Polarized Preferences In Homegrown Value Auctions

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Abstract

Incentive compatible auction experiments, often referred to as homegrown value auctions, have become a popular tool for exploring how controversial product attributes and knowledge of these attributes affect consumer willingness to pay. A common observation in these experiments is a prevalence of zero bids and bimodal bid distributions. One possible explanation is that individuals have polarized preferences: find all products with a particular attribute desirable (positive polarization) or undesirable (negative polarization). The purpose of this paper is to explore three questions. Do polarized preferences exist? If they do exist, can they be identified? If they can be identified, does their identification provide useful information? To answer these questions, polarized preferences are theoretically formalized. This theory is used to discuss bidding behavior and how common experimental design features can facilitate or hinder the identification of polarization. The weaknesses of common econometric models used to analyze auction bids are reviewed in the context of polarization and a new model is proposed. Finally, the new model is tested using data from a home grown value auction. The results of this analysis suggest that polarized preferences do exist and that accounting for them can improve estimates of willingness to pay and likelihood that a product is valued at all.

Economic experiments, often referred to as homegrown value auctions, have become a popular tool for determining how a product's attributes and knowledge of its attributes affect desirability. These experiments use incentive compatible auctions to elicit the maximum willingness to pay (WTP) for a product or to trade one product for another.[1] Examples include packaging (Menkhaus et al. 1992; Hoffman et al. 1993), food quality (Melton et al. 1996), food safety (Hayes et al. 1995; Roosen et al. 1998; Brown, Cranfield and Henson 2005; Ward, Bailey and Jensen 2005), and production methods (Buhr et al. 1993; Fox et al. 1994; Fox et al. 1998; Lusk et al. 2001; Fox, Hayes and Shogren 2002; Alfnes and Rickertsen 2003; Huffman et al. 2003; Lusk et al. 2004; Wachenheim, Lambert and VanWechel 2007).

A common observation in these auctions is a prevalence of zero bids (e.g., Table 1). Evidence of bimodal bid distributions has also been reported (Fox at al. 1994; Alfnes and Rickertsen 2003; Huffman et al. 2003). Intuitively, these observations are not too surprising given the controversial nature of many attributes (e.g., irradiated, genetically modify, and pesticide treated). For example, consider an experiment that asks individuals to bid on an irradiated food item. Irradiation substantially reduces the risk of food borne illness due to bacterial or pathogen contamination, however, its long-term health consequences are unknown. If all an individual cares about is food borne illness, he might always value irradiated foods and be willing to pay something for them. If all an individual cares about are unknown long-term health consequences, he might always find irradiated foods undesirable and never be willing to pay anything for them. If an individual cares about both food borne illness and unknown long-term health consequences, he might value some irradiated foods (e.g., foods more likely to be contaminated), but not others (e.g., foods less likely to be contaminated) such that he is willing to pay for some, but not others. This example describes three distinct motives for irradiated food preferences. The first can be referred to as positive polarization — all products with an attribute are desirable. The second can be referred to as negative polarization — all products with an attribute are undesirable. The third can be referred to as unpolarized — some products with an attribute are desirable, while others are undesirable. If individuals fall into these distinct preference categories, a prevalence of zero bids and multimodal bid distributions might be expected.

The purpose of this article is to explore three questions. Do polarized preferences exist? If they do exist, can they be identified? If they can be identified, is this information useful? To answer these questions, the notion of polarized preferences is theoretically formalized. This theory is used to discuss how common experimental design features can facilitate or hinder identification. The weaknesses of common econometric models used to analyze auction data are briefly discussed in the context of identifying polarization before an alternative model is proposed. Finally, the alternative model is demonstrated with data from an auction of invasive and noninvasive plants.

The results and discussion indicate that some experimental design features facilitate, while others hinder, polarization identification. Common econometric models are generally not flexible enough to characterize polarization. The proposed alternative provides additional flexibility. There is evidence of polarization with respect to a plant's invasive and noninvasive attributes. Accounting for polarization yields more accurate estimates of observed behavior.

The contributions include the formal development of preference polarization, an econometric model for identifying polarization, and the presentation of evidence of and value of accounting for polarization. These contributions are important because they provide more refined tools for interpreting behavior in homegrown value auctions, which can help marketers better understand demand for novel product attributes, and policy makers and others better craft information campaigns to encourage or discourage consumption of products with socially desirable or undesirable attributes.

Theoretical Model & Implications

Polarization can be defined in the context of classical theory. Let $x \in \mathbb{R}_+^L$ be a vector of products. Assume preferences are rational, locally nonsatiated, and continuous so they can be represented by a continuous real valued utility function $U(x)$. Partition x into products that share polarizing attributes such that $x = (x_1, \ldots, x_J)$ where J is the number of attributes of interest and $x_j = \left(x_{1j}, \ldots, x_{L_j j}\right)$ for L_j equal to the number of products with attribute j. Note that attributes are chosen to be mutually exclusive ($\sum_{j=1}^{J} L_j = L$). Let $x_{\sim s} = (x_1, \ldots, x_{s-1}, x_{s+1}, \ldots, x_L)$ be a vector of products excluding x_s. Define an attribute j as negatively polarized if for all $x, x' \in \mathbb{R}_+^L$ and all $k = 1, \ldots, L_j$, $x_s = x_{kj} > x_{kj}' = x_s'$ and $x_{\sim s} = x_{\sim s}'$ imply $U(x') > U(x)$. Define an attribute j as positively polarized if for all $x, x' \in \mathbb{R}_+^L$ and all $k = 1, \ldots, L_j$, $x_s = x_{kj} > x_{kj}' = x_s'$ and $x_{\sim s} = x_{\sim s}'$ imply $U(x) > U(x')$. An attribute is unpolarized if it is not negatively and not positively polarized. Intuitively, products with a negatively polarized attribute are always undesirable, while products with a positively polarized attribute are always desirable.

The potential for identifying polarization in an incentive compatible auction can be explored with modest embellishments to the consumer's problem while recognizing that opportunities "in the field" (i.e. outside the lab experiment) can influence behavior within an experiment (Harrison, Harstad, and Rutstrom 2004). Let $m^o > 0$ be income and $p \in \mathbb{R}^L_{++}$ be prices in the field. Assume an individual is endowed with some product $x_s^a \geq 0$ and some income $m^a \geq 0$ inside the lab. Assume x_s^a cannot be resold, but m^a can be spent in the field. Also assume there are no transaction costs. Harrison, Harstad, and Rutstrom (2004) discuss how these last two assumptions can be relaxed if necessary.

Consumption opportunities depend on whether x_s^a is freely disposable. In some experiments, individuals are not required to consume an acquired product and can dispose of them after leaving the experiment. In others, individuals are required to consume an acquired product before leaving the experiment. With free disposal consumption opportunities are

(1) $$B(x_s^a, p, m) = \{x \in \mathbb{R}^L_{+}: p \cdot x \leq m + p_s\, x_s^a \text{ and } p_{\sim s} \cdot x_{\sim s} \leq m\}$$

where $p_{\sim s} = (p_1, \ldots, p_{s\text{-}1}, p_{s+1}, \ldots, p_L)$ and $m = m^o + m^a$. The first constraint in equation (1) captures the notion that an individual can consume more products given the endowment x_s^a. The benefits of this extra consumption are limited however if the individual is not interested in x_s — the second constraint. Without free disposal, opportunities are

(1') $$B(x_s^a, p, m) = \{x \in \mathbb{R}^L_{+}: p \cdot x \leq m + p_s\, x_s^a \text{ and } x_s \geq x_s^a\}.$$

The first constraint in equation (1') again captures the notion that an individual can consume more products given the endowment x_s^a. Now, however, the second constraint implies that the individual is required to consume at least x_s^a of x_s.

Optimal consumption is defined by

(2) $x(x_s^a, p, m) = \{x \in B(x_s^a, p, m): U(x) \geq U(x') \text{ for all } x' \in B(x_s^a, p, m)\}$,

which says an individual maximizes utility. Since $B(x_s^a, p, m)$ is a compact, convex set and $U(x)$ is a continuous real valued function, this problem has a solution. With this solution, the indirect utility function is $V(x_s^a, p, m) = U(x)$ for any $x \in x(x_s^a, p, m)$. With free disposal, $V(x_s^a, p, m)$ satisfies[2]

(i) $V(x_s^a, p, m') > V(x_s^a, p, m)$ for $m' > m > 0$,

(ii) $V(x_s^a, p, m) = V(0, p, m)$ for all $x_s^a > 0$ if $x_s = x_{kj}$ where j is a negatively polarized attribute, and

(iii) $V(x_s^{a\prime}, p, m) > V(x_s^a, p, m)$ for all $x_s^{a\prime} > x_s^a \geq 0$ if $x_s = x_{kj}$ where j is a positively polarized attribute.

Intuitively, (i) says that more income increases utility (Walras Law). Property (ii) says that endowing an individual with a product that has a negatively polarized attribute does not affect utility because undesirable products can be freely disposed. Property (iii) says that endowing an individual with a product that has a positively polarized attribute increases utility. Without free disposal, $V(x_s^a, p, m)$ satisfies

(i') $V(x_s^a, p, m') > V(x_s^a, p, m)$ for all $m' > m > 0$ if $x_{\sim s}$ satisfies local nonsatiation or $x_s = x_{kj}$ where j is a positively polarized attribute,[3] (ii') $V(x_s^a, p, m) > V(x_s^{a'}, p, m)$ for all $x_s^{a'} > x_s^a \geq 0$ if $x_s = x_{kj}$ where j is a negatively polarized attribute, and

(iii') $V(x_s^{a'}, p, m) > V(x_s^a, p, m)$ for all $x_s^{a'} > x_s^a \geq 0$ if $x_s = x_{kj}$ where j is a positively polarized attribute.

Property (i') again says that more income increases utility. This property requires the additional innocuous assumption that there are products other than x_s that are desirable or that x_s is positively polarized. Property (ii') says that endowing an individual with a product that has a negatively polarized attribute decreases utility, while property (iii) says that endowing an individual with a product that has a positively polarized attribute increases utility. This indirect utility function and its properties facilitate the evaluation of optimal bidding behavior in an incentive compatible auction.

Single Product Auctions Without An Endowment

The simplest auction is a single product auction. Assume the product is $x_{kj}^a > 0$. Let $b_{kj}(t_j)$ be the optimal bid for x_{kj}^a given preferences for j equal to t_j where $t_j = +$ for positively polarized, $t_j = –$ for negatively polarized, and $t_j = \pm$ for unpolarized preferences. The optimal bid is

(3) $$b_{kj}(t_j) = \max\{w \geq 0: V(x_{kj}^a, p, m - w) \geq V(0, p, m)\},$$

which says to choose the largest possible non-negative bid such that acquiring the product does not decrease welfare. The assumption that bids cannot be negative is consistent with the vast majority of the homegrown auction literature. This bid depends on polarization, but not free disposal. If preferences are positively polarized, $b_{kj}(+) > 0$ by property (i) and (iii) or (i') and (iii'). For negatively polarized preferences, $b_{kj}(-) = 0$

by property (i) and (ii) or (i') and (ii'). For unpolarized preferences, $b_{kj}(\pm) \geq 0$ depending on whether the product is desirable. The only distinguishing characteristic of these bids in terms of polarization is whether they are equal to or greater than zero. A positive bid signals positively polarized or unpolarized preferences, while a zero bid signals negatively polarized or unpolarized preferences. Therefore, an individual's bid conveys some information on polarization, but does not precisely discern it.

Single Product Auctions With An Endowment

In many auctions, individuals are endowed with one product and given the opportunity to trade for another. In these instances, an individual's optimal bid depends on free disposal. With free disposal, the optimal bid is

(3') $b_{kjk'j'}(t_j,\ t_{j'}) = \max\ \{w \geq 0: V(x_{kj}^a, p, m - w) \geq V(x_{k'j'}^a, p, m)\}$

where $x_{kj}^a > 0$ is the product offered for trade, $x_{k'j'}^a > 0$ is the endowed product, and t_j and $t_{j'}$ are polarization for attributes j and j'. As with equation (3), equation (3') says to choose the largest possible non-negative bid such that trading $x_{k'j'}^a$ for x_{kj}^a does not decrease welfare. This bid will depend on polarization for both products.

There are nine possible types of polarization: $(t_j,\ t_{j'}) \in$ {(+,+),(±,+),(-,+),(+,±),(±,±),(-,±),(+,-),(±,-),(-,-)}. Properties (i) – (iii) imply a zero bid may result when $(t_j,\ t_{j'}) \in$ {(-,+), (-,±), (-,-), (+,+), (±,+), (+,±), (±, ±), (±,-)}, while a positive bid may result when $(t_j,\ t_{j'}) \in$ {(+,+), (±,+), (+,±), (±, ±), (±,-), (+,-)}. Therefore, a positive bid signals that preferences are not negatively polarized for the auctioned product, and a zero bid signals that preferences are not positively polarized for the auctioned product and negatively polarized for the endowed product. An individual's bid again provides some information on polarization, but does not discern it precisely.

Without free disposal, there are still nine types of polarization and the optimal bid is defined by equation (3'). However, equation (3') does not capture the full extent of the choice set because an individual is expected to consume any product acquired during the experiment. Ethical standards for experimental research preclude forced consumption, so an individual may refuse. Still, refusal typically results in the forfeiture of some monetary payment, say m^a. If

(4) $V(0, p, m^o) > V(x_{kj}^a, p, m^o + m^a)$ and $V(0, p, m^o) > V(x_{k'j'}^a, p, m^o + m^a)$,

refusal is optimal. Therefore, there are three observable behaviors that signal polarization: a positive bid, a zero bid, and refusal to complete the experiment.

With free disposal, completion of the experiment is always optimal because the individual can always keep m^a and throw away $x_{k'j'}^a$. Without free disposal, properties (i') – (iii') imply that refusal to complete the experiment will only occur if both *j* and *j'* are negatively polarized. Negative polarization for *j* and *j'* is not sufficient for refusal because giving up m^a may be more distasteful than consuming x_{kj}^a or $x_{k'j'}^a$. If an individual does not refuse, properties (i') – (iii') imply that a zero bid may result when $(t_j, t_{j'}) \in$ {(-,+), (+, +), (±,+), (+,±), (±, ±), (-,±), (±,-), (-,-)}, while a positive bid may result when $(t_j, t_{j'}) \in$ {(+,+), (±,+), (+,±), (±, ±), (-,±), (±,-), (-,-), (+,-)}. Therefore, refusal signals preferences that are negatively polarized with respect to the auctioned and endowed product, a zero bid signals preferences that are not positively polarized for the auctioned product and negatively polarized for the endowed product, and a positive bid signals preferences that are not negatively polarized for the auctioned product and positively polarized for the endowed product. Bids do not provide a completely informative signal for polarization, though refusal is completely informative.

It is useful to summarize the informational implications of an individual's experimental behavior in a single product auction. Table 2 summarizes the information sets implied by the three possible signals based on the auction's characteristics and conditioning information.

Comparing an auction without an endowment, regardless of free disposal, to an auction with an endowment and free disposal, an auction without an endowment is always at least as informative, and typically more informative, about the polarization of the auctioned product. This is true because the information set implied by a positive or zero bid is at least as refined (e.g., {+, ±} and {–, ±} as compared to {+, ±} and {–, ±} or {+, ±} and {+, –, ±}). The only case when an auction without an endowment is not strictly more informative is when the endowed product is known to be negatively polarized a priori. While one might consider endowing an individual with a product of unknown polarization and having them trade for a product with known polarization, such a strategy does not improve the information conveyed by bids and may in fact make them less informative (e.g., if the auctioned product is known to be unpolarized).

Comparing an auction without an endowment to an auction with an endowment and without free disposal is generally ambiguous because the completely informative refusal signal is introduced with the endowment. For individuals who would choose to complete the experiment with an endowment, an auction without an endowment will always be more informative because it produces more refined information sets (e.g., {+, ±} and {–, ±} as compared to {+, ±} and {+, –, ±} or {+, –, ±}). Therefore, there is an informational tradeoff between auctions without an endowment and auctions with an endowment and without free disposal. Without an endowment, more information can be obtained on positively polarized and unpolarized preferences. With an endowment, more information can be obtained on negatively polarized preferences.

A more disturbing revelation from these results is how little information about polarization is conveyed by an individual's bid in a single product auction. This paucity of information typically makes it impossible to use individual bids to discriminate between positively polarized and unpolarized preferences, and negatively polarized and unpolarized preferences.

Simultaneous Product Auction Without An Endowment

Many experiments in the literature have simultaneously auctioned multiple products either with or without an endowment. To avoid demand reduction bias (List and Lucking-Reilly 2000), these experiments randomly select one auction as binding. Since the theory above suggests auctions without endowments are more informative in terms of polarization when an individual does not refuse to complete the experiment, the discussion of simultaneous product auctions focuses on the case without endowments. Showing that these informational advantages are true for simultaneous product auctions is not difficult, just tedious.

Suppose an individual participates in L_j^s incentive compatible auctions simultaneously where $L_j \geq L_j^s > 1$. Let the auctioned products be denoted by x_{kj}^a for k =

1,…, L_j^s. The optimal bid $b_{kj}(t_j)$ for each x_{kj}^a is defined by equation (3). Observationally, there are L_j^s bids, one for each product. In terms of polarization, whether a bid for a particular product is zero or positive is the only distinguishable characteristic. For L_j^s auctions, there are $2^{L_j^s}$ bid patterns that can be used to make inferences. If $b_{kj}(t_j) = 0$ for all $k = 1,\ldots, L_j^s$, j is not positively polarized. If $b_{kj}(t_j) > 0$ for all $k = 1,\ldots, L_j^s$, j is not negatively polarized. For the remaining $2^{L_j^s} - 2$ possible bid combinations where $b_{kj}(t_j) = 0$ and $b_{k'j}(t_j) > 0$ for some $k, k' \in \{1,\ldots, L_j^s\}$ and $k \neq k'$, j is unpolarized — the signal is completely revealing. Therefore, simultaneous auctions without an endowment are at least as informative and can be more informative than single product auctions without an endowment. Indeed, if $L_j^s = L_j$ such that all products with the potentially polarizing attribute j are represented in the auction, the auction will be completely informative.

Econometric Implications

The goal of many homegrown value auctions is to estimate the WTP or changes in the WTP when factors like the information provided to individuals changes. Since the primary purpose of this article is to assess the existence of polarized preferences and possibilities for identification, econometric implications are explored in the context of estimating the WTP. Implications for assessing changes in the WTP are left for future work. The scope is also limited to auctions without endowments, since these types of auctions tend to be more informative for polarization.

For an econometric perspective, suppose n individuals participate in an auction experiment and let $N = \{1,\ldots, n\}$. Let $V_i(x_{kj}^a, p, m_i)$ be the ith individual's indirect utility function where m_i is individual income. An individual's WTP for the product x_{kj}^a is defined by $W_{ikj} = \max\{w \in \mathbb{R}: V_i(x_{kj}^a, p, m_i - w) \geq V(0, p, m_i)\}$. With free disposal, $p_{kj}x_{kj}^a \geq W_{ikj} \geq 0$, while without free disposal $p_{kj}x_{kj}^a \geq W_{ikj}$ if j is positively polarized or $x_{\sim s}$ for $x_s = x_{kj}^a$ is locally nonsatiated (see proof in the appendix). Equation (3) and property (i) imply that this WTP can be written as

(5) $W_{ikj} = m_i - V_i^{-1}(x_{kj}^a, p, V_i(0, p, m_i))$ or

(6) $W_{ikj} = \mu_{kj} + e_{ikj}$

where μ_{kj} is the expected WTP for all individuals and e_{ikj} is how the individual's WTP differs from the expected value due to unobservable differences in preferences.

The estimation of equation (6) is complicated by several factors. First, the properties of the distribution of e_{ikj} depend on whether there is free disposal. Second, the properties of the distribution of e_{ikj} also depend on polarization, which is not directly observable. To better understand these complications, let $w_{ikj}{}^{\tau}$ be an individual's WTP, $\mu_{jk}{}^{\tau}$ be the expected WTP, and $f_{jk}{}^{\tau}(\varepsilon_{ikj}{}^{\tau})$ be the density of $\varepsilon_{ikj}{}^{\tau}$ conditional on polarization t. With free disposal, positively polarized preferences imply $p_{kj}x_{kj}^a \geq w_{ikj}^+ = \mu_{kj}^+ + e_{ikj}^+ > 0$ such that $f_{jk}{}^{+}(\varepsilon_{ikj}{}^{+}) > 0$ for $\varepsilon_{ikj}{}^{\tau} \in (-\mu_{kj}^+, p_{kj}x_{kj}^a - \mu_{kj}^+]$ and zero otherwise. Negatively polarized preferences imply $w_{ikj}^- = 0$ such that $\mu_{kj}^- = 0$ and $f_{jk}{}^{-}(\varepsilon_{ikj}{}^{-}) = 1$ for $e_{ikj}^- = 0$ and zero otherwise. Unpolarized preferences imply $p_{kj}x_{kj}^a \geq w_{ikj}^{\pm} = \mu_{kj}^{\pm} + e_{ikj}^{\pm} \geq 0$ such that $f_{jk}{}^{\pm}(\varepsilon_{ikj}{}^{\pm}) > 0$ for $e_{ikj}^{\pm} \in [-\mu_{kj}^{\pm}, p_{kj}x_{kj}^a - \mu_{kj}^{\pm}]$ and zero otherwise. Without free disposal, the implications of positively polarized preferences do not change, while negatively polarized preferences imply $w_{ikj}^- = \mu_{kj}^- + e_{ikj}^- < 0$ such that $f_{jk}{}^{-}(\varepsilon_{ikj}{}^{-}) > 0$ for $e_{ikj}^- \in [-\infty, -\mu_{kj}^-)$ and zero otherwise, and unpolarized preferences imply $p_{kj}x_{kj}^{a} \geq w_{ikj}^{\pm} = m_{kj}^{\pm} + e_{ikj}^{\pm}$ can be positive or negative such that $f_{jk}^{\pm}(\varepsilon_{ikj}^{\pm}) > 0$ for $e_{ikj}^{\pm} \in [-\infty, p_{kj}x_{kj}^{a} - m_{kj}^{\pm}]$ and zero otherwise. Unconditionally, Bayes rule implies

$$(7) \qquad f_{kj}(\varepsilon_{ikj}) = q_j^{+}f_{jk}^{+}(\varepsilon_{ikj}) + q_j^{\pm}f_{jk}^{\pm}(\varepsilon_{ikj}) + q_j^{-}f_{jk}^{-}(\varepsilon_{ikj})$$

for $e_{ikj} \in [-\infty, \infty]$ where $q_j^{\tau} \geq 0$ is the probability of polarization t such that $q_j^{+} + q_j^{\pm} + q_j^{-} = 1$.

Figure 1 illustrates the implications of polarization on the WTP distribution with and without free disposal for alternative degrees of polarization: (a) $q_j^{+} = q_j^{\pm} = q_j^{-} = 1/3$, (b) $q_j^{+} = q_j^{\pm} = ½$ and $q_j^{-} = 0$, (c) $q_j^{\pm} = q_j^{-} = ½$ and $q_j^{+} = 0$, and (d) $q_j^{+} = q_j^{-} = ½$ and $q_j^{\pm} = 0$. The figure assumes that x_{kj}^{a} is not traded outside the lab ($p_{kj} = \infty$). The figure also assumes that without free disposal positively polarized preferences come from a truncated normal distribution with mean 4 and variance 1, negatively polarized preferences come from a truncated normal distribution with mean -2 and variance 1, and unpolarized preferences come from a normal distribution with mean 1 and variance 1. A key insight is that polarized preferences can produce unimodal, bimodal, or even multimodal WTP distributions, which raises the question of whether the econometric models commonly used to analyze auction data are adequate for identifying polarization and accounting for its implications.

The most commonly used econometric models are the tobit and double hurdle models. With the tobit, w_{ikj} in equation (6) is treated as a latent variable and e_{ikj} is assumed to be normally distributed with a mean 0 and variance σ_{kj}^{2}. The likelihood of a positive bid b_{ikj} is $\Pr_{ikj}(b_{ikj}) = f(b_{ikj} - m_{kj}, \sigma_{kj}^{2})$ where $f(e, s^{2})$ is the normal density with mean 0 and variance σ^2. The probability of a zero bid is $\Pr_{ikj}(0) = \Phi(-\,m_{kj} / \sigma_{kj})$ where $\Phi(e)$ is the cumulative standard normal. Under free disposal, the tobit implies

$$(8) \qquad f_{kj}(\varepsilon_{ikj}) = \begin{cases} 0, & \text{for } \varepsilon_{ikj} < -\mu_{jk} \\ \Phi\left(\frac{-\mu_{jk}}{\sigma_{jk}}\right), & \text{for } \varepsilon_{ikj} = -\mu_{jk} \\ \phi(\varepsilon_{ikj}, \sigma_{jk}^2), & \text{otherwise} \end{cases},$$

while without free disposal

$$(8') \qquad f_{kj}(\varepsilon_{ikj}) = \phi(\varepsilon_{ikj}, \sigma_{jk}^2) \text{ for } e_{ikj} \in [-\infty, \infty].$$

Equation (8) is a unimodal or bimodal density like panel (c) or (d) in Figure 1 with free disposal. It is not however flexible enough to produce densities similar to those observed in panels (a) and (b). Equation (8') is a unimodal density that is not flexible enough to capture the multimodal patterns exhibited in Figure 1 without free disposal.

Double hurdle models break up estimation into two parts. The first part estimates the probability that an individual is unwilling to trade or bids zero, while the second part captures the probability of a bid given the individual is willing to trade or has submitted a positive bid. Generally, the probability of a zero bid can be written as $\Pr_{ikj}(b_{ikj} = 0) = G(-m_{kj})(1 - Q) + Q$ where $G(\cdot)$ is a cumulative distribution and Q is the probability that an individual is unwilling to trade, while the probability of a positive bid can be written as $\Pr_{ikj}(b_{ikj}) = g(b_{ikj} - m_{kj})\,(1 - Q)$ where $g(\cdot)$ is the density of $G(\cdot)$. In these models, the probability Q is somewhat analogous to the probability of negatively polarized preferences in equation (7). There is however no differentiation between positively polarized or unpolarized preferences. This lack of differentiation means the WTP distribution implied by a double hurdle model is unimodal like panel (c) or bimodal like panel (d) in Figure 1 when there is free disposal and $g(\cdot)$ is unimodal as typically specified (e.g., normal or truncated normal). The standard models do not produce the types of bimodal or multimodal densities seen in Figure 1 without free disposal or the bimodal densities seen in panels (a) and (b) with free disposal.

There may be cases where a tobit or double hurdle model reasonably approximate the WTP distribution when preferences are polarized. However, Figure 1 suggests that there are also cases where these models will not adequately describe variation in the WTP across individuals, which raises the question of whether it is possible to develop an alternative more flexible model. Equation (7) is suggestive. Like the double hurdle model, the estimation problem can be conceptualized in two parts. In the first part, the probability that an individual has positively polarized, negatively polarized, or unpolarized preferences is considered. In the second part, the individual's WTP given polarization is considered. In this context where ${F_{kj}}^{\pm}(\cdot)$ is the cumulative distribution of ${f_{kj}}^{\pm}(\cdot)$, the probability of a zero bid is

(9) $$\Pr_{ikj}(0) = {q_j}^{\pm}{F_{kj}}^{\pm}\left(-{\mu_{jk}}^{\pm}\right) + {q_j}^{-},$$

which is the probability that an individual with unpolarized preferences bids zero plus the probability that an individual has negatively polarized preferences. The probability of a positive bid is

(10) $$\Pr_{ikj}(b_{ikj}) = {q_j}^{+}{f_{kj}}^{+}\left(b_{ikj} - {\mu_{jk}}^{+}\right) + {q_j}^{\pm}{f_{kj}}^{\pm}\left(b_{ikj} - {\mu_{jk}}^{\pm}\right),$$

which is the probability that an individual with positively polarized preferences bids b_{ikj} plus the probability that an individual with unpolarized preferences bids b_{ikj}. In an auction with a single product, these probabilities yield the log-likelihood function

(11) $L = \sum_{i=1}^{N} ln\left(d_{ikj}\text{Pr}_{ikj}(b_{ikj}) + (1 - d_{ikj})\text{Pr}_{ikj}(0)\right)$

where $d_{ikj} = 1$ for $b_{ikj} > 0$ and zero otherwise.

Equations (9) – (11) consider the analysis of bids from a single product auction. With simultaneous auctions where products share an attribute j, there is additional information to be taken into account. Recall that for L_j^s auctions where $j = 1, \ldots, L_j^s$, there are $2^{L_j^S}$ bidding patterns to convey polarization information. Define $\Omega_j^+ = \{i \in N$: $b_{ikj} > 0$ for all $j = 1, \ldots, L_j^s\}$ as the set of individuals whose preferences are not negatively polarized, $\Omega_j^- = \{i \in N$: $b_{ikj} = 0$ for all $j = 1, \ldots, L_j^s\}$ as the set of individuals whose preferences are not positively polarized, and $\Omega_j^{\pm} = \{i \in N$: $i \notin \Omega_j^+$ and $i \notin \Omega_j^-\}$ as the set of individuals whose preferences are definitely unpolarized. The probability of observing all positive bids is

(12) $\text{Pr}_{ikj}^+ = q_j^+ \prod_{k=1}^{L_j^S} f_{kj}^+(b_{ikj} - \mu_{jk}^+) + q_j^{\pm} \prod_{k=1}^{L_j^S} f_{kj}^{\pm}(b_{ikj} - \mu_{jk}^{\pm})$.

The first term on the right-hand-side of equation (12) has two parts: the probability that an individual has positively polarized preferences and the probability of observing $b_{ikj} > 0$ for all k given positively polarized preferences. The second term also has two parts: the probability that an individual has unpolarized preferences and the probability of observing $b_{ikj} > 0$ for all k given unpolarized preferences. The probability of observing all zero bids is

(13) $\text{Pr}_{ikj}^- = q_j^{\pm} \prod_{k=1}^{L_j^S} F_{kj}^{\pm}(-\mu_{jk}^{\pm}) + q_j^-$.

For equation (13), the first term on the right-hand-side again has two parts: the probability that an individual has unpolarized preferences and the probability of observing $b_{ikj} = 0$ for all k given unpolarized preferences. The second term simply reflects the probability of negatively polarized preferences, since the probability $b_{ikj} = 0$ for all k given negatively polarized preferences is one. The probability of observing some positive and some zero bids is

$$(14) \quad \Pr_{ikj}^{\pm} = q_j^{\pm} \prod_{k=1}^{L_j^S} \left(d_{ikj} f_{kj}^{\pm}(b_{ikj} - \mu_{jk}^{\pm}) + (1 - d_{ikj}) F_{kj}^{\pm}(-\mu_{jk}^{\pm}) \right).$$

Equation (14) also has two parts, but the second part can be further divided into another two parts. The first part reflects the probability that an individual has unpolarized preferences. The first term in the second part captures the probability of positive bids given unpolarized preferences, while the second term captures the probability of zero bids given unpolarized preferences. Equation (12) – (14) can be combined into the log-likelihood

$$(15) \quad L = \sum_{\tau \in \{+,\pm,-\}} \sum_{i \in \Omega_j^{\tau}} ln\left(\Pr_{ikj}^{\tau}\right).$$

Taxonomically, equations (12)-(15) describe a censored-finite-mixture model. With free disposal, this model can capture the types of densities seen in figure 1. Without free disposal, the model will not capture the peaks to the left in panels (a), (c), and (d), which are attributable to negative polarization. These peaks are missed because individuals with negatively polarized preferences always bid zero, which yields no information on $f_{jk}^{-}(\varepsilon_{ikj}^{-})$, but this issue is not unique to the proposed model.

An Example

The existence of polarization was explored by using the econometric model proposed in equations (12) – (15) to analyze data from an ornamental plant auction that was conducted in St. Paul, Minnesota, during April of 2007. The primary purpose of the experiment was to determine if labeling plants based on their invasive and noninvasive, and native and nonnative attributes affected an individual's WTP. Since invasive plants can cause environmental or economic harm, or harm to human health (Executive Order 13112), it was hypothesized that labeling plants as invasive would make them less desirable. This hypothesis is supported by survey data where 98% of the respondents said they would not buy plants labeled as invasive (Reichard and White 2001).

In the experiment, a two round, 2^{nd} – price Vickery auction was used to elicit the WTP for ten different ornamental plants. The plants were paired such that plants in a pair were almost identical in appearance. Different pairs of plants differed notably in appearance. The plants also differed in terms of their invasive attribute, with one invasive and one noninvasive plant in each pair. The native and nonnative attribute was not varied systematically. Individuals were not told about these attributes in the first round, while they were told in the second. More details regarding the experiment are reported in Yue, Hurley, and Anderson (2011), as is an analysis of how bids changed from one round to the next. As expected, the authors found that individuals discounted plants with the invasive attribute. What the authors did not discern is the proportion of individuals who would likely never buy invasive plants.

Econometric Implementation

The econometric model was used with second round bids to simultaneously estimate WTP distributions for the five invasive plants. The analysis was also conducted separately with the five noninvasive plants. First round bids were not analyzed because individuals had not been told which plants were invasive. The invasive attribute was explored because negatively polarization seemed likely. The noninvasive attribute was analyzed because positive polarization seemed likely.

Implementing the model requires $f_{kj}{}^{+}(b_{ikj} - \mu_{jk}{}^{+})$ and $f_{kj}{}^{\pm}(b_{ikj} - \mu_{jk}{}^{\pm})$ to be explicitly defined. For $f_{kj}{}^{+}(b_{ikj} - \mu_{jk}{}^{+})$, the log-normal distribution was used because it constrains the WTP to be greater than 0. For $f_{kj}{}^{\pm}(b_{ikj} - \mu_{jk}{}^{\pm})$, a normal density was used. The log-likelihood function was programmed in STATA and optimized using STATA's ml command. Since it is not uncommon for finite-mixture models to have local optima (Titterington et al. 1985), a range of starting values were used.

Polarization tests were conducted by estimating three restricted models in addition to the unrestricted model, referred to as Model 1. Model 2 assumed no negative polarization such that $q_j{}^{-} = 0$ and $q_j{}^{+} \geq 0$. Model 3 assumed no positive polarization such that $q_j{}^{-} \geq 0$ and $q_j{}^{+} = 0$. Model 4 assumed no negative and no positive polarization such that $q_j{}^{-} = 0$ and $q_j{}^{+} = 0$, which is analogous to a tobit analysis. While Models 2 - 4 are nested in Model 1, statistical tests based on the likelihood ratio statistic and the χ^2 distribution are not valid because the null hypothesis implies restrictions on the boundary of the parameter space (Titterington et al. 1985). Therefore, model comparisons were accomplished using the parametric bootstrapping method described in Schlattmann (2009). For example, to compare Models 1 and 2, estimates for Model 2 were used to simulate 499 replicates of the experimental data. The original data was used for the 500th replicate. Model 1 and 2 were then estimated for each replicate in order to calculate the distribution of the likelihood ratio statistic and its *p*-value assuming Model 2 was the true model. The procedure was repeated to compare Model 1 with Model 3 and 4.

Results

Table 3 provides estimates and standard errors calculated using the bootstrapping method described above for the mean WTP for positively polarized and unpolarized preferences. Table 4 provides estimates and standard errors for the standard deviation of the WTP. Table 5 provides estimates and 90% confidence intervals calculated using the bootstrapping method described above for the probability of positive and negative polarization. It also reports the maximized log-likelihood, and comparisons Model 1 to Models 2 - 4 based on the log-likelihood ratio statistic and its boot strapped distribution.

Three results are immediately clear from Tables 3 and 4. First, assuming positive and negative polarization, the mean WTPs and standard deviations for positively polarized preferences are higher than for unpolarized preferences. While this result might be expected because individuals with positively polarized preferences always value the attribute, it need not be the case if other attributes are not valued as highly as they are for unpolarized individuals. Second, the mean WTPs and standard deviations for unpolarized preferences are larger when no positive polarization is assumed, which is to be expected given the first result. Finally, assuming no positive polarization, estimates for the mean WTPs and standard deviations for unpolarized preferences are nearly identical regardless of whether negative polarization is also assumed — all of the differences in the unpolarized parameter estimates between Models 3 and 4 are within \$0.02. When positive polarization is assumed, whether negative polarization is also assumed appears to matter more — 80% of the difference in the unpolarized parameter estimates between Models 1 and 2 are greater than \$0.03, with more than half greater than \$0.08.

The results in Table 5 explore which assumptions regarding polarization are best. The estimates and confidence intervals for the probabilities of positively and negatively polarized preferences support the existence of both for the invasive attribute. While positive polarization is also supported for the noninvasive attribute, negative polarization is not. Comparisons based on the log-likelihood ratio statistic favor Model 1 over Models 2 – 4 for both the invasive and noninvasive attribute. While these results may seem to contradict the results based on estimates of the probability of negative polarization for the noninvasive attribute, this difference can be explained by how accounting for negative, as well as positive, polarization effects estimates of the mean and standard deviation of the WTP for positively polarized and unpolarized preferences. The individual parameter test for negative polarization ignores these differences, while the likelihood ratio test does not.

The weight of evidence overwhelming supports negatively polarized, positively polarized, and unpolarized preferences. To better understand the more practical implications of these results, model predictions of observable bidding behavior were compared. Figure 2 shows the observed average bids and standard deviations by plant and plant attribute along with each models' estimates. The figure also shows 90% confidence intervals based on the parametric bootstrapping method described above. All four models produce similar estimates for the observed average bid. Furthermore, all of the observed average bids fall within the confidence intervals for all the model estimates. While the observed standard deviations are also all within the confidence intervals for Models 1 and 2, this is not the case for Models 3 and 4. For Model 3, the observed standard deviations fall outside the confidence intervals for the second and fifth noninvasive plant. For Model 4, the observed standard deviations fall outside the confidence intervals for the first, second, and fifth noninvasive plant. Figure 3 shows the observed average of positive bids and standard deviations with each model's estimates and 90% confidence intervals. For Models 1 and 2, the observed averages and standard deviations are all within the confidence intervals. This is not the case for Models 3 and 4. For Model 3, 70% of the observed averages and 90% of the standard deviations fall outside the confidence intervals. For Model 4, 50% of observed averages and 80% of the standard deviations fall outside the confidence intervals. Figure 4 shows the observed proportion of zero bids with each model's estimates and 90% confidence intervals. For Models 1 and 2, all but one of the observed proportions fall within the confidence intervals. Again, this is not the case for Models 3 and 4 with 50% of the observed proportions falling outside the confidence intervals.

The results in Figure 2 suggest that reasonable estimates of the mean WTP may still be obtained without accounting for polarization. However, ignoring polarization, may not lead to reasonable estimates of the variation in the WTP. Figures 3 and 4 suggest that accounting for polarization, particularly positive polarization, can provide additional insights that cannot be confidently obtained by ignoring polarization. For example, the average WTP and its variation for individuals who value a particular product, and the proportion of individuals who do not value a particular product.

Summary & Conclusions

Home grown value experimental auctions have served as a useful tool for understanding the effect of different product attributes and knowledge of these attributes on preferences. Often the attribute of interest is controversial such that some individuals find it quite desirable or quite distasteful. Unfortunately, it is often difficult to predict a priori whether individuals will find an attribute desirable or distasteful. It is also often the case that preferences vary widely. Some individuals find an attribute desirable, while others find it distasteful. A challenge that has emerged from such varied and unpredictable preferences is the prevalence of difficult to interpret zero bids and multimodal bid distributions. A variety of experimental design and econometric strategies have been employed to address this challenge.

The purpose of this article was to propose a theoretical framework for conceptualizing such varied preferences and an econometric model for better characterizing their implications. The theoretical framework categorizes individuals as positively polarized, negatively polarized, or unpolarized in terms of their preference for an attribute. Positively polarized individuals always value products with the attribute, while negatively polarized individuals never value these products. Unpolarized individuals value some of these products, but not others depending on other product attributes. The econometric model is a censored-finite-mixture model. With this model, it is possible to estimate the probability that an individual has positively polarized, negatively polarized, and unploarized preferences. It is also possible to estimate the WTP distribution conditional on an individual having positively polarized or unpolarized preferences.

Using the proposed econometric model, evidence of preference polarization was found in data from a home grown value auction for invasive and noninvasive ornamental plants. Accounting for this polarization, particularly positive polarization, produced better estimates of observed bidding behavior. While accounting for negative polarization in addition to positive polarization did not substantially improve estimates of observed behavior, it did improve the fit of the econometric model based on the log-likelihood ratio statistic. There are also more pragmatic reasons for identifying the probability of negative polarization. For example, suppose a regulatory agency is considering the mandatory labeling of invasive ornamental plants to discourage individuals from purchasing them. Assuming no polarization, it is easy to conclude that about one in three consumers would no longer purchase these labeled plants. Accounting for polarization, it is difficult to conclude that more than one in six would no longer purchase these plants. The difference in these assessments of the policy is that about one in six found some invasive plants are undesirable, while others are desirable. For these consumers, the policy is unlikely to eliminate invasive plant purchases because they may simply purchase those invasive plants that they find desirable. Accounting for negative polarization, the regulator can be more confident about the labeling policy's effect.

More work remains to fully understand the utility of using polarization to better understand individual preferences. Polarization provides one mechanism for accounting for correlation in individual bidding behavior attributable to a particular product attribute. Another alternative is to estimate a simultaneous equation tobit or double hurdle model with correlated errors, though it is unclear that such models would be capable of capturing the types of multimodal bid distributions observed in previous experiments. While the work in this manuscript focuses on estimating the WTP, many homegrown value auction experiments are interested in estimating differences in the WTP. The implications of polarization in terms of estimating these differences remain to be explored. As noted in the discussion on auctions with endowments, the challenge to understanding these implications is the potential for differences in polarization to confound the interpretation of observed differences in the WTP. The proposed model does not make it possible to estimate the distribution of the WTP for negatively polarized individuals when there is no free disposal and non-negative bids are not permitted. Further research might explore how this limitation might be overcome. One possibility is to allow individuals to submit negative bids like Parkhurst, Shogren, and Dickinson (2004). Another is to further explore the implications of conducting simultaneous auctions with an endowment. Here the challenge is to determine if it is possible to structure the auctions to reduce confounding nature of the endowment or to be able to unequivocally determine the polarization of the endowed product.

Footnotes

[1] Alternatively, some experiments elicit the minimum willingness to accept to make a trade.

[2] Proof of these properties and other important results are available from the authors upon request.

[3] For $x = (x_s, x_{\sim s}) \in \mathbb{R}_+^L$, $x_{\sim s} \in \mathbb{R}_+^{L-1}$ satisfies the property of local nonsatiation if for every $x_s \in \mathbb{R}_+$, $x_{\sim s} \in \mathbb{R}_+^{L-1}$ and $e > 0$, there is $y_{\sim s} \in \mathbb{R}_+^{L-1}$ such that $\|y_{\sim s} - x_{\sim s}\| \leq e$ and $(x_s, y_{\sim s})$ **f** $(x_s, x_{\sim s})$.

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Table 1. Prevalence of zero bids in experimental auctions.

Study	Zero Bids (%)	Comments
Buhr et al. (1993)	15 - 87	Reported only for selected trials. Varied by trial and treatment.
Fox et al. (1994)	53	Reported only for the 20th trial.
Hayes et al. (1995)	0	
Fox et al. (1998)	12 - 50	Depended on treatment.
Roosen et al. (1998)	35 - 72	Depended on trial and product attributes.
Lusk et al. (2001)	63 - 83	Depended on type of auction.
Fox et al. (2002)	32 - 40	Depended on trial.
Alfnes and Rickertsen (2003)	0.4 - 25	Depended on product attribute and treatment.
Huffman et al. (2003)	8 - 26	Depended on trial and product attributes.
Lusk et al. (2004)	11	
Brown et al. (2005)	8 - 10	Depended on treatment and product attributes.
Wachenheim et al. (2007)	3.5	

Table 2. Information sets implied by the signal, auction characteristics, and conditioning information in a single product auction.

Auction Characteristics		**Signal**		
Endowment	Free Disposal	Positive Bid	Zero Bid	Refusal
			Auctioned Product	
No	Yes	{+, ±}	{–, ±}	
No	No	{+, ±}	{–, ±}	
Yes	Yes	{+, ±}	{+, –, ±}	
Yes	No	{+, –, ±}	{+, –, ±}	{–}
			Endowed Product	
Yes	Yes	{+, –, ±}	{+, –, ±}	
Yes	No	{+, –, ±}	{+, –, ±}	{–}
			Auctioned & Endowed Products	
Yes	Yes	{(+, +), (+, –), (+, ±), (±, +), (±, –), (±, ±)}	{(+, +), (+, ±), (–, +), (–, –), (–, ±), (±, +), (±, –), (±, ±)}	
Yes	No	{(+, +), (+, –), (+, ±), (–, –), (–, ±), (±, +), (±, –), (±, ±)}	{(+, +), (+, ±), (–, +), (–, –), (–, ±), (±, +), (±, –), (±, ±)}	{(–, –)}
			Auctioned Product Given + Polarized Endowment	
Yes	Yes	{+, ±}	{+, –, ±}	
Yes	No	{+, ±}	{+, –, ±}	{–}
			Auctioned Product Given – Polarized Endowment	
Yes	Yes	{+, ±}	{–, ±}	
Yes	No	{+, –, ±}	{–, ±}	{–}
			Auctioned Product Given Unpolarized Endowment	
Yes	Yes	{+, ±}	{+, –, ±}	
Yes	No	{+, –, ±}	{+, –, ±}	{–}
			Endowed Product Given + Polarized Auctioned Product	
Yes	Yes	{+, –, ±}	{+, ±}	
Yes	No	{+, –, ±}	{+, ±}	{–}
			Endowed Product Given – Polarized Auctioned Product	
Yes	Yes		{+, –, ±}	
Yes	No	{–, ±}	{+, –, ±}	{–}
			Endowed Product Given Unpolarized Auctioned Product	
Yes	Yes	{+, –, ±}	{+, –, ±}	
Yes	No	{+, –, ±}	{+, –, ±}	{–}

Notes: + indicates positively polarized preferences. - indicates negatively polarized preferences. ± indicates unpolarized preferences. (t_j, $t_{j'}$) indicates polarization t_j for the auctioned product and $t_{j'}$ for the endowed product.

Table 3. Mean (standard error)[a] of the WTP distribution for positively polarized and unpolarized preferences

	Invasive Attribute				Noninvasive Attribute			
Plant	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 1**	**Model 2**	**Model 3**	**Model 4**
				Positively Polarized Preferences				
1	1.66	1.41			2.64	2.53		
	(0.36)	(0.25)			(0.57)	(0.51)		
2	2.22	1.82			3.47	3.35		
	(0.36)	(0.25)			(0.48)	(0.44)		
3	3.50	3.07			3.92	3.87		
	(0.53)	(0.52)			(0.47)	(0.44)		
4	3.90	3.29			6.42	6.23		
	(0.42)	(0.54)			(0.77)	(0.71)		
5	3.76	3.18			5.95	5.75		
	(0.40)	(0.41)			(0.87)	(0.78)		
				Unpolarized Preferences				
1	0.32	0.21	1.03	1.03	0.55	0.51	1.40	1.40
	(0.06)	(0.06)	(0.13)	(0.13)	(0.08)	(0.08)	(0.13)	(0.13)
2	0.36	0.25	1.22	1.21	0.81	0.76	2.00	2.00
	(0.08)	(0.08)	(0.15)	(0.15)	(0.13)	(0.12)	(0.18)	(0.18)
3	0.52	0.28	1.79	1.77	1.08	1.00	2.29	2.28
	(0.12)	(0.09)	(0.21)	(0.21)	(0.18)	(0.17)	(0.20)	(0.20)
4	0.29	0.17	1.90	1.89	2.10	1.99	4.20	4.19
	(0.06)	(0.05)	(0.23)	(0.24)	(0.28)	(0.29)	(0.33)	(0.33)
5	0.36	0.21	1.96	1.95	1.95	1.86	4.10	4.10
	(0.08)	(0.06)	(0.25)	(0.24)	(0.26)	(0.25)	(0.37)	(0.37)

[a] Calculated based on 500 bootstrapped replicates.

Table 4. Standard deviation (standard error)[a] of the WTP distribution for positively polarized and unpolarized preferences

	Invasive Attribute				Noninvasive Attribute			
Plant	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 1**	**Model 2**	**Model 3**	**Model 4**
				Positively Polarized Preferences				
1	1.91	1.59			2.51	2.36		
	(0.78)	(0.51)			(1.02)	(0.87)		
2	1.93	1.60			2.15	2.07		
	(0.59)	(0.40)			(0.63)	(0.55)		
3	2.97	3.43			2.20	2.12		
	(0.82)	(1.00)			(0.52)	(0.48)		
4	2.13	3.23			3.35	3.25		
	(0.46)	(0.95)			(0.83)	(0.77)		
5	2.10	2.66			3.78	3.63		
	(0.44)	(0.64)			(1.00)	(0.92)		
				Unpolarized Preferences				
1	0.58	0.52	1.18	1.17	0.65	0.64	1.22	1.22
	(0.08)	(0.10)	(0.10)	(0.11)	(0.06)	(0.06)	(0.10)	(0.10)
2	0.72	0.69	1.31	1.31	0.91	0.90	1.66	1.66
	(0.11)	(0.15)	(0.11)	(0.12)	(0.08)	(0.09)	(0.12)	(0.12)
3	1.06	0.79	1.89	1.91	1.34	1.29	1.76	1.76
	(0.17)	(0.17)	(0.14)	(0.16)	(0.13)	(0.14)	(0.12)	(0.12)
4	0.50	0.42	2.03	2.03	2.07	2.07	2.84	2.84
	(0.07)	(0.09)	(0.17)	(0.18)	(0.16)	(0.17)	(0.21)	(0.21)
5	0.64	0.53	2.16	2.16	1.91	1.91	3.22	3.22
	(0.09)	(0.11)	(0.19)	(0.21)	(0.14)	(0.14)	(0.25)	(0.25)

[a] Calculated based on 500 bootstrapped replicates.

Table 5. Estimated probabilities of alternative preference types [90% confidence intervals][a], maximized log-likelihood, and likelihood ratio statistics for comparisons to Model 1

	Model 1	Model 2	Model 3	Model 4
		Probability of Positively Polarized Preferences		
Invasive Attribute	0.384	0.517		
	[0.275, 0.495]	[0.403, 0.630]		
Noninvasive Attribute	0.310	0.336		
	[0.195, 0.430]	[0.220, 0.454]		
		Probability of Negatively Polarized Preferences		
Invasive Attribute	0.144		0.144	
	[0.065, 0.223]		[0.065, 0.223]	
Noninvasive Attribute	0.013		0.013	
	[0.000, 0.039]		[0.000, 0.039]	
Invasive Log-Likelihood	-577.18	-601.43	-671.31	-710.08
Noninvasive Log-Likelihood	-716.13	-722.43	-826.56	-830.94
Parameters Estimated	22	21	11	10
Observations	76	76	76	76
***Null Hypothesis*:**		***No Negatively Polarized Preferences***	***No Positively Polarized Preferences***	***No Positively or Negatively Polarized Preferences***
			Invasive Attribute	
Likelihood Ratio Statistic		48.51	188.26	265.80
p-value[b]		< 0.002	< 0.002	< 0.002
			Noninvasive Attribute	
Likelihood Ratio Statistic		12.61	220.86	229.62
p-value[b]		0.004	< 0.002	< 0.002

[a] Calculated based on 500 bootstrapped replicates.

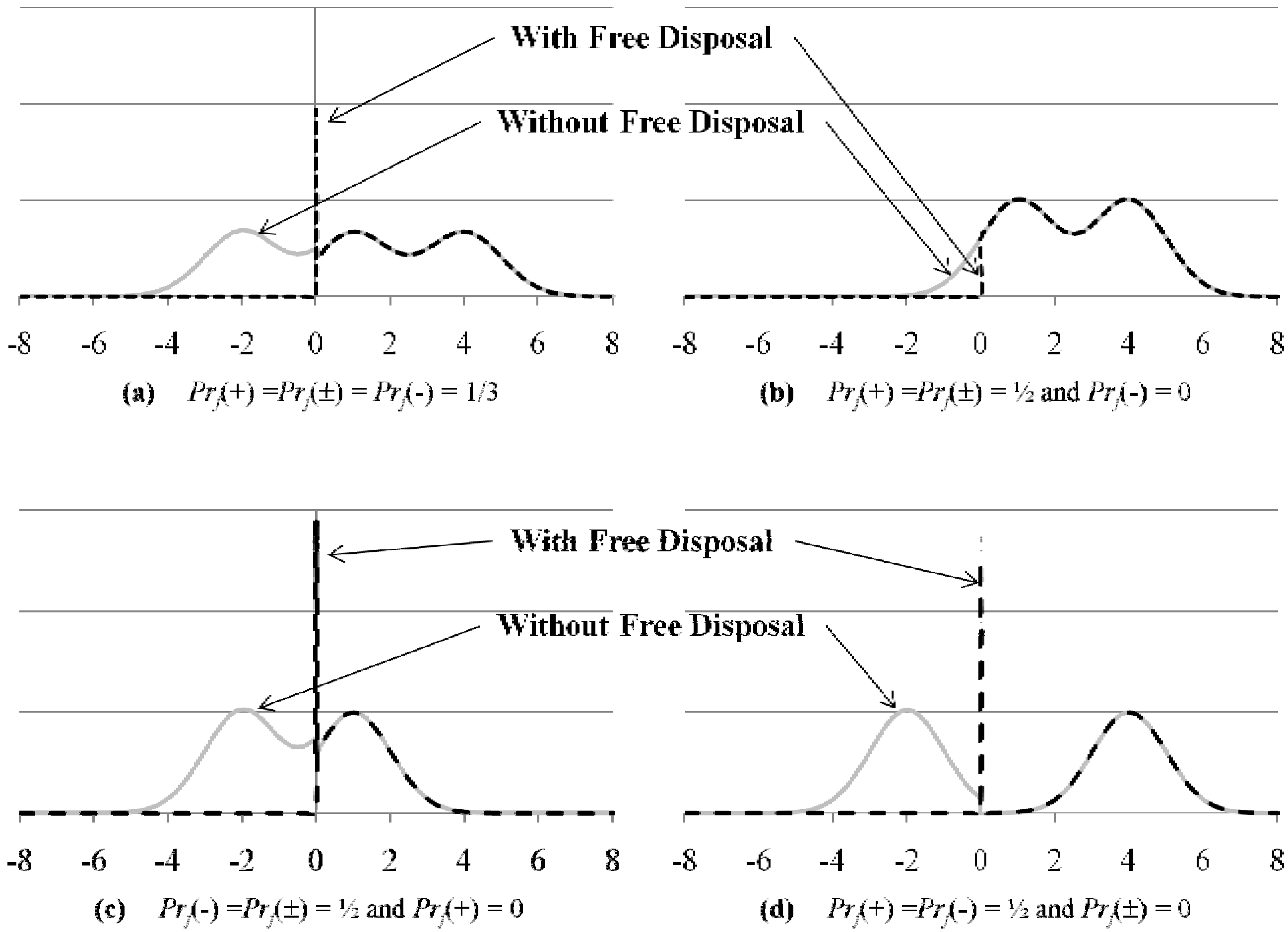

Figure 1. Implications of polarized preferences on the WTP distribution with and without free disposal assuming positively polarized preferences without free disposal come from a truncated normal distribution with mean 4 and variance 1, negatively polarized preferences without free disposal come from a truncated normal distribution with mean -2 and variance 1, and unpolarized preferences without free disposal come from a normal distribution with mean 1 and variance 1 given levels of polarization (a) 1/3, (b) ½ and = 0, (c) ½ and = 0, and (d) ½ and 0

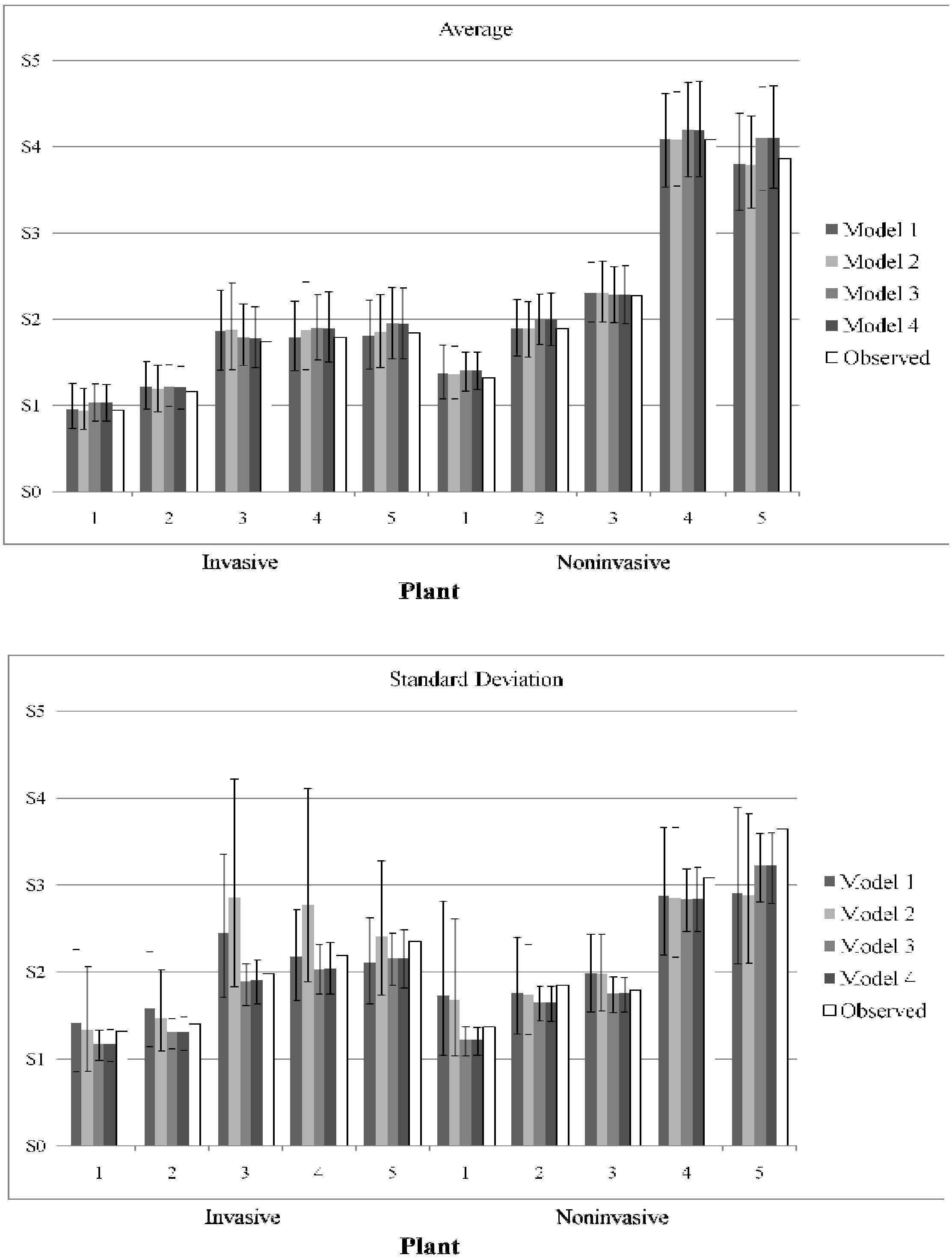

Figure 2. Estimated and observed average and standard deviation of bid by plant and plant attribute with 90% confidence intervals

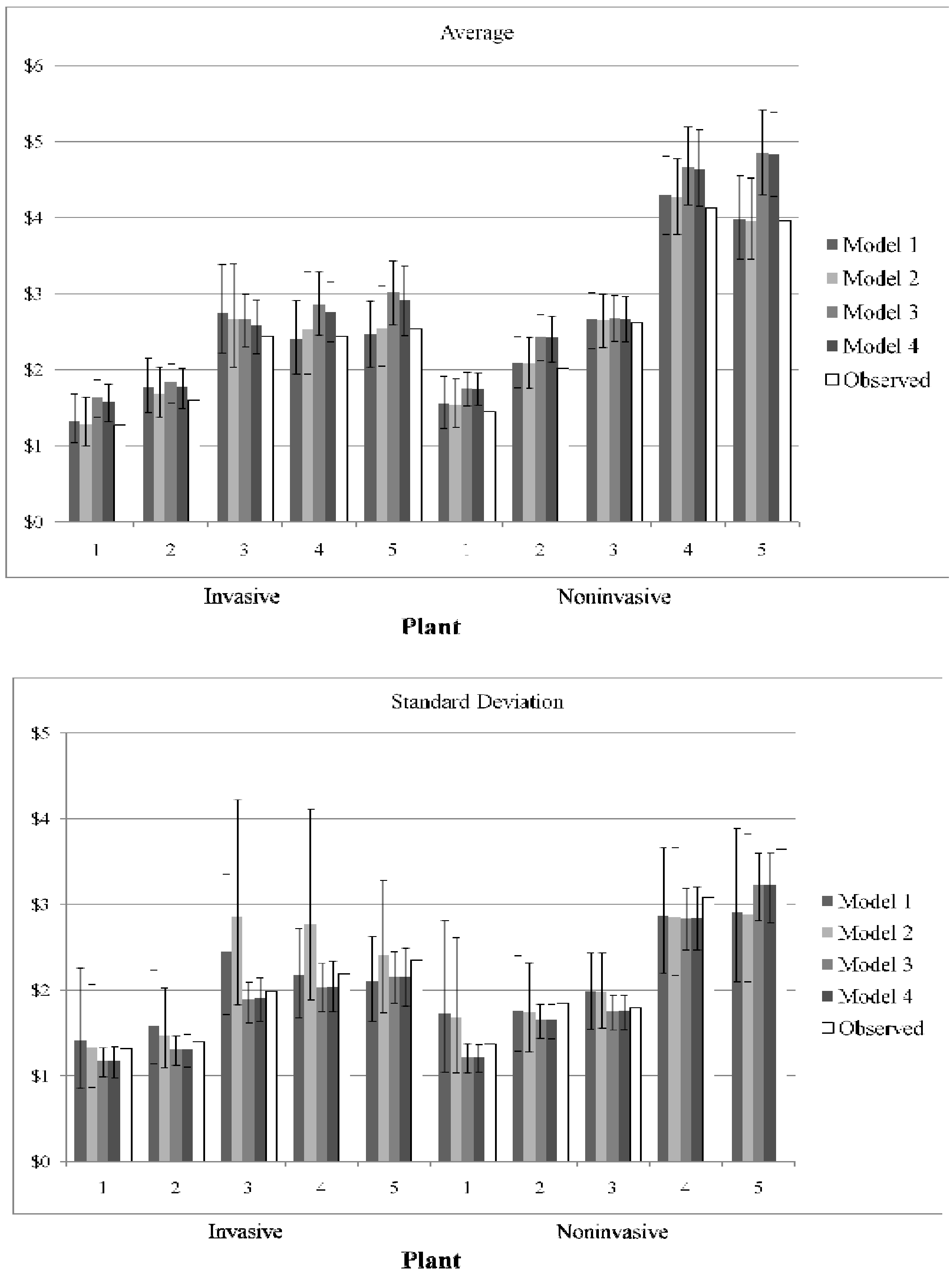

Figure 3. Estimated and observed average and standard deviation of positive bids by plant and plant attribute with 90% confidence intervals

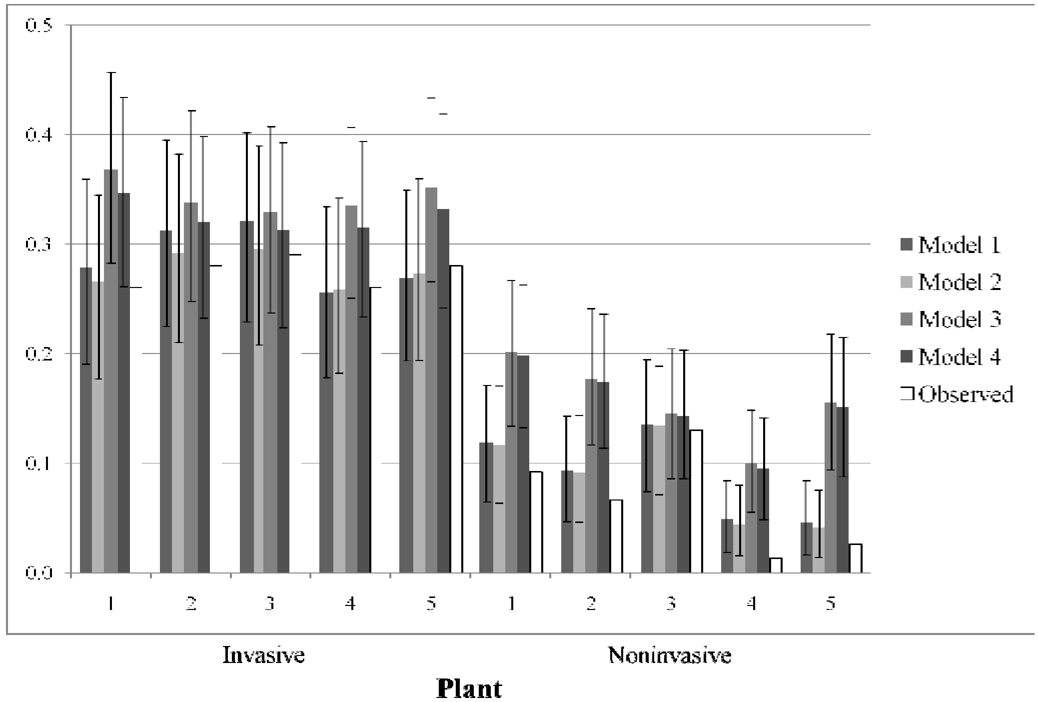

Figure 4. Estimated and observed proportion of zero bids by plant and plant attribute with 90% confidence intervals