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# **Bayesian estimation of non-stationary Markov models combining micro and macro data**

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## Objective:

- (1) Develop a Bayesian estimation framework for non-stationary Markov models that allows combining micro and macro data based estimation techniques previously considered as alternatives only
- (2) Evaluate the derived estimator using Monte Carlo Simulations

## Relevance: Farm Structural Change

- General aims:** (1) Identify and quantify factors that determine farm structural change (2) Predict structural change in response to these factors
- Often modeled as a Markov Process (Zimmermann et. al 2009)
  - But current estimation techniques do not allow using available micro and macro data in a satisfying way

## Background on Markov processes

- A Markov process allows to model the movement of individuals between a finite number of predefined states,  $i=1, \dots, k$ , as a stochastic process
- A Markov process is characterized by a transition probability matrix  $P_t$
- The vector  $n_t$  denotes the number of individuals in each state and develops over time,  $t$ , according to a (first order) Markov process:

$$n_t = P_t' n_{t-1}$$

## Specification of $P_t$

- $P_t$  is assumed to be a function of explanatory variables
- We propose two different specifications for ordered and unordered Markov states based on the multinomial logit model and the ordered logit model
- Main differences: (1) Multinomial logit model requires assumption of iid errors which might be inappropriate with ordered Markov states (2) Ordered logit model requires less parameter to be estimated

## Combining...

### macro data...

#### Definition:

- The number of individuals in each class,  $n_t$ , is observed over time
- Individual transitions are not observed and many different transitions could result in the observed data
- $P_t$  needs to be estimated

#### Data availability:

- (Usually) good
- Example: For the analysis of EU farm structural change it is available from the Farm Structure Survey at population level

#### Example: Macro data

State	A	B	C
Size	Small	Medium	Large
Farms in $t=0$	60	30	10
Farms in $t=1$	40	40	20

### with micro data...

#### Definition:

- The movements between classes is observed for each individual over time
- The micro transition matrix give the number of individuals transitioning from a specific state in  $t-1$  to a state in  $t$
- $P_t$  can be calculated directly

#### Data availability:

- (Usually) limited
- Example: For the analysis of EU farm structural change a sample is available from the Farm Accountancy Data Network (FADN)

#### Example:

#### Micro transition matrix

to \ from	A	B	C	$\Sigma$
A	30	10	10	60
B	10	10	10	30
C	0	10	0	10
$\Sigma$	40	40	20	

## Posterior

$\propto$

## Likelihood

Macro data based likelihood:  $n_t$  are distributed as weighted sum of independent multinomials (MacRae 1977)

$$L(n_1, \dots, n_T | \beta) = \prod_{t=1}^T \sum_{H_t \in \mathbb{H}_t} \prod_{i=1}^k n_{i,t-1}! \left( \prod_{j=1}^k P_{ijt}^{n_{ij,t}} / n_{ij,t}! \right)$$

A large sample approximation is employed (Brown and Payne 1986)

$\times$

## Prior

A micro data based likelihood function is specified for the prior weights

Micro transitions are multinomial distributed with size equal the number of individuals in the corresponding class in  $t-1$  and probabilities of  $P_t$

$$p(\beta) = \prod_{t=1}^T \prod_{i=1}^k n_{i,t-1}! \left( \prod_{j=1}^k P_{ijt}^{n_{ij,t}} / n_{ij,t}! \right)$$

## in Bayesian estimation of Markov models

## Computation

- A sample from the posterior density is obtained via a random walk Metropolis Hastings algorithm
- The posterior mean, which is the optimal Bayesian estimator under squared error loss, is approximated by the mean of the posterior sample

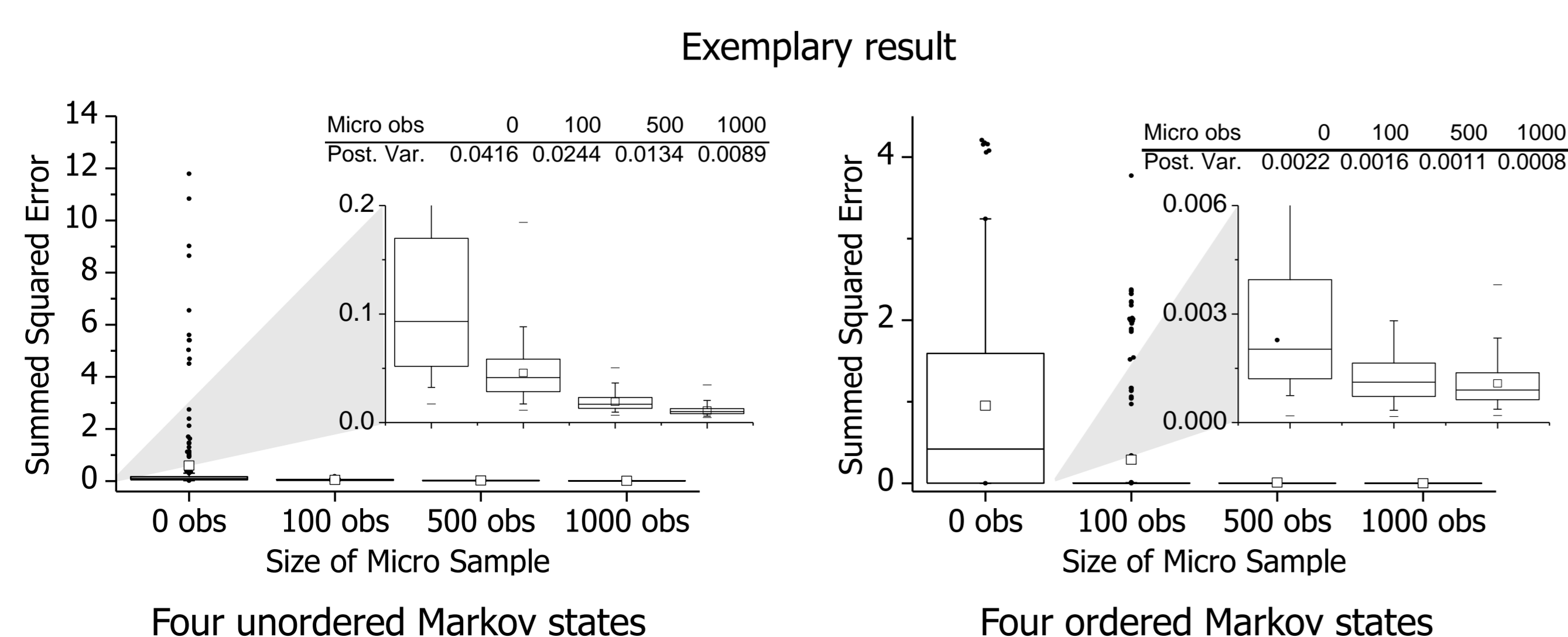
## Monte Carlo Simulation

- **Aim:** Analyse influence of prior on posterior and estimator performance
- Separate simulation for ordered/unordered Markov states
- 10 true models with 20 repetitions each

Scenarios	(1)	(2)	(3)	(4)
Size of micro sample	0	100	500	1000
Markov States (k)	3,4,5			
Size of macro sample	10,000			
Time periods	100			
Expl. variables ( $n_z$ )	6 (incl. a constant)			

## Monte Carlo Results

Box-Whisker plots of the (summed) squared deviation from the true values in the 200 simulation runs and (summed) variance of the posterior density



## Conclusions

- Inclusion of micro data as prior information reduces Mean Square Error (MSE) and posterior variance
- Improvement stronger the more Markov states are considered
- Prior information increases numerical stability of the estimation

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