Permanence of Carbon Sequestered in Forests under Uncertainty

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C.S. Kim, J. Lewandrowski, R. Sands, and R. Johansson[1]

Numerous economic studies have demonstrated that agricultural land owners could mitigate significant quantities of greenhouse gas (GHG) emissions by shifting areas of cropland and pasture into trees – a process called afforestation (Lewandrowski et al. (2004), Lubowski et al. (2006), McCarl and Schneider (2001), Sands and Kim (2009), and US EPA (2005)). To realize the GHG mitigation potential of afforestation it is necessary for some mechanism to exist by which farmers can convert increases in carbon stored in trees and biomass to income. Three possible mechanisms are the establishment of a carbon market (such as would happen under a state, regional, or national cap-and –trade program), the creation of a direct government payment for the adoption of specific carbon sequestering practices (analogous to the payments farmers receive under USDA's Conservation Reserve Program), and the development of voluntary carbon-related contracts between two or more private parties.

For afforestation to result in GHG mitigation requires that the associated carbon remain stored in soils or biomass for a long time (viewpoints range from 20 to over 100 years). As an example, the Forest project protocol developed by the Climate Action Reserve for use in the California climate program requires that reforestation projects occur on lands not in forest cover for the previous 10 years and must remain in reforested cover for 100 years (see Climate Action reserve 2010). In policy and scientific settings this is referred to as the "permanence" issue.

Permanence has a practical implication for the design of carbon sequestration incentives and the structure of associated carbon trading frameworks. Specifically, incentives must accommodate both the possibility and the uncertainty, that carbon already sequestered and credited within the context of a carbon trading framework will be prematurely released at some point in the future. Such releases could be unintentional (as in the case of a future fire event or a pest/disease outbreak) or deliberate (as in the case of a landowner decision to harvest timber prior to a previously agreed on date).

In the context of a carbon trading framework, the premature release of carbon from a parcel of afforested land would likely create an obligation to either replace the released carbon or to compensate the buyer - since it would already have been paid for and, presumably, used to meet the GHG mitigation commitment of the buyer. Conceptually, the obligation to replace carbon released prematurely from afforested lands could rest with either the buyer or seller. We assume it rests with the seller because as the land owner, the seller has direct control of how the afforested lands are actually managed. Additionally, buyers would largely be entities looking to meeting specific emissions reductions targets. If carbon sequestered through afforestation came with significant uncertainty regarding its permanence, these entities would likely look to alternative suppliers of GHG mitigation or to opportunities to reduce emissions within their own operation. Or the buyer may request that the supplier provide insurance to cover potential losses in expected carbon sequestration amounts. For example, the aforementioned forestry protocol for California's climate program requires that projects build up a buffer pool as a function of reversal risk due to probability of fire or pest infestation (Climate Action Reserve 2010).

In this paper we examine the issue of permanence in the context of sequestering carbon through afforestation. We develop a dynamic nested optimal control model of carbon

sequestration associated with the decision to afforest a tract of land given there are uncertainties associated with fire and insect/disease hazards. Conceptually, these potential hazards are similar in that their occurrence at any time t is uncertain and landowners can take specific actions – although generally different actions - in any time period t to reduce the probability of sustaining losses related to them. The hazards differ, however, in that fire represents a large loss in carbon at a moment in time, while insect/disease infestations are more likely to be reflected in a period of significant slowing of the rate of carbon accumulation than was anticipated followed by a sustained period of slowly decreasing carbon losses. The nature of these losses will influence the design of incentives under GHG mitigation frameworks that require carbon losses to be replaced as well as the strategies farmers adopt to deal with the uncertainties associated with these events occurring.

Forest Production, Risk Management, and Quasi-fixed input

Existing dynamic forest products models typically focus on production of timber and assume a point-of-input and point-of-output structure. Some models have added production of sequestered carbon with annual payments to landowners but assume a risk-free environment. Therefore, existing models of timber production need some modification to reflect production of carbon sequestration given risks associated with forest fire and disease/pest outbreaks.

We start with the model presented by van Kooten et al. (1995) to maximize the present value of the timber and carbon sequestration benefits (PVB) over a rotation of length *T* as follows (all variables are explained in Table 1):

(1) $\mathrm{PVB}_i = P_i Y_i(X_i(T))e^{-rT} + \int_0^T S_i Y_i'(X_i(t))e^{-rt}dt$

where, timber production is expressed in volume rather than by age. Equation (1) can be

rewritten as follows:

(2) $\mathrm{PVB}_i = \left(\int_0^T P_i Y_i'(X_i(t))e^{-rt}dt - r\int_0^T P_i Y_i(X_i(t))e^{-rt}dt \right) + \int_0^T S_i Y_i'(X_i(t))e^{-rt}dt$

$$= \int_0^T (P_i + S_i)Y_i'(X_i(t))e^{-rt}dt - r\int_0^T P_i Y_i(X_i(t))e^{-rt}dt$$

Since $\Delta t = 1$, the derivative of Y_i with respect to time variable t can be rewritten by $\dfrac{dY_i(X_i(t))}{dt} \equiv$

$Y_i(X_i(t)) - Y_i(X_i(t-1))$, and therefore, equation (2) is rewritten as follows:[2]

(3) $\mathrm{PVB}_i \equiv \int_0^T (P_i + S_i)[Y_i(X_i(t)) - Y_i(X_i(t-1))]e^{-rt}dt - r\int_0^T P_i Y_i(X_i(t))e^{-rt}dt$

$$= \int_0^T [(P_i(1-r) + S_i)Y_i(X_i(t)) - (P_i + S_i)Y_i(X_i(t-1))]e^{-rt}dt$$

Our model of timber and sequestered carbon production given risks and uncertainties associated

with forest fire and tree disease/pest outbreaks is based on several modifications to van Kooten's

model as presented in equation (3).

Following Kim et al. (2011), we assume the time path of carbon sequestration in a newly

planted forest will follow a logistic growth function (i.e., passing through intervals where carbon

accumulation increases, sequentially, very slowly, increases at an increasing rate, increases at a

decreasing rate, slowly approaches and reaches a new higher equilibrium level).

We treat all capital equipment as quasi-fixed inputs (see Epstein, 1981; and Vasavada and Chambers,1986) and we use hazard functions (see Kieffer, 1988; Kim et al., 2010) to capture uncertainties associated with the timing of fire events and disease/pest outbreaks. We explicitly consider the properties at the optimum - of both the preventive and control measures taken to reduce the likelihood of carbon losses related to disease/pest outbreaks and preventive measures taken to reduce losses related to fire.

Historically, responses to the threat of forest fires have consisted of both preventive measures before forest fire evolves and eradication (i.e., suppression) activities once fires are detected. Preventive measures include remote sensing for early detection, satellite and aerial monitoring, and controlled burnings. Suppression measures include a host on ground- and aerial-based suppression systems. Therefore, we use a modified hazard function approach to capture risks and uncertainties associated with the timing of forest fires (Kamien and Schawartz, 1971; Kieffer, 1988; Kim et al., 2010)

We first define $M_i(t)$ to be the probability that forest fire occurs by time t at the ith site, with $M_i(t=0) = 0$, as follows:

$$(4) \qquad M_i(t) = 1 - exp[-\alpha_i m_i(F_i(t))]t, \quad m_i(F_i(t=0)) = 0, \quad \frac{\partial m_i}{\partial F_i} < 0, \quad \alpha_i = \frac{1}{1+\varepsilon_i};$$

where, $\varepsilon_i = (\frac{t}{m_i})(\frac{\partial m_i}{\partial F_i}\frac{\partial F_i}{\partial t}) < 0$ is the time elasticity of the conditional probability of forest fire. In equation (4), both the probability that forest fire occurs by the time t and the conditional

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[2] When one assumes that $\frac{dY_i(X_i(t))}{dt} \equiv Y_i(X_i(t+1)) - Y_i(X_i(t))$, $Y_i(X_i(t+1))$ can be represented by $(1+v(t))Y_i(X_i(t))$, where $v(t)$ is the rate of timber growth following the Weibull distribution (see van Kooten et al., 1995).

probability that forest fire will occur during the next year would decline as fire preventive

measures are adopted.

Using equation (4), the probability density function of the time for forest fire occurrence,

$\dfrac{\partial M_i(t)}{\partial t}$, can be presented as the following state equation:

(5) $\dfrac{\partial M_i(t)}{\partial t} = m_i(F_i(t))[1 - M_i(t)],$ where $m_i(F_i(t=0)) = 0,$ $\dfrac{\partial m_i}{\partial F_i} < 0;$

Insect pests and diseases are part of all forest ecosystems. However, landowners can

reduce both the likelihood of incurring a pest/disease outbreak at a given point in the future and

the damage done to standing trees if an outbreak does occur by implementing various preventive

measures (these include selecting pest/disease resistant seeding and prophylactic spraying and

other treatments to discourage pests and diseases from taking hold). We define $N_i(t)$ to be the

probability that discovery of tree disease or insect has occurred by time t, with $N_i(t=0) = 0$, at the

ith site and presented as follows:

(6) $N_i(t) = 1 - exp[-\beta_i n_i(E_i^b(t))]\, t,$ $n_i(E_i^b(t=0)) = 0,$ $\dfrac{\partial n_i}{\partial E_i^b} < 0,$ $\beta_i = \dfrac{1}{1 + \delta_i};$

where, $\delta_i = (\dfrac{t}{n_i})(\dfrac{\partial n_i}{\partial E_i^b}\dfrac{\partial E_i^b}{\partial t}) < 0$ is the time elasticity of the conditional probability of

discovering forest disease / insects. Equation (6) states that the adoption of the preventive

measures reduces the probability of discovering forest disease/pest at time t and the conditional

probability of discovering forest disease/pest during the next year would decline as the adoption

of the preventive measures increases.

Equation (6) can be rewritten as a state equation as follows:

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(7) $\quad \dfrac{\partial N_i(t)}{\partial t} = n_i(E^b_i(t))[1 - N_i(t)], \qquad n_i(E^b_i(t{=}0)) = 0, \quad \dfrac{\partial n_i}{\partial E^b_i} < 0;$

where $\dfrac{\partial N_i(t)}{\partial t}$ is the probability density function of the time for first discovery of a disease or

pest.

Once disease/pest is discovered in a tract of forest, landowners can implement control

measures, (*i.e.*, $E^a_i(t)$), to reduce damages from diseases (e.g., more aggressive spraying and

removing infected and nearby trees). Populations of pest and disease species are assumed to

follow a logistic growth function (Eiswerth and Johnson, 2002; Huffaker and Cooper, 1995).

When control measures are implemented, we adjust logistic growth function of pest/disease

population as follows (Kim et al., 2007):

(8) $\quad \dfrac{\partial a_i(t)}{\partial t} = g_i(1 - k_i(E^a_i(t)))a_i(t)\left[1 - \dfrac{(1 + k_i(E^a_i(t)))a_i(t)}{A_i}\right],$

where $\dfrac{\partial k_i}{\partial E^a_i(t)} > 0$ for $i = 1, 2, \ldots, l.$

It should be emphasized that risk or uncertainty associated with forest fire and tree

disease/pest differs in modeling carbon sequestration through afforestation. While diseases and

insects gradually slow down and then reduce the production of both timber and sequestered

carbon, fire results in a large instantaneous destruction of timber and release of sequestered

carbon.

Finally, our tree production function includes the variable inputs, $X_i(t)$, and the quasi-

fixed input, $K_i(t)$, given that the investment, $I_i(t)$, such that the marginal timber product of

investment is negative to reflect the adjustment costs associated with the investment (Epstein 1981). The rate of increase of quasi-fixed input (i.e., capital stock) over time is represented by:

$$(9) \qquad \frac{\partial K(t)}{\partial t} = I(t) - \varepsilon K(t), \quad K(t) > 0,$$

where ε is the rate of depreciation.

The Model

The net economic benefits resulting from timber production and carbon sequestration vary depending largely on whether forest fire occurs, whether tree diseases spread, and whether farmers purchase hazard insurance(s) for forest fire and/or tree diseases. We assume that financial compensation to a landowner, in the case of a forest fire event, depends exclusively on whether a landowner purchased fire insurance policies. Table 2 shows profits in any given year under 10 different scenarios. The profit function for timber production and carbon sequestration through afforestation, which incorporates the quasi-fixed input and risk associated with forest fire and tree disease is then represented by a nested optimal control model as follows:

$$(10) \quad J(w, P_i, r, S, F, E^b, E^a, R, K_0) = \underset{I,X,E,F}{Max} \int_0^T e^{-rt} \left\{ \sum_{i=1}^{I} A_i \left((1 - M_i(t)) \left[(1 - N_i(t)) [((1-r)P_i + S_i)Y_i \right. \right. \right.$$

$(X_i(t), K(t), I(t)) - (P_i + S_i)Y_i(X_i(t-1), K(t-1), I(t-1)) - w'X_i(t) - C_b(F_i(t), E_i^b(t)) - \theta_i C_1(f_i) -$

$\phi_i C_2(e_i)] + N_i(t)\big((1 - \phi_i)[((1-r)P_i^* + S_i)Y_i^*(X_i^*(t), K(t), I(t), a_i(t)) - (P_i^* + S_i)Y_i^*(X_i^*(t-1), K(t-1), I(t-1), a_i(t-1)) - w'X_i^*(t)] - C_a(F_i(t), E_i^a(t)) - \theta_i C_1(f_i) - \phi_i C_2(e_i) + (\phi_i - 1)d_2 + \phi_i [((1-r) P_i$

$+ S_i) Y_i(X_i(t), K(t), I(t)) - (P_i + S_i)Y_i(X_i(t-1), K(t-1), I(t-1)) - w'X_i(t)]\big) \big] + M_i(t)\big[\theta_i [((1-r)P_i + S_i)Y_i(X_i(t), K(t), I(t)) - (P_i + S_i)Y_i(X_i(t-1), K(t-1), I(t-1)) - w'X_i(t) - C_1(f_i)] + (\theta_i - 1)d_1\big]\big) - R'(t)I(t) \right\} dt + V(K,T).$

The profit function in (10) is maximized subject to the state equations in (5), (7), (8), and (9). The Hamiltonian equation is then represented as follows:

$$(11) \quad H = e^{-rt} \Bigg\{ \sum_{i=1}^{I} A_i \Bigg((1 - M_i(t)) \Big[(1 - N_i(t)) [((1-r)P_i + S_i)Y_i(X_i(t), K(t), I(t)) - (P_i + S_i)Y_i(X_i(t-1),$$

$$K(t-1), I(t-1)) - w'X_i(t) - C_b(F_i(t), E_i^b(t)) - \theta_i C_1(f_i) - \phi_i C_2(e_i)] + N_i(t)\big((1 - \phi_i)[((1-r) P_i^*$$

$$+ S_i)Y_i^*(X_i^*(t), K(t), I(t), a_i(t)) - (P_i^* + S_i)Y_i^*(X_i^*(t-1), K(t-1), I(t-1), a_i(t-1)) - w'X_i^*(t)] - C_a(F_i(t),$$

$$E_i^a(t)) - \theta_i C_1(f_i) - \phi_i C_2(e_i) + (\phi_i - 1)d_2 + \phi_i[((1-r)P_i + S_i)Y_i(X_i(t), K(t), I(t)) - (P_i + S_i)Y_i(X_i(t-1), K(t-$$

$$1), I(t-1)) - w'X_i(t)]\big) \Big] + M_i(t)\Big[\theta_i [((1-r)P_i + S_i)Y_i(X_i(t), K(t), I(t)) - (P_i + S_i)Y_i(X_i(t-1), K(t-1), I(t-$$

$$1)) - w'X_i(t) - C_1(f_i)] + (\theta_i - 1)d_1 \Big] \Bigg) - R'(t)I(t) \Bigg\} + V(K,T) + \lambda_1 m_i(F_i(t))[1 - M_i(t)] + \lambda_2 n_i$$

$$(E_i^b(t))[1 - N_i(t)] + \lambda_3 g_i(1 - k_i(E_i^a(t)))a_i(t)[1 - \frac{(1 + k_i(E_i^a(t)))a_i(t)}{A_i}] + \lambda_4[I(t) - \varepsilon K(t)],$$

where, the preventive measures adopted per acre to prevent forest fire (F_i), the preventive measures adopted per acre to prevent forest diseases (E_i^b), the control measures adopted per acre to control tree diseases after the first discovery (E_i^a), and per-acre capital investment (I_i) are control variables; the variable inputs, X_i, are decision variable; the probability of forest fire occurrence, $M_i(t)$, the probability of discovery of fungal diseases or insects, $N_i(t)$, the acres infested by fungal diseases or insects, $a_i(t)$, and quasi-fixed inputs, $K(t)$, are state variables; and λ_i ($i = 1. . , 4$) are adjoint variables associated with the state $M_i(t)$, $N_i(t)$, $a_i(t)$, and $K(t)$, respectively. The necessary conditions for optimum are presented in Appendix A.

The economic properties of the optimal conditions for control variables presented in equations (A1) through (A4) in Appendix I are better served by investigating the adjoint variables λ_i ($i = 1. . , 4$). Since there are many time-varying variables in the differential equations (A6) through (A9), it is very difficult, if not impossible, to solve these equations for the adjoint variables. Alternatively, the adjoint variables $\lambda_i(t)$ ($i = 1. . , 4$) measures the marginal effects

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(i.e., shadow values) of the state variables, $M_i(t)$, $N_i(t)$, $a_i(t)$, and $K(t)$, respectively, on the profit function in equation (10). By differentiating the objective function J in equation (10) with respect to the state variables, $M_i(t)$, $N_i(t)$, $a_i(t)$, and $K(t)$, respectively, as shown in Appendix B, we obtain the followings:

(12-1) $\lambda_1(t) < 0$;   (12-2) $\lambda_2(t) < 0$;   (12-3) $\lambda_3(t) < 0$;   (12-4) $\lambda_4(t) > 0$.

As the probability that forest fire occurs in year t increases, the net economic benefits would decline so that the adjoint variable $\lambda_1(t)$ is expected to be negative. Similarly, as the probability of discovering acres infested with tree diseases increases, the net economic benefits are expected to decline so that the adjoint variable $\lambda_2(t)$ is expected to be negative. As the acres infested with tree diseases increase, the net economic benefits would be reduced so that the adjoint variable $\lambda_3(t)$ is expected to be negative. Meanwhile, an increase in quasi-fixed inputs (i.e., capital stocks) would increase the net economic benefits, and therefore, the adjoint variable, $\lambda_4(t)$, associated with capital stock is expected to be positive. Results in equation (12) also indicate that it is more beneficial for land owners to allocate more resources for the preventive measures to protect trees from diseases than to protect from infestation of tree diseases.

The optimality condition (A1) indicates that the marginal net benefits from the reduction in the forest-fire hazard rate by adopting preventive measures equal the marginal costs (shadow values) of adopting forest-fire preventive measures. Equation (A2) indicates that the marginal net benefits from the reduction in the tree disease hazard rate by adopting preventive measures equal the marginal costs (shadow values) of adopting tree disease preventive measures before tree diseases spread. Equation (A3) explains that the marginal costs of adopting control measures after discovery of tree diseases equal the marginal benefits resulting from the reduction

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of the infested acre's intrinsic growth rate. Equation (A4) indicates that the shadow values of capital equal the sum of the expected adjustment costs and rental value of capital stock. Finally, equation (A5) indicates all variable inputs are used up to the point where the expected marginal value products of each input equal its unit price.

Carbon Sequestration

Given carbon credit payment for afforestation, an important question is how much carbon will be sequestered at the end of the terminal time (T) under various scenarios. Following van Kooten et al. (1995), the quantity of carbon sequestration during the planning horizon in risk-free environment (q_T) is given by:

$$(13) \quad q_T = \int_0^T A_i \alpha_i [Y_i(X_i(t), K(t), I(t)) - Y_i(X_i(t-1), K(t-1), I(t-1))] dt$$

where α_i is tons of carbon per volume of timber biomass at the ith site. When the forest fire and tree disease hazards are considered as shown in equation (10), the quantity of carbon sequestered during the planning period (Q_T) is represented by:

$$(14) \quad Q_T = \int_0^T A_i (1 - M_i(t)) \{(1 - N_i(t))\alpha_i [Y_i(X_i(t), K(t), I(t)) - Y_i(X_i(t-1), K(t-1), I(t-1))]$$

$$+ N_i(t) \alpha_i [Y_i^*(X_i^*(t), K(t), I(t), a_i(t)) - Y_i^*(X_i^*(t-1), K(t-1), I(t-1), a_i(t-1))]\} dt$$

Comparing equations (13) and (14), the quantity of carbon sequestration which contributed to the risks associated with forest fire and tree disease hazards is represented by:

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(15) $\quad q_T - Q_T = \int\limits_0^T A_i \left\{ \left[M_i(t) + N_i(t)(1 - M_i(t)) \right] \alpha_i \left[Y_i \left(X_i(t), K(t), I(t) \right) - Y_i \left(X_i(t\text{-}1), K(t\text{-}1), I(t\text{-}1) \right) \right] \right.$

$\left. - N_i(t)[1 - M_i(t)] \alpha_i \left[Y_i^* \left(X_i^*(t), K(t), I(t), a_i(t) \right) - Y_i^* \left(X_i^*(t\text{-}1), K(t\text{-}1), I(t\text{-}1), a_i(t\text{-}1) \right) \right] \right\} dt$,

which is positive. Equation (15) indicates that the quantity of carbon sequestration through afforestation would be overestimated if risks associated with forest fire and tree diseases are not considered in the model. As the probability that forest fire occurs in year t approaches one (i.e., $M_i(t) = 1$) in equation (15), the quantity of carbon sequestration overestimated equals the entire carbon sequestration, when risk and uncertainties are not considered, as presented in equation (13). When the risk associated with tree disease infestation is not considered, meanwhile, overestimation is less than those in the case with forest fire hazard. These results are expected because the quantity of carbon sequestration is reduced as trees are infested with diseases, while all carbon sequestered is wiped out by forest fire.

Conclusions

Our research addresses two issues largely neglected in the modeling of land use change to afforestation for carbon sequestration: (1) capital equipment is an important determinant of changing land use to afforestation; and (2) the effects of tree disease, insects, and fire hazard on the net carbon sequestration. Results indicate that the net economic benefits and the quantity of carbon sequestration through afforestation would be overstated if risks associated with forest fire and tree diseases, as well as quasi-asset fixity, are not considered in the model. Indeed, such

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contributions are included in recent protocols for eligible carbon offset projects in the case of

California's climate program, but heretofore, the theoretical model that might allow a

comparison of the comparative statics has been lacking. Future research might well link the

concept and empirical applications for a region.

Table 1. Variable Description.

Y	The volume of timber growing on a stand.
P	The per unit price of timber.
X	A vector of inputs.
S	Carbon payment per unit of tree volume.
T	Terminal time.
r	The rate of discount.
$M(t)$	The probability that forest fire occurs by time t.
$F(t)$	Preventive/suppression measures adopted before the forest fire occurs.
$m(F(t))$	The conditional probability that forest fire will occur during the next time unit, given that forest fire has not occurred at time t.
$N(t)$	The probability that discovery of tree disease or insect has occurred by time t.
E^b	Preventive measures adopted before the first discovery of tree disease or insect.
E^a	Control measures adopted to reduce damages from tree diseases.
$n(E^b(t))$	The conditional probability that discovery of tree disease or insect will occur during the next time unit, given that disease has not been discovered at time t.
$a(t)$	acres covered by tree disease in time t.
A_i	Acres with the ith-site characteristics.
g	The growth rate of pest population.
k	A fractional coefficient.
Y^*	The tree production at the presence of tree diseases.
K	Capital stock.
I	Investment.
C_b	Costs for adopting preventive measures to protect forest from fire and disease.
C_a	Costs for adopting preventive measures to protect forest from fire and adopting control measures to control tree diseases after discovery of tree diseases.
$C_1(f)$	Yearly amortized forest fire insurance premium.
$C_2(e)$	Yearly amortized forest disease insurance premium.
θ	Dummy variable such that $\theta=1$ if forest fire insurance is purchased and $\theta = 0$ otherwise.
ϕ	Dummy variable such that $\phi=1$ if tree diseases and $\phi = 0$ otherwise.
d_1	Payback of carbon payment when carbon sequestered is released to atmosphere by forest fire.
d_2	payback of carbon payment when carbon sequestered is released to atmosphere by tree diseases.
V	Salvage value of heavy equipments.
$U(T)$	Costs associated with timber harvest and replanting trees at the terminal time, T.

Table 2. Net economic benefits in year t under alternative scenarios.

M	N	θ	ϕ	Net Economic Benefits
1	0	1	0	$((1-r)P_i+\text{s})Y_i(t) - (P_i+\text{s})Y_i(t\text{-}1) - w'X_i(t) - C_1(f_i) - R'(t)I(t)$
1	0	0	0	$-d_1 - R'(t)I(t)$
0	1	1	1	$((1-r)P_i+\text{s})Y_i(t) - (P_i+\text{s})Y_i(t\text{-}1) - w'X_i(t) - C_a(F_i(t), E_i^a(t)) - C_1(f_i) - C_2(e_i) - R'(t)I(t)$
0	1	1	0	$((1-r)P_i^*+\text{s})Y_i(t) - (P_i^*+\text{s})Y_i(t\text{-}1) - w'X_i^*(t) - C_a(F_i(t), E_i^a(t)) - C_1(f_i) - d_2 - R'(t)I(t)$
0	1	0	1	$((1-r)P_i+\text{s})Y_i(t) - (P_i+\text{s})Y_i(t\text{-}1) - w'X_i(t) - C_a(F_i(t), E_i^a(t)) - C_2(e_i) - R'(t)I(t)$
0	1	0	0	$((1-r)P_i^*+\text{s})Y_i(t) - (P_i^*+\text{s})Y_i(t\text{-}1) - w'X_i^*(t) - C_a(F_i(t), E_i^a(t)) - d_2 - R'(t)I(t)$
0	0	1	1	$((1-r)P_i+\text{s})Y_i(t) - (P_i+\text{s})Y_i(t\text{-}1) - w'X_i(t) - C_b(F_i(t), E_i^b(t)) - C_1(f_i) - C_2(e_i) - R'(t)I(t)$
0	0	1	0	$((1-r)P_i+\text{s})Y_i(t) - (P_i+\text{s})Y_i(t\text{-}1) - w'X_i(t) - C_b(F_i(t), E_i^b(t)) - C_1(f_i) - R'(t)I(t)$
0	0	0	1	$((1-r)P_i+\text{s})Y_i(t) - (P_i+\text{s})Y_i(t\text{-}1) - w'X_i(t) - C_b(F_i(t), E_i^b(t)) - C_2(e_i) - R'(t)I(t)$
0	0	0	0	$((1-r)P_i+\text{s})Y_i(t) - (P_i+\text{s})Y_i(t\text{-}1) - w'X_i(t) - C_b(F_i(t), E_i^b(t)) - R'(t)I(t)$

Appendix A. Necessary conditions for optimality:

(A1) $\quad \dfrac{\partial H}{\partial F_i} = 0$ implies

$$
e^{-rt} A_i \left\{ - \frac{\partial M_i(t)}{\partial F_i} \left(\left[(1 - N_i(t)) \left[((1-r)P_i + S_i) Y_i(X_i(t),\, K(t),\, I(t)) - (P_i + S_i) Y_i(X_i(t-1),\, K(t-1),\, I(t-1)) \right. \right. \right. \right.
$$

$$
- w'X_i(t) - C_b(F_i(t),\, E_i^b(t)) - \theta_i C_1(f_i) - \phi_i\, C_2(e_i) \left] + N_i(t) \left((1 - \phi_i) \left[((1-r)P_i^* + S_i) Y_i^* (X_i^*(t),\, K(t),\, I(t), \right. \right. \right.
$$

$$
a_i(t)) - (P_i^* + S_i) Y_i^* (X_i^*(t-1),\, K(t-1),\, I(t-1),\, a_i(t-1)) - w'X_i^*(t) \left] - C_a(F_i(t),\, E_i^a(t)) - \theta_i C_1(f_i) - \right.
$$

$$
\phi_i\, C_2(e_i) + (\phi_i - 1)d_2 + \phi_i \left[((1-r)P_i + S_i) Y_i(X_i(t),\, K(t),\, I(t)) - (P_i + S_i) Y_i(X_i(t-1),\, K(t-1),\, I(t-1)) - \right.
$$

$$
w'X_i(t) \left] \right) \right] - \left[\theta_i \left[((1-r)P_i + S_i) Y_i(X_i(t),\, K(t),\, I(t)) - (P_i + S_i) Y_i(X_i(t-1),\, K(t-1),\, I(t-1)) - w'X_i(t) - \right. \right.
$$

$$
C_1(f_i) \left] \right] + (\theta_i - 1)d_1 \right] \right) - (1 - M_i(t)) \left[(1 - N_i(t)) \left(\frac{\partial C_b(F_i,\, E_i^b)}{\partial F_i} \right) + N_i(t) \left(\frac{\partial C_a(F_i,\, E_i^a)}{\partial F_i} \right) \right] \right\} = -
$$

$$
\lambda_1(t) \left[(1 - M_i(t)) \left(\frac{\partial m_i}{\partial F_i} \right) - m_i \left(\frac{\partial M_i}{\partial F_i} \right) \right].
$$

(A2) $\quad \dfrac{\partial H}{\partial E_i^b} = 0$ implies

$$
e^{-rt} A_i \left\{ - \left(\frac{\partial N_i(t)}{\partial E_i^b} \right) \left((1 - M_i(t)) \left[\left[((1-r)P_i + S_i) Y_i(X_i(t),\, K(t),\, I(t)) - (P_i + S_i) Y_i(X_i(t-1),\, K(t-1),\, I(t-1)) \right. \right. \right. \right.
$$

$$
- w'X_i(t) - C_b(F_i(t),\, E_i^b(t)) - \theta_i C_1(f_i) - \phi_i\, C_2(e_i) \left] - \left((1 - \phi_i) \left[((1-r)P_i^* + S_i) Y_i^* (X_i^*(t),\, K(t),\, I(t), \right. \right. \right.
$$

$$
a_i(t)) - (P_i^* + S_i) Y_i^* (X_i^*(t-1),\, K(t-1),\, I(t-1),\, a_i(t-1)) - w'X_i^*(t) \left] - C_a(F_i(t),\, E_i^a(t)) - \theta_i C_1(f_i) - \right.
$$

$$
\phi_i\, C_2(e_i) + (\phi_i - 1)d_2 + \phi_i \left[((1-r)P_i + S_i) Y_i(X_i(t),\, K(t),\, I(t)) - (P_i + S_i) Y_i(X_i(t-1),\, K(t-1),\, I(t-1)) - \right.
$$

$$
w'X_i(t) \left] \right) \right] - (1 - M_i(t))(1 - N_i(t)) \left(\frac{\partial C_b(F_i,\, E_i^b)}{\partial E_i^b} \right) \right\}
$$

$$
= - \lambda_2(t) \left[(1 - N_i(t)) \left(\frac{\partial n_i}{\partial E_i^b} \right) - n_i \left(\frac{\partial N_i}{\partial E_i^b} \right) \right].
$$

(A3) $\quad \dfrac{\partial H}{\partial E_i^a} = 0$ implies

$$
e^{-rt} A_i (1 - M_i(t)) N_i(t) \left(\frac{\partial C_a(F_i(t),\, E_i^a)}{\partial E_i^a} \right) = \lambda_3\, g_i a_i \left(1 - \frac{2k_i a_i}{A_i} \right) \left(\frac{\partial k_i}{\partial E_i^a} \right).
$$

(A4) $\quad \dfrac{\partial H}{\partial I} = 0$ implies

$$- e^{-rt}A_i\left\{\left[(1-M_i)\Big([1-N_i(1-\phi_i)]((1\text{-}r)P_i+S_i)(\frac{\partial Y_i}{\partial I}) + N_i(1-\phi_i)((1\text{-}r)P_i^*+S_i)(\frac{\partial Y_i^*}{\partial I})\Big)\right.\right.$$

$$\left.+ M_i\theta_i((1\text{-}r)P_i+S_i)(\frac{\partial Y_i}{\partial I}) - R\right\} = \lambda_4, \text{ where } \frac{\partial Y_i}{\partial I} < 0 \text{ and } \frac{\partial Y_i^*}{\partial I} < 0.$$

(A5) $\quad \dfrac{\partial H}{\partial X} = 0$ implies

$$\left[1 - (1-\theta_i)M_i(t) - (1-\phi_i)N_i(t)(1-M_i(t)\right]((1\text{-}r)P_i+S_i)(\frac{\partial Y_i}{\partial X_i}) = w.$$

(A6) $\quad -\dfrac{\partial H}{\partial M_i} = \dfrac{\partial \lambda_1}{\partial t}$ implies

$$L_1 + m_i(F_i(t))\lambda_1 = \frac{\partial \lambda_1}{\partial t},$$

where $L_1 = e^{-rt}A_i\left\{\left[(1-N_i(t))\left[((1\text{-}r)P_i+S_i)Y_i(X_i(t), K(t), I(t)) - (P_i+S_i)Y_i(X_i(t\text{-}1), K(t\text{-}1), I(t\text{-}1))\right]-\right.\right.$

$w'X_i(t) - C_b(F_i(t), E_i^b(t)) - \theta_iC_1(f_i) - \phi_i C_2(e_i)\big] + N_i(t)\Big((1-\phi_i)\big[((1\text{-}r)P_i^*+S_i)Y_i^*(X_i^*(t), K(t), I(t),$

$a_i(t)) - (P_i^*+S_i)Y_i^*(X_i^*(t\text{-}1), K(t\text{-}1), I(t\text{-}1), a_i(t\text{-}1)) - w'X_i^*(t)\big] - C_a(F_i(t), E_i^a(t)) - \theta_iC_1(f_i) - $

$\phi_i C_2(e_i) + (\phi_i-1)d_2 + \phi_i\big[((1\text{-}r)P_i+S_i)Y_i(X_i(t), K(t), I(t)) - (P_i+S_i)Y_i(X_i(t\text{-}1), K(t\text{-}1), I(t\text{-}1)) - $

$w'X_i(t)\big]\Big)\Big] - \Big[\theta_i\big[((1\text{-}r)P_i+S_i)Y_i(X_i(t), K(t), I(t)) - (P_i+S_i)Y_i(X_i(t\text{-}1), K(t\text{-}1), I(t\text{-}1)) - w'X_i(t) - $

$\theta_iC_1(f_i)\big] + (\theta_i-1)d_1\Big]\right\}.$

(A7) $\quad -\dfrac{\partial H}{\partial N_i} = \dfrac{\partial \lambda_2}{\partial t}$ implies

$$L_2 + n_i(E_i^b(t))\lambda_2 = \frac{\partial \lambda_2}{\partial t},$$

where $L_2 = e^{-rt}A_i\left\{(1-M_i(t))\left[\big[((1\text{-}r)P_i+S_i)Y_i(X_i(t), K(t), I(t)) - (P_i+S_i)Y_i(X_i(t\text{-}1), K(t\text{-}1), I(t\text{-}1))\right.\right.$

$- w'X_i(t) - C_b(F_i(t), E_i^b(t)) - \theta_iC_1(f_i) - \phi_i C_2(e_i)\big] - \Big((1-\phi_i)\big[((1\text{-}r)P_i^*+S_i)Y_i^*(X_i^*(t), K(t), I(t),$

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$a_i(t)\big) - (P^*_i + S_i)Y^*_i\left(X^*_i(t\text{-}1),\ K(t\text{-}1),\ I(t\text{-}1),\ a_i(t\text{-}1)\right) - w'X^*_i(t)\big] - C_a(F_i(t),\ E^a_i(t)) - \theta_i C_1(f_i) -$

$\phi_i\, C_2(e_i) + (\phi_i - 1)d_2 + \phi_i\left[((1\text{-}r)P_i + S_i)Y_i\left(X_i(t),\ K(t),\ I(t)\right) - (P_i + S_i)Y_i\left(X_i(t\text{-}1),\ K(t\text{-}1),\ I(t\text{-}1)\right) -\right.$

$\left.w'X_i(t)\right]\Big)\Big]\Big\}$.

(A8) $-\dfrac{\partial H}{\partial a_i} = \dfrac{\partial \lambda_3}{\partial t}$ *implies*

$$L_3 - \lambda_3\left[\left(g_i(1 - k_i) - \frac{2g_i(1 - k_i^2)a_i}{A_i}\right)\right] = \frac{\partial \lambda_3}{\partial t},$$

where $L_3 = -e^{-rt}A_i(1 - M_i)\,N_i(1 - \phi_i)\left[((1\text{-}r)P^*_i + S_i)\left(\dfrac{\partial Y^*_i}{\partial a_i}\right)\right] > 0.$

(A9) $-\dfrac{\partial H}{\partial K} = \dfrac{\partial \lambda_4}{\partial t}$ *implies*

$$L_4 + \varepsilon\lambda_4 = \frac{\partial \lambda_4}{\partial t},$$

where $L_4 = -e^{-rt}A_i\left\{(1 - M_i)\left[[1 - N_i(1 - \phi_i)]((1\text{-}r)P_i + S_i)\left(\dfrac{\partial Y_i}{\partial K}\right) + N_i(1 - \phi_i)((1\text{-}r)P^*_i + S_i)\left(\dfrac{\partial Y^*_i}{\partial K}\right)\right]\right.$

$$+ M_i\,\theta_i((1\text{-}r)P_i + S_i)\left(\frac{\partial Y_i}{\partial K}\right)\Bigg\} < 0.$$

(A10) $\dfrac{\partial H}{\partial \lambda_1} = \dfrac{\partial M_i}{\partial t}$ *implies*

$$\frac{\partial M_i(t)}{\partial t} = m_i(F_i(t))[1 - M_i(t)]$$

(A11) $\dfrac{\partial H}{\partial \lambda_2} = \dfrac{\partial N_i}{\partial t}$ *implies*

$$\frac{\partial N_i(t)}{\partial t} = n_i(E^b_i(t))[1 - N_i(t)]$$

(A12) $\dfrac{\partial H}{\partial \lambda_3} = \dfrac{\partial z_i}{\partial t}$ implies

$$\frac{\partial a_i}{\partial t} = g_i (1 - k_i) a_i \left[1 - \frac{(1 + k_i) a_i}{A_i} \right].$$

(A13) $\dfrac{\partial H}{\partial \lambda_4} = \dfrac{\partial K}{\partial t}$ implies

$$\frac{\partial K}{\partial t} = I - \varepsilon K$$

(A14) $\displaystyle \lim_{t \to T} \lambda_1 = 0$, $\displaystyle \lim_{t \to T} \lambda_2 = 0$, $\displaystyle \lim_{t \to T} \lambda_3 = 0$. $\displaystyle \lim_{t \to T} \lambda_4 = \frac{\partial V(K,T)}{\partial K}$, and $\displaystyle \lim_{t \to T} \lambda_4 K = 0$.

Appendix B.

(B1) $\lambda_1 \equiv \dfrac{\partial J}{\partial M_i} = -\displaystyle\int_0^T e^{-rt} L_1 \, dt < 0.$

(B2) $\lambda_2 \equiv \dfrac{\partial J}{\partial N_i} = -\displaystyle\int_0^T e^{-rt} L_2 \, dt < 0.$

(B3) $\lambda_3 \equiv \dfrac{\partial J}{\partial a_i} = -\displaystyle\int_0^T e^{-rt} L_3 \, dt < 0.$

(B4) $\lambda_4 \equiv \dfrac{\partial J}{\partial K} = -\displaystyle\int_0^T e^{-rt} L_4 \, dt > 0.$

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