A New Measure of the Producer Welfare Effects of Technological Change

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May 3, 2011


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* Many thanks to Dimitrios Dedakas for spurring my thinking on this topic, and to Julian Alston and Stelios Katranidis for discussions.
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It is well recognized that the statistical reliability of the conventional method of estimating the effects of technological change on producer welfare is often quite poor. I present a method that enhances the statistical reliability of such estimates. I emphasize that when measuring the welfare effects of technological change, valuable information can be gleaned from data on input prices and quantities. This type of data is often available, but the conventional measure typically does not take full advantage of its availability.

Difficulties in Measuring the Change in Producer Welfare Due to Technological Change

The Conventional Measure

Letting $T^0$ be some initial level of technology and $T^1$ be a subsequent level, the conventional measure of producer welfare change due to a technology change is defined in equation (1), and illustrated in figure 1 by area $B - area A$.\(^1\)

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\(^1\) How «producer surplus» should be interpreted in (1) depends on how prices in markets linked vertically or horizontally to the market in question are treated in the analysis. For example, since input prices $w$ do not appear in (1), we may presume either that input prices are assumed not to change, or that they are assumed to change in market equilibrium as $p$ changes. Unless the empirical analysis specifically includes input price data in its estimation of the supply curve, the supply curve being estimated is actually an equilibrium supply curve, not an ordinary supply curve. Changes in geometric «producer surplus»-type areas behind equilibrium supply curves measure not only the welfare change of producers of the good $q$, but can include changes in the welfare of suppliers and input demanders in related markets. If some but not all input prices are explicitly included in the estimation of the supply function, then the implicit assumption is that the prices of the non-included inputs either do not change or are being allowed to change as the equilibrium changes. Then the geometric area behind the estimated supply curve will represent the welfare of the producers of $q$ and of some of the suppliers and demanders in linked markets whose prices have not been included in the estimation of $S(\cdot)$. In the same way, when using our methodology the proper interpretation of who the «producers» are depends on which prices are included in the empirical estimation of the supply function. In the context of price changes (not from technological changes), Just, Hueth, and Schmitz (2004, chapter 9) discuss these issues in detail.
\[ \Delta PS = \int_0^{p^f} S(p, T^f) dp - \int_0^{p^0} S(p, T^0) dp . \]

In (1), \( S(\cdot) \) is a supply function for some market’s good, and \( p \) is a variable representing the price of that good. The standard “producer surplus,” defined as the geometric area under the price and behind the supply curve, is used to measure welfare. The change in welfare is measured as the difference in the producer surpluses under the two (price, technology) scenarios.

**Challenges Faced when Using the Conventional Measure**

Attempts to use the measure in (1) to gauge the producer welfare effects of changes in technology date back to Griliches’s (1957, 1958) studies of the economics of hybrid corn technology. Since then, scores of articles have reported estimates of the costs and returns of research and resultant technology change. (A few fairly recent examples are Moschini, Lapan and Sobolevsky 2000; Gotsch and Burger 2001; Perrin and Fulginiti 2001; Demont, Oehmke and Tollens 2006; Frisvold, Reeves, and Tronstand 2006; and Hareau, Mills, and Norton 2006.) The standard reference to the welfare economics of technological change is Alston, Norton, and Pardey (1995), which offers a thorough presentation of both the theory that underlies use of the conventional measure, and of how to apply that theory. In this article I build upon their treatment of the subject.

The conventional method of estimation of the producer welfare effects of a technology change is burdened by a well-recognized difficulty: the method usually requires extrapolation of the econometric estimation of the supply function to regions outside the range the data (Scobie 1976; Lindner and Jarett 1978; Rose 1980; Voon and Edwards 1991). Just, Hueth, and Schmitz (2004, pp. 284-290) provide a helpful discussion, which we draw upon in figure 2. The small circles in figure 2 represent observed (quantity, price) data points, and the means of the quantity
and price data are shown at \( q \) and \( p \). (Note that we have price on the horizontal axis and quantity on the vertical axis.) The linear functional form is assumed, and the estimated supply curve is \( \hat{S}(p) = \hat{\alpha} + \hat{\beta}p \). But \( \hat{\alpha} \) and \( \hat{\beta} \) are estimates, and subject to statistical error. Curves \( B_L \) and \( B_U \) represent the boundaries of a confidence interval for the supply curve, and the distance between these bounds increases as price moves array from \( p \). For example, given the estimated supply curve in figure 2, for a producer price of \( p^0 \), if the supply curve were estimated with perfect accuracy, meaning that the true curve is \( \hat{S}(p) \), the producer surplus would be area \( aec \). But if the true curve is \( S(p) = \alpha + \beta p \), which lies within the confidence bounds, then the true producer surplus is much smaller, area \( dbc \). As drawn, the estimated producer surplus is about twice the actual producer surplus. Clearly, the confidence interval of the estimate of the whole of producer welfare can be quite wide. The econometrics cannot work magic. When no observations have been made in which the price is very low, it is simply not possible to estimate the vertical intercept of the supply curve with much statistical confidence. There is a large and involved literature discussing how supply should be assumed to shift, whether in a parallel, pivotal, or some other fashion (c.f., Lindner and Jarret 1978, 1980; Rose 1980; Wise and Fell 1980, Norton and Davis 1981; Voon and Edwards 1991; Haung and Sexton 1996; Edwards and Voon 1997; Wohlgenant 1997). Alston, Norton, and Pardey (1994, pp. 63-64) recognize and discuss the challenge of making the proper assumption about the character of the supply shift:

*There has been a great deal of discussion in the literature about the effects of different types of research-induced supply shifts on the size and distribution of research benefits, and rightly so. This choice in the analysis is crucially important... Unfortunately, economic theory is not informative about either the functional form of supply and demand or the functional form (parallel, pivotal,
proportional, or otherwise) of the research-induced supply shift. ... We might hope to obtain plausible estimates of elasticities at the data means, but definitive results concerning functional forms are unlikely and it is impossible to get statistical results that can be extrapolated to the price or quantity axes (i.e., the full length of the function) with any confidence.

Strong critiques about the dependence of producer welfare measures on assumptions about supply shifts have appeared in the literature. Beattie (1995, p. 1065) was in general complimentary in his review of Alston, Norton, and Pardey (1994), but he also wrote,

*If total benefits from a research-induced supply shift are halved when that shift is deemed to be pivotal rather than parallel, and if producer benefits disappear when the supply shift is pivotal against an inelastic demand, then it seems to me that we have a rather big problem here.*

The assumed functional form of the supply curve is of ultimate importance in the conventional measure of producer welfare change. It easily may be the case that an assumed functional form fits the data well locally, i.e., in its range, and therefore passes all goodness-of-fit statistical tests, but that the estimate is poor globally. If the global fit is poor, then the estimate using the conventional measure of welfare change is likely to be poor.

In the following, I derive and discuss a new measure of the change in producer welfare due to technology change. The measure does not generally require estimation of the supply curve far beyond the range of the data, and therefore when using this new measure, increased statistical confidence can be placed on the estimation of the change in producer welfare. The key to the new method is to use data from input markets in the measurement of
producer welfare change. Since a supply curve reflects marginal costs of production, it is natural to take advantage of available data by estimating input costs in input markets, instead of ignoring input market data and attempting to measure input costs in the output market.

Figure 1. The conventional measure of producer welfare change due to a technology change is $B - A$
Figure 2. Statistical reliability of supply curve estimates and of estimates of producer surplus.
Why Are We Doing Any of This? Why Is this an Economics Problem, and Not Just and Accounting Problem?

A change in profits equals the change in revenues minus the change in costs. Say that in the presence of some initial technology, we observe an output price of $p^0$, an output quantity of $q^0$, input prices of $w^0 = (w^0_1, ..., w^0_L)$ and input quantities of $x^0 = (x^0_1, ..., x^0_L)$. Under some subsequent technology, say that we observe $p^1$, $q^1$, $w^1$, $x^1$. Then measuring the change in profits due to the technology change is a simple matter of accounting: $\Delta\Pi = (p^1 q^1 - w^1 x^1) - (p^0 q^0 - w^0 x^0)$. But there are reasons why the problem at hand is an economics problem and not an accounting problem. Often, reliable data on input and output prices and quantities are not available. In some nations, detailed and reliable data on prices and quantities simply are not kept. In other nations, such data are kept, but only for purchased inputs. It can be hard to place a value on inputs that are not traded in markets, for example of a producer’s own labor, or, even more difficult, the cost of accumulating human capital. The accounting measure above keeps track of the change in profits due to technology change only if no other factors (such as weather) affect quantities and prices of inputs and outputs. If everything but the technology is not held equal, then the challenge becomes to parse out the effects of the technology change from the effects of other changes. Finally, often the question being asked is an *ex-ante* question. The question is not how an existing technology change has affected producer profits, but rather, how some potential technology change (or research that will create a technology change) will affect producer profits. Data on future prices and quantities data cannot be available in the present. In any of these cases, the traditional approach is for economists to use estimated systems of supply and demand equations to estimate the welfare effects of technology changes. The method, in general, is to use price and quantity data from the past, and create an econometrically estimated
model of pertinent markets, and to use “producer surplus”-type welfare measures to estimate the impact of the technology change.

**An Introductory Example: Output Supply and Input Demand Functions All Observable**

To illustrate the gist of the new measure, I start with the simplest case, in which it is assumed that the economist has been able to estimate a system of output supply and all input demand functions in markets affected by a technological change to the production technology. First I examine the case of a single, competitive firm, and show how its change in profits is affected by the change in technology and the accompanying changes in output and input prices. (Aggregation across many firms is straightforward.)

**Basic Framework and Some Notation**

Let a competitive firm produce a single output $q$ using a vector of inputs, $x_A = (x_{A1}, \ldots, x_{AL})$ purchased at prices $w_A = (w_{A1}, \ldots, w_{AL})$ per unit. The firm’s production technology is defined by a neoclassical production function $f(x_A, T)$, where $T$ is a technology parameter. Letting the variable $p$ represent the output price, the definition of the producer’s profit function is

\[ \Pi(p, w_A, T) = \max_{x_A} pf(x_A, T) - w_A x_A. \]

---

2 Of course, if the firm’s output supply function and input demand functions are all known, and the level of technology and all prices are observable before and after the technological change, then there may be actually no need to use traditional geometric welfare measures to find *ex post* the change in profits. The change in profits can be calculated directly, as a matter of simple accounting, using initial and subsequent prices and quantities of inputs and outputs. But the upcoming example is pertinent to *ex ante* estimation of future effects of technology change, and presenting this simplest case will prove useful as an introduction to the more general methodology.

3 The subscript $A$ in this section is used in this section only to keep the notation consistent with the next section. In the next section, $A$ signifies that the input is purchased, and that the economist has sufficient data to estimate its demand function.
The vector of input demand functions, denoted \( \mathbf{x}_A^d(p,w_A,T) = (x_{A1}^d(p,w_A,T),...,x_{AL}^d(p,w_A,T)) \), is defined as that which solves the profit maximization problem above. The supply function is \( S(p,w_A,T) \equiv f(x_A^d(p,w_A,T)) \). Now say that the value of the vector of parameters in (2) changes from some initial level \( (p_0, w_A^0, T_0) \) to a subsequent level \( (p_1, w_A^1, T_1) \). A researcher who knows the supply and input demand functions could directly calculate the resultant change in profits as 

\[
\Delta \Pi = \int_M \frac{\partial \Pi(p,w_A,T)}{\partial p} dp + \frac{\partial \Pi(p,w_A,T)}{\partial w_A} dw_A + \frac{\partial \Pi(p,w_A,T)}{\partial T} dT
\]

where we are defining 

\[
\frac{\partial \Pi(p,w_A,T)}{\partial w_A} dw_A = \sum_{i=1}^{L} \frac{\partial \Pi(p,w_A,T)}{\partial w_{Ai}} dw_{Ai}
\]

The integral on the right-hand side of (3) is a line integral, with \( M \) being an arbitrary piecewise smooth path of integration in \( \mathbb{R}^{L+2} \), with endpoints \( (p_0, w_A^0, T_0) \) and \( (p_1, w_A^1, T_1) \) (Kaplan 1984, pp. 292-293, especially equation (5.48)). Hotelling’s lemma implies that we may rewrite (3):

\[
\Delta \Pi = \int_M \left[ S(p,w_A,T) dp - x_A^d(p,w_A,T) dw_A + \frac{\partial \Pi(p,w_A,T)}{\partial T} dT \right]
\]
In this specific example, the conventional measure of profit change is the change in “producer surplus,” \( \Delta PS \), defined much like in (1) as

\[
(5) \quad \Delta PS = \int_{p^0}^{p^1} S(p, w_A, T^1) dp - \int_{p^0}^{p^0} S(p, w_A, T^0) dp.
\]

As already discussed, applied research using the measure in (5) presents significant statistical difficulties. But there are alternative paths of integration that in many cases will present fewer difficulties. We denote one such path \( N \), where \( N \) is defined using real numbers: \( t^1 < \hat{t} < \ldots < \hat{t}^{L+2} < \hat{t}^{L+3} \) and the functions \( \hat{p}(t) \), \( \hat{w}_A(t) \), \( \hat{T}(t) \), where \( \hat{T}(t^1) = T_0 \), \( \hat{T}(t) \) is continuous and differentiable on \([t^1, \hat{t}^2]\), and \( \hat{T}(t) = T^4 \) on \([\hat{t}^2, \hat{t}^{L+3}]\); \( \hat{p}(t) \) is constant at \( p^0 \) on \([t^1, \hat{t}^2]\), is continuous and differentiable in \([\hat{t}^2, t^3]\), and takes on the value \( p^1 \) on \([t^3, \hat{t}^{L+3}]\); for \( l = 1, \ldots, L \), \( \hat{w}_A(t) \) is constant at \( w^0_A \) on \([t^1, \hat{t}^{l+2}]\), is continuous and differentiable in \([\hat{t}^{l+2}, \hat{t}^{l+3}]\), and \( \hat{w}_A(t) = w^1_A \) when \( t \geq \hat{t}^{l+3} \). For \( L = 1 \), path \( N \) is illustrated, along with functions that define it, in figure 3.

Following Kaplan (1984, p. 309, especially equation (5.63)), we can convert the line integral in (4), taken over the specific path \( N \), to a definite integral:

\[
(6) \quad \Delta \Pi = \int_{t^1}^{t^{L+3}} \left( \frac{\partial \Pi(\hat{p}(t), \hat{w}_A(T), \hat{T}(t))}{\partial T} \frac{d\hat{T}(t)}{dt} + \frac{\partial \Pi(\hat{p}(t), \hat{w}_A(T), \hat{T}(t))}{\partial \hat{p}(t)} \frac{d\hat{p}(t)}{dt} \right. \\
+ \left. \sum_{l=1}^{L} \frac{\partial \Pi(\hat{p}(t), \hat{w}_A(T), \hat{T}(t))}{\partial \hat{w}_A} \frac{d\hat{w}_A(t)}{dt} \right) dt.
\]

\(^4\) Here we follow the terminology of Just, Hueth, and Schmitz (2004, pp. 54-58) in defining the term “producer surplus” as the geometric area behind the supply curve and below the output price. This area represents economic profits in some, but not all, circumstances.
By design, over various subintervals of \([t^1, t^{L+3}]\), the functions \(\hat{T}(t), \hat{p}(t), \hat{w}_{A1}(t), \ldots, \hat{w}_{AL}(t)\) are constant; therefore their derivatives \(\frac{d\hat{T}(t)}{dt}\), \(\frac{d\hat{p}(t)}{dt}\), and \(\frac{d\hat{w}_{al}(t)}{dt}\) are zero, allowing us to write,

\[
\Delta \Pi = \int_{t^1}^{t^3} \partial \Pi_p \left( p^0, w^0_A, \hat{T}(t) \right) d\hat{T}(t) \frac{d\hat{T}(t)}{dt} + \int_{t^1}^{t^3} \partial \Pi_p \left( \hat{p}(t), w^0_A, T^1 \right) d\hat{p}(t) \frac{d\hat{p}(t)}{dt} \]

(7)

Changing variables of integration in (7), we have,

\[
\Delta \Pi = \int_{\hat{t}^1}^{\hat{t}^3} \partial \Pi \left( p^0, w^0_A, T^1 \right) d\hat{T}(t) + \int_{\hat{t}^1}^{\hat{t}^3} \partial \Pi \left( p^0, w^0_A, T^1 \right) d\hat{p}(t) \frac{d\hat{p}(t)}{dt} + \sum_{l=1}^{L} \int_{\hat{t}^1}^{\hat{t}^3} \partial \Pi \left( p^1, w^1_{A1}, w^0_{A1}, \ldots, w^0_{n}, T^1 \right) \frac{dw_{A1}(t)}{dt} dt.
\]

(8)

By applying Hotelling’s lemma and referring to the definitions of \(\hat{T}(t), \hat{p}(t), \hat{w}_{A1}(t), \ldots, \hat{w}_{AL}(t)\), we then obtain,

\[
\Delta \Pi = \int_{p^0}^{p^1} S(p, w^0_A, T^1) dp - \sum_{l=1}^{L} \int_{w^0_{A1}}^{w^1_{A1}} x^d_{A1} \left( p^1, w^1_{A1}, w^0_{A1}, \ldots, w^0_{n}, T^1 \right) dw_{A1} + \int_{t^1}^{t^3} \partial \Pi \left( p^0, w^0_A, T^1 \right) d\hat{T}(t).
\]

(9)

The integrals on right-hand side of (9) taken behind the output supply and input demand curves are standard “welfare trapezoids,” discussed at length in Just Hueth and Schmitz (2004, pp. 52-62, 75-82) for the case in which prices change but no technology change is considered.

The final integral on the right-hand side of (8) is the most interesting for our purposes and must be the central focus of this article. From (2) and the envelope theorem, we know that

\[
\frac{\partial \Pi}{\partial T} \left( p^0, w^0_A, T \right) \equiv p^0 \frac{\partial f}{\partial T} \left( x^d_A \left( p^0, w^0_A, T \right) , T \right)
\]

(10)

where we define
Substituting (14) into (10), we have,

\[
\frac{\partial f \left(x^d_A (p^0, w^0_A, T), x^d_B (p^0, w^0_B, T), T \right)}{\partial T} \equiv \lim_{\Delta T \rightarrow 0} \frac{f \left(x^d_A (p^0, w^0_A, T), T + \Delta T \right) - f \left(x^d_A (p^0, w^0_A, T), T \right)}{\Delta T}.
\]

We already defined the supply function by the identity \( S(p, w_A, T) = f \left(x^d_A (p, w_A, T), T \right) \), and therefore we can write \( S(p^0, w^0_A, T) = f \left(x^d_A (p^0, w^0_A, T), T \right) \). Taking the derivative of both sides of this identity with respect to \( T \), multiplying both sides by \( p^0 \), then rearranging, we have

\[
p^0 \frac{\partial f \left(x^d_A (p^0, w^0_A, T), T \right)}{\partial T} \equiv p^0 \frac{\partial S \left(p^0, w^0_A, T \right)}{\partial T} - \sum_{l=1}^{L} p^0 \frac{\partial f \left(x^d_A (p^0, w^0_A, T), T \right)}{\partial x^d_A (p^0, w^0_A, T)} \frac{\partial x^d_A (p^0, w^0_A, T)}{\partial T}.
\]

First-order conditions of the original profit-maximization problem require the following:

\[
p^0 \frac{\partial f \left(x^d_A (p^0, w^0_A, T), T \right)}{\partial x^d_A (p^0, w^0_A, T)} \equiv w^0_A, \quad l = 1, ..., L..
\]

Substituting (13) into (12), we have,

\[
p^0 \frac{\partial f \left(x^d_A (p^0, w^0_A, T), T \right)}{\partial T} \equiv p^0 \frac{\partial S \left(p^0, w^0_A, T \right)}{\partial T} - \sum_{l=1}^{L} w^0_A \frac{\partial x^d_A (p^0, w^0_A, T)}{\partial T}.
\]

Substituting (14) into (10), we have,

\[
\frac{\partial \Pi (p^0, w^0_A, T)}{\partial T} \equiv p^0 \frac{\partial S \left(p^0, w^0_A, T \right)}{\partial T} - \sum_{l=1}^{L} w^0_A \frac{\partial x^d_A (p^0, w^0_A, T)}{\partial T}.
\]

Integrating on both sides of (15), we obtain,

\[
\int_{T_0}^{T_1} \frac{\partial \Pi (p^0, w^0_A, T)}{\partial T} dT = p^0 \int_{T_0}^{T_1} \frac{\partial S \left(p^0, w^0_A, T \right)}{\partial T} dT - \sum_{l=1}^{L} w^0_A \int_{T_0}^{T_1} \frac{\partial x^d_A (p^0, w^0_A, T)}{\partial T} dT,
\]

which, by the Fundamental Theorem of Calculus implies,

\[
\int_{T_0}^{T_1} \frac{\partial \Pi (p^0, w^0_A, T)}{\partial T} dT = p^0 \left[ S \left(p^0, w^0_A, T_1 \right) - S \left(p^0, w^0_A, T_0 \right) \right] - \sum_{l=1}^{L} w^0_A \left[ x^d_A (p^0, w^0_A, T_1) - x^d_A (p^0, w^0_A, T_0) \right].
\]

Finally, substituting (17) into (9), we have
\[
\Delta \Pi = \int_{p^0}^{p^1} S(p, w^0_A, T^1) dp - \sum_{i=1}^{L} \int_{w_{i-1}^0}^{w_i^1} x^d_A \left( p^1, w^0_{i-1}, w^0_{i+1}, ..., w^0_n, T^1 \right) dw_{Al}
\]

Equation (18) is key. The right-hand side of (18) offers a new measure of the producer welfare effect of a technological change. For the case of \( L = 1 \) input, figure 4 this new welfare measure. The first term on the right-hand side of (18) is shown by (the negative of) “trapezoid” \( A \) in the output market panel of the diagram. The second term is shown by “trapezoid” \( C \) in the input market panel. The third and fourth term is shown by rectangles \( B \) and \( D \). Using the new measure, then, the change in profits is represented as \(-A + B + C + D\).

To explain the potential advantages of the new measure over the conventional measure (shown in (5)), let us break down the new measure into its components. First, consider the term
\[
\int_{p^0}^{p^1} S(p, w^0_A, T^1) dp ,
\]
and compare it to the traditional measure’s term \( \int_{0}^{p^1} S(p, w^1_A, T^1) dp \). Whereas the latter requires integration along the entire length of the supply curve, from 0 to \( p^1 \), the former only requires integration in the smaller interval, \([p^0, p^1]\), and therefore estimation of
\[
\int_{p^0}^{p^1} S(p, w^0_A, T^1) dp \]
will be more statistically reliable and less dependent on assumed functional form than will be estimation of \( \int_{0}^{p^1} S(p, w^1_A, T^1) dp \).
Figure 3. A path of integration, and functions that parametrically define it.
Second, consider the term \( p^0 \left[ S(p^0, w_A^0, T^1) - S(p^0, w_A^0, T^0) \right] \), illustrated by rectangle \( C \) in figure 4. The base of this rectangle is of length \( S(p^0, w_A^0, T^1) - S(p^0, w_A^0, T^0) \), or the distance \( h_i \). This is a type of the “\( K \)-shift” calculated and used in numerous studies, including Peterson (1967), Schmitz and Seckler (1970), Mansfield, et al. (1977), and Bresnahan (1986), and discussed in detail Alston, Norton, and Pardey (1995, pp. 65, 210, 304, 327, 397):

The size of the research-induced supply shift — the \( K \)-factor — is a crucial determinant of the total benefits from research. The accuracy of the estimate of \( K \) ... will determine the accuracy and validity of the estimates of research benefits and any research priorities that are derived, based on those estimates (Alston, Norton and Pardey 1995, p. 327)

Estimating the \( K \)-shift is a central challenge to the measurement of the producer welfare effects of technology change, whether one uses the conventional or new measure. As will be explained, the new measure requires that \( K \)-shifts in input market demand curves also be estimated. But, in summary, if data or other information are available to estimate these additional \( K \)-shifts, the benefit of the new measure is that its application does not require estimating geometric areas behind entire supply curves. The information provided by the full-length supply curve is instead gleaned from input market data, without extrapolation far outside the data’s range. Such data is often available but not used in studies of the welfare impacts of technological change.\(^6\)

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\(^5\) Various notation has been used in the literature to denote the horizontal and vertical shifts in the supply curve that are brought about by a change in technology. In part, the notation used depends upon whether a vertical shift or a horizontal shift is considered. The horizontal \( K \)-shift in the output market that I refer to here is analagous to the vertical \( k_1 \)-shift in Alston, Norton and Pardey (1995, p. 328-332).

\(^6\) The case is similar to the one of multiple exogenous price changes, addressed by Just, Hueth, and Schmitz (2004, pp. 61-66, 76-78, 291-292), in which measures estimating welfare effects of price changes in many related markets are more statistically reliable than those which estimate the welfare impacts on many markets simply by examining one market.
Figure 4. An application of the new measure of producer welfare change: \( \Delta\Pi = -A - (-B) + C - (-D) \).
A Simulative Example

To demonstrate potential benefits of the new measure of producer welfare change presented in (18), next I present results from a Monte Carlo simulation. In the simulation, I assumed the presence of one hundred heterogeneous profit-maximizing producers participating in a competitive output market and a competitive input market. For \( j = 1, \ldots, 100 \), firm \( j \)'s product \( q_j \) was created according to a quadratic production function:

\[
q_j = f(A_{1j}, T, \Delta W, b_j) = b_{1j} x_{A1} + b_{11j} x_{A1}^2 + b_{1Tj} x_{A1} T + b_{1wj} x_{A1} \Delta W,
\]

where \( x_1 \) is a variable representing the amount of an input used in production, \( T \) is a parameter representing an initial technology, and \( \Delta W \) is a “shifter” variable, whose value is assumed to be known by the producer when he/she makes input decisions. The production function in (19) implies that the input demand function for firm \( j \) is,

\[
x_{A1j}^d(p, A_{1j}, T, \Delta W) = \arg \max_{x_{A1j} \geq 0} \{ pf(x_{A1j}, T, \Delta W, b_j) - w_1 x_{A1j} \} =
\]

\[
\begin{cases}
\frac{w_{A1}}{p} - b_{1j} - b_{1Tj} T - b_{1wj} \Delta W, & \frac{w_{A1}}{2b_{11j}} - b_{1j} - b_{1Tj} T - b_{1wj} \Delta W \leq 0 \\
0, & \frac{w_{A1}}{2b_{11j}} - b_{1j} - b_{1Tj} T - b_{1wj} \Delta W \geq 0,
\end{cases}
\]

where variable \( p \) denotes output price and \( w_1 \) denotes input price. Aggregate input demand is defined as

\[
X_{1j}^d(p, A_{1j}, T, \Delta W, e_{1dt}) = \sum_{j=1}^{100} x_{A1j}^d(p, A_{1j}, T, \Delta W) + e_{1dt},
\]

where \( e_{1dt} \) is a random disturbance term.

Firm \( j \)'s output supply function is

\[
s_j(p, A_{1j}, T, \Delta W) = f(x_{A1j}^d(p, A_{1j}, T, \Delta W), T, \Delta W, b_j),
\]
where the production function coefficients are listed in vector $b_j = (b_{ij}, b_{11j}, b_{1Tj}, b_{1Wj})$, which differs across all firms. Market supply is

$\sum_{j=1}^{100} S_j(p, w_{A1}, T, \Delta W, e_{st}) = e_{st}$

where $e_{st}$ is a random disturbance term.

Demand for the product is assumed to take a linear functional form:

$D(p, \Delta Y, \Delta R, e_{dt}) = a_0 + a_1 p + a_Y \Delta Y + a_R \Delta R + e_{dt}$

with $a_0 = 40$ and $a_1 = -48$, $a_Y = 0.1$, and $a_R = 0.1$. $\Delta Y$ is the value that a demand shifter (which might be thought of as “change in income from its expected value”) takes on in a period $t = 1, \ldots, 40$. $\Delta R$ is another demand shifter, of the same type as $\Delta Y$. Input supply is also assumed to take on a linear functional form:

$X_1(w_{A1}, \Delta L, e_{1st}) = t_0 + t_1 w_{A1} + t_2 \Delta L + e_{1st}$

with $t_0 = 5.85641$, $t_1 = 21.9615$ and $t_2 = 1$. $\Delta L$ is the value that an input supply shifter takes on in period $t$.

In each Monte Carlo run, each firm’s production coefficients $b_j$ were drawn randomly from a multivariate normal distribution with a diagonal variance-covariance matrix. The mean values of the distribution were (-0.10718, -0.1, 0.2, -0.2), and the standard deviations of $b_{ij}$, $b_{11}$, and $b_{1Tj} b_{1Wj}$ were one tenth of their mean values. For each period in each Monte Carlo run, the standard deviations of $\Delta W$, $\Delta Y$ and $\Delta R$ were 0.25, 2.5, and 2.5. The standard deviation of $\Delta L$ was fifteen percent of the baseline equilibrium quantity of the input. The “baseline model” set the shifters and disturbances in (21), (23), (24) and (25) to zero: $\Delta Y = \Delta R = \Delta W = \Delta L = e_{1d} = e_{1s}$

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7 The purpose of including shift variables in the model was to allow for identification of the supply and demand functions using the 3SLS estimation procedure. The values of the standard deviations of the model’s shift variables and error terms were chosen to permit an accurate (local) econometric estimate of the model. Of course, if supply and input demand functions cannot be estimated with adequate accuracy, then neither the new measure nor the conventional measure will provide statistically reliable measures of welfare change.
= e_d = e_s = 0, and set the technology level to T^0 = 4. Baseline equilibrium prices clear both the input and output markets of the baseline model. The mean values in b_j and parameter values a_0, a_1, t_0 and t_1 were chosen so that at those values the baseline equilibrium prices were p^0 = 1.00, w_1^0 = 0.40, and so that at those prices the elasticity of the baseline output supply function, input supply function, and consumer demand function were 1, 1.5 and -8. All shifters and disturbance terms were drawn from independent normal distributions with mean zero. The standard deviations of e_1d, e_dt and e_s were equal to five percent of their equations’ respective baseline equilibrium quantities.

The Monte Carlo experiment was conducted using Mathematica (Wolfram Research, Inc 2008). At the beginning of each Monte Carlo run, one hundred “firms” were created by drawing values of the coefficient vectors b_1, \ldots, b_{100} from the distribution of the production coefficients vector. This created one hundred production functions, firm input demand functions, and firm supply functions, according to (20) and (22). Aggregate input demand and output supply functions were then established by (21) and (23). Each Monte Carlo run had forty periods, and in each period t = 1, \ldots, 40 values of the random shifter variables ∆L_t, ∆W_t, ∆Y_t and ∆R_t and the disturbance terms e_{dt}, e_{dt}, e_{1dt}, and e_{1dt} were drawn from their distributions. In each run, market prices and quantities were determined in two equilibria, with the initial equilibrium generated assuming the value of the technology parameter to be T^0 = 4.0, and with the subsequent generated with T^1 = 4.5.

For each Monte Carlo run, the procedure described above generated a simulated “data set” with forty observations: p^*_t, w^*_A_t, Q^*_t, X^*_t, ∆L_t, ∆W_t, ∆Y_t and ∆R_t for t = 1, \ldots, 40. Taking on

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8 Demand was chosen to be very elastic so that the change in technology led to an increase in producer profits, not a decrease. The qualitative results of the simulation do not depend on this assumption, but most applied studies report a producer gain from technological change, and I thought that the simulative example would be easier to explain and follow if the producer welfare change was positive instead of negative.
the role of a researcher having access to the run’s data set, but not knowing the true model that
generated it, I assumed the following linear functional forms for supplies and demands:

\[(26) \bar{X}_{A1}'(p,w_{A1},T,\Delta W,u_{1dr}) = \beta_{10} + \beta_{1p}p + \beta_{1w}w_{A1} + \beta_{1T}T + \beta_{1W} \Delta W + u_{1dr},\]

\[(27) \bar{S}(p,w_{A1},T,\Delta W,u_{sl}) = \beta_{s0} + \beta_{sp}p + \beta_{sw}w_{A1} + \beta_{st}T + \beta_{sw} \Delta W + u_{sl},\]

\[(28) \bar{D}(p,\Delta Y_t,\Delta R_t,u_{dt}) = \beta_{dt0} + \beta_{dp}p + \beta_{dy} \Delta Y_t + \beta_{dt1} \Delta R_t + u_{dt},\]

\[(29) \bar{X}_{A1}'(w_{A1},\Delta L,u_{1st}) = \beta_{1s0} + \beta_{1sw}w_{A1} + \beta_{1st} \Delta L + u_{1st}.\]

For each Monte Carlo run, I estimated the coefficients in (26)-(29) using three-stage least
squares, restricting \(\beta_{1p} = -\beta_{1s}\) because of the symmetry of the profit function’s Hessian matrix
(Varian 1992, p. 46). Then I measured the true change in profits and the estimated change in
profits caused by the technological change, first by using the model’s true supply and demand
functions, and then using its estimated supply and demand functions. Both the conventional
measure and the new measure were used, and then the differences between the true change in
profits and the estimated change in profits were calculated for both measures, in order to
compare the statistical reliability of each. In every Monte Carlo run, as expected, when the true
supply and demand functions were used, the change of profits was calculated exactly using the
conventional measure or the new measure. But when the estimated functions were used, the
mistaken assumptions about global functional form caused much smaller errors in estimation
when the new measure was used than when the traditional measure was used.

Figures 5 and 6 illustrate my procedures on the first of the 1000 Monte Carlo runs, which
was typical. The Mathematica (Wolfram Research, Inc., 2008) program for the Monte Carlo
experiment can be found in an on-line appendix (Bullock 2010). In figures 5 and 6, the solid-
lined curves are baseline curves. That is, they assume that the shifters and disturbances all take
on their expected values of zero. The first Monte Carlo run’s true baseline equilibrium prices under the initial technology are $p^*(T^0) = 0.989$ and $w_1^*(T^0) = 0.408$. These prices clear the input and output markets, given that all disturbances and shift variables are set to zero, and given the initial technology level. Prices $p^*(T^1) = 0.936$ and $w_1^*(T^1) = 0.459$ are the true baseline prices under technology level $T^1$. Forty years of equilibrium $(Q, p)$ and $(X_{A1}, w_{A1})$ “data” points generated in the first Monte Carlo run are shown as a scatter plots in figures 5 and 6. Also generated in each run were forty years of data on shifters $\Delta L$, $\Delta R$, $\Delta W$, and $\Delta Y$. For each run, a restricted 3SLS procedure was applied to this data, in order to obtain the run’s estimates of the parameters in (25)-(29). Calling the functions thus estimated $\hat{X}_{A1}^0(p, w_{A1}, T, \Delta W)$, $\hat{S}(p, w_{A1}, T, \Delta W)$, $\hat{D}(p, \Delta Y, \Delta R)$, and $\hat{X}_{A1}^1(w_{A1}, \Delta L)$, estimated equilibrium price functions $\hat{p}^*(T)$ and $\hat{w}_{A1}^*(T)$ were derived from the estimated linear model, assuming market clearing and setting all shifters, $\Delta L$, $\Delta R$, $\Delta W$, and $\Delta Y$ to zero. As shown in the figures, the estimated equilibrium prices in the first Monte Carlo run were $\hat{p}^*(T^0) = 0.988$, $\hat{w}_{A1}^*(T^0) = 0.409$, $\hat{p}^*(T^1) = 0.941$, and $\hat{w}_{A1}^*(T^1) = 0.460$.

We let $\Delta H^0 = 0$. For each Monte Carlo run, the traditional measure and the new measure defined in equations (5) and (18) were applied to the estimated linear model to estimate the aggregate change in expected producer profits emanating from the technology change.\footnote{In figure 5, point $z = \left(\hat{S}(p, w_{A1}^*(T^0), T^0, \Delta W^0, \epsilon_1^0), p^*(T^0)\right) = \left(D(p^*(T^0), \Delta Y^0, \Delta R^0, \epsilon_2^0), p^*(T^0)\right)$, = (843.71, 0.9890) is indistinguishable from point $z' = \left(\hat{D}(\hat{p}^*(T^0), \Delta Y^0, \Delta R^0, \hat{p}^*(T^0)\right) = (837.059, 0.9893)$. Similarly, point $a = (0, 0.9890)$ is indistinguishable from $a' = (0, 0.9893)$.}
(30) \[ \Delta \hat{P}_S = \int_0^{\hat{P}_i(T^1)} \hat{S}(p, \hat{w}_{i1}^*(T^1), \Delta W^0, T^1) dp - \int_0^{\hat{P}_i(T^0)} \hat{S}(p, \hat{w}_{i1}^*(T^0), \Delta W^0, T^0) dp = 155.47. \]

\[ \Delta \hat{I}_{NM} = \int_{\hat{p}^*(T^1)}^{\hat{p}^*(T^0)} \hat{S}(p, \hat{w}_i^*(T^0), \Delta W^0, T^1) dp - \int_{\hat{p}^*(T^0)}^{\hat{w}_i^*(T^0)} \hat{X}_i^d(\hat{p}^*(T^1), w_i, \Delta W^0, T^1) dw_i \]

\[ + \hat{p}^*(T^0) \left[ \hat{S}(\hat{p}^*(T^0), \hat{w}_i^*(T^0), \Delta W^0, T^1) - \hat{S}(\hat{p}^*(T^0), \hat{w}_i^*(T^0), \Delta W^0, T^0) \right] \]

\[ - \hat{w}_i^*(T^0) \left[ \hat{X}_i^d(\hat{p}^*(T^0), \hat{w}_i^*(T^0), \Delta W^0, T^1) - \hat{X}_i^d(\hat{p}^*(T^0), \hat{w}_i^*(T^0), \Delta W^0, T^0) \right] = 31.253. \]

The (true) baseline profits for the first Monte Carlo run can be calculated directly or as a change in producer surplus as measured behind the baseline true supply curves, or by using the new measure with the true curves. Since there is no estimation error to the true curves, all three methods produce the same result, which is that the technology change leads to a change in baseline profits of 24.191. Using the conventional measure, this change in profits is shown in figure 5 by area \( dtv \) – area \( azv \):

(32) \[ \Delta PS = \int_0^{\hat{P}_i(T^1)} S(p, w_{i1}^*(T^1), \Delta W^0, T^1) dp - \int_0^{\hat{P}_i(T^0)} S(p, w_{i1}^*(T^0), \Delta W^0, T^0) dp = 24.191. \]

Thus, the error in using the new measure to estimate the change in producer welfare in this particular Monte Carlo run was 31.253 – 24.191 = 7.062, whereas the error from using the conventional measure in the estimate was 155.473 – 24.191 = 131.

The increased statistical reliability provided by the new measure is illustrated by the histograms in figure 7, which show the distribution of producer welfare estimates from the one thousand Monte Carlo runs of the simulation. The upper panel shows the results from making estimates using the new measure, and the lower panel shows the results from the conventional
measure. Because in every Monte Carlo run a new set of one hundred firms was drawn, the true producer welfare change was different in every run. But the differences were small; the mean of producer welfare changes was 30.79, and the standard deviation was only 0.49. The mean of the new measure’s estimates was 33.88, and their standard deviation was 11.41. In contrast, the histogram from the conventional measure’s estimates has a mean of 186.47, a standard deviation of 53.29, and lies entirely to the right of the true mean. (Source: New Measure 2-Mkt Simulation FINAL NOTRASH 28Oct10.nb and .xls., and New Measure TEMP 2-Mkt Simulation FINAL NOTRASH LAST TEN 28Oct10.nb).

Figures 5, 6 and 7, and equations (30) – (32) illustrate the central point of this article. If the researcher knows exactly the actual supply function, then the conventional measure, which is the change in producer surplus measure, provides the exact change in producer profits that result from a change in market equilibrium brought about by a technology change. In the Monte Carlo run illustrated in figures 5 and 6, this change was 30.79. Note that in figure 5, in the neighborhood of the data, the estimates of supply and demand are extremely accurate. The baseline and estimated baseline demand curves, $D(p,\Delta Y^0,\Delta R^0,\epsilon^0_d)$ and $\hat{D}(p,\Delta Y^0,\Delta R^0)$, are almost indistinguishable in the range of observed $(Q,p)$ points. Similarly, in that range of data the true baseline supply curves are very near their estimated counterparts. But away from the range of the observed data, the true and estimated supply curves differ greatly. This divergence of the true and estimated supply functions outside the range of the data results, of course, from a mistaken assumption about functional form in the econometric analysis. But given how “good” the local fit of the linear model is, it would prove very difficult to reject the linear model using a statistical test. Given the conventional measure, the researcher is forced, then, to assume various functional forms, and compare the results, hoping to obtain a “ballpark” estimate of the producer welfare change. But since determining what the supply function actually is and how it shifts
(whether in a pivotal or parallel fashion, etc.) has usually proved impossible in applied studies, then Beattie’s (1995, p. 1065) aforementioned critique is applicable to the use of the conventional measure: “we have a rather big problem here.”

By making use of input market data, the new measure can help solve this problem. *The new measure finds cost changes not by integrating behind supply curves, but rather by examining “rectangles” associated with input demand curves and output supply curves.* The bases of such rectangles require estimation of $K$-shifts in input demand and output supply. In the conventional method, only the $K$-shift in output supply need be estimated. But, if data from input markets is available, and reasonable estimates of the input demand shifts are obtainable, the new measure can ameliorate the statistical unreliability endemic in the conventional measure’s literature.
Figure 5. Effects of a change in technology on the output market.
Figure 6. Effects of a change in technology on the input market.
Figure 7. From the two-market model, histograms of the estimated changes in producer welfare under the new measure and the conventional measure. From a Monte Carlo analysis with 1000 runs. (Source: New Measure 2-Mkt Simulation.nb and New Measure TEMP 2-Mkt Simulation FINAL NOTRASH LAST TEN 28Oct10.nb and New Measure 2-Mkt Simulation FINAL NOTRASH 28Oct10.xls.)
Figure 8. From the three-market model, histograms of the estimated changes in producer welfare under the new measure and the conventional measure. From a Monte Carlo analysis with 1000 runs. (Source: New Measure 3-Mkt 19Nov10 NOTRASH, in Mathematica version 6, and New Measure 3-Mkt Histogram 19Nov10 NOTRASH in Mathematica version 7.)
Figure 8 illustrates the results of similar Monte Carlo experiment run on a similar model, which had three inputs instead of two. It was assumed that the researcher had price and quantity data on the output and two of the inputs, and that a third input was an “owned input,” not traded in markets, and therefore not providing price and quantity data. We do not estimate the $K$-shifts for the three-market model, but rather assume that when running the econometric estimations, the researcher knows these values for certain.\textsuperscript{10} The Mathematica (Wolfram Research, Inc., 2008) program for this Monte Carlo experiment can be found in an on-line appendix (Bullock 2010). The results are much the same as in the two-input model: the mean of the estimates calculated with the new measure were 28.7927, giving an estimation error of only 4.3%, as the true mean of the change in producer welfare was 27.6047. In contrast, the mean of the estimates calculated using the conventional measure were 76.1906, almost triple the true mean.

\textsuperscript{10} We do this to focus on the error that comes about because a mistaken functional form is assumed, and not commingling it with the error that comes from mis-estimating the $K$-shifts. As we stated, the new measure increases the reliability of the estimate of producer welfare change because it does not require accurate global estimation of the supply curves. It does, however, depend just as much on the accuracy of the estimates of the $K$-shifts as does the conventional measure.
References


Bullock, D.S “A New Measure of the Producer Welfare Effects of Technological Change: Mathematical Appendix


