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The Impact of Price-Induced Hedging Behavior on Commodity Market Volatility

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Abstract

The utility maximization problem of a grain producer is formulated and solved numerically under prospect theory as an alternative to expected utility theory. Conventional theory posits that the optimal hedging position of a producer should not increase solely due to increases in the level of futures prices. However, a strong degree of positive correlation is apparent in the data. Our results show that with prospect theory serving as the underlying behavioral framework, the optimal hedge of a producer is affected by changes in futures price levels. The implications of this price-induced hedging behavior on spot prices and volatility are subsequently considered.

Key Words: futures markets, hedging, prospect theory, risk preferences

JEL codes: D03, D81, G11, Q13

1. Introduction

Agricultural commodity markets have seen significant increases in volatility in recent years. Although there has been the occasional run-up over the years, the recent swings in volatility of corn, wheat, and soybeans have been particularly noteworthy. For each of these commodities, volatilities spiked to near-record levels during mid-2008. Subsequently, volatilities declined somewhat but began edging higher in 2010 along with upward price movements.

Following the 2008 price spikes, futures market speculators have received increasing criticism for their role in exacerbating upward commodity price movements and increased volatility.⁴ This has fueled a debate as to the extent that commodity derivatives markets should be free of regulation due to the risks associated with higher volatility. Despite several studies having concluded that there is rather weak evidence that speculators do contribute to upward price movements (Irwin, Sanders, & Merrin, 2009; Robles, Torero, & von Braun, 2009), the discussion has continued unabated.

It is generally accepted that futures markets are beneficial to commercial entities with a physical position in the underlying commodity which serves as an invaluable risk management tool. In this regard, it is possible for a producer who is naturally long in a given commodity to hedge a portion of his future output by taking a short position in the futures market and subsequently offsetting his position as harvest approaches. In the case of grains, a common scenario is for a producer to sell his future output forward through a contract with a local elevator. The elevator then manages its corresponding risk by taking a (possibly equivalent) short futures position. This hedging approach allows a producer to

⁴ Speculators are referred to here as non-commercial entities with no physical position in the underlying asset.

effectively lock in a price for his crop well in advance of harvest. The producer is then obligated to deliver his grain to the elevator at harvest.

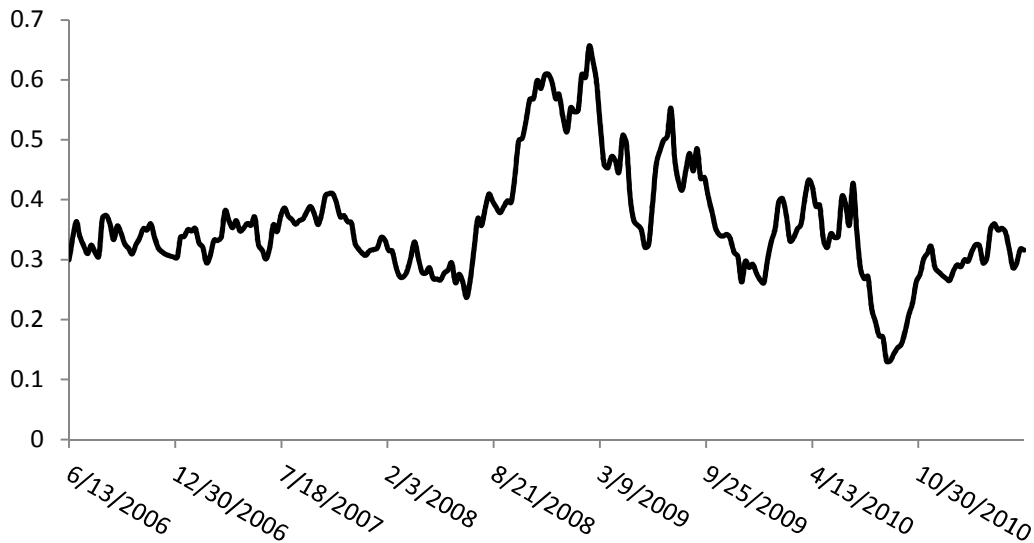
There are two main contributions of this paper. The first is to outline a mechanism drawing on prospect theory whereby the amount hedged by grain producers in short futures contracts is dependent on the level of futures prices at a given point in time. This is a behavior that deviates from what conventional expected utility (EU) theory would predict. The second goal of the paper is to illustrate the effect this price-level-dependent futures hedge has on spot prices and price volatility at harvest. Specifically, a scenario will be presented in which prices and volatility at harvest increase as the amount of output hedged in futures markets increases during a crop year due to rising futures prices. *Ceteris paribus*, the larger the share of output that a producer is committed to through the use of forward markets, the less output available at harvest to respond to potential negative supply shocks as harvest approaches, i.e., bad weather events. This is because a portion of the crop has already been sold.⁵ It should be emphasized here that this result would not be obtained if speculators were the only group of traders taking the opposing futures trade. As illustrated in figure 1, however, grain purchasers who are naturally short the underlying commodity typically account for between 30% and 60% of futures contracts sold by producers.

Conventional EU theory posits that the existence of an unbiased futures market allows a risk averse farmer to determine his optimal level of production as a function of only his marginal cost and the current futures price, referred to as the “separation result” (Sandmo, 1971; Holthausen, 1979; Feder, Just, & Schmitz, 1980). This result holds when there is price

⁵ In the case where a producer sells his crop to an elevator as an intermediary, it will be assumed that the elevator simply hedges in futures contracts precisely the amount that is sold forward by the producer.

uncertainty and no production uncertainty. Moreover, with only price uncertainty (and no basis risk), the optimal hedge is the complete hedge, effectively removing all exposure to risk. Allowing for output uncertainty complicates the determination of this optimal hedge somewhat by requiring knowledge of the interaction between revenue and prices (Rolfo, 1980; Grant, 1985). Allowing for basis risk adds yet another dimension to consider (Lapan & Moschini, 1994; Myers & Hanson, 1996).

Figure 1: Ratio of Producer Long to Short Contracts



Source: Commodity Futures Trading Commission (CFTC) Disaggregated Commitment of Traders Report

In an EU framework where there is no basis risk and futures markets are unbiased, the optimal amount of output hedged through futures contracts is determined by two factors: yield variability and the price-yield correlation. The presence of yield variability causes the optimal hedge to be less than the complete hedge observed when there is only price

uncertainty.⁶ This is because the “natural hedge,” due to a negative correlation between prices and yields, partially replaces the need for a futures hedge. As the price-yield correlation becomes more strongly negative, the natural hedge becomes relatively more effective and the amount hedged in futures markets decreases. Therefore, when prices and yields are uncorrelated and yield variability is held fixed, EU theory predicts that the optimal hedge does not increase as the level of futures prices increases. Prospect theory shows that even when prices and yields are uncorrelated and yield variability is constant, an increase in futures prices will cause the amount hedged to increase.

As time progresses toward harvest, it is reasonable to assume that the degree of yield uncertainty diminishes. As shown by Lapan and Moschini (1994), this “time decay” is the only factor which would cause the optimal hedge to adjust under EU theory when prices and yields are uncorrelated. Given that the optimal hedge is the complete hedge when there is only price uncertainty, this causes the optimal hedge to increase as output uncertainty is resolved during a crop year. This aspect, however, is unrelated to a change in futures prices. Thus, the conclusion that changes in futures price levels do not affect the optimal hedge under EU theory is still maintained.

The empirical reality, however, is that we observe a fairly strong correlation between producer hedging activity and the futures price level. Figure 2 provides an illustration. The dashed line, read from the left vertical axis, shows the aggregate corn producer hedge ratio from June 2006 to December 2010. This ratio is determined as the aggregate quantity of grain held by producers in short futures contracts divided by expected production plus

⁶ It is possible that the presence of yield variability could cause the optimal amount hedged to be more than the complete hedge, but this would require a positive correlation between prices and yields which is not realistic.

inventories.⁷ The solid line displays nearby futures prices, read from the right vertical axis. Immediately apparent in figure 2 is the strong degree of correlation between the two series, particularly during price increases observed in 2006, 2008, and late 2010. It should be noted that forward contracts and options are not included in this figure, which could cause the amount of crop committed in advance to be substantially higher.

Figure 2: Producers' Hedge Ratio Based on Expected Production and Current Inventories

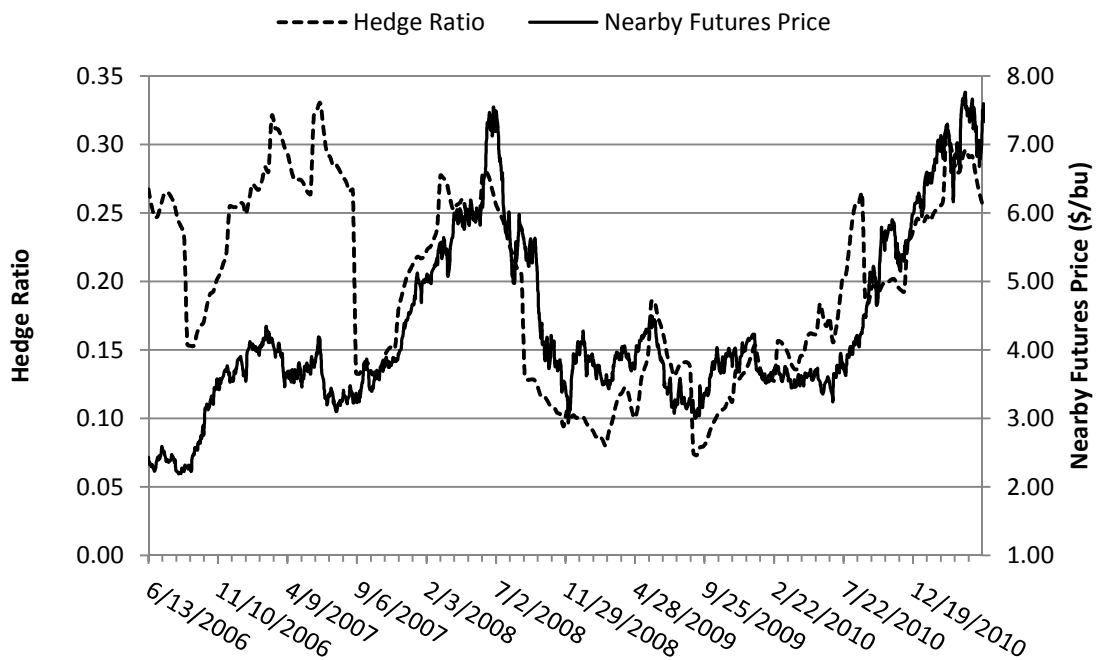


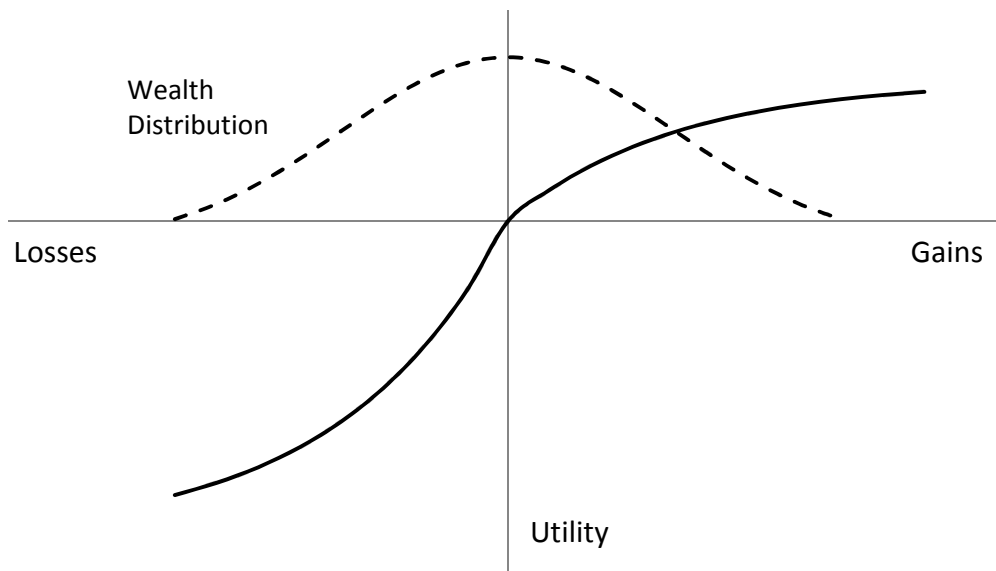
Figure 2 thus provides the motivation for considering a model in which hedging activity is a function of the futures price level and the implications this would present for spot prices and volatility at harvest. If futures prices rise during a crop year, for example due to increased demand for grain, this demand shock would induce farmers to sell a larger share

⁷ Data on futures positions are obtained from the Commodity Futures and Trading Commission (CFTC) Disaggregated Commitment of Trader Reports. Production and inventory data are obtained from the United States Department of Agriculture (USDA).

of their crop in advance, reducing the flexibility to respond to subsequent negative supply shocks at harvest by reducing the uncommitted quantity available.

In order to allow for a non-expected utility theory outcome in which futures price levels affect the optimal hedge ratio determination, this paper draws on a rapidly growing body of literature in behavioral finance referred to as prospect theory. Prospect theory was originally conceptualized in a seminal paper by Kahneman and Tversky (1979) and has become an empirically grounded alternative frequently used in behavioral economic models. At its core, prospect theory suggests that agents exhibit behavior that is inconsistent with the efficient markets hypothesis of expected utility under rational expectations. Specifically, it posits that agents tend to be risk averse over gains and risk seeking over losses as illustrated by the solid curve in figure 3.

Figure 3: An Example of Utility Based on Prospect Theory



Whereas traditional EU theory suggests a utility function that is everywhere concave, the function based on prospect theory in figure 3 is convex in the domain of losses

(negative changes in wealth) and concave in the domain of gains. A secondary component of prospect theory, referred to as loss aversion, also recognizes that agents are more sensitive to losses than to gains, seen by the differing slopes of the utility function in figure 3 for changes in wealth of equal magnitude on either side of the vertical axis.

Intuitively, we would expect that an agent who is risk averse over some domain and risk seeking over some other domain should behave differently than an agent who is everywhere risk averse. Within the context of this paper, a grain producer under prospect theory seeking to determine how much of his output to hedge in the futures market would take a different action than the same producer in an expected utility framework. By definition, risk seeking means to prefer a gamble relative to a certain outcome. A gamble in this context would entail a producer adding to his risk by taking a long position in the futures market in addition to his natural long position in the physical market. A risk averse agent, on the other hand, would take a short position in the futures market to mitigate his exposure to risk. Thus, if the representative producer's probability distribution of wealth contains mass on both sides of the vertical axis, the producer will exhibit both risk seeking and risk aversion tendencies.

The producer's wealth distribution is determined by price and output distributions. For a wealth distribution initially centered at the vertical axis, as shown in figure 3, a rightward shift in the distribution of prices will consequently shift the wealth distribution rightward resulting in the agent becoming more risk averse and less risk seeking. Recalling the fact that a risk seeking producer would choose to take a long futures position and a risk averse producer a short position, this rightward shift in the price distribution causes the producer

to take a larger short futures position. An increase in demand that shifts the wealth distribution rightward, then, causes the optimal futures hedge to increase.

To our knowledge, this is the first paper to propose a mechanism by which the optimal hedge ratio increases due solely to changes in the level of futures prices, thereby contributing to otherwise higher price levels and volatility at harvest in the event of a subsequent supply shock. The remainder of the paper is organized as follows. In section 2, we provide a review of the relevant literature regarding futures hedging in an agricultural context and a brief review of literature on prospect theory. In section 3, we present a simple model that incorporates prospect theory into a producer's hedging decision. In section 4, we present numerical simulations to show the effect of prospect theory on the determination of the optimal hedge and ultimately its impact on spot prices and volatility. Section 5 provides concluding remarks.

2. Literature Review

Optimal Producer Futures Hedging

Some of the first studies developing the theory of hedging in commodity markets in an EU framework are those of Sandmo (1971), Holthausen (1979), and Feder, Just and Schmitz (1980). Considering only price uncertainty, these papers present implications associated with hedging in futures markets and the effect of futures markets on the production decision. These papers propose a two period model in which a producer makes his production and hedging decision in the first period by maximizing a given utility function of second period profit. In addition to the result that a producer's output decision is made separate from the evolution of cash prices, it is also shown that the optimal hedge

is to hedge all output, which is considered to be known with certainty. Turnovsky (1983) and Kawai (1983) seek to expand on this by determining market clearing spot and futures prices in a rational expectations framework. The two-period model of these early papers was subsequently generalized by Anderson and Danthine (1983) into three periods and also by Ho (1984) in an intertemporal context.

Rolfo (1980), Grant (1985), and Losq (1982) generalize these models that consider only price uncertainty to allow for output uncertainty as well. Assuming a mean-variance representation of utility, Rolfo, Grant, and Losq show that the optimal hedge consists of two components, a pure hedging component and a pure speculative component.⁸ In unbiased futures markets, the speculative component vanishes, leaving only the hedging component. The hedge is determined as the ratio between the covariance of revenue with futures prices and the variance of futures prices. In most other utility representations, such analytically appealing results are often difficult to obtain.

Adding another dimension of risk, Lapan and Moschini (1994) construct a model to allow for the determination of the optimal hedge under joint price, output, and basis risk. In their model, Lapan and Moschini assume a constant absolute risk aversion (CARA) utility function to obtain analytical results characterizing the optimal hedge ratio. Myers and Hanson (1996) also consider the additional effects of basis risk in their study.

Various functional forms for utility are employed in the literature. Whereas some representations are more tractable, others may be more capable of capturing empirical realities. Perhaps the simplest form used to derive the optimal futures hedge is the minimum variance utility representation (Johnson, 1960; Ederington, 1979). Mean-

⁸ Speculative here refers to taking a position in the futures market in an attempt to gain from changes in futures price movements.

variance utility provides another widely used form through which analytical results are easily obtained (Turnovsky, 1983; Rolfo, 1980). Constant absolute risk aversion (CARA) represents a unique form of utility that is often used in the literature due to its tractability in generating closed form analytical solutions (Lapan & Moschini, 1994; Lence, 1995; Lien, 2001; Mattos, Garcia, & Pennings, 2008). When closed form expressions are unattainable, one must employ numerical estimation procedures to determine the optimal futures hedge (Cecchetti, Cumby, & Figlewski, 1988; Baillie & Myers, 1991).

While there have been many studies that have focused on commodity futures market hedging behavior within an expected utility framework, very few have considered the implications in a non-expected utility environment. Three papers that have attempted to do so are those of Albuquerque (1999), Lien (2001), and Mattos et al. (2008).

Albuquerque (1999) applies prospect theory in the context of a loss averse firm seeking to determine an optimal currency hedge, specifically the implications of managing downside risk, compared with a conventional firm that is not loss averse. Lien (2001) uses a two-period model of grain production to show how the optimal hedge of a commodity producer differs under loss aversion as opposed to a producer who maximizes mean-variance utility. One of the main results is that loss aversion has no effect when futures markets are unbiased. If markets are in either backwardation or contango, this is not true. Lien (2001) goes on to show how, and in which direction, the optimal hedge is influenced in these generalized cases. Mattos et al. (2008) expand on this result by also allowing for subjective probability weighting together with loss aversion. Employing numerical simulations, Mattos et al. (2008) show that the optimal hedge ratio decreases as the degree

of loss aversion increases, as risk seeking behavior increases, or as the parameter of subjective probability weighting decreases.

Prospect Theory

Prospect theory was introduced by Kahneman and Tversky (1979) as an attempt to better explain observed psychological behavior, with the violation of Allais' paradox serving as a well known example. There have been numerous other theories proposed to explain deviations from traditional expected utility theory such as weighted-utility theory (Hong, 1983), disappointment aversion (Gul, 1991), regret theory (Bell, 1982; Loomes & Sugden, 1982), and rank-dependent utility (Quiggin, 1982). Prospect theory is often espoused as the most promising due to its ability to capture observed behavior by relaxing only the independence axiom among the von Neumann Morgenstern (VNM) expected utility axioms. The other theories cited here require the weakening of additional axioms beyond independence.

Maintaining much of the structure of the standard VNM axioms has allowed prospect theory to become a relatively accepted alternative to the efficient markets hypothesis in behavioral economics. For this reason, prospect theory and loss aversion was chosen as the model specification in this paper although a brief mention will be made of how regret theory, another seemingly plausible alternative in our context, could also be applied to obtain similar results.

3. The Model

Consider a three-period model of corn production. In the context of this model, period 1 can be thought of as a pre-planting period (March), period 2 as an intermediate (July) period, and period 3 as harvest (October). A grain consumer makes a utility maximizing demand decision facing expected spot market prices in period 3. All consumption is assumed to occur in this terminal period. Aggregate demand will be specified by the following isoelastic demand function:

$$q_3 = \delta_0 \left(p_3^s \right)^{-\delta_1} \varepsilon_2 \quad (1)$$

In equation (1), δ_i are exogenously specified parameters, q_3 is the quantity of grain demanded in period 3, p_3^s is the terminal spot price, and ε_2 is taken to be a demand shock variable such that $E_1[\varepsilon_2]=0$. This formulation explicitly allows for the possibility of a demand shock occurring in period 2, which will affect both the period-2 futures price as well as the period-2 conditional expectation of the period-3 spot price.

In period 1, a representative corn producer must determine how much output to produce and how much of this output to hedge in the futures market. The producer faces both price and output risk. It is assumed that there is no basis risk. Futures markets are assumed to be unbiased and for simplicity this model does not allow for inventory holdings. The producer determines his optimal output and optimal futures hedge by maximizing the following utility as a function of terminal wealth:

$$U(\tilde{W}) = U\left(\tilde{p}_3^s \tilde{y}a + x_1(p_1^f - \tilde{p}_2^f) + x_2(\tilde{p}_2^f - \tilde{p}_3^f) - ca\right) \quad (2)$$

In the above, \tilde{p}_i^f denotes the futures price in period i , \tilde{y} denotes the crop yield at harvest, and c represents the marginal cost of an acre of land, assumed to be constant.

Tildes indicate random variables unknown to the producer in period 1 for which there is a known distribution. In period 1, decision variables include a , the amount of acreage the producer chooses to allocate to crop production and x_1 , the quantity of grain hedged in futures contracts (where short positions are represented by positive values). In period 2, acreage is fixed and only the quantity hedged, x_2 , may be adjusted.

In a 3 period model, maximization of equation (2) is solved recursively. In period 2, the representative producer, taking a and x_1 as fixed, chooses x_2 optimally according to:

$$\max_{x_2} E_2 U \left(\tilde{p}_3 \tilde{y} a + x_1 (p_1 - p_2) + x_2 (p_2 - \tilde{p}_3) - ca \right) \quad (3)$$

In equation (3), E_2 represents the expectations operator in period 2. Given the assumptions of unbiasedness, no basis risk, and no inventory holdings, the futures price will be equal to the spot price in each period, allowing us to drop the s and f superscripts. With x_2 optimally chosen (denoted as x_2^*), the producer faces a similar maximization problem then in period 1 with a and x_1 as choice variables. Thus, solving equation (4) results in solutions to each of the decision variables, a , x_1 , and x_2 from the perspective of period 1.

$$\begin{aligned} & \max_{a, x_1} E_1 U \left(\tilde{p}_3 \tilde{y} a + x_1 (p_1 - \tilde{p}_2) + x_2^* (\tilde{p}_2 - \tilde{p}_3) - ca \right) \\ & = \max_{a, x_1} E_1 U \left(\tilde{p}_3 \tilde{y} a + x_1 (p_1 - \tilde{p}_2) - ca \right) \end{aligned} \quad (4)$$

Hedging Under Expected Utility Theory

In an EU framework, $U(\cdot)$ is typically taken to be some increasing and concave function, $U' > 0$ and $U'' < 0$, stemming from risk aversion. If this were the case, equation (4) would be solved as the following integral:

$$\max_{a, x_1} \iint U(\tilde{p}_3 \tilde{y} a + x_1 (p_1 - \tilde{p}_3) - ca) f(\tilde{p}_3, \tilde{y}) d\tilde{p}_3 d\tilde{y} \quad (5)$$

Here $f(\tilde{p}_3, \tilde{y})$ represents the joint distribution of prices and yields. Equation (5) also makes use of the fact that from the perspective of period 1, the expectation of period-3 prices is the same as the expectation of period-2 prices, i.e., $\tilde{p}_2 = \tilde{p}_3$. As shown by Grant (1985), first order conditions would be specified as:

$$\frac{dE[U(\cdot)]}{da} = \iint U'(\cdot)(\tilde{p}_3 \tilde{y} - c) f(\tilde{p}_3, \tilde{y}) d\tilde{p}_3 d\tilde{y} = 0 \quad (6)$$

$$\frac{dE[U(\cdot)]}{dx_1} = \iint U'(\cdot)(p_1 - \tilde{p}_3) f(\tilde{p}_3, \tilde{y}) d\tilde{p}_3 d\tilde{y} = 0 \quad (7)$$

It is assumed that the presence of risk neutral speculators ensures that futures markets clear and are unbiased, where speculators' futures positions are denoted as z_1^* . Additionally, following from grain consumer utility maximization that gives rise to equation (1), consumers also choose an optimal futures position denoted by v_1^* . Assuming N producers and optimal solutions to equations (6) and (7) denoted by a^* and x_1^* , first period market clearing conditions are specified as:

$$q_3 = N\bar{y}a^* \quad (8)$$

$$x_1^* + v_1^* + z_1^* = 0 \quad (9)$$

where equation (8) represents spot market clearing, with $E[\tilde{y}] = \bar{y}$, and equation (9) represents the futures market clearing condition. It should be emphasized here that due to consistent, unchanging risk preferences on the part of the producer (and consumer by assumption), period-2 market clearing conditions can be expected to take precisely the form of equations (8) and (9) in the absence of any new information revealed in period 2.

Upon choosing the optimal production, a^* , this amount cannot be changed in period 2. From the perspective of a producer, only x_2^* could potentially differ from x_1^* . In an attempt to further characterize the optimal hedging decision, equation (7) can be rewritten as:

$$E[U'(\cdot)(p_1 - \tilde{p}_3)] = E[U'(\cdot)]E[(p_1 - \tilde{p}_3)] - \text{cov}[U'(\cdot), \tilde{p}_3] \quad (10)$$

Given that futures markets are unbiased, equation (10) reduces to:

$$E[U'(\cdot)(p_1 - \tilde{p}_3)] = -\text{cov}[U'(\cdot), \tilde{p}_3] = 0 \quad (11)$$

In general, a closed-form expression for the optimal hedge, x_i^* is unobtainable. However, it can be seen from equation (11) that determining x_i^* amounts to the determination of $\text{cov}(U', \tilde{p}_3)$. This covariance term is dependent upon the correlation between prices and output (or yields) as well as output (or yield) variability as explained earlier. When prices and yields are uncorrelated, an increase in the level of futures prices has no effect on this covariance term and thus no effect on the optimal hedge, x_i^* .

Moving from period 1 to period 2, a portion of uncertainty surrounding crop yields at harvest is resolved. Assuming prices and yields are uncorrelated, this is the only factor which will cause the optimal hedge to change from period 1 to period 2 under EU theory. Allowing for negative correlation between prices and yields does not change the key result of this paper. An increase in the level of futures prices still causes the optimal hedge to increase initially before declining somewhat.⁹ For this reason, we make the simplifying assumption that prices and yields are uncorrelated, but model both cases and illustrate

⁹ As with price-yield correlation, the presence of yield variability may also cause the optimal hedge to decline slightly under expected utility. However, yield variability must be rather high for the magnitude of this effect to be non-trivial. Moreover, this also does not affect the main results of this paper and will be ignored for expositional purposes but included in the numerical results.

how the results change when allowing for correlation. As this assumption is not crucial to our main results, the exposition from this point forward considers only the case when prices and yields are uncorrelated. This is done to isolate and accentuate the effects of the underlying behavioral framework.

Therefore, a demand shock realized in period 2, which causes futures prices to rise, will not cause the optimal hedge to change. Intuitively, this is because producers expect this same (higher) price to prevail in the next period as well and risk preferences, embodied in the utility function, have not changed due to the shock. In this paper, we show that the optimal hedge is affected by a change in the level of futures prices due to changing risk preferences inherent under prospect theory. In conventional EU theory, a change in the level of futures prices would have no impact on the change in the volatility of spot prices, or prices themselves, at harvest. Under prospect theory, this is not so. A change in futures prices will result in a change in volatility of spot prices as well as a change in price levels at harvest.

Hedging Under Prospect Theory

The approach to modeling market conditions under prospect theory is quite similar to the approach shown above under traditional EU theory that posits $U' > 0$ and $U'' < 0$ everywhere. In prospect theory, however, we have $U' > 0$, $U'' > 0$ for $\tilde{W} < 0$ and $U' > 0$, $U'' < 0$ for $\tilde{W} > 0$ where $\tilde{W} = \tilde{p}_3 \tilde{y} a + x_1 (p_1 - \tilde{p}_2) + x_2 (\tilde{p}_2 - \tilde{p}_3) - ca$, or the change in wealth from period 1 to period 3. Thus, in our model, equation (5) must be written as:

$$\max_{a, x_1} \int_{-\infty}^0 U_1(\tilde{W}) f(\tilde{W}) d\tilde{W} + \int_0^{\infty} U_2(\tilde{W}) f(\tilde{W}) d\tilde{W} \quad (12)$$

In equation (12), U_1 is a convex function reflecting risk seeking behavior in the domain of losses and U_2 is a concave function indicating risk averse behavior in the domain of gains. The distribution of \tilde{W} is determined by the distributions of \tilde{p} and \tilde{y} .

If the distribution of \tilde{W} were known, equation (12) could be solved separately for the optimal period-1 futures position with x_1^L corresponding to the solution for the term on the left and x_1^G the solution for the term on the right. The optimal hedge could then be determined as $x_1^* = \alpha x_1^G + (1-\alpha)x_1^L$ where α equals the fraction of \tilde{W} such that $W \geq 0$. The optimal hedge would be a weighted average of risk seeking behavior and risk averse behavior where the weights are determined by the area under the probability density function for random variable \tilde{W} as shown in figure 3.

Now suppose that equation (12) were solved first by assuming that the producer was everywhere risk seeking, i.e., $U_1 = U_2, U' > 0, U'' > 0$. This would be the case if the entire distribution of \tilde{W} were to reside to the left of the vertical axis in figure 3. For a producer who is naturally long in the underlying physical commodity, this would imply taking a long position in the futures market, clearly a more risky proposition. The solution to this problem would be $x_1^{L*} = -\infty$. A strictly risk seeking producer would prefer the largest gamble possible. As a practical matter, a producer would clearly face some wealth constraint preventing him from taking such a long position. Therefore, let the solution to this problem be given by $x_1^{L*} = L$ where L is some real negative number arising due to the wealth constraint.

Likewise, suppose that equation (12) were solved with the producer everywhere risk averse, i.e., $U_1 = U_2, U' > 0, U'' < 0$. This would correspond to the case in which the entire distribution of \tilde{W} lies to the right of the vertical axis in figure 3. The solution in this case would correspond to the standard EU outcome presented earlier, where the producer takes a short position in the futures market to mitigate exposure to risk. Let this solution be denoted by $x_1^{G*} = G$, where $G > 0$. Thus, in the case where $\alpha = 1$, we have $x_1^* = x_1^{G*} = G$ and for $\alpha = 0$, we have $x_1^* = x_1^{L*} = L$. The general solution to this approach can be represented as the following:

$$\begin{cases} x_1^* = x_1^{L*} & \text{for } \alpha = 0 \\ x_1^* = x_1^{G*} & \text{for } \alpha = 1 \\ x_1^* = \alpha x_1^{G*} + (1 - \alpha) x_1^{L*} & \text{for } 0 < \alpha < 1 \end{cases} \quad (13)$$

It should be clear that since $L < G$, there is increasing weight placed on the risk averse solution and decreasing weight placed on the risk seeking solution as the distribution of \tilde{W} shifts from left to right. This would occur with a period-2 demand shock such that $\varepsilon_2 > 0$. Thus, we have the result that $x_2^* > x_1^*$ whenever $p_2 > p_1$ and $0 < \alpha < 1$. It is important to emphasize here that the only change required to cause an increase in the optimal hedge is an increase in the price level. As of period 1, the producer has sold in advance an amount of output equal to x_1^* . In the absence of hedging, the amount of unsold output is $a^* N(\bar{y} - x_1^*)$.¹⁰

Once period-1 decisions have been made, the producer faces an optimization problem similar to equation (12) in period 2 as follows:

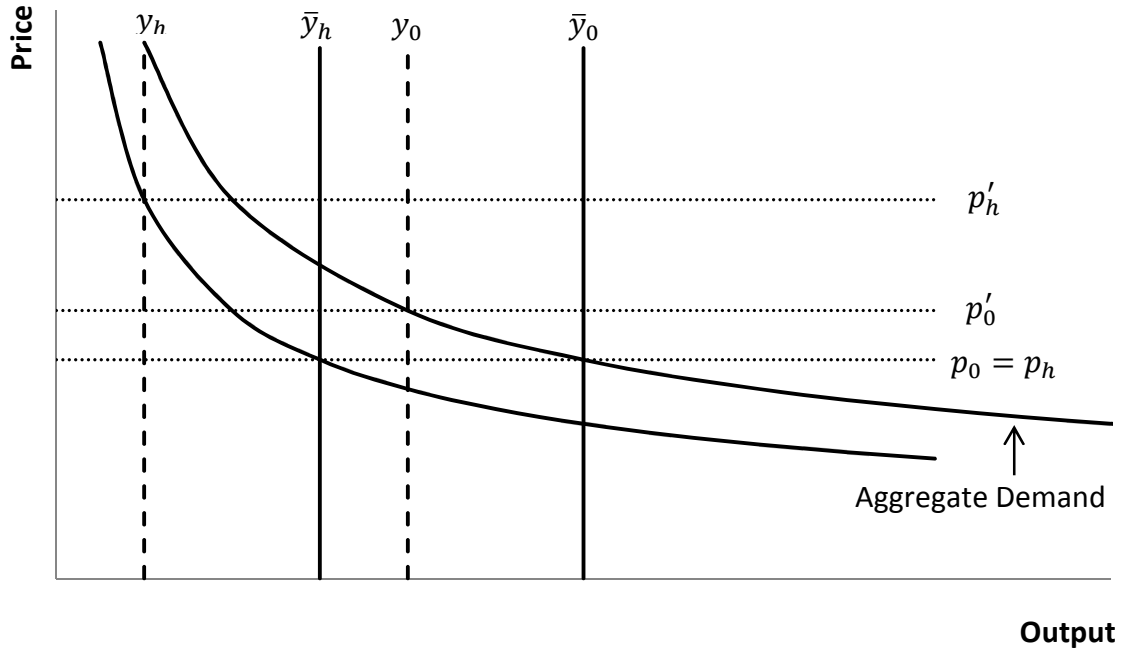
¹⁰ Again for expositional purposes, this supposes that only commercial end-users take the opposing trade to producers' short positions. The presence of speculators would reduce the amount of committed production in that speculators do not typically intend to take delivery of the underlying commodity.

$$\max_{x_2} \int_{-\infty}^0 U_1(\tilde{W}) f(\tilde{W}) d\tilde{W} + \int_0^{\infty} U_2(\tilde{W}) f(\tilde{W}) d\tilde{W} \quad (14)$$

In this case, $\tilde{W} = \tilde{p}_3 \tilde{y} a^* + x_1^* (p_1 - p_2) + x_2 (p_2 - \tilde{p}_3) - c a^*$ as before with x_1^* and a^* fixed at values optimally chosen in period 1 and period-2 prices no longer uncertain. As acreage is no longer a choice variable in period 2, the only decision variable that a producer can adjust is the amount of output hedged in futures contracts. Assuming for the moment that there is no output uncertainty resolved from period 1 to period 2, the producer would choose his period-2 hedge, x_2^* such that $x_2^* = x_1^*$. Consider, however, the case in which $\varepsilon_2 > 0$. An increase in demand will cause $p_2 > p_1$ and $x_2^* > x_1^*$ as more weight is placed on the risk averse solution and less weight on the risk seeking solution. In this case, the amount of unsold output available at harvest will be $a^* N(\bar{y} - x_2^*)$, which is less than the amount previously available in period 1, $a^* N(\bar{y} - x_1^*)$.

Moving forward to the terminal period, consider the effect of a negative supply shock, possibly unfavorable weather near harvest, such that $y < \bar{y}$. The negative supply shock affects the entire harvest. However, a portion of this harvest has already been sold in advance. As the supply curve shifts leftward, there is a larger increase in the spot price in the presence of futures market hedging than if there had not been any crop sold in advance. Moreover, the increase in the level of futures prices in period 2, which caused the amount hedged to increase, will exacerbate this price increase even further. Figure 4 provides the intuition behind this result.

Figure 4: Effect of Terminal Period Supply Shock on Price Volatility



In figure 4, aggregate demand is taken to be the sum of two (equal) individual grain consumer demand curves, here shown to be each equal to half of the aggregate demand. Group 1 is interpreted as consumers purchasing grain through forward (futures) markets, intending to take delivery at harvest. Group 2 purchases only in the spot market at harvest. In the absence of any supply shocks at harvest, realized aggregate supply is equal to expected aggregate supply \bar{y}_0 . In this case, the price paid by group 1, p_0 is equal to the price paid by group 2, p_h . In equilibrium, both groups purchase grain in the amount of \bar{y}_h since they are equal in size.

Now consider the effect of a terminal period supply shock in which realized aggregate supply is reduced to y_0 . In this case, grain producers have committed to supplying grain to group 1 in the amount contractually agreed upon, \bar{y}_h . Subtracting this amount from aggregate realized supply gives $y_0 - \bar{y}_h = y_h$, the amount of uncommitted grain available to

respond to the negative shock at harvest. Now the equilibrium price at harvest, based on the demand curve of group 2, still in need of grain, is p_h' . As illustrated in figure 4, this price is greater than what the equilibrium price would have been if there had been no futures trading, p_0' . This simple illustration shows that an increase in the amount of output hedged by producers in the futures market reduces the amount of output available at harvest to respond to negative (or positive) supply shocks. As a result, cash prices and volatility at harvest are necessarily higher. Thus, in the context of our paper, any factor that increases futures prices (such as an increase in demand) within a crop year will cause spot prices and the volatility of prices at harvest to increase. In the case of isoelastic demand, this increase increases dramatically as the severity of the supply shock at harvest increases, or as the amount of crop sold in advance increases.

4. Numerical Results

This section provides numerical results to support the theory presented in the previous section to help provide a better understanding of two key points: the extent to which the optimal hedge is affected by upward price movements under prospect theory and the ensuing impact on spot prices and volatility.

For the purposes of the numerical simulations, a CARA utility representation will be used to obtain results due to its ability to be parameterized to encompass prospect theory as well as its general prevalence in the hedging literature alluded to in section 2. Utility is thus specified as follows:

$$U = \begin{cases} -\varphi \left[1 - \exp(\theta_L \tilde{W}) \right] & \text{for } W < 0 \\ 1 - \exp(-\theta_G \tilde{W}) & \text{for } W \geq 0 \end{cases} \quad (15)$$

In equation (15), $\tilde{W} = \tilde{p}_3 \tilde{y} a + x_1 (p_1 - \tilde{p}_2) + x_2 (\tilde{p}_2 - \tilde{p}_3) - ca$ as before, θ_G (θ_L) defines the measure of risk averse (risk seeking) behavior over gains (losses), and φ allows for the possibility of a higher sensitivity to losses than gains (loss aversion). This would be the case when $\varphi > 1$. Estimates for φ are found to be between 2.25 and 2.5 (Kahneman & Tversky, 1992; Pennings & Smidts, 2003). Given the specification of utility in equation (15), the producer's first-period maximization problem can be written as:

$$\begin{aligned} \max \int_{-\infty}^0 & -\varphi \left[1 - \exp\left(\theta_L (\tilde{p}_3 \tilde{y} a + x_1 (p_1 - \tilde{p}_3))\right) \right] f(\tilde{W}) d\tilde{W} + \\ & \int_0^{\infty} \left[1 - \exp\left(-\theta_G (\tilde{p}_3 \tilde{y} a + x_1 (p_1 - \tilde{p}_3))\right) \right] f(\tilde{W}) d\tilde{W} \end{aligned} \quad (16)$$

Prices, \tilde{p}_3 , are drawn from a lognormal distribution with a (period 1) mean and standard deviation of 4.19 and 1.28 respectively. As explained previously, yields are assumed to be uncorrelated with prices and are drawn from a four-parameter beta distribution. For robustness, results will also be presented for a case in which prices and yields are negatively correlated with a correlation coefficient of -0.47. The mean and variance of the yield distribution in period 1 is 150 and 15.4 respectively. From these distributions, a distribution for revenue per acre, $\tilde{p}_3 \tilde{y}$ is constructed. For simplicity, and without loss of generality, the demand equation given by (1) is calibrated by adjusting δ_0 so as to generate a market clearing price equal to the mean of the given price distribution for $a = 1$. The elasticity of demand, δ_1 , is set equal to 0.5. The cost parameter c , is set to ensure that there is an approximately equal probability of realizing a loss as a gain.

For the purposes of simulations, φ is set equal to 2.25 as cited in the literature, but the value of this parameter is not crucial to the key results. Likewise, values of risk aversion

and risk seeking parameters, θ_G and θ_L are set equal to each other at 0.01. The values for these parameters were chosen so as to allow for sufficient non-degenerate curvature over the range of the distribution of \tilde{W} . The relatively arbitrary values chosen for these parameters are also not crucial to the key results, but provide some ease in numerical simulations by preventing extremely large or small numbers, given the range of \tilde{W} .

In most circumstances, as discussed earlier, it can be expected that a producer who has a natural long position in an underlying commodity will take a short position in the futures market to manage price and production risk. Recalling the fact that a risk seeking producer will choose a long futures position as large as possible without a wealth constraint, a lower bound was placed on the size of the long position taken. This bound can be interpreted as a wealth constraint which ensures that the amount hedged by the producer will be non-negative, thereby conforming to empirically observed data. Ultimately, we are interested in how the optimal hedge changes due to an increase in the level of prices, so the exact magnitude of this bound is of little importance.

As mentioned in section 3, after period-1 decisions on production (acreage) and hedging have been made, the only choice variable in period 2 is the hedging decision. Moreover, there is an assumed stickiness in the hedging decision. Once a portion of output is sold in short futures contracts, it cannot be “unsold.” Thus, the only real decision is whether to hedge additional output in the second period beyond what was hedged in the first period. In the second period, we allow for a demand shock of varying intensity (i.e. $\varepsilon_2 > 0$) that causes producers to hedge additional output under prospect theory due to a price level increase.

Figure 5a illustrates this result for two cases. The solid curve represents the case in which no uncertainty is resolved moving from period 1 to period 2. The dashed curve represents the case in which the variance of prices and yields is reduced by 25%. In figure 5a, the period-2 conditional expectation of the period-3 spot price is displayed on the horizontal axis. This is the mean of the price distribution as of period 2. The curves then represent the optimal hedge ratio read from the vertical axis. As prices increase, risk averse behavior takes over and a larger short position is maintained. The optimal hedge ratio eventually flattens out at a level corresponding to the solution in which the producer is strictly risk averse.

Figure 5b presents the case when there is negative correlation between prices and yields. As can be seen from the figure, the result that the optimal hedge initially increases as the weight on risk averse behavior increases is still maintained. In contrast to figure 5a, however, the optimal hedge reaches a peak before subsequently declining as the effectiveness of a natural hedge is increased.

Finally, in period 3 we consider the effect of a supply shock in order to emphasize the pronounced effect on volatility and spot prices at harvest due to producers having sold a greater amount in the futures market between periods 1 and 2. Intuitively we would expect that as producers sell a larger share in advance, the effect of the supply shock in the terminal period will become amplified. This is illustrated in figure 6 for supply shocks ranging from -10% to 10%. In this figure, the solid curve represents the case in which the hedge ratio remains unchanged from period 1 to period 2 (baseline hedge). The dashed curve represents the case in which there is a period-2 demand shock of 30 bu/acre.

Figure 5a: Hedge Ratio as a Function of Price Levels (No price-yield correlation)

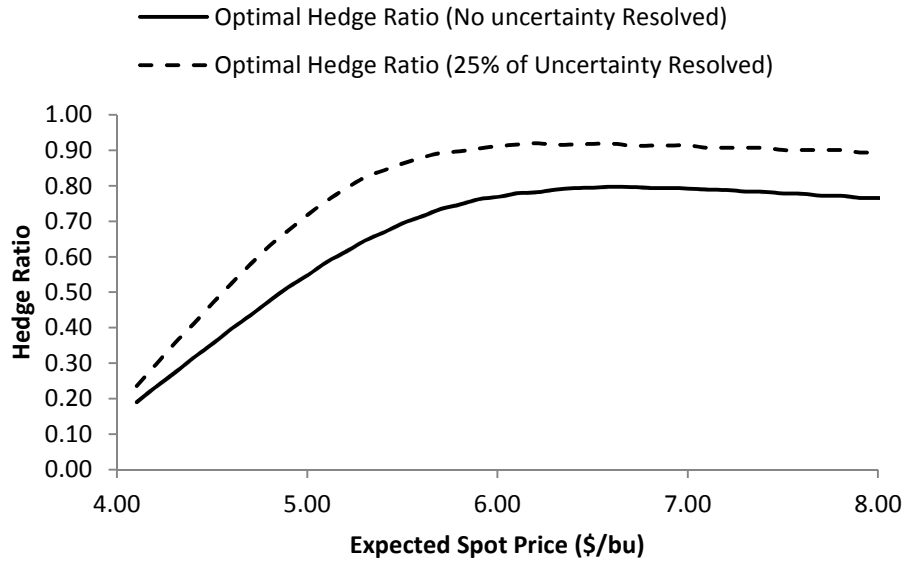


Figure 5b: Hedge Ratio as a Function of Price Levels (Negative price-yield correlation)

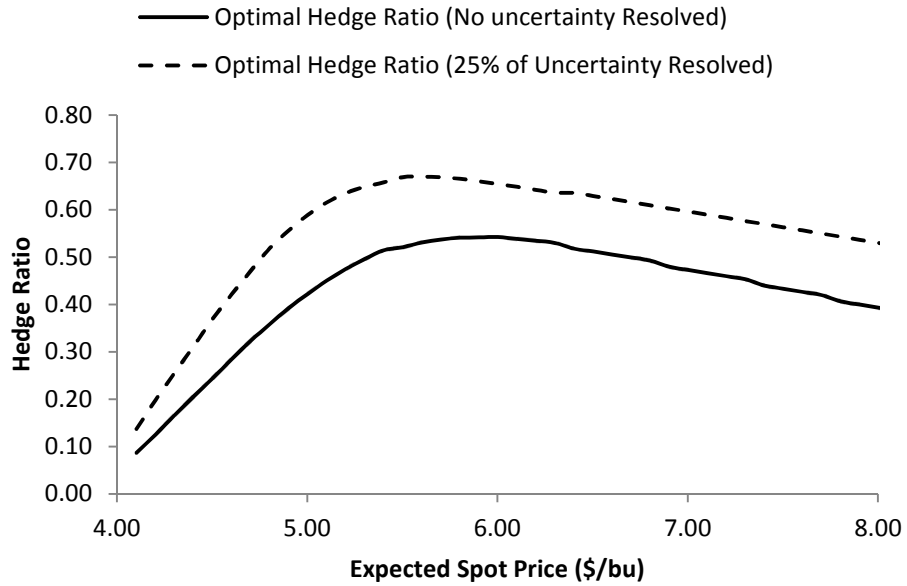
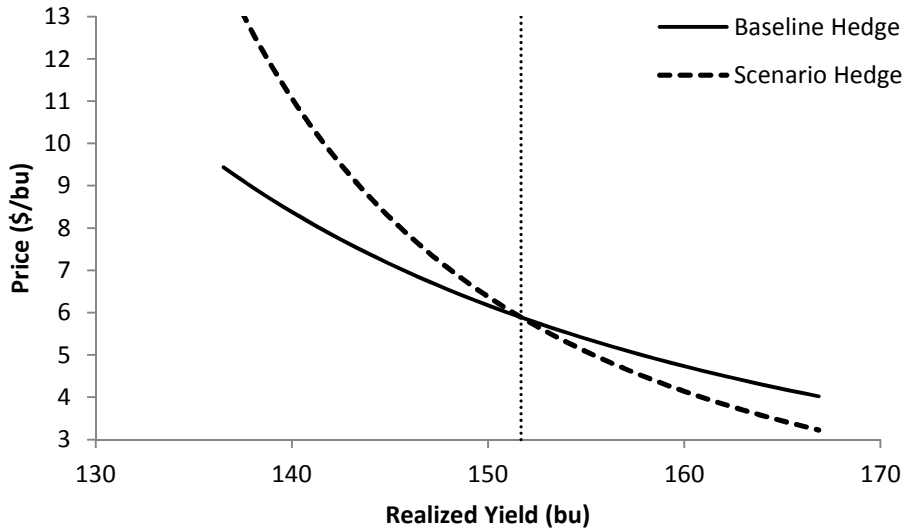


Figure 6: Effect on Spot Price Levels and Volatility due to Increased Hedging



With period-2 expected yields of approximately 152 bu/acre (assuming the 25% uncertainty resolution case), a -10% supply shock corresponds to a realized yield of 137 bu/acre. As expected, increased hedging activity results in a higher variation of equilibrium prices. This effect increases non-linearly as the magnitude of the period-3 supply shock increases. As illustrated, changes in price volatility are minimized when the amount of output hedged in period 2 remains unchanged from the amount hedged in period 1, i.e. the baseline hedge. It is also apparent in figure 6 that observed spot prices in the scenario hedge case increase due to a negative period-3 supply shock above those price levels that would have been observed in the baseline hedge case. It should be clear from figure 6, and intuitively so, that less unsold grain available at harvest (due to higher period-2 hedge ratios) directly translates to a higher conditional variance in spot prices.

5. Concluding Remarks

Using prospect theory as an alternative to conventional expected utility theory, this paper considers the corresponding utility maximization problem of a representative grain producer. There are two key findings. First, under prospect theory futures price level increases can lead to a higher share of output being hedged in futures markets. This is a result that conventional expected utility theory does not generate, but is observed empirically. Second, as more output is sold in advance, there is less available at harvest to respond to a potential supply shock, resulting in greater spot price volatility as well as higher spot price levels.

In connection with these findings, there are several qualifications and limitations that must be addressed. The first qualification is the degree to which output sold in advance in futures markets is considered unavailable in the terminal period. The numerical results of this paper present a case in which grain consumers, intending to take delivery at harvest, take the opposing positions for all of the output hedged by producers in short futures contracts. As was mentioned in the paper, the ratio of commercial long contracts to commercial short contracts typically lies within a range of 0.3 and 0.6 with non-commercial traders (speculators) accounting for the remainder of opposing positions. Thus, the numerical results on price level effects and volatility effects at harvest due to increased short hedging are biased upward somewhat. These effects should be scaled by the ratio of commercial long-to-short contracts to better capture the extent to which future output is sold to a buyer intending to take delivery, thereby rendering this output unavailable at harvest.

A second point that deserves some mention is the extent to which the optimal hedge under expected utility theory differs from the optimal hedge under prospect theory in magnitude. Since expected utility theory assumes producers are everywhere risk averse, the corresponding optimal hedge will be an upper bound for the hedge under prospect theory. This is because prospect theory incorporates some degree of risk seeking behavior, which would imply taking a long position in futures markets rather than a short position. It might seem then that the effects under expected utility theory at harvest would always dominate the effects under prospect theory if more output is hedged under the former. The focus of this paper, however, is not necessarily on the magnitude of the optimal hedge ratio in any given period, which is somewhat arbitrary, but rather the direction and magnitude of the change in this hedge ratio from one period to the next.

A final comment to be made concerning the results of this paper is in regards to the specific behavioral assumption being made: that grain producers are risk averse over gains and risk seeking over losses. A limitation of this paper is that this is not necessarily the only behavioral assumption that would give rise to increased short hedging activity due to an increase in futures prices. Regret theory would also generate similar results. In this case, as the distribution of prices rises above some reference, a producer would experience regret if prices subsequently fall below this level. This would induce the producer to sell more when prices are above the reference in order to avoid regret later. Modeling this or other behavioral assumptions could be done as a possible extension to the current paper.

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