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# How Policy Affects Incentives and Contract Duration in Biomass Production

(Preliminary)

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Abstract: Policies such as Biomass Crop Assistance Program (BCAP) which aim to assist farmers with biomass production may act as a double-edged sword. On one hand, they lure farmers to adopt biomass production in the short term. On the other hand, they can't be irresponsible for farmers' abandonment of biomass production in the long run. The paper sharpens this idea in a principal-agent setting and argues that by offering a timely loyalty premium the agents' take-and-run behavior can be mitigated. Moreover, the model shows that effort and investment in human capital are increasing in loyalty premium when agents decide to continue providing biomass after testing-water contract expires.

*Key words: BCAP, incentive, contract duration, Take-and-Run*

## **1 Introduction**

To reduce the dependence on foreign oil and alleviate the environmental pollution, the U.S. government has mandated the development of renewable substitutes of oil (Thorsell et al. 2004). The Energy Independence and Security Act of 2007 have mandated that 36 billion gallons per year of ethanol be produced in the U.S. by 2022, with 21 billion gallons per year from cellulosic feedstock other than corn (U.S. Congress 2007).

Different from the traditional corn-based ethanol production, cellulosic biorefinery faces supply issues. Besides high production and transportation costs identified from production literature, Altman, Sanders and Boessen (2007) found that asset specificity exists in the biomass supply chain. Biomass producers invest in specialized equipment for harvest, collect, storage and transport. Some biomass such as switchgrass is dedicated to bioenergy production. Alternative use is limited to be burnt to power electricity plant. Due to the absence of a spot market for switchgrass, uncertainty associated with production and potential hold-up by the biorefinery, farmers are not willing to produce switchgrass on their land. On the other hand, the biorefinery invests in asset specific processing facility, power plant, and processing technology. For example, specialized enzymes are developed to break down certain biomass. Switching to other biomass would require high “re-pooling” of the current enzymatic process. Farmers who realize that the biorefinery who invests in huge sunk cost must have enough switchgrass for year round operation may gauge switchgrass price and act opportunistically. Gow and Swinnen (2001) found that large firms and co-ops are more likely to breach the contract though the transaction cost of dealing with them is lower than with small firms. In short, both the

biomass suppliers and the biorefinery can behave opportunistic due to the investment in specific assets and uncertainty.

Klein and Crawford (1978) suggested vertical integration and contract can be used to solve the opportunistic behavior. Currently, the Department of Energy and private investments granted 19 integrated biorefineries in 15 states. In some state level and private-owned biorefineries, contracting is prevalent. In 2008, University of Tennessee (UT) Switchgrass Farmer Incentive Program offered farmers with \$450 per acre per year for switchgrass production on 3-year contract. Being offered a fixed rate contract based on acreage, farmers are fully insured against price risks; however, in the meantime, they are induced to implement less effort such as using the less productive land for switchgrass production. POET, the largest ethanol producer in the world, signed four-year contract with farmers for corn cobs to a pilot-scale cellulosic ethanol plant in Emmetsburg, Iowa. Short-term contract can't solve the opportunistic behavior as the vertical integration or long-term contract does (Klein and Crawford, 1978). Dubois and Vukina (2009) studied the effects of contract duration on agents' behavior in a principal-agent setting. They argued that increase in contract duration increases both investment and effort, and consequently production. This finding echo of research of Joskow (1987) who pointed out that long term contract is used when large quantity (a feature of dedicated asset) is transacted in the contractual relationship. However, they didn't explain the reasons of the change from short term contract to long term contract.

In 2008, USDA Farm Service Agency initiated the Biomass Crop Assistance Program (BCAP) to assist owners and operators of agricultural and non-industrial private forest land to establish, produce, and deliver eligible biomass feedstocks to biomass conversion

facility. The BCAP offers up to five years establishment payment and annual payments and up to two years matching payments for eligible annual and non-woody perennial crops. Song, Zhao and Swinton (2009) used a two-way conversion model to study farmer's decision to adopt switchgrass and found that government subsidies such as BCAP on motivating conversion to switchgrass production reduce the land shares for switchgrass in the long run. BCAP acts as a double-sided sword to induce biomass production. On one hand, it along with other subsidies lures farmers to adopt biomass production. On the other hand, it may foster farmers to switch back to traditional enterprises when the subsidy effects are gone. Bull (1987) studied explicit long term contract in labor market. He found that pension and retirement bonus acts as self-enforcement capital are crucial for the existence of such explicit long term contract.

In this study, I'm providing additional support to Song, Zhao and Swinton's argument that policies such as BCAP may discourage biomass production in the long run, but only under certain conditions. Moreover, I argue that not only the amount of subsidies matters, but the distribution/timing of subsidies matter in promoting biomass production. I'll first illustrate it in a simple engine oil change example. I have two options for engine oil change. I can either go to Ms. Wang or Dr. Qin. Assume that I only need five times oil change for the life of my car. Ms. Wang offers first time customer a \$10 off coupon and no ending bonus. The regular price is \$35 per oil change at Ms. Wang. Dr. Qin has flat rate \$30 per oil change for all periods. Optimal strategy is to go to Ms. Wang in period 1 and switch to Dr. Qin for the following four oil change. This is what I call Take-and-Run behavior. Now, Ms. Wang offers a free oil change in the fifth service. Optimal strategy becomes to go to Ms. Wang in all periods for oil change. However, if I only change the

distribution/timing of Ms. Wang's pricing, the result will change again. Swapping Ms. Wang's first period price and last period price as shown in Table 1, Take-and-Run occurs again. Now suppose I need six times oil change for the life of my car. Ms. Wang offers first time customer a \$10 off coupon and a free oil change in the sixth oil change service. Dr. Qin still has flat rate of \$30/service. I would still go to Ms. Wang for all six times. I keep adding the times of oil change I need for my car and I keep going to Ms. Wang for all times until I need nine times of service for my car. As can be seen, the duration I go to Ms. Wang for oil change not only depends on the amount of the discount, but the distribution/timing of the discount.

The BCAP has similar features with Ms. Wang's new customer discount. Without long term incentives such as ten-year loyalty premium or twenty-year retirement bonus, farmers may take advantage of the BCAP and switch to other production when BCAP effect vanishes. What should the biomass conversion facility do if farmers take and run when BCAP expires? One possible strategy is to add an ending loyalty premium or redistribute some of the initial subsidies as credible loyalty premium to induce them to enduringly supply the biomass conversion facility. Another question is when to provide the ending bonus.

The rest of the paper proceeds as following: Section 2 develops a Take-and-Run Model to demonstrate the conditions for underinvestment in both human capital and physical assets when take-and-run occurs. Section 3 shows the timing of ending subsidy. Section 4 proposes testable hypothesis. Section 5 summarizes results and conclusion.

## 2 Take-and-Run Model

The model is based on Dubois and Vukina (2009). Two types of investments - unobservable human capital  $i$  and observable physical capital  $\varphi$  - are defined in their principal-agent setting. The investment in physical capital such as a baler is discrete.  $\varphi = 1$  if the agent invests in a baler while  $\varphi = 0$  if the agent doesn't invest in a baler. This investment is irreversible and only incurs a fixed cost normalized to 1 and paid when acquired. The unobservable investment  $i$  in specific knowledge increases the stock of specific knowledge  $k$  and costs  $C(i) = i^2$ . The stock of knowledge depreciates at rate  $\mu$  but increases additively with investment  $i$  such that  $k = \mu k_o + i$ , where  $k_o$  is the previous period investment.

In addition to these two types of investments, the agent also supplies an unobservable productive effort  $e$ . The cost of effort depends on knowledge  $k$  as given by  $G(e, k) = \frac{e^2}{2k}$ , which implies that specific knowledge  $k$  reduces the marginal cost of effort. Physical investment  $\varphi$  raises the productivity of effort such that the production function is given by  $q(e, \varphi) = \pi(\varphi)e\varepsilon$ , where  $\varepsilon$  is a production shock unknown at the time where efforts are exerted, with  $\pi(1) > \pi(0) > 0$ . The contract payment  $w(q)$  is assumed to be linear in principal predetermined parameters  $\alpha$  and  $\beta$ , i.e.,  $w(q) = \alpha q + \beta$ , where  $\alpha$  is the piece rate payment and  $\beta$  is the base payment when no output is delivered.

Based on this setting, I add two types of government subsidies in the analysis: initial fixed costs subsidy and piece rate subsidy and ending loyalty premium. Hipple and Duffy (2002) found that farmers are hesitating to make long term commitment before testing the water. Hence, instead of dynamic setting, I assume that there're only two stages of



biomass production. The decision to enroll in biomass production in short term and the decision to continue in biomass production are separate so that they can be solved independently. In the first stage, farmers choose whether to enroll in a short-term biomass production. In the second stage, they choose whether to renew.

The expected utility in Stage I is given by:

$$V(e_1, k_1, \varphi_1) = \{e_1, k_1, \varphi_1 \in [0,1]\} \overset{Max}{\alpha\pi(\varphi_1)e_1 + \beta - \frac{\gamma}{2}\alpha^2\pi(\varphi_1)^2e_1^2\sigma^2 - \frac{e_1^2}{2k_1} - (k_1 - \mu k_o)^2 - \varphi_1 + \lambda + \theta\pi(\varphi_1)e_1}, \quad (1)$$

FOCs:

$$\frac{\partial V(e_1, k_1, \varphi_1)}{\partial e_1} = \alpha\pi(\varphi_1) - \gamma\alpha^2\pi(\varphi_1)^2e_1^*\sigma^2 - \frac{e_1^*}{k_1^*} = 0 \Rightarrow$$

$$\frac{e_1^*}{k_1^*} = \frac{(\alpha+\theta)\pi(\varphi_1)}{1+k_1^*\gamma\alpha^2\pi(\varphi_1)^2\sigma^2}, \quad (2)$$

$$\frac{\partial V(e_1, k_1, \varphi_1)}{\partial k_1} = \frac{e_1^{*2}}{2k_1^{*2}} - 2(k_1^* - \mu k_o) = 0 \Rightarrow$$

$$\frac{e_1^{*2}}{k_1^{*2}} = 4(k_1^* - \mu k_o), \quad (3)$$

$$(2) \text{ and } (3) \Rightarrow \left[ \frac{(\alpha+\theta)\pi(\varphi_1)}{1+k_1^*\gamma\alpha^2\pi(\varphi_1)^2\sigma^2} \right]^2 = 4(k_1^* - \mu k_o), \quad (4)$$

By fixed point theorem, there exists a unique solution for  $k_1^*$ .

From now on, I consider the risk neutral agent. Risk attitude complicate the analysis and I'll deal with in future work if possible.

Optimal investment in human capital and effort are derived from (2) and (3):

$$k_1^* = \mu k_o + \frac{(\alpha + \theta)^2 \pi(\varphi_1)^2}{4}, \quad (5)$$

$$e_1^* = (\alpha + \theta) \pi(\varphi_1) \left[ \mu k_o + \frac{(\alpha + \theta)^2 \pi(\varphi_1)^2}{4} \right], \quad (6)$$

$$\frac{\partial k_1^*}{\partial \varphi_1} = \frac{(\alpha + \theta)^2 \pi(\varphi_1) \pi'(\varphi_1)}{2} > 0, \text{ and } \frac{\partial e_1^*}{\partial \varphi_1} = \frac{4(\alpha + \theta) \mu k_o \pi'(\varphi_1) + 3(\alpha + \theta)^3 \pi(\varphi_1)^2 \pi'(\varphi_1)}{4} > 0. \text{ This isn't}$$

inconsistent with the observation. Farmers with large farms can be considered as risk neutral. They invest large amount of money in equipment as well as in human capital such as learning how to operate the machine to increase productivity. They may gain economies of scale and scope, but they spend more effort on management and taking care of large acreage. Farmers with small farms may or may not invest in equipment.

Expected utilities from investing in physical asset and from not investing in physical asset are:

$$\begin{aligned} V^*(e_1, k_1, \varphi_1) &= \frac{(\alpha + \theta)^2 \pi(\varphi_1^*)^2 k_1^*}{2} - \frac{(\alpha + \theta)^4 \pi(\varphi_1^*)^4}{16} + \beta - \varphi_1^* + \lambda \\ &= \frac{(\alpha + \theta)^2 \pi(\varphi_1^*)^2 \mu k_o}{2} + \frac{(\alpha + \theta)^4 \pi(\varphi_1^*)^4}{16} + \beta - \varphi_1^* + \lambda, \end{aligned} \quad (7)$$

$$V^*(e_1, k_1, 1) = \frac{(\alpha + \theta)^2 \pi(1)^2 \mu k_o}{2} + \frac{(\alpha + \theta)^4 \pi(1)^4}{16} + \beta - 1 + \lambda, \quad (8)$$

$$V^*(e_1, k_1, 0) = \frac{(\alpha + \theta)^2 \pi(0)^2 \mu k_o}{2} + \frac{(\alpha + \theta)^4 \pi(0)^4}{16} + \beta + \lambda, \quad (9)$$

To enroll in the BCAP, the expected utility from optimal actions must be at least equal to the reservation utility such as growing corn or CRP, i.e.,  $V^*(e_1, k_1, \varphi_1) \geq \bar{u}$ .

$$k_1 \geq \frac{16(\bar{u} + 1_{\{\varphi_1 > 0\}} - \beta - \lambda) + (\alpha + \theta)^4 \pi(\varphi_1)^4}{8(\alpha + \theta)^2 \pi(\varphi_1)^2}, \text{ or}$$

$$k_o \geq \frac{16(\bar{u} + 1_{\{\varphi_1 > 0\}} - \beta - \lambda) - (\alpha + \theta)^4 \pi(\varphi_1)^4}{8\mu(\alpha + \theta)^2 \pi(\varphi_1)^2}, \quad (10)$$

**Proposition 1:** To enroll in BCAP when the sum of the base payment and the actual fixed cost (fixed cost minus corresponding subsidy) don't exceed the reservation utility, the minimum initial human capital stock ( $k_o$ ) is decreasing in  $\alpha, \beta, \theta, \lambda, \mu$ , and  $\pi(\cdot)$ , and increasing in  $\bar{u}$ . Optimal stock of specific knowledge and effort in Stage I are increasing in  $\mu, k_o, \alpha, \theta$ , and  $\pi(\cdot)$ .

**Proof of Proposition 1:** see Appendix.

The result is very intuitive. Expected utility is increasing in  $k_o, \alpha, \beta, \theta, \lambda, \mu$ , and  $\pi(\cdot)$ , so large payment, subsidy, discount factor on specific knowledge and productivity compensate for small initial specific knowledge. However, initial specific knowledge should be high enough to raise expected utility to outweigh reservation utility. Stock of specific knowledge in Stage I is increasing in  $\mu$  and  $k_o$  by its definition. Agents would like to invest more in specific knowledge when piece rate payment and subsidy and productivity are increasing.

To induce investment on physical asset, i.e.,  $V(e_1, k_1, 1) > V(e_1, k_1, 0)$ ,

$$k_o > \frac{16 - (\alpha + \theta)^4 [\pi(1)^4 - \pi(0)^4]}{8\mu(\alpha + \theta)^2 [\pi(1)^2 - \pi(0)^2]}, \quad (11)$$

**Proposition 2:** To induce investment on physical asset, the minimum initial human capital stock is decreasing in  $\alpha, \theta, \mu$ , and  $\pi(1)$ , but increasing in  $\pi(0)$ .

**Proof of Proposition 2:** see Appendix.

Expected utility is increasing in  $k_o, \alpha, \beta, \theta, \lambda, \mu,$  and  $\pi(\cdot)$ , but expected utility with investment on physical asset increases more for the same amount increase in  $\alpha, \theta$  and  $\mu$  when initial specific knowledge is above some threshold.

Stage II consists of finite repeated renewal periods. I'm interested in the point when agents stop renewal and before which the probability to renew is assumed to be 1 in each period. If agents don't renew after Stage I, they take and run.

The expected utility at the ending point is:

$$V(e_2, k_2, \varphi_2 | \varphi_1^*) = \underset{\{e_2, k_2, \varphi_2 \in [0,1]\}}{\text{Max}} n \left[ \alpha \pi(\varphi_2) e_2 + \beta - \frac{e_2^2}{2k_2} - (k_2 - \mu k_1)^2 \right] - 1_{\{\varphi_2 > \varphi_1\}} + \chi \pi(\varphi_2) e_2, \quad (12)$$

Differentiating with respect to  $k_2$  and  $e_2$  and rearrange to obtain optimal investment in human capital and effort:

$$k_2^* = \mu k_1^* + \frac{(n\alpha + \chi)^2 \pi(\varphi_2)^2}{4}, \quad (13)$$

$$e_2^* = (n\alpha + \chi) \pi(\varphi_2) \left[ \mu k_1^* + \frac{(n\alpha + \chi)^2 \pi(\varphi_2)^2}{4} \right], \quad (14)$$

$$\begin{aligned} V^*(e_2, k_2, \varphi_2 | \varphi_1) = & -5\pi(\varphi_2)^4 \alpha^4 n^5 + \left( \frac{\pi(\varphi_2)^4 \alpha^4 - 80\pi(\varphi_2)^4 \alpha^3 \chi}{4} \right) n^4 + \left( \frac{3\pi(\varphi_2)^4 \alpha^3 \chi - \pi(\varphi_2)^2 \alpha^2 \mu k_1^* - 60\pi(\varphi_2)^4 \alpha^2 \chi^2}{2} \right) n^3 + \\ & \left( \pi(\varphi_2)^2 \alpha^2 \mu k_1^* + \frac{5\pi(\varphi_2)^4 \alpha^2 \chi^2}{2} - 20\pi(\varphi_2)^4 \alpha^3 \chi - \pi(\varphi_2)^2 \alpha \mu k_1^* \chi \right) n^2 + \left( 2\pi(\varphi_2)^2 \alpha \mu k_1^* \chi + \right. \\ & \left. \frac{\pi(\varphi_2)^4 \alpha \chi^3}{4} - \frac{\pi(\varphi_2)^2 \alpha^2 \mu k_1^*}{2} - 5\pi(\varphi_2)^4 \chi^4 + \beta \right) n + \pi(\varphi_2)^2 \mu k_1^* \chi^2 - 1_{\{\varphi_2 > \varphi_1\}}, \quad (15) \end{aligned}$$

If agents don't renew in the first period ( $n = 0$ ) in Stage II, a Take-and-Run behavior shows up. In Stage II, the expected utility in the 1<sup>st</sup> period is not larger than the reservation utility in the 1<sup>st</sup> period, i.e.,  $V^*(e_2, k_2, \varphi_2 | \varphi_1) < \bar{u}$ , equivalently,

$$1_{\{\varphi_2 > \varphi_1\}} > \beta - \bar{u} - \pi(\varphi_2)^4 \Delta + 2\mu k_1^* \chi \pi(\varphi_2)^2 + \mu k_1^* \chi^2, \quad (16)$$

where  $\Delta = 5x^4 - .75x^3 + 27.5\alpha^2 x^2 + 38.5\alpha^3 x + 4.75\alpha^4$ .

**Result 1:**

(1) When there's no loyalty subsidy, i.e.,  $\chi = 0$ , agents take-and-run after Stage I if

$1_{\{\varphi_2 > \varphi_1\}} > \beta - \bar{u} - 4.75\pi(\varphi_2)^4 \alpha^4$ . As can be seen, the likelihood the Take-and-Run behavior will happen is the highest when no investment in physical capital in Stage I but in Stage II, the lowest when no investment in physical capital in either stage, and in the middle when investment occurs in Stage I.

(2) When  $\chi > 0$ , and  $\chi$  is so large that  $\Delta > 0$ , the likelihood the Take-and-Run behavior will happen is larger when no investment in physical capital in Stage I but in Stage II than when investment occurs in Stage I.

(3) Investment in both specific knowledge and effort is increasing in  $n$ . Take-and-Run behavior causes underinvestment.

**Timing of the Loyalty Premium:**

One implication from the oil change decision example is that initial subsidy should be high enough to attract agents, but can't be too high to encourage Take-and-Run. The ending bonus should be large enough to induce long term business.

According to the Fundamental Theorem of Algebra, every polynomial with odd degrees has some real root. Hence, there's at least a solution for  $n$ . I want to show that  $n$  is monotonic increasing in  $\chi$  under certain conditions. Transferring some initial subsidy to the loyalty premium so that the expected utility is just larger than the agents' reservation utility to adopt biomass and stay longer in the biomass industry.

### **Testable Hypotheses:**

1. The likelihood to adopt biomass is high when agents have high initial specific knowledge in biomass production, when agents are eligible for initial subsidies, when contract payment is high, when agents previously invest in physical capital and etc.
2. The likelihood to abandon biomass production is high when agents don't have equipment to harvest, store and transport biomass under high harvest, storage and transportation cost.

### **Results and Conclusions:**

Under the principal-agent setting, I studied farmers' biomass adoption decision under subsidies. I found that farmers who have initial specific knowledge in biomass production, are eligible for initial subsidies and previously invest in physical assets needed in

biomass production are more likely to adopt biomass. Increasing piece rate payment increases farmers' willingness to adopt biomass and induces higher investment in specific knowledge and effort.

More importantly, I found that the first group to quit biomass production is the ones who don't have equipment to harvest, store and transport biomass when the costs associated with harvest, storage and transportation are high. Under certain conditions, a timely loyalty premium can lengthen the contract duration and alleviate the Take-and-Run behavior.

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## Appendix

Proof of Proposition 1:

Assume the reservation utility exceeds the sum of the real fixed cost and base payment,

$$\text{i.e., } \nabla = \bar{u} + 1_{\{\varphi_1 > 0\}} - \lambda - \beta > 0,$$

$$\text{Let } \dot{k}_o \equiv \frac{16(\bar{u} + 1_{\{\varphi_1 > 0\}} - \beta - \lambda) - (\alpha + \theta)^4 \pi(\varphi_1)^4}{8\mu(\alpha + \theta)^2 \pi(\varphi_1)^2} = \frac{2\nabla}{\mu(\alpha + \theta)^2 \pi(\varphi_1)^2} - \frac{(\alpha + \theta)^2 \pi(\varphi_1)^2}{8\mu},$$

$$\frac{\partial \dot{k}_o}{\partial(\alpha + \theta)} = -\frac{4\nabla}{\mu(\alpha + \theta)^3 \pi(\varphi_1)^2} - \frac{2(\alpha + \theta)\pi(\varphi_1)^2}{8\mu} < 0,$$

The same for both  $\mu$  and  $\pi(\varphi_1)$ .

Proof of Proposition 2:

$$\text{Let } \ddot{k}_o \equiv \frac{16 - (\alpha + \theta)^4 [\pi(1)^4 - \pi(0)^4]}{8\mu(\alpha + \theta)^2 [\pi(1)^2 - \pi(0)^2]} = \frac{2}{\mu(\alpha + \theta)^2 [\pi(1)^2 - \pi(0)^2]} - \frac{(\alpha + \theta)^2 [\pi(1)^2 - \pi(0)^2]}{8\mu},$$

$$\frac{\partial \ddot{k}_o}{\partial(\alpha + \theta)} = -\frac{2}{\mu(\alpha + \theta)^3 [\pi(1)^2 - \pi(0)^2]} - \frac{(\alpha + \theta)^2 [\pi(1)^2 - \pi(0)^2]}{4\mu} < 0,$$

The same procedure for  $\mu$ ,  $\pi(1)^2$  and  $\pi(0)^2$ .

Table 1. Optimal Oil Change Path and Distribution/Timing of Ending Bonus

Dealer	Ending Bonus	1	2	3	4	5	6
	Yes	25	35	35	35	0	-
Ms. Wang	Yes	0	35	35	35	25	-
	Yes	25	35	35	35	35	0
	No	25	35	35	35	35	-
Dr. Qin	No	30	30	30	30	30	30