Acreage Decision under Price & Yield Uncertainty

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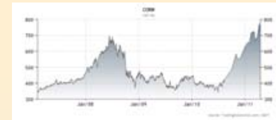

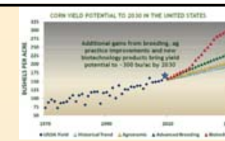

INTRODUCTION

The acreage allocation decision is related to fixed input and *quasi*-fixed output information. Uncertainty is related to price and yield in the output market. The randomness of price is represented by $p_i = \bar{p}_i + e_i$ where $e_i = h(x, \alpha)$, $E[e_i] = 0$ and $E[p_i] = \bar{p}_i$. The variable x represents a set of infinite demand shifting variables and α represents a set of passive parameters of the variables. Therefore, price will deviate from expected price if $\sum_i \alpha_i \neq 0$ in a crop year. One example of price deviation could come from changes in consumer tastes and/or population diversity. Similarly, the randomness of yield is represented by $y_i = \bar{y}_i + \varepsilon_i$ where $\varepsilon_i = f(z, \beta)$, $E[\varepsilon_i] = 0$, and $E[y_i] = \bar{y}_i$. The variable z represents a set of input factors uncontrolled by the farm household and β represents a set of passive parameters of the variables. Therefore, yield will deviate from expected yield if $\sum_i \beta_i \neq 0$ in a crop year. One example of yield deviation could come from unusual and uncontrollable events of nature in a crop year.

Previous studies provide increasing evidence showing that risk and risk preference are important factors in agricultural production decisions. Chavas and Pope (1985) examined expected utility maximizing conditions in allocating input factors under output price uncertainty for any risk preference and probability structure. In their study, they indicated that risk responsive behavior under uncertainty influences output supply and input factor demand and those effects are critical if firms are non-risk-neutral. Chavas and Holt (1990) developed an acreage supply response model using a specified expected utility function. In their study, they indicated that risk and wealth variables play an important role in determining acreage allocations.

OBJECTIVE

By taking these previous studies as a starting point, this study tries to develop a system of acreage allocation under price and yield uncertainty in order to identify acreage response to changes in wealth and risk. As previous studies have indicated, wealth and risk effects on acreage response will be related to the risk attitude of a particular farm household. Therefore, any such system that is developed to such an end should show how wealth and risk effects on acreage response depend upon a farm household's risk attitude. Also, a system derived from expected utility maximizing procedure has to be examined under classical microeconomic theory: symmetry, positive semi-definiteness, and homogeneity.

MODEL DEVELOPMENT

Model development starts with defining a share (or percentage) acreage. The total acreage of a farm household is defined as $L = \sum_{i=1}^{n} A_i$ where A_i is the number of acres devoted to crop i and L is the number of total acres available for producing n crops. Usually, individual farm size is different so that the value of L is different depending upon the specifications for each individual farm household. In order to eliminate this difference among farm households, the acreage constraint can be modified into percentage form and expressed as $\sum_{i=1}^{n} a_i = 1$ where a_i is acreage share of crop i ($a_i = A_i/L$). The sum of share acreage allocated to n crops will be equal to one regardless of differences in individual farm households. Also, the acreage share allows that the acreage constraint is defined as follows:

(1) $\left(\sum_{i=1}^{n} a_i\right)^k = 1$.

Consider a farm household producing n crops where y_i is yield of the ith crop per acre and p_i is the corresponding market price, $i = 1, \ldots, n$, then share acreage revenue is given by

(2) $r = \sum_{i=1}^{n} p_i y_i a_i$.

Denoting the cost of production per acre of the ith crop as c_i, then the total share acreage cost of agricultural production is expressed as

(3) $c = \sum_{i=1}^{n} c_i a_i$.

Since output prices $p = (p_1, \ldots, p_n)$ and crop yields $y = (y_1, \ldots, y_n)$ are not observed by a farm household when production decisions are made, acreage share revenue (r) is a risky variable. In contrast, total cost is fixed because input prices are known at the time crop acreages are allocated. Now, let the budget constraint of a farm household be represented by:

(4) $G = w + \sum_{i=1}^{n} \pi_i a_i$,

where G represents all goods purchased by the farm household, w denotes wealth, and $\pi_i = p_i y_i - c_i$ is net profit of crop i. And G, w, and π_i (p_i and c_i) are assumed to be numéraire normalized by a consumer price index, q. Equation (4) states that wealth plus farm profit is equal to consumption expenditures.

If a farm household recognizes that the expected net profit for crop i is greater than that for crop j at planting, then a profit maximizing farm household will allocate all arable acreage to crop i. Therefore, we need an assumption in order to identify the effects of the random components of price and yield on acreage decisions that the expected net profit of crops are equal to each other, $\bar{\pi}_i = \bar{\pi}_j$. However, *real* net profit is different, $\pi_i \neq \pi_j$, because of the random nature of the components of price and yield. In addition, let us assume that a farm household's risk preferences are represented by a von Neumann-Morgenstern utility function (denoted $U(G)$) satisfying the necessary condition of concavity. If a farm household maximizes expected utility under competition, then the decision model is then expressed as

(5) $L = EU(\Pi) + \lambda\left[1 - \left(\sum_{i=1}^{n} a_i\right)^k\right]$,

where E is the expectation operator over the random variables and $\Pi = w + \sum_{i=1}^{n} \pi_i a_i$ represents wealth.

This formulation illustrates that an acreage decision is made under uncertainty because of the random nature attributed to p and y with given probability distributions. Consequently, E in (5) is taken over the uncertain variables p and y and is based on the information available to a farm household at the time of planting. The utility function is assumed to be monotonically increasing in wealth at a decreasing rate. Also, under the assumption of competitiveness, the decision variable (a) does not influence the probability distributions of p and y.

MODEL DEVELOPMENT CONT'D

The economic optimization problem in equation (5) for the acreage allocation decision implies that it explains (i) risk and the attitude to risk as having a tangible effect on the acreage allocation process, and (ii) it examines the system of acreage allocation equations according to classical microeconomic theory which include symmetry, positive semi-definiteness, and homogeneity . From equation (5), the n first-order conditions to a farm household expected utility maximization problem are given by

(6.1) $L_{a_i} = \frac{\partial L}{\partial a_i} = E(U_\Pi \Pi_{a_i}) - \lambda k a_i \left(\sum_{i=1}^{n} a_i\right)^{k-1} = 0$, and

(6.2) $L_\lambda = \frac{\partial L}{\partial \lambda} = 1 - \left(\sum_{i=1}^{n} a_i\right)^k = 0$.

Then, n optimal acreage equations are defined as follows:

(7) $a_i^* = \frac{E(U_\Pi \Pi_{a_i})}{E\left(\sum_{i=1}^{n} U_\Pi \Pi_{a_i}\right)}$, $i = 1, \ldots, n$.

1. Wealth Effect

Sandmo (1971) and Chavas and Holt (1990) have examined the relationship between wealth effects, $\partial a^* / \partial w$, and the nature of risk preference. Their studies indicated that a zero wealth effect corresponds to constant absolute risk aversion, while non-zero wealth effect corresponds to non-constant absolute risk aversion. In order to examine their findings, equation (7) can be differentiated in terms of wealth (w),

(8) $\frac{da_i^*}{dw} = \frac{E(U_{\Pi\Pi}\Pi_{a_i}\Pi_w) E\left(\sum_{j \neq i}^{n} U_\Pi \Pi_{a_j}\right)}{E\left(\sum_{j=1}^{n} U_\Pi \Pi_{a_j}\right)^2}$,

where $E(U_{\Pi\Pi}\Pi_{a_i}\Pi_w) = (\bar{p}_i \bar{y}_i - c_i)\bar{U}_{\Pi\Pi} + (\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)(U_{\Pi\Pi} - \bar{U}_{\Pi\Pi})$.
$\bar{U}_{\Pi\Pi}$ represents a change in the expected marginal utility of wealth (Π) when w changes and $U_{\Pi\Pi}$ represents a change in the marginal utility of wealth (Π) when w changes. The probability of w having a specific value in is $0 \leq \phi(h) \leq 1$ and $\int_0^h \phi(w) dw = 1$.

Proposition 1. ***A zero wealth effect will be satisfied if a farm household is constantly absolute risk averse.***

Proposition 2. ***A zero wealth effect will be satisfied under non-constant absolute risk averse if disturbances of price and yield are zero. Conversely, a zero effect of wealth will not be satisfied under non-constant absolute risk averse if disturbances of price and yield are not zero.***

Proof.
If a farm household is constant absolute risk averse, then $E(U_{\Pi\Pi}\Pi_{a_i})$ will be zero regardless of risk of price and yield and market condition. Therefore, $\partial a_i^* / \partial w = \partial a_j^* / \partial w = 0$ in equation (8). Also, when e and ε are zero with $py = c$ in competitive market, $E(U_{\Pi\Pi}\Pi_{a_i}\Pi_w)$ will be zero regardless whether a farm household is constant or non-constant absolute risk aversion. That is, $\partial a_i^* / \partial w = \partial a_j^* / \partial w = 0$ in equation (8). As a result, a zero effect of wealth (w) on acreage decisions will depend on farm household's risk preference, risk of the output market, and output market structure.

2. Farm Effect

In equation (7), $E(U_\Pi \Pi_{a_i}) = \bar{U}_\Pi \bar{\pi}_i + (U_\Pi - \bar{U}_\Pi)\bar{p}_i \varepsilon_i + (U_\Pi - \bar{U}_\Pi)\bar{y}_i e_i + (U_\Pi - \bar{U}_\Pi) e_i \varepsilon_i$. $\bar{U}_\Pi$ represents the expected marginal utility of wealth (Π), which is expressed as $\bar{U}_\Pi = U_\Pi \int_0^h \phi(w) dw$. U_Π represents the marginal utility of wealth (Π) as w changes from h_0 to h_1. Therefore, n differential acreage allocation equations can be obtained from differentiating equation (7) in terms of $\bar{\pi}$, e, ε, and $e\varepsilon$ and is illustrated mathematically as follows:

(9) $da_i^* = \sum_{j=1}^{n} \gamma_{ij} d\bar{\pi}_j + \sum_{j=1}^{n} \phi_{ij} de_j + \sum_{j=1}^{n} \varphi_{ij} d\varepsilon_j + \sum_{j=1}^{n} \psi_{ij} de_j \varepsilon_j$.

The own and cross parameters of γ in equation (9) are as follows:

(10.1) $\gamma_{ii} = \frac{\bar{U}_\Pi \left(\sum_{j=1}^{n} \Phi_j - \bar{U}_\Pi \bar{\pi}_i\right)}{\left(\sum_{j=1}^{n} \Phi_j\right)^2}$, and

(10.2) $\gamma_{ij} = \frac{-\bar{U}_\Pi \bar{\pi}_i \bar{U}_\Pi}{\left(\sum_{j=1}^{n} \Phi_j\right)^2}$,

where $\Phi_j = \bar{U}_\Pi \bar{\pi}_j + (U_\Pi - \bar{U}_\Pi)\bar{p}_j \varepsilon_j + (U_\Pi - \bar{U}_\Pi)\bar{y}_j e_j + (U_\Pi - \bar{U}_\Pi) e_j \varepsilon_j$.

Symmetry and positive semi-definite restrictions with respect to optimization for equation (5) are related to compensated wealth acreage effect, assuming that one holds utility constant. The compensated wealth acreage effect takes the form as follows:

(11) $\frac{da_i^c}{d\bar{\pi}_j} = \frac{da_i^*}{d\bar{\pi}_j} - \frac{da_i^*}{dw} \cdot a_i^*$, $i, j = 1, \ldots, n$.

where $\partial a_i^c / \partial \bar{\pi}_j$ is the wealth compensated acreage effect of crop j on crop i maintaining constant utility of a farm household. The matrix of compensated effects is symmetric and positive semi-definite (Chavas, 1987). As Chavas and Holt (1990) indicated, equation (11) also implies that the slope of the uncompensated function $\partial a_i^* / \partial \bar{\pi}_j$ can be decomposed as the sum of two terms: the compensated slope (or substitution effect) $\partial a_i^c / \partial \bar{\pi}_j$ which maintains a given level of utility plus the wealth effect $\left(\partial a_i^* / \partial w \cdot a_i^*\right)$.

A. Symmetry

The cross compensated acreage effects are derived by using equations (7), (8), and (10.2) as follows:

(12) $\frac{da_i^c}{d\bar{\pi}_j} = \frac{-\bar{U}_\Pi \bar{\pi}_i \bar{U}_\Pi}{\left(\sum_{j=1}^{n} \Phi_j\right)^2} - \frac{(\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)(U_{\Pi\Pi} - \bar{U}_{\Pi\Pi})\left(\sum_{j \neq i}^{n} \Phi_j\right)\Phi_i}{\left(\sum_{j=1}^{n} \Phi_j\right)^2}$.

Equation (12) indicates that cross compensated acreage effects will depend upon the level of uncertainty and the expected values for price and yield. Therefore, the general condition of symmetry under uncertainty cannot be satisfied because disturbances are different for price and yield. However, if *Proposition 1* or *2* holds, then the symmetry condition would be satisfied.

Proof.
Since the expected net profit of crop i is equal to the expected net profit of crop j, $\gamma_{ij} = \frac{-\bar{U}_\Pi \bar{\pi}_i \bar{U}_\Pi}{\left(\sum_{j=1}^{n} \Phi_j\right)^2} = \frac{-\bar{U}_\Pi \bar{\pi}_j \bar{U}_\Pi}{\left(\sum_{i=1}^{n} \Phi_i\right)^2} = \gamma_{ji}$.
Given condition, $\frac{da_i^c}{d\bar{\pi}_j} = \frac{da_j^c}{d\bar{\pi}_i}$ will be satisfied because $(\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)$ is zero when *Proposition 1* holds or Φ_i is zero when *Proposition 2* holds. Therefore, any violation of the symmetry condition related to uncertain output market at planting time might stem from that 1) a farm household has a different expectation with respect to crop yield and price, 2) price and yield risk for each crop is different, and/or 3) an individual farm household has a different risk preference.

B. Positive Semi-definite

Own compensated acreage effects are derived by using equations (7), (8), and (10.1) as follows:

(13) $\frac{da_i^c}{d\bar{\pi}_i} = \frac{\bar{U}_\Pi \left(\sum_{j \neq i}^{n} \Phi_j + (\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)\right)}{\left(\sum_{j=1}^{n} \Phi_j\right)^2} + \frac{(\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)(U_{\Pi\Pi} - \bar{U}_{\Pi\Pi})\left(\sum_{j \neq i}^{n} \Phi_j\right)\Phi_i}{\left(\sum_{j=1}^{n} \Phi_j\right)^2}$.

Therefore, the general condition of positive semi-definiteness under uncertainty indicates that the non-negativity of own compensated acreage effects will depend upon 1) price and yield risk (direction and magnitude), 2) expected values of price and yield, and 3) the relative strength between $U_{\Pi\Pi}$ and $\bar{U}_{\Pi\Pi}$. Regardless of a farm household's risk preference , violation of the non-negativity condition implies that the acreage allocated for crop i can decrease even when the expected net profit for crop i increases because price and yield risks are high (toward negative direction) and a change in the marginal utility of wealth of a farm household is relatively small compared to a change in expected marginal utility of wealth. However, if *Proposition 1* or *2* holds, then the non-negativity condition of own compensated acreage effect will be satisfied.

Proof.
Under *Proposition 1* or *2*, the own compensated acreage effects will be reduced into the form $\frac{da_i^c}{d\bar{\pi}_i} = \frac{\bar{U}_\Pi \left(\sum_{j \neq i}^{n} \bar{U}_\Pi \bar{\pi}_j\right)}{\left(\sum_{j=1}^{n} \bar{U}_\Pi \bar{\pi}_j\right)^2}$.
We know that expected marginal utility is positive $\left(\bar{U}_\Pi = U_\Pi \int_0^h \phi(\pi_i) d\pi_i \geq 0\right)$. Also, in order to allocate acreage to crop j, requires the assumption that expected revenue in producing crop j should be greater than or equal to the total cost in producing crop j, implying $\bar{\pi}_j \geq 0$. Therefore, $\frac{da_i^c}{d\bar{\pi}_i} \geq 0$ when *Proposition 1* or *2* holds.

C. Homogeneity

Regardless of a farm household's risk preference, the homogeneity condition is defined as:

(14) $\sum_{j=1}^{n} \frac{da_i^c}{d\bar{\pi}_j} = \frac{\bar{U}_\Pi \sum_{j=1}^{n} (\bar{y}_j e_j + \bar{p}_j \varepsilon_j + e_j \varepsilon_j)}{\left(\sum_{j=1}^{n} \Phi_j\right)^2} = 0$.

The general condition of homogeneity depends, therefore, on e_j, ε_j, and $e_j\varepsilon_j$. This implies that production decisions can be affected by risks associated with price and yield even when all input and output prices change proportionally. In this case, homogeneity would not be satisfied without *Proposition 1*.

CONCLUSIONS

This study is intended to describe a system of acreage allocation under price and yield uncertainty so as to identify the role of output market uncertainty in acreage decisions. This study adopted expected utility as developed by Chavas and Holt. The major findings of this study are as follows: 1) a zero effect of wealth in acreage decisions would depend on not only the risk preference of farm households but also the risk and structure of the output market, 2) violation of symmetry might come from (i) different expectation about yield and price, (ii) risk difference in the price and yield for each crop, and/or (iii) different risk preference of farm household, 3) the non-negativity of own compensated acreage effects would be satisfied if *Proposition 1* or *2* holds, and 4) production decisions would be affected by risk of price and yield even when all input and output prices change proportionally in which case homogeneity would not be satisfied without *Proposition 1*.

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