \qquad \forall H', H'' \in [0, \hat{H}], \quad k' \in \left[\bar{k}, k^*\right] \tag{83}$$

Of course, for the household to participate in migration, this all must occur with an value of $\hat{H}$ defined by the condition:

$$\frac{u'(c_1)}{e^{-r}u'(c_2)} = \frac{\frac{1}{p^m}\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) - \frac{\partial f}{\partial H}\left(\hat{H},\bar{k}+I\right)}{\frac{\partial f}{\partial H}\left(\hat{H},\bar{k}\right) - \frac{1}{p^m}\left[\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) - \frac{\partial \nu}{\partial H}\left(\bar{H}-\hat{H}\right)\right]} \tag{84}$$

or the conditions:

$$\frac{\partial f}{\partial H}\left(\hat{H},\bar{k}\right) = \frac{1}{p^m}\left[\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) - \frac{\partial \nu}{\partial H}\left(\bar{H}-\hat{H}\right)\right] \tag{85}$$

and

$$\frac{\partial f}{\partial H}\left(\hat{H},\bar{k}+I\right) = \frac{1}{p^m}\frac{\partial m}{\partial H}\left(\bar{H}-\hat{H}\right) \tag{86}$$

The second set of conditions suggests a particular relationship between migration costs and investment, while the first condition would relate both of these to the marginal rate of intertemporal substitution.

2.6.7 Households that participate only in migration in both periods

Another possibility is that the returns to migration for a household could be large enough that the household does not engage in any other economic activities. If this were the case we would expect that the marginal returns to migration over the two periods were greater than any possible from either the home production activity or the local labor market. We would expect this

$$\left(1+e^{-r}\right)\frac{\partial m}{\partial H}\left(\bar{H}\right) - \frac{\partial \nu}{\partial H}\left(\bar{H}\right) > \frac{\partial f}{\partial H}\left(H',\bar{k}\right) + e^{-r}\frac{\partial f}{\partial H}\left(H'',k^*\right) \qquad \forall H',H'' \in [0,\bar{H}] \tag{87}$$

to be true, as well as this:

$$\left(1+e^{-r}\right)\frac{\partial m}{\partial H}\left(\bar{H}\right) - \frac{\partial \nu}{\partial H}\left(\bar{H}\right) > \frac{\partial g}{\partial H}\left(H'\right) + e^{-r}\frac{\partial g}{\partial H}\left(H''\right) \qquad \forall H',H'' \in [0,\bar{H}] \tag{88}$$

which could be the result of particularly lucrative migration possibilities, or low potential in either of the other activities. This suggests a bifurcated drive towards the adoption of migration as an exclusive or dominant activity. Those who face low returns in home production and the local labor market may see higher returns in a distant labor market, perhaps where the returns to education are lower but wages are higher and may move to obtain those returns. On the other hand, those with access to better migration networks, or higher levels of human capital that can pay off in a distant and lucrative market will also be driven to migrate. This also brings up another point: not all migration opportunities or destinations will be the same. In our model, this is represented by potential variations in the function m.

2.7 Modeling the household-specific function coefficients

In the model above we suggest that each household can exhibit different values for the coefficients in the production and earnings functions. The effects of household heterogeneity are the key feature of the model, but so far we have not delved into the mechanics of how this might work. Many factors that fit under the general category of 'unobservables' will certainly also fit into a list of things that help determine household ability and productivity in these activities, but there are also many things that can be observed that we can use to model these coefficients.

2.7.1 Coefficients in the home production activity production function

This is obviously the largest and most complex of the functions that we have. Both human capital and physical capital enter into this production function, and each in multiple ways. Without worrying about functional form for the moment we can think about what characteristics of the household and of the village might help determine the values that these coefficients take on. Both physical capital and human capital are rough, broad categories. The characteristics of the actual capital possessed by the household could play an important part. For instance, again assuming that the home production activity is an agricultural one, characteristics of household plots (soil types, inclination) could have a serious impact on productivity. Characteristics of non-land capital can also be important, such as age of agricultural machinery. Relevant characteristics of human capital could include age, education, and experience. Household characteristics could include information networks (does the household know a lot of other farmers?) as well as some geographical factors like proximity to other communities.

2.7.2 Coefficients in the wage labor and migration earnings functions

These two functions are very similar, and so we can look at modeling them in the same way. Each is essentially a wage function, so education and experience should be factors. Also, we can think of network effects that help to secure more lucrative positions in both of these types of activities.[...]

3 Policy and migration

The proposed model can be used to think about the role of government in migration. The tradeoffs which we have suggested that the household faces in making migration decisions are replete with opportunities for government intervention. These opportunities can flow from direct financial avenues, such as wage policy and price policy, and less direct, technological avenues.

Government expenditure on agricultural research and development could have a myriad of effects in our model. Targeted funding could be used to try to impact any of the production function coefficients. This can be consistent with several different goals of government policy. For instance, research and training that attempts to increase the productivity of labor in agricultural production could be a tool to discourage migration. On the other hand, attempts to increase the productivity of capital in agriculture could encourage investment, and through that have a push effect on migration.[...]

4 Empirical analysis

The model above hypothesizes that household and village characteristics are important determinants of household migrations decisions. To understand more about the relationship between these characteristics and decisions we now turn to empirical analysis. There are several approaches that can advance our understanding.

The true goal is to fit the dynamic structural model using data. This is a fairly involved process, and will take a lot of time. In order to begin to test our basic hypotheses we can start with something simpler before investing in the full structural model. One possibility is to examine how household and regional characteristics relate to decisions to participate in different production and labor market activities. In the model above we have defined a set of different types that households can be classified into, determined by their activity choices.

4.1 Data

In order to begin empirical analysis, we have to start matching data to our theoretical model. The data we will be using for this project come from the ENHRUM (La Encuesta Nacional a Hogares Rurales de México, or National Rural Household Survey of Mexico), which is currently a two period panel of about 1500 households throughout rural Mexico. This data comes from very detailed surveys that cover the spectrum of household economic activity. Information about demographics, physical and financial assets and family work history is also covered. In short, the data contains virtually everything we could want for this project.

Using our model and the data, we can define the activity choices faced by households in rural Mexico. The single most common home production activity is agriculture. Migration activities originating in rural Mexico can be dropped into two distinct bins: internal and international. In the internal category are people who travel to Mexico City and Puebla and other commercial centers for industrial jobs as well as people who work in these places or in resort areas in unskilled or low skilled service industries. International migration from rural Mexico is dominated by the United States. Jobs in the United States tend to feature low returns to human capital, and are often in the agricultural labor, construction or service industries. Many researchers have hypothesized that migration within Mexico exhibits higher relative returns to human capital, so it makes sense to consider these two broad migration destinations as two different activities.

Another important division that can be made in our data is among wage labor markets. Here, we want to capture the important differences between agricultural labor and other types of labor. Essentially, the agricultural worker is subject to a type of uncertainty that the non-agricultural laborer does not usually face. Also, many agricultural laborers are either farmers in their own right, or else landless rural residents. Outside of agriculture, the range of skills and expectations is much different than inside agriculture.

4.2 Reduced form models of policy functions

In addition to the classification of households into types, our model makes certain predictions about the relationship between policy functions and household characteristics. So a first step we can make in this direction is to put together a reduced form model that relates the policy variable to household characteristics. For example, starting with agricultural home production, we can infer from our model above that the level of investment in agricultural production should be determined by household and regional characteristics. In the model above we talk about ideal levels of capital being obtained through investment. However, it is realistic to think that there could be significant adjustment costs to obtaining agricultural capital in rural Mexico. this means that it would be tenuous to imagine that we observe the ideal capital levels. We can observe the change in capital over time, or the slope of the policy function, quite easily.

Starting form the condition above for investment, we can construct a reduced form model of investment. Ideal capital level for a household involved in home production will be determined by the parameters of the production function that the household uses to produce. If the household is involved in other activities, the human capital alotment towards home production and through that the ideal capital level, will also be related to the parameters of other production or wage functions that the household is using. This is implicitly assuming some kind of imperfections in the labor market, although the variation in wages observed in rural Mexico suggests that this is a good assumption.

Since what we observe in the data is actually the change in I over time, we can think about what in our production functions can change over time to get an idea of what to include in our reduced form analysis. We want to include characteristics of the household that can impact the coefficients in the production and wage functions, as these are interacting with the different types of capital, both of which can change over time. Depending on how our data are measured, we may also need to control for inflation. One way to approach this is to use the share of total production cost that is spent on capital as our dependent variable, instead of just a nominal value. Essentially, this just means that we are deflating by the total production expenditures. This can also be considered as a measure of capital intensity of production, which is certainly of interest as we think about rural production and investments.

We can estimate an equation relating the change in capital share of expenditure to household and regional characteristics to investigate our hypotheses about household characteristics and production. a very basic statement of this model can be made in the form of:

$$\Delta y \approx y_{i1} - y_{i2} = \beta x_{i1} + \gamma z_{j1} + \theta + u_i \tag{89}$$

To be clear, we are hypothesizing in the empirical model presented here that the rate of change of capital expenditure share of household i in village j is a function of household characteristics in period one (x_{i1}), village characteristics in period one (z_{j1}) state level fixed effects (θ) and an error term u_i. Because we are measuring characteristics in period one, this is similar to saying that we are viewing the rate of change between the periods as an approximation of the slope in the first period. Since our main goal with this initial empirical analysis is to confirm the existence of some of the types of relationships that we hypothesize, this is an okay assumption.

4.3 Estimation and results

The household characteristics that are used to estimate the slope of the policy function are summarized in table one. It should be noted at this point that this model is being run on a selected sample of the rural households in our data, notably those households who are involved in agricultural production in both the first and the second period. This leaves a possibility for some concern about the process of selection into this category being correlated in some way with the determination of capital expenditure share. In order to investigate this concern, we estimated a heckman model to correct for the possible selection bias. However, the results of the heckman model tell us that the process of selection into this category is not correlated with the determination of capital expenditure share change, and so we instead proceed with just the second stage equation.

Data used in the estimation are summarized in table 1, and the results of our estimation are presented in table 2. We have four coefficients in our model with statistical significance, and each can be related to the theoretical model presented above. While it certainly wouldn't be bad to find more significant coefficients, finding the ones we have here in this simple reduced form analysis does suggest that our hypotheses are worth investigating more thoroughly

On a dummy variable that represents whether or not the household head speaks an indigenous language, we see a positive coefficient. This tells us that in the case where the household head speaks an indigenous language, the household will exhibit a higher rate of change of capital expenditure share than a similar household whose head does not speak an indigenous language. This result could come from a few different directions. One possibility is that indigenous households have less access to labor markets or non agricultural investment opportunities, and so are more likely to devote their investments and their human capital into agriculture. This lack of access could be geographic (if indigenous languages survive better in remote areas) or they could stem from some kind of language barrier or other disadvantage that these households suffer from. Another possibility is that communities where indigenous languages survive are closely knit communities where agricultural production knowledge is shared easily. The first possibility is the equivalent of having lower than average marginal productivity in wage markets or in other production activities, whereas the second is akin to higher marginal products in the agricultural production activity.

Households with more people who are highly educated (six or more years of education) are adding less to their capital expenditures over time than otherwise similar households. This is most likely the result of highly educated people having better opportunities in non-agricultural production, wage work, or migration. Reaching back into our theoretical model, this would be the same as saying that marginal productivity in non-agricultural production, in wage markets or in migration are moving human capital away from agricultural home production and through that movement lowering the ideal level of capital in agricultural production, which decreases the rate of capital expenditure share change.

Form of property rights also appears to make a difference in capital levels in agricultural production. Our empirical results suggest that households who hold a larger percentage of their land in ejidal arrangements exhibit higher rates of capital share change. This requires a bit of thought. Ejidal rights are not quite as strong as private property rights, but they may be stronger than other communal property rights. More ejidal lands may also be connected to stronger ejido organizations at the village level, which could both ease adjustment costs for increasing capital in agriculture or increase marginal productivity through higher levels of information sharing in the community. This could also be related to ejidal landholding being the most secure form that some households might have access to.

The tortillerias variable suggests that the economic level of activity in the village is also an important consideration in the determination of agricultural capital use. Tortillas are a staple of consumption in rural Mexico. More commercialized tortilla makers suggests higher levels of market consumption, higher demand for commercially produced goods and services and generally larger amounts of economic activity. In terms of the theoretical model, this suggests that the marginal product of human capital may be higher in wage work than it is in towns with fewer tortillerias, due to a higher demand for labor. Similarly, marginal products in service provision or in non-agricultural production could also be higher in villages with more active economies.

One thing that we do not see in this reduced form model is a relationship between household migration experience and the rate of change in capital expenditure share. There are many possible explanations for this. One is that the state level fixed effects (which are not reported in the tables but are included in the model) are absorbing a lot of the migration effects. Another possibility is that the impacts of migration on capital investment may be mixed and dependent on other factors in a non-linear fashion. This would basically imply that we are experiencing some kind of mis-specification bias in our simple reduced form model, that we are not able to capture the conditional nature of the relationship in this very simple model. The next steps of this project will attempt to delve more into this relationship.

What we do have so far is confirmation that some household characteristics are related to investment in agricultural capital. Importantly, these characteristics are ones that will vary across villages, and so we can expect that the aggregate investment and production effects will vary quite a bit across different regions. Educational access, indigenous languages and systems of land tenure vary regionally in rural Mexico, and so the (untested) hypothesis that different regions can experience different net effects of migration due to the aggregation of household level investment decisions is something that we can continue to look into with some expectation of interesting results.

Table 2: Estimation Coefficients

Variable	Coefficient	Standard errors	t	p-value
total land	0.03	0.02	1.45	0.15
indigenous language	7.50	3.21	2.34	0.02
English language	-2.39	7.48	-0.32	0.75
age	0.09	0.08	1.16	0.25
high education (# of people in hh with)	-1.80	0.98	-1.84	0.07
Medium education	-0.60	0.81	-0.74	0.46
Experience in local labor markets (yrs)	-0.11	0.26	-0.43	0.67
Experience in int'l migration	0.35	0.31	1.14	0.25
experience in internal migration	0.04	0.16	0.27	0.79
percent of land ejidal	0.05	0.03	2.03	0.04
percent of land with high quality soil	-0.03	0.02	-1.30	0.19
bracero dummy	-4.99	5.73	-0.87	0.38
post office in town	3.00	6.50	0.46	0.65
tortillerias in town	-4.76	1.62	-2.93	0.00
irrigation dummy	-1.83	3.23	-0.57	0.57
phone booths	0.09	0.62	0.14	0.89

Table 1: Summary Statistics

Variable	Mean	Std. Dev.
Change in capital share of expenditure	-0.04	0.28
total land	8.15	32.22
indigenous language	0.33	0.47
English language	0.02	0.15
age	51.33	14.57
high education (# of people in hh with)	0.68	1.32
Medium education	1.40	1.58
Experience in local labor markets (yrs)	19.55	4.93
Experience in int'l migration	1.60	4.11
experience in internal migration	4.14	6.37
percent of land ejidal	62.08	47.56
percent of land with high quality soil	34.86	46.30
bracero dummy	0.33	0.47
post office in town	0.20	0.40
tortillerias in town	0.63	0.82
irrigation dummy	0.21	0.41
phone booths	1.50	1.97

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