A Dynamic Optimisation of Summer/Autumn Feeding Strategies for Wool Production in the Mediterranean Environment of Western Australia

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To counteract declining quantity and poor quality of paddock feed over summer/autumn, farmers in Western Australia must decide how much of which supplement to provide to grazing sheep to optimise the use of supplements. The solution is complicated by the interaction of biological and economic factors and the intertemporal nature of the problem. A dynamic programming approach was used to determine optimal feeding strategies for different starting conditions. Lupin seed was preferred to oats for weaners when paddock feed was abundant, while oats was preferred for weaners and adults when paddock feed was scarce. The optimal feeding rate usually starts low and then gradually increases to higher rates as the autumn break approaches.

1. Introduction

The mediterranean environment in Western Australia (WA) is characterized by cool wet winters followed by hot dry summers. The annual rainfall in the agricultural regions varies from 250 to 1000 mm and decreases from the south west coastal corner to the north east. The summer/autumn season is generally restricted to the November-April period and is typified by hot, dry conditions. Some unreliable rainfall can be expected in this period, commonly from isolated thunderstorms. The winter/spring season generally starts in May and ends in October. This period is characterised by more reliable rainfall which is associated with the movement of frontal systems across the south west of the Australian continent. The consequence of this is that pasture growth occurs over the winter/spring period. Dry pasture or cereal stubbles are the major feed available over summer with a limited amount of green pick following some summer rainfall events. Both the quantity and quality of dry feed decline over the summer season, and rainfall accelerates the process through decay or leaching of the soluble nutrients (Purser 1983).

Over the summer/autumn period the main decisions facing farmers wishing to maximise wool income are the timing, the type and the amount of supplements required by grazing sheep. These decisions are difficult because the outcome of the summer/autumn feeding strategies are determined by many factors. They range from the starting quantity and quality of paddock feed, stocking rates, initial sheep liveweight and type of sheep, the economic prospect for wool prices including penalties for low staple strength, supplementary feed costs, the length of the summer/autumn period, likelihood of summer rainfall, and the farmers' personal goals of running the property, etc. Furthermore, the problems inherent in the supplementary feeding decision are dynamic in nature and cannot be solved easily.

When making an economic feeding decision, the current costs of supplementary feeding must be offset against future profits due to the current feeding. A feeding regime resulting in an excessive consumption of paddock feed may lead to a scarcity of dry feed for later use; while, on the contrary, a feeding regime which reduces current consumption of paddock feed may result in poor dry feed utilization because the quantity and quality of dry feed declines quickly whether it is used or not. This pattern of decline usually leads to a loss in sheep liveweight and wool production when a feed shortage occurs. The loss in wool production can also cause a significant reduction in the potential value of the fleece when the effect of poor nutrition affects the strength of the wool fibre. Therefore, most woolgrowers in WA provide supplementary feed to prevent large liveweight losses, particularly

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for weaners and pregnant ewes over the summer/ autumn period (Rowe *et al.* 1989).

Supplementary feeding can improve sheep intake and utilization of dry pasture or stubble (Gardner and Doyle 1991). Thus, one of the objectives of supplementary feeding is to utilize the dry pasture efficiently when its quantity and quality are still high. Once the feed quantity and quality are limiting, the objective of supplementary feeding is usually to maintain or minimise the rate of loss of sheep body condition. Sheep receiving supplements during the summer period usually have higher wool growth rates entering winter (Rowe *et al.* 1989). This carryover effect on wool production is strengthened if sheep liveweight can be maintained or increased during the summer/autumn period. Apart from increased animal production, supplementary feeding in late autumn can reduce sheep dry pasture feed intake significantly. This will extend paddock feed supplies, thus reducing the risk of soil erosion which is also a major concern in WA.

The adoption of these strategies has occurred during times of high wool and sheep prices under the Reserve Price Scheme. However, it is still not clear whether supplementary feeding is profitable and what supplementary feeding strategies should be adopted with current low wool prices. From 31 May 1990, the minimum reserve price was reduced to 700 c/kg clean from 870 c/kg clean and the floor price for each micron grade was reduced by 19.5 per cent. In February 1991 the Reserve Price Scheme was removed and the wool price dropped to a record low of around 450 c/kg clean. After the initial shock of the suspension of the floor price, the wool price returned to an average of 557 c/kg clean over the 1991/1992 season, still substantially lower than the previous 700 c/kg clean floor price (Australian Wool Corporation 1992).

Historically, most of the research done in determining summer/autumn feeding strategies was by biologists through various experimental designs (Rowe and Ferguson 1986; Butler *et al.* 1987; Rowe *et al.* 1989; Thompson and Curtis 1990; Gardner and Doyle 1991; among others). The common feature of these studies was the analysis of various supplementary feeding strategies by comparison with a no supplementary feeding strategy. The major limitation of these studies lay in their inability to identify the optimal decision rule efficiently: the experimental decision rules can be set in an infinite number of ways and the experimental results depend on the specific values of the environmental and other variables during the experiment. By contrast, economic studies have applied various optimisation techniques in analysing grazing and/ or drought strategies (for example, Mauldon and Dillon 1959; Dillon and Lloyd 1962; Officer and Dillon 1965; Bartlett *et al.* 1974; Thatcher and Lloyd 1975; Hunter *et al.* 1976; Toft and O'Hanlon 1979; Kingwell and Pannell 1987). However, to the authors' knowledge, no study has yet given the same attention to the problem considered here, i.e., the optimal summer/autumn supplementary feeding strategies. Therefore, the objective of this study is to improve the economic efficiency of supplementary feeding practices and to derive the general guidelines for woolgrowers with regard to what to feed, how much to feed and when to feed over the summer/autumn period.

There are a number of techniques which could be used for solving deterministic intertemporal optimisation problems, ranging from various forms of mathematical programming, such as linear, separable, quadratic and non-linear programming, optimal control and dynamic programming (DP). A DP approach was chosen over other optimisation techniques for the following reasons. First, the optimisation problem can be defined by DP in a free format in contrast to the matrix formulation needed for most mathematical programming techniques. Second, DP optimisation can have flexibility in both the return function and the biological model used for the simulation of summer/autumn sheep grazing. It does not require linear or continuous relations to represent the problems in contrast to those of multi-period linear programming or optimal control. Third, while multi-period linear programming formulations increase in size to accommodate piece-wise approximations of non-linear constraints, DP addresses such constraints directly. Furthermore, DP can narrow the solution space to specific values of state variables for different periods while multi-period linear programming cannot. Fourth, in DP models, the rate of change of the state variables can be parameterised as a function of previous state variable values, state variable interactions, other system variables as well as time. In multi-period mathematical programming the rate of change of the state variables, in general, can be varied only with respect to time. Finally, DP optimisation models can yield a sequence of optimal feedback decision rules useful to decision makers, while most other dynamic optimisation techniques generate only single optimal time paths for state or control variables.

In adopting DP for the study, it is recognised that DP may not be the most appropriate technique for solving other dynamic optimisation problems, particularly those with a large number of state variables due to the so-called curse of dimensionality (Kennedy 1986). However, DP is a natural technique to choose for solving dynamic problems involving a small number of state variables (generally less than three or four).

2. A Dynamic Programming Model

It is recognised that cash flow and/or peak debt limit may be important to some woolgrowers particularly when wool prices are low. However, this constraint is more appropriately incorporated in a whole farm context rather than the paddock-level model described below. Thus, the woolgrower is assumed to have a goal of maximisation of net income from wool production over a six-month summer/autumn period subject to the transition of two state variables. These are the starting quantity of paddock feed and average sheep liveweight. A holistic approach to the problem may involve five state variables: the length of the summer/autumn period, average sheep liveweight, flock size, dry matter on-offer and its quality (i.e. digestibility). To keep the dynamic model formulation manageable, a fixed stocking rate is used and a constant decline rate is applied for the quantity and quality of the paddock feed. A constant stocking rate is justified because (a) most farmers maintain stock numbers through to winter, (b) stocking rate increases are not feasible because of the shortage of feed, and (c) stock prices are usually low during the summer making selling decisions unprofitable.

A constant decline rate was used because there were no data for determining a more appropriate pattern. A six-month summer period is used for analytical simplicity, but any other lengths of run can be dealt with in a similar manner by varying the planning horizon of the decision problem. Thus, only two state variables are left for the simplified model: the initial sheep liveweight (LW) and the initial level of dry matter on-offer (DM). The state classification for sheep liveweight ranged from 36 to 74 kg for adult sheep and 18 to 46 kg for weaners, with 2 kg intervals. The level of dry matter on-offer ranged from 0 to 5000 kg/ha with 200 kg intervals.

In formulating the problem, seven stages are designated from pasture senescence to the autumn break. The beginning of stage 7 is only considered as the outcome of stage 6. A stage t is defined as a 30-day period. At the beginning of each stage, based on observations of the condition of animal and pasture, a supplementary feeding regime (SUP) is selected and applied for that period. The feeding regime chosen, along with the current sheep liveweight and dry matter on-offer determine the state of the pasture and animal at the start of the following stage, which in turn affects the next feeding decision. Mathematically, a dynamic optimisation model for the above decision process results in the following recursive equation:

$$(1)\quad V_t(LW_t,DM_t)=\mathrm{Max}[\Pi(LW_t, DM_t,SUP_t) + V_{t+1}(LW_{t+1}, DM_{t+1})] \quad (t=1,2...,6)$$

where $V_t(LW_t, DM_t)$ are the optimal values from following the optimal feeding decisions since stage t and $\Pi(LW_t, DM_t,SUP_t)$ is the stage return function expressed as a function of the sheep liveweight, paddock feed supply and the feeding decision. Discounting is not used since wool income can only be acquired through wool harvest which normally occurs once a year. Maximisation in (1) utilises 61 different feeding decisions. Decision 1 is nil feeding. Decisions 2 to 31 involve various rates of feeding of lupin seed ranging from 25 g/hd per day to 750 g/hd per day with 25 g intervals. Decisions 32 to 61 are the counterparts for oat grain supplementation.

A summer/autumn sheep grazing model, SUMMERPACK (Orsini 1990), was modified to simulate the data required for this study. SUMMERPACK was developed by the Department of Agriculture WA to assist farmers and agricultural advisers with the management of sheep over the summer/autumn period in a mediterranean environment. It provides predictions through time of sheep liveweights, sheep intake, quality and quantity of paddock feed on-offer. The modification of the model for this study included the incorporation of wool production and its economic value, and thus allows the comparison of alternative feeding strategies.

Two types of sheep are distinguished in the modified model. These are weaner and adult dry sheep. It is a paddock-level model with the assumption of an average liveweight for each sheep and a subclover dominant pasture. The model runs on a daily time step.

A modified flow chart of the SUMMERPACK model is outlined in Figure 1. Sheep intake is the central variable of the model; it drives and is driven by most of the other variables. Dry matter intake for each sheep is first determined by the sheep liveweight and the quantity and quality of paddock feed. It is then adjusted for supplements through substitution and supplementation effects (see below). Total dry matter intake for the flock is thus calculated from the intake of each sheep and the number of sheep in the flock.

The amount of dry matter on-offer over time is calculated from its natural decline rate and from the amount eaten by the flock. The natural decline rate of herbage is assumed to be a constant proportion of the amount on offer. The total digestible intake is calculated as the sum of the digestible intake of herbage and of supplement. Sheep liveweight change is derived from the energy balance through an energy conversion mechanism which is driven by sheep liveweight. A more detailed description of the model is given by Orsini (1990).

Wool production is predicted from sheep liveweight, liveweight change and the use of supplements. The income from wool production is calculated from the amount of wool produced and its market price. The income together with the costs of feeding and other on-farm and off-farm costs determine the economic value of any stocking rate and supplementary feeding strategy over the summer/autumn period.

Stage return function Π ($/ha) is calculated as follows:

(2) $\Pi(LW_t, DM_t, SUP_t) = (PW.CW_t - 30FC_t - 30OFC)SR$

where PW is the net wool price, which was $4.01/kg clean for adult sheep and $4.86/kg for weaners; CW_t is the wool produced per head per decision stage (kg clean); FC is the feeding costs per day which is dependent on the rates of supplements (in which $0.187/kg for lupin and $0.137/kg for oats were used); OFC is the other on-farm variable costs per day which is the annual variable cost (e.g. shearing, vaccinations, drenching) spread out evenly over the year ($4.81/hd per annum) and SR is the number of sheep per hectare (SR=12). The values for wool prices were the predicted net prices for 1992 which are net of 12 per cent wool tax, 4 per cent selling cost (brokers charge, insurance and testing), and an estimated 18.3 c/kg clean for freight and other costs (Western Farmer and Grazier 1992). The value for feeding costs and other on-farm variable costs were the estimated regional figures in WA for 1992 (J. Young, pers. comm. August 1991). This return function can be viewed as the returns to the fixed factor (e.g. land, machinery and manager's expertise) since no fixed costs are included in the calculation.

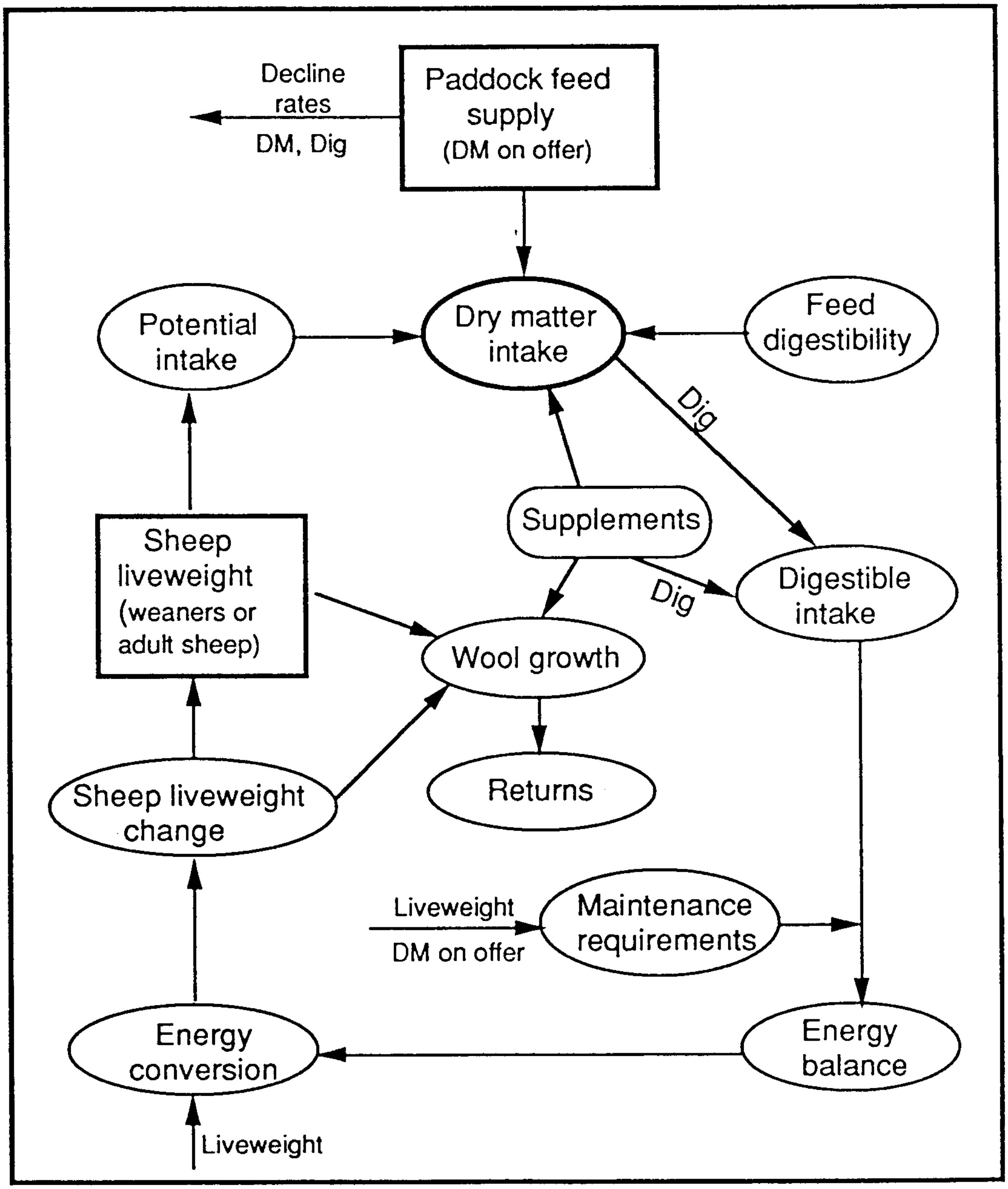
Terminal value function V_7 is determined by the sheep liveweight and feed availability at stage 7. Adult sheep are assumed to be dead below 35 kg and weaners below 18 kg, corresponding to a condition score of 0 (Orsini 1990). Paddock feed must be greater than 1000 kg DM/ha because there is a risk of serious soil erosion when paddock feed levels drop below this level (Carter 1990; Carter *et al.* 1992). A penalty of $0.187/kg DM, equivalent to the price of lupins, is used if the level of dry matter falls below this level. This forces overgrazing to be less profitable than feeding lupins. Thus

(3) $V_7 = RETW$
If ($LW_7 >= 35$ or 18 and $DM_7 >= 1000$)

$= -PS \cdot SR$
If ($LW_7 < 35$ or 18 and $DM_7 >= 1000$

$= -PS \cdot SR - 0.187(1000 - DM_7)$
If ($LW_7 < 35$ or 18 and $DM_7 < 1000$)

Figure 1: The Modified SUMMERPACK Model Structure



$= RETW - 0.187(1000-DM_7)$
If (LW_7>=35 or 18 and DM_7<1000)

where RETW is the expected value of wool produced per hectare for an average winter/spring season (kg/ha clean) and PS is the price of sheep ($5 for adult dry sheep and $10 for weaners (Western Farmer and Grazier 1992)). The expected wool production in winter is calculated from an expected winter daily wool growth rate (g/hd/day clean), WWGday, and the length of the winter season (i.e. 185 days). Thus,

(4) RETW=[185PW(WWGday/1000)-185OFC]SR

The expected daily wool growth rate during the winter is determined by the final sheep liveweight LW_7 as follows:

(5) $WWGday = AWGday + b1(LW_7-LW2)^{0.75}$
if $LW_7 >= LW2$
$= AWGday - b1(LW2-LW_7)^{0.75}$
if $LW_7 < LW2$

where the average winter daily wool growth rate AWGday (set to 14 g/hd for adult dry sheep and 11 g/hd for weaners) is adjusted by the gap between the terminal liveweight LW_7 and the base liveweight LW2 for a condition score 2 animal (set to 50 kg for adult sheep and 30 kg for weaners). Parameter b1 measures the carry-over effect of a per unit increase of metabolic size (liveweight$^{0.75}$) on wool growth rate during the winter. It is set to 0.48 for adult dry sheep (K. Curtis, pers. comm. September 1991) and 0.68 for weaners (Allden 1979).

The average liveweight of a sheep at stage t +1 in equation (1) depends on the initial average liveweight in the previous stage plus net liveweight change LWC_t. The net liveweight change over stage t is a function of liveweight, the quantity and quality of dry matter on-offer and supplements supplied. It is calculated from the summation of daily net liveweight changes over the 30-day period. Thus

(6) $LW_{t+1}=LW_t+LWC_t(LW_t,DM_t,SUP_t,Dig_t,DigSUP_t)$

where Dig_t and $DigSUP_t$ are the digestibility for paddock feed and supplements respectively. The digestibilities of supplements are constant and set at 85 per cent for lupins and 75 per cent for oats, respectively. The detailed mathematical specification of daily liveweight change functions are given in Orsini (1990).

Dry matter on-offer at stage t+1, DM_{t+1}, in equation (1) is defined by the following equation:

(7) $DM_{t+1}=DM_t-DME(LW_t, DM_t,SUP_t)-DMD_t$

where DM_t is the initial dry matter on-offer at stage t; DME is the total sheep intake over stage t which is the summation of daily sheep dry matter intake over the 30-day period and DMD_t is the total amount of dry matter reduced through natural decline over stage t.

Daily sheep dry matter intake, DME_i, is first derived from a potential intake, PI, which is a function of sheep liveweight at day i, and then adjusted for two scaling factors: pasture availability factor, AF, and feed digestibility factor, DF, which are functions of dry matter on-offer DM_i and dry matter digestibility Dig_i at day i, respectively. These two factors limit the extent to which the potential intake can be realised. Dry matter intake for each sheep is also adjusted if supplements are provided. Small amounts of high protein supplements (i.e. lupins) will increase the intake and utilization of dry pasture. Such effects are only likely when lupins are given to stock with high energy demands (i.e. weaners) (Gardner and Doyle 1991). Cereal grain (i.e. oats) are assumed to substitute for dry feed at any levels according to a substitution rate, SubR. The substitution rate increases with decreasing herbage availability and digestibility. Thus,

(8) $DME_i = PI(LW_i)\cdot AF(DM_i)\cdot DF(Dig_i)$
for no supplements or for small amounts of lupins (<= 50 g/hd) fed to weaners;

(9) $= PI(LW_i)\cdot AF(DM_i)\cdot DF(Dig_i) - SUP_i\cdot SubR(DM_i,Dig_i)$
for cereal grain supplements; and

(10) $=(PI(LW_i)-SUP_i)\cdot AF(DM_i)\cdot DF(Dig_i)$
for all other situations.

The numerical functions in the above equations are given in Orsini (1990). When supplements are given, they are assumed to be completely consumed first as a supplement to dry matter intake according to equation (8) or as a substitute for dry matter intake according to equations (9) and (10). Equation (10) reflects the sheep preference for supplements and limits the consumption of paddock feed by the extent to which potential intake has already been satisfied from supplements.

Clean wool production at stage t, CW_t, is the total clean wool grown at stage t which is the summation of daily clean wool growth, $SWGDay_i$, over the 30-day period. The daily clean wool growth is predicted from sheep liveweight, liveweight change, type of sheep and the level of supplementary feeding according to the data from Curtis (pers. comm. September 1991) and Allden (1979) as follows:

(11) $SWGday_i = Max(1, -0.25 + 0.45LW_i^{0.75} + 0.028LWCday_i)$
for adult sheep and $SUP_i > 0$;

$= Max(1, 0.21LW_i^{0.75} + 0.005LWCday_i)$
for adult sheep and $SUP_i = 0$;

$= Max(1, -0.829 + 0.522LW_i^{0.75} + 0.035LWCday)$
for weaners.

Feeding costs at stage t, FC_t, is the summation of daily feeding costs FC_i over a 30-day period. The daily feeding cost was calculated from the price of supplements ($0.187/kg for lupin and $0.137/kg for oats), its level of use and the sheep numbers, SR. Therefore

(12) $FC_i = 0.187 SUP_i SR/1000$
if lupin supplements are fed;

$= 0.137 SUP_i SR/1000$
if cereal grain supplements are fed.

The quality of dry feed is assumed to decline according to a constant decline rate, 0.3 per cent per day. Therefore, the following equation is used for the calculations of digestibility at each stage.

(13) $Dig_t = Dig0(1-0.003)^{30(t-1)}$

where Dig0 is the initial digestibility (set to 62 per cent) at stage 1.

The recursive equation (1) was solved for adult dry sheep and weaners, respectively, by backward induction with a two-dimensional linear interpolation method to approximate V_{t+1} at each stage (Kennedy 1986). Furthermore, due to natural decay, after stage 1 the states which can not be reached from any state at the previous stage are deleted. This greatly reduces the computing time required to solve the model. The redundant states are those states with the level of forage on-offer greater than the stage maximum available forage, $DMMax_t$. The maximum available forage at stage t is estimated as follows:

(14) $DMMax_t = 5000(1-0.003)^{30(t-1)}$

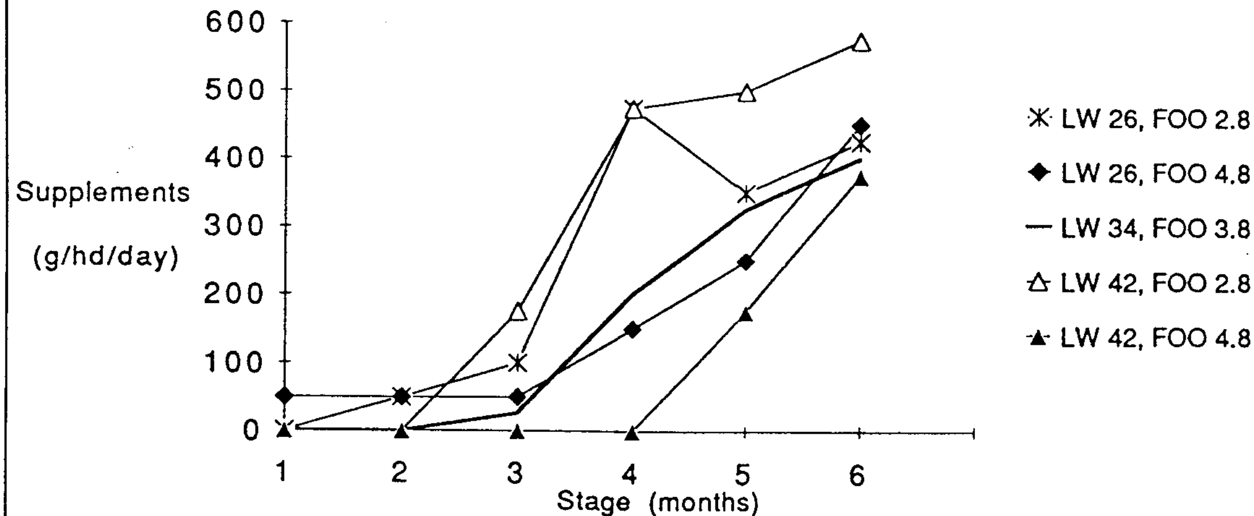
where 0.003 is the daily natural decline rate of forage on-offer and 5000 (kg/ha) is the maximum level of paddock feed at the start of stage 1. The value of 0.30 per cent/day used is consistent with that adopted by Orsini (1990). Although a constant decline rate is commonly experienced in the region, under heavy grazing (i.e. a very high stocking rate) or after heavy rainfall events, the decline rates may increase significantly.

3. Results and Discussion

There are many possible starting combinations of sheep liveweight and dry matter on offer. To illustrate the general patterns of the optimal feeding strategies and to simplify the presentation of the results, five representative states covering the range of possible starting values in the field were selected. These five states represented the following combinations at the beginning of summer: light weight weaners (26 kg) with a limited amount of forage on-offer (2.8 t DM/ha), light weight weaners with abundant forage (4.8 t DM/ha), medium weight weaners (34 kg) with reasonable forage (3.8 t DM/ha), heavy weight weaners (42 kg) with a limited amount of forage and heavy weight weaners with abundant forage.

Optimal supplementary feeding strategies for a weaner flock derived from the dynamic optimisation model, i.e. equation (1), are presented in Figure 2. Optimal time paths of average liveweight and for-

FIGURE 2
Optimal Feeding Strategies for a Weaner Flock



Note: Lupin is used when the rates of supplements are less than 175g; the rest is oats.

FIGURE 3
Optimal Time Paths of Average Liveweight and Optimal Values for a Weaner Flock

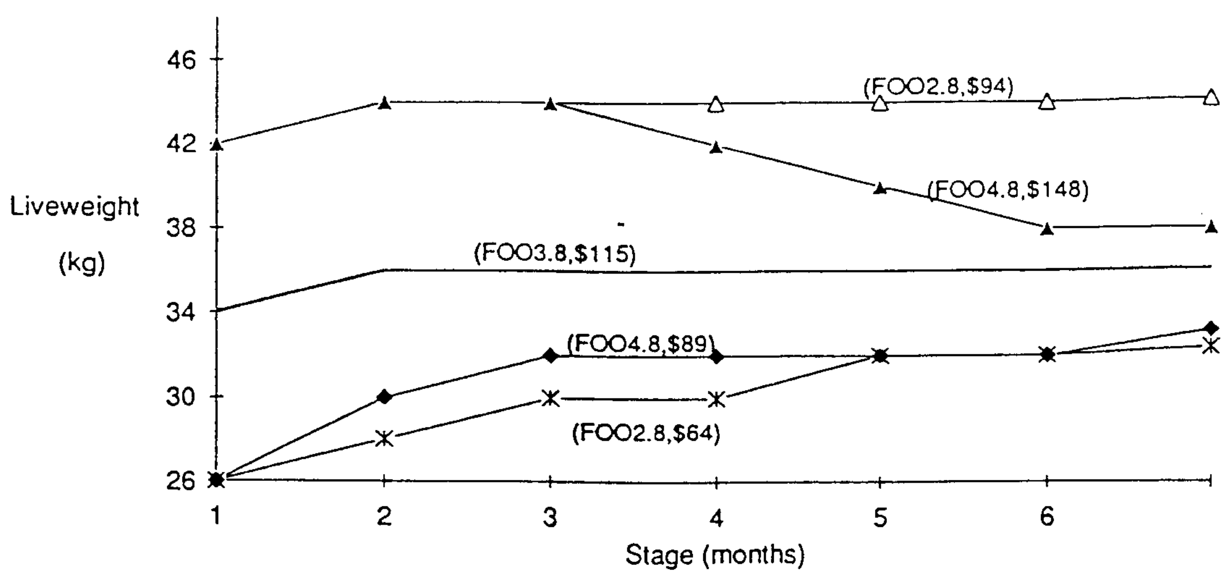
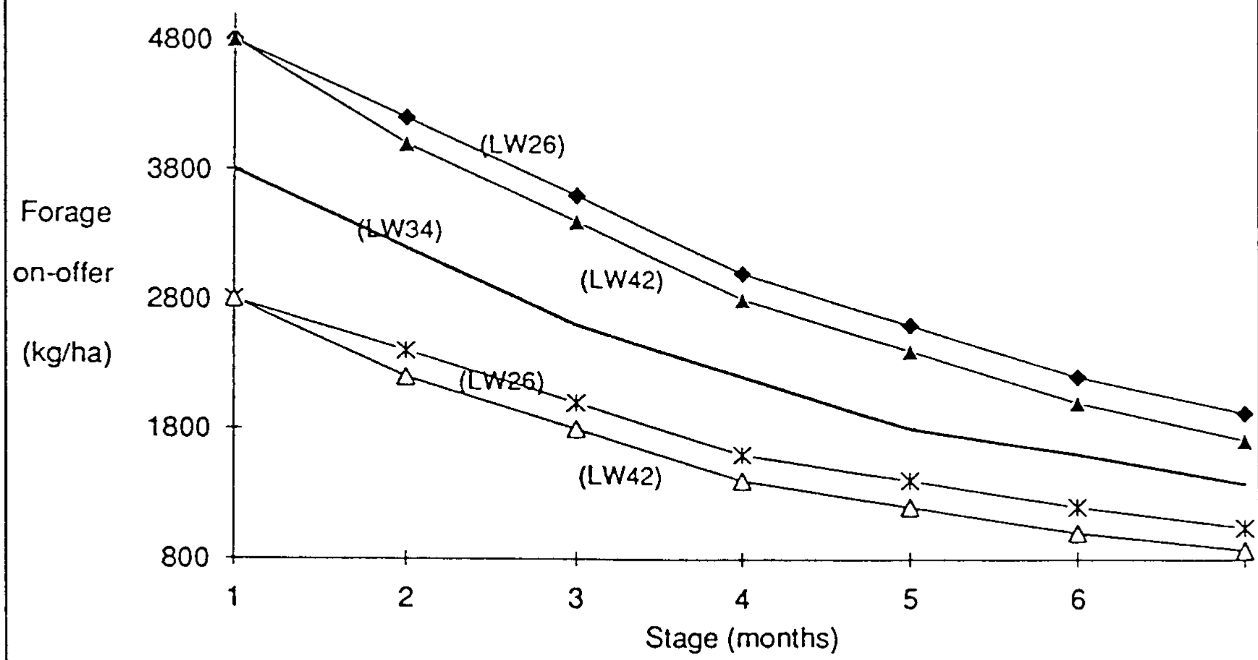


FIGURE 4
Optimal Time Paths of Forage On-offer for a Weaner Flock



age on-offer under the optimal feeding strategies are depicted in Figures 3 and 4.

For light weight weaners (26 kg) combined with a low amount of forage on-offer (2.8 t DM/ha), a zero supplementary feeding strategy was most economical for stage 1. There was just sufficient dry matter to carry the weaners over summer without dropping below the 1000 kg DM erosion risk threshold. Any supplementary feeding in the first month of summer will substantially increase the sheep liveweight and result in a higher level of dry feed intake later on. This would result in a feed shortage at later stages.

Supplementary feeding starts with a low rate of lupins (50 g/hd per day) at stage 2 changing to 100 g/hd lupins at stage 3. Lupins were used instead of oats because of its high energy content as well as its supplementation effects at the low rates of feeding, thus improving the utilization of dry feed. After stage 3, high rates of oats were selected until the autumn break at the end of stage 6. High rates of oats were selected for late summer due to the relatively cheap price and its substitution effect for dry paddock feed, thus reducing the consumption of paddock feed, and so soil erosion risks were avoided. The oscillating pattern of feeding rates during stages 4 to 6 may be due to the discrete values used for state classifications of the model, which is of little importance in practice. The outcome of supplementary feeding for this situation was therefore increasing sheep body condition (i.e. liveweight) at the earlier stages (see Figure 3) to utilise the limited amount of dry feed efficiently to avoid erosion or pasture deterioration.

For light weight weaners (26 kg) combined with plentiful forage on-offer (4.8 t DM/ha), the optimal feeding policy was different from the above. Nil feeding was no longer optimal because there was no risk of erosion under the condition of abundant forage on-offer. The optimal feeding strategy was to feed small amounts of lupins (50 g/hd/day) from pasture senescence (stage 1) to late summer (stage 3). This will improve the dry feed consumption through the supplementation effects (i.e. equation (8)) as well as through the increasing liveweight which will increase the sheep potential intake. After stage 3, the optimal decision was supplementary feeding for maintenance (Figure 2) with the most cost effective feed. This strategy involved feeding 150 g/hd/day lupins in stage 4 and then 250 g/hd/day and 450 g/hd/day of oats at stage 5 and 6, respectively. The increasing amount of supplements after stage 4 reflected the high maintenance requirement for weaners when pasture quality deteriorates despite a substantial amount of feed in the paddock (>2 t DM/ha, Figure 4). The feeding strategy for this situation was thus to improve the utilization of dry pasture when its quality was high so that the liveweight can be built up as soon as possible (Figure 3). Once the quality of pasture became limiting, the strategy changed to feeding to achieve maintenance.

For medium weight weaners (34 kg) with reasonable forage on-offer (3.8 t), supplementary feeding was not necessary in early and mid summer (i.e. stage 1 and 2) when both the quantity and quality of dry feed was non-limiting. After mid summer the feeding strategy was to maintain the sheep liveweight as illustrated in Figure 2. This feeding for maintenance policy was similar to that for state 2. The objective of feeding in this situation was to maintain the sheep body condition throughout the summer/autumn with minimum feed costs (Figure 3).

The optimal feeding strategy for heavy weight weaners (42 kg) combined with a limited amount of forage on-offer (2.8 t) from senescence was similar to that for medium weight weaners. However, a much higher rate of supplementation was required from late summer (stage 3) until the autumn break. No lupins were used throughout the summer/autumn period as oats substitute for dry paddock feed, thus extending paddock feed supplies and reducing the erosion risks as indicated in Figure 4. The very high rate of oat supplementation indicated both the prominent risk of erosion and the high maintenance requirements. The optimal supplementary feeding strategy for this state was to maintain sheep liveweight as well as to avoid soil erosion (Figure 3).

With heavy weight weaners (42 kg) facing abundant forage on-offer at senescence (4.8 t), a zero supplementary feeding rate was optimal throughout summer and early autumn (stage 4). After stage 4 when quality of paddock feed was low, the optimal strategy was to feed for maintenance with rates of 175 g/hd/day oats in stage 5 and 375 g/hd/day in stage 6. The extended nil feeding strategy allowed gradual loss of body weight thus reducing the cost of feeding. The optimal feeding policy for this state was to allow sheep survival at minimum feeding costs.

The optimal values from following the optimal feeding policies increased with higher levels of forage and liveweight, as indicated in Figure 3. For instance, the values increased from $64/ha to $89/ha and from $94/ha to $148/ha when feed increased from 2.8 t DM/ha to 4.8 t DM/ha at initial liveweights of 26 kg and 42 kg, respectively.

Related to the optimal values were the shadow prices which represent the marginal value of the product for a unit increase in state variables. Thus, a higher shadow price for paddock dry feed, 2.7 c/kg DM (=($148-$94)/2000) vs 1.25 c/kg, applies when weaners were in good body condition (42 kg) compared to weaners in poor condition (26 kg). The lower shadow price for dry feed when weaners are in poor condition is not unusual since the response in wool growth is impaired by poor body condition. The sheep intake which contributes the nutrients for wool growth is also limited by the poor body condition (Orsini 1990). The shadow price for sheep liveweight was also higher, 3.69 c/kg (=($148-$89)/16) vs 1.88 c/kg, when paddock feed was plentiful compared to when it was scarce. This indicated a higher demand for dry feed when sheep were in good condition and a higher demand for liveweight when paddock feed supply was abundant under the set stocking assumptions.

The optimal feeding strategies for adult dry sheep and their related time paths of average liveweight and paddock dry feed on-offer are presented in Figures 5, 6 and 7, respectively. The optimal values are also presented in Figure 6. Five states similar to those chosen for weaners were selected for presentation. The starting liveweights for light, medium and heavy weight adult dry sheep were 44 kg, 54 kg and 64 kg respectively.

FIGURE 5
Optimal Feeding Strategies for an Adult Dry Sheep Flock

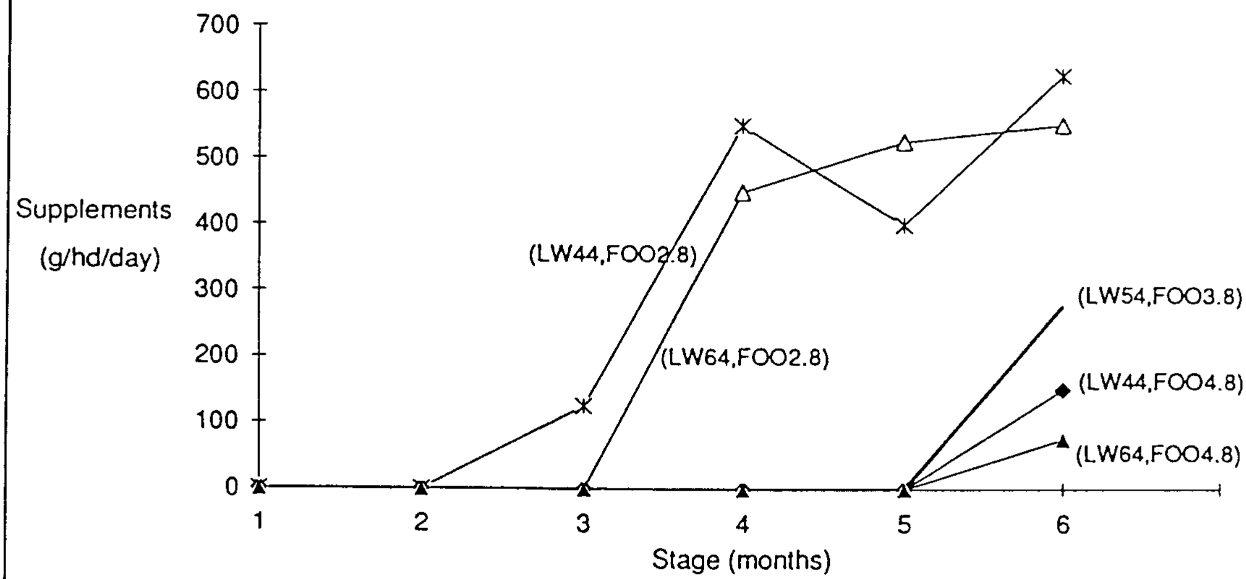


FIGURE 6
Optimal Time Paths of Average Liveweight and Optimal Values for an Adult Dry Sheep Flock

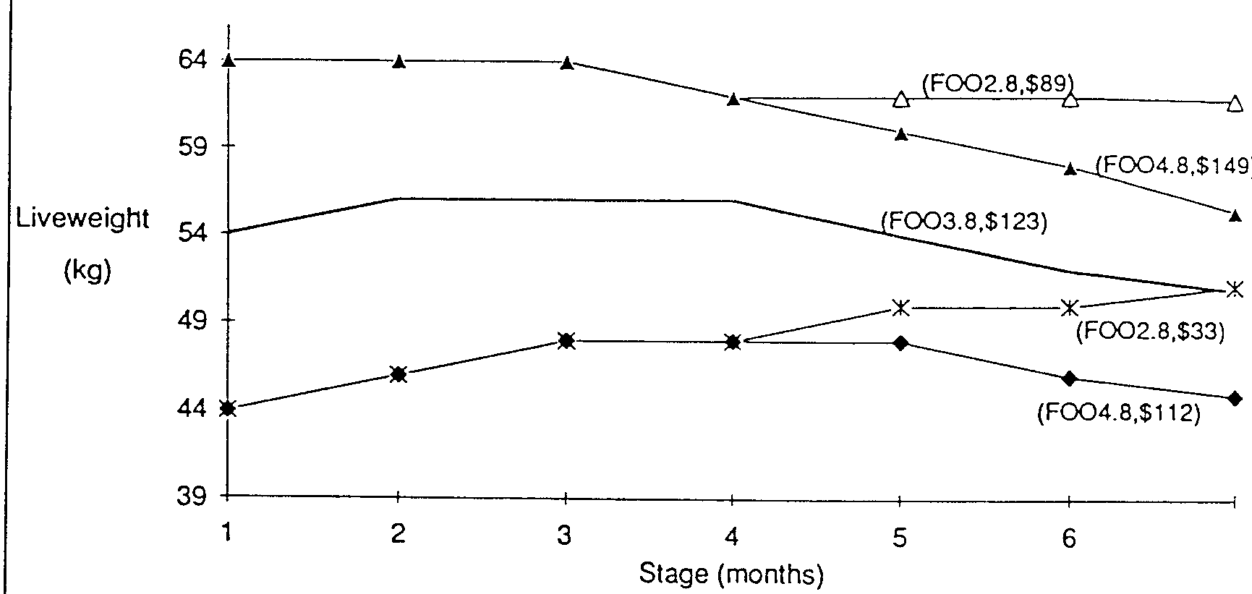
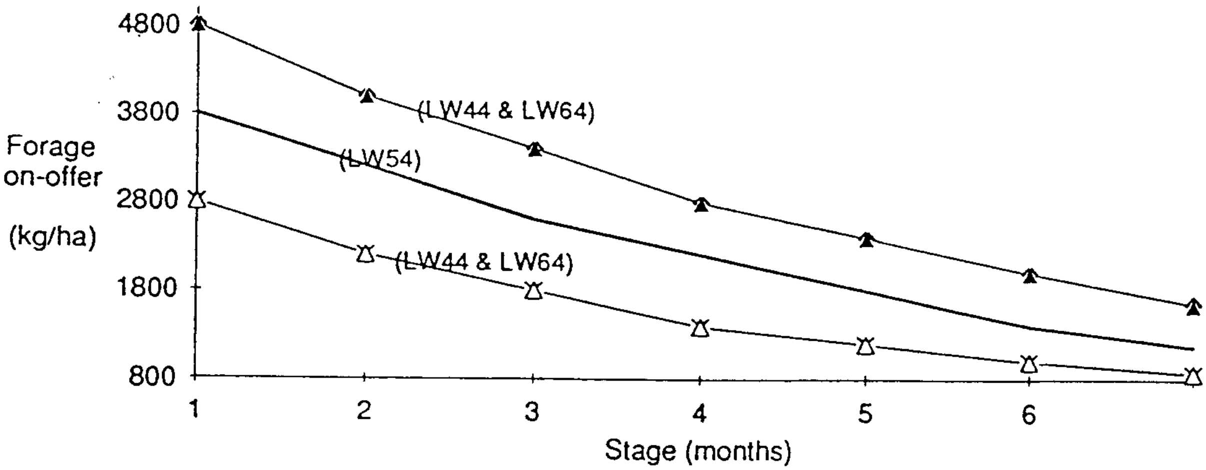


FIGURE 7
Optimal Time Paths of Forage On-offer for an Adult Dry Sheep Flock



As depicted graphically in Figure 5, the nil feeding strategy was optimal from pasture senescence to mid summer (stage 2) irrespective of initial liveweight and paddock dry feed supply. Unlike those selected for weaners, low rates of lupins were no longer economically efficient at the early stages because of the disappearance of supplementation effects with adult sheep. Direct supplementation effects on dry feed only occur when low rates of lupins (<=50 g/hd/day) are given to stock with high energy demands.

If the initial paddock dry feed supplies were above average (>=3.8 t DM/ha), as indicated in Figure 5, the nil feeding strategy was extended to stage 5 before adopting a maintenance or survival feeding strategy in late autumn (stage 6) . This described a strategy which allowed adult dry sheep to build up weight in the case of 44 kg and 54 kg sheep (Figure 6) and to maintain weight in the case of 64 kg sheep during the initial stages when both the quantity and quality of paddock dry feed were non-limiting. Then gradual loss of weight was allowed until late autumn. Feeding started only when sheep lost liveweight too rapidly causing low rates of wool growth or the increasing risk of death.

When paddock feed was not sufficient to carry the sheep through the summer/autumn period, supplementary feeding was essential. Feeding was started at an early stage before sheep started to lose weight in the case of 44 kg sheep, but was postponed for one month to allow slight loss of weight in the case of heavy adult dry sheep (Figures 5 and 6). Very high rates of oats (averaging more than 500 g/hd/ day) were selected throughout the autumn to avoid erosion (Figure 7), since oats have a strong substitution effect on dry feed intake. This could also help to maintain or improve sheep liveweight, as indicated in Figure 6, for heavy and light weight adult dry sheep.

It is also worth mentioning that the optimal feeding strategy called for oats only. This indicated that oats was economically superior to lupins at the assumed market conditions as far as dry sheep were concerned, although lupins had a higher nutritional value. This was due to the lower cost and the higher substitution rate of oats compared to lupins. The model selected oats as a means of conserving paddock feed.

The optimal values for the optimal feeding strategies increased with higher initial levels of forage on-offer and liveweight, as indicated in Figure 6. However, shadow prices were contrary to those observed for weaners. A lower shadow price of paddock dry feed, 3 c/kg DM (=($149-$89)/2000) vs 3.95 c/kg DM (=($112-$33)/2000), applied when adult dry sheep were in good body condition. The shadow price for sheep liveweight was also lower, $1.85/kg (=($149-$112)/20) vs $2.80/kg (=($89-$33)/20), when paddock feed was plentiful. This was not unusual since a higher demand for dry feed at increasing liveweight only occurs with adult dry sheep less than 50 kg. Once sheep are above 50 kg, the potential sheep intake starts to decrease with each additional unit of weight (Orsini 1990). Thus, the demand for dry feed will decrease rather than increase if dry sheep weigh more than 50 kg. Similarly, when the feed supply was plentiful a heavy weight adult dry sheep had a lower shadow price because it could not utilize the dry feed efficiently.

The actual feeding strategy adopted by graziers varies from district to district and from flock to flock (e.g. pregnant ewes and weaners vs adult dry sheep, fine wool vs broad wool). However, under low wool prices a so-called famine-feast strategy which delays the start of supplementary feeding until late in summer is often practised. This strategy is similar to the optimal feeding strategies for adult sheep although its start of feeding may be somewhat late. Feeding small amount of lupins to weaners when feed is abundant is seldom practised by graziers. This may be due to the lack of information on the supplementation effect of lupin seed with paddock dry feed or may be due to the grazier's reluctance to feed sheep when abundant feed is available. The graziers also prefer to use the supplements produced on-farm rather than buy in additional feed.

The famine-feast strategy may minimise the cost of feeding if the autumn break comes early and it may also have a small impact on reducing wool fibre diameter. However, higher death rates and weak or rotten wool (low staple strength) may result from this strategy if the autumn break is not early. Also, under this strategy graziers seem to start feeding after sheep lose some weight, while the model starts feeding before or as soon as sheep lose weight if paddock feed is limiting. Thus, the optimal feeding strategies will maintain a safe body condition for sheep and therefore carry a higher wool growth rate into winter.

4. Sensitivity Analysis

Different patterns of decline rates of dry matter on-offer and digestibility were examined using a sensitivity analysis. The values of decline rates used range from 0.05 to 0.5 per cent per day which is the maximum range observed in the region (B. Warren, pers. comm. March 1992). The range of decline rates tested did not alter the pattern of feeding for either adults or weaners though the actual feeding rates shifted to compensate for the higher or lower levels of dry matter on-offer and digestibility. At low decline rates (0.05-0.10 per cent), less supplements were selected while higher feeding rates were chosen for the higher decline rate (0.5 per cent). The differences were small (less than 75 g/hd/day) and occurred only when the starting paddock feed was low. A varying rate of decline rate was also tried (starting with 0.5 per cent at stages 1 and 2, followed by 0.3 per cent at stages 3 and 4 and 0.05 per cent at stages 5 and 6), and similar optimal feeding patterns for weaners and adult sheep were selected. However, feeding lupins (50 g/hd/day) to light and medium weight weaners at the first three stages followed by oats at later stages was found optimal. A slightly lower rate of oats at stages 5 and 6 was optimal for heavy weight weaners.

Removal of the paddock feed constraint (penalties for levels less than 1000 kg DM/ha) means that the model no longer chooses feeds that substitute in order to conserve paddock dry matter. However, the impact of soil erosion resulting from overgrazing is likely to be a severe drop in pasture productivity in following seasons. If the paddock feed constraint is removed and the starting level of paddock feed is low (2.8 t DM/ha), sensitivity analysis showed that the model made greater use of paddock feed. To achieve this, the model (a) chose lower rates of feeding, (b) started feeding later, and (c) used the cheapest supplement source of energy (i.e. mostly oats in the study).

Sensitivity analysis was also applied to wool prices with a variation of 10 per cent tested. The same patterns of optimal feeding strategies were sustained except for minor changes in the feeding rates (0-100 g/hd/day range). For adult sheep this occurred in the last one or two stages but for weaners the changes occurred from as early as stage one. The greatest changes occurred when initial dry matter on-offer was limiting (2.8 t DM/ha). For weaners, the model started feeding later/earlier if the price was reduced/increased. The direction of the change in the feeding rate was positively correlated with the price movement. The change in net returns from the wool price variations was about 16 per cent for the adult sheep flock and 24 per cent for the weaner flock when paddock feed levels were abundant. However, with a limited amount of initial dry matter on-offer (2.8 t DM/ha), the net returns varied from 26 per cent to 67 per cent as the initial sheep liveweight of the adult sheep dropped from 64 kg to 44 kg. For weaners the variation in net returns ranged from 28 per cent to 33 per cent as the initial weaner liveweight changed from 42 kg to 26 kg. The loss and benefit due to price variations were roughly symmetrical with only a slightly higher figure occurring on the loss side.

5. Concluding Comments

This study evaluated only lupin seed and oat grain as possible feed supplements. Each was used differently by the model. When paddock feed was abundant, lupins were preferred for weaners because they supplemented the sheep intake and increased the utilisation of the paddock feed. When the amount of pasture on-offer was low, additional use of available pasture was not possible, and so the model chose oats to maintain the sheep while conserving more of the paddock feed. For adult sheep, oats were always the preferred supplement irrespective of the level of paddock feed on-offer. These results indicated that the supplementation vs. substitution effect of supplementary feed is very important.

This analysis also demonstrated that, by keeping other variables constant, through a deterministic dynamic programming model, the optimal feeding strategy can be derived as a function of the most reliable variables in the field, i.e. paddock feed supply and sheep liveweight. These two variables can be monitored objectively during the summer/autumn period and thus can be used as indicators when making feeding decisions. However, caution is needed in extending the current results since other variables are usually not invariate. If changes do occur in the decline rates of the paddock feed, stocking rates or wool market conditions, the quantitative results derived in this study would not hold. However, the qualitative results should still apply. The most efficient way to determine the possible changes in optimal feeding pattern under different conditions is to run the SUMMERPACK simulation model and use the general patterns in this study as a reference to facilitate the design of possible management options. Furthermore, the same dynamic optimisation approach could be used to estimate the optimal stocking rate over summer if stock adjustment is possible. This may require flock size to be included as an extra state variable in the formulation.

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