REVIEW OF MARKETING AND AGRICULTURAL ECONOMICS
VOL. 46, No. 2 (August, 1978)

Estimation of a General Class of Demand Functions for Meat in Australia

L. W. Johnson*

Australian demand for meat and beef equations are estimated using a generalised functional form developed by Box and Cox. On the surface, results do not allow rejection of the usual double-log specification commonly used, but possible reasons for this are suggested. More encouraging is that estimated income elasticities for beef are seen generally to decline over the sample period, a result more in line with *a priori* expectations. Finally, possible extensions of our work are suggested.

1 Introduction

In most empirical investigations concerning Australian demand for meat functions, the relevant variables are assumed to enter the demand equations either linearly or in logarithmic form.[1] It is well known that the choice of functional form can impose restrictions on the results of an empirical analysis of demand (*e.g.* a double-log specification assumes constant elasticities throughout the sample period). Although there may be good reasons for assuming, say, constant elasticities, this point is rarely, if ever, discussed in the literature. However, Chang [2] suggests that sufficient evidence exists to imply that income elasticities for meat decline over time in developed countries. Furthermore, if they do decline and the substitution effect of a price change is small, price elasticities should also be decreasing over time.

Fortunately, a procedure has been developed by Box and Cox [1] to simultaneously estimate the regression parameters in a model and the optimal (in a maximum likelihood sense) functional form from a certain class of functional forms of which the linear and double-log specifications are special cases. The so-called Box-Cox transform has been successfully applied to the demand for money [10, 11, 13, 14, 15], the demand for electricity [6] and to the estimation of density gradients [5].

The purpose of this paper is to investigate Australian demand for meat equations using the Box-Cox procedure to test for functional form, paying particular attention to movements of income elasticities over time. We are restricting ourselves here to single equation methods to focus on the specification problem even though Spitzer [11] has recently extended the use of the Box-Cox generalised functional form to systems of simultaneous equations. In the next section, we briefly outline the Box-Cox procedure, in Section

* Department of Economics, Macquarie University. Partial financial support was received from a Macquarie University Research Grant. I would like to thank David Farmer for his excellent research assistance and C. D. Throsby, S. Durbin and two anonymous referees for their comments on an earlier draft.

1. A quick look at Table 1, page 197 in Main, Reynolds & White [7] which surveys 13 relevant studies will provide the desired information.

3 the equations and data used are specified, followed by a discussion of empirical results in Section 4. We conclude with some general comments on the problem of specification of functional form and suggest extensions of our basic work in the analysis of demand for meat in Australia.

2 The Box-Cox Transform

The Box-Cox transformation is defined for a positive variable X as

$$X^* = \begin{cases} (X^\lambda - 1)/\lambda & \text{if } \lambda \neq 0 \\ ln\ X & \text{otherwise.} \end{cases} \quad (1)$$

That is, the parameter, λ, is defined to be non zero, but as λ approaches zero it can be shown that $(X^\lambda - 1)/\lambda$ approaches $ln\ X$. We now define a stochastic linear relationship between a dependent variable, Y^*, and a number of explanatory variables, X_i^*, of the form

$$Y_t^* = \beta_o + \beta_1 X_{t1}^* + \ldots + \beta_k X_{tk}^* + \mu_t \quad (2)$$

where the random error, μ_t, is assumed to be normally distributed with zero mean and constant variance σ^2. It is easy to see that a linear relationship between the untransformed variables, Y and X_i, obtains when $\lambda = 1$, and as λ approaches 0 we get the usual double-log relationship.

The elasticity of Y with respect to X_k for a statistical model is the value of the random variable, $(\partial Y_t/\partial X_{tk})/(X_{tk}/Y_t)$, evaluated at the conditional expectation, $E(Y_t)$. For equation (2), this can be shown to be

$$E_{yk} = \beta_k (Y/X_k)^{-\lambda} \quad (3)$$

Note that elasticities depend on the value of λ. For example, if $\lambda = 0$ (log form), $E_{yk} = \beta_k$.

The parameters to be estimated in (2) are $\beta_o, \ldots, \beta_k$ and λ, the unknown transformation parameter. Under our assumptions on the error terms, μ_t, it can be shown [1, p.215] that for a given value of λ, the maximised log likelihood for (1) can be written

$$L(\hat{\lambda}) = c - (n/2)\ ln\ \hat{\sigma}^2(\hat{\lambda}) + (\hat{\lambda} - 1) \sum_t ln\ Y_t \quad (4)$$

where c is a constant not dependent on the parameters and $\hat{\sigma}^2(\hat{\lambda})$ is the estimated error variance of the regression of Y_t^* on all the X_t^*'s in (2). Therefore, to find the overall maximum likelihood, all we need do is to regress Y^* on the X^*'s for a range of $\hat{\lambda}$'s and choose that $\hat{\lambda}$ which maximises (4). It is then a simple task to calculate confidence intervals for λ or to test hypotheses about λ by using the likelihood ratio method (*e.g.* [2, 15]), although tests often prove inconclusive if the likelihood function is rather flat, as we shall see below.

3 Equations and Data

Since the main purpose of this paper is to illustrate the use of the Box-Cox transform in demand for meat studies, we have concentrated on the aggregrate demand for meat and the demand for beef. Preliminary equations for mutton, lamb, and pork were also estimated but will not be discussed here.

It is postulated that per capita apparent consumption of meat (Q_m) is dependent upon real per capita disposable income (Y), the price of meat (P_m), and, since other foods may be substitutes for meat, the price of other foods (P_F). The demand for beef (Q_B) is assumed to depend on income (Y), beef

Table 1: Demand for Meat Using Box-Cox Technique
(t statistics in parentheses)

Regression Number	Constant	Meat Price (P_m)	Income (Y)	Food Price (P_F)	Meat Price/ Food Price (P_m/P_F)	1st Q (D_1)	2nd Q (D_2)	3rd Q (D_3)	$\hat{\lambda}$	$\bar{R}^2$	D—W
1	—7.571 (—.695)	—.2814 (—4.123)	.0567 (1.667)	.9845 (1.947)		.153 (.873)	.489 (2.99)	.303 (1.83)	.5	.4770	2.576
2	2.896 (1.05)	—.7651 (—4.796)		.6938 (1.073)		—.005 (—.270)	.051 (2.58)	.024 (1.19)	—.1	.4705	2.422
3	6.526 (11.49)	—.7301 (—5.882)	—.0182 (—.252)			—.012 (—.404)	.066 (2.24)	.030 (.986)	.0	.4572	2.349
4	3.170 (6.59)		.0492 (.849)		—.9523 (—6.133)	.005 (.112)	.108 (2.69)	.058 (1.39)	.1	.4746	2.454

Table 2: Detailed Empirical Results for Preferred Demand for Meat Equation
(*t* statistics in parentheses)

λ	Constant	P_m	Y	P_F	D_1	D_2	D_3	$\bar{R}^2$	$D—W$	$L(\hat{\lambda})$
1.0	—25.107	—.131	.067	.523	.848	2.41	1.51	.4768	2.585	—24.179
	(—.912)	(—4.00)	(2.06)	(2.25)	(1.04)	(3.167)	(1.93)			
0.8	—14.956	—.178	.027	.674	.428	1.27	.793	.4769	2.581	—24.079
	(—.813)	(—4.05)	(1.90)	(2.13)	(.973)	(3.10)	(1.89)			
0.6	—9.345	—.241	.045	.868	.216	.673	.418	.4770	2.578	—24.030
	(—.729)	(—4.10)	(1.74)	(2.01)	(.906)	(3.03)	(1.85)			
0.5	—7.571	—.281	.057	.984	.153	.489	.303	.4770	2.576	—24.026
	(—.695)	(—4.12)	(1.67)	(1.95)	(.873)	(2.99)	(1.83)			
0.4	—6.262	—.328	.072	1.12	.108	.355	.220	.4771	2.575	—24.035
	(—.666)	(—4.15)	(1.59)	(1.89)	(.841)	(2.96)	(1.81)			
0.2	—4.605	—.446	.116	1.43	.054	.188	.116	.4771	2.572	—24.099
	(—.627)	(—4.19)	(1.45)	(1.78)	(.778)	(2.89)	(1.77)			
0.0	—3.771	—.605	.186	1.84	.027	.099	.062	.4770	2.570	—24.223
	(—.612)	(—4.23)	(1.32)	(1.68)	(.720)	(2.83)	(1.74)			
—0.4	—3.431	—1.111	.476	3.07	.007	.028	.017	.4760	2.565	—24.673
	(—.628	(—4.30)	(1.09)	(1.50)	(.618)	(2.71)	(1.68)			
—1.0	—4.995	—2.743	1.96	6.77	.001	.004	.003	.4718	2.555	—25.901
	(—.662)	(—4.35)	(.832)	(1.32)	(.515)	(2.57)	(1.64)			
—2.0	—20.514	—12.067	24.66	29.43	.00004	.0002	.0001	.4538	2.519	—29.568
	(—.670	(—4.28)	(.643)	(1.23)	(.501)	(2.46)	(1.69)			

price (P_B), prices of other meats (P_A) and the price of other foods (P_F)[2]. Therefore, our two basic equations are

$$Q_m^* = \beta_o + \beta_1 Y^* + \beta_2 P_m^* + \beta_3 P_F^* + \mu \qquad (5)$$

$$Q_B^* = \gamma_o + \gamma_1 Y^* + \gamma_2 P_B^* + \gamma_3 P_A^* + \gamma_4 P_F^* + w \qquad (6)$$

However, we have experimented with slightly different formulations for both equations by using (P_m/P_F) in (5) and either (P_B/P_A) or (P_B/P_F) in (6) instead of entering the variables separately.

Our data involves quarterly observations from the first quarter of 1962 to the second quarter of 1975, a total of 54 observations. To account for seasonal effects, three (untransformed) quarterly dummies were included in all regressions. The main source of data was Main, Reynolds and White [7, pp. 210-11]. In addition, a food price index was taken from the *A.B.S. Consumer Price Index* and adjusted to represent the price level of non-meat food (P_F)[3]. Consumption of meat (Q_m) is a simple sum of per capita consumption of beef (and veal), lamb, mutton and pork (not ham or bacon) given in [7]. The meat price index (P_m) was calculated as a quantity weighted sum of the prices of beef, lamb, mutton and pork again from [7]. Adjusted meat price (P_A) is an index of meat prices adjusted for the price of beef in much the same way as P_F.

4 Empirical Results

There have been several empirical studies concerned with Australian demand for meat produced in the past two decades. These studies have provided estimates of all kinds of price (own- and cross-) and income elasticities for various meats for either Australia or various states or cities. A good summary of most of this work may be found in a recent article in this *Review* by Richardson [8, pp.80-85] so we will not provide a summary here. Suffice it to say that the vast majority of this work has relied on ordinary least squares estimation of either linear or loglinear demand equations with essentially no consideration of other functional forms.

Our results[4] for the meat function may be found in Table 1 while the beef equation estimates are in Table 3. We initially iterated over λ from +1. to —2. by increments of 0.1. We then determined a much smaller range within which the maximum log likelihood fell and iterated by .01 but found empirical results essentially unchanged. Therefore, only results from the initial iterations are reported. The best equation for the demand for meat seems to be regression number 1 in Table 1 where P_m, Y and P_F are all included. More detailed results for this preferred equation for various values of $\hat{\lambda}$ are presented in Table 2. Note that parameter estimates are not directly comparable since a different value of $\hat{\lambda}$ implies a different set of transformed variables. The estimated optimal $\hat{\lambda}$ was 0.5 which is *not* significantly different from zero using the approximate chi-square likelihood ratio test even though 0.5 seems to be far away from zero. As a matter of fact, it is also not significantly different from one, apparently because the likelihood function is quite flat.[5] This could be due to the degree of aggregation in the data.

2. All prices have been deflated using the Consumer Price Index.
3. From the CPI breakdowns published in the *ABS Consumer Price Index* it can be (roughly) calculated that meat contributes between 33-35% of the food groups contribution with a high degree of consistency. Our adjusted food price index was constructed by attempting to extract this component from food prices as a whole, but we admit that our approach was rather arbitrary.
4. All computations were done using a modified version of a computer program written by H. S. Chang [3].
5. The problem of a fairly flat likelihood function arises frequently in applied situations making the search for a global maximum very difficult. The present situation is a case in point.

Ignoring for the moment that $\hat{\lambda} = 0.5$ is not significantly different from either 0 or 1, the income elasticity of demand for meat calculated at the sample means with $\hat{\lambda} = 0.5$ is 0.2259. For the same equation with $\lambda = 0$, it is 0.1862 while in the linear case ($\lambda = 1$), 0.2667. Although the alternative elasticity estimates are not radically different from one another, we tend to prefer the estimate arrived at through use of the generalised functional form which, in this case, resulted in an income elasticity about half way between the estimates from the linear and double-log forms.

The disturbing fact that emerges from our meat results is that for $\hat{\lambda} = 0.5$, estimated income elasticities for meat are rising over the sample period[6], contrary to our argument above that income elasticities for food should be declining over time in a developed economy. One possible complication could be our method of aggregation, although it is more likely that the problem has arisen because of a combination of our choosing to ignore the simultaneity aspect as well as the dynamic nature of the demand for meat.

Turning now to the demand for beef results in Table 3, the most reasonable estimates, in our opinion, are given by regression number 6 with P_B, Y and P_A as explanatory variables. More detailed results for this equation are given in Table 4. All coefficients are significant at the 5% level and have the correct sign. The estimated $\hat{\lambda}$ is —0.3, although as in the meat regression, the hypothesis that $\lambda = 0$ cannot be rejected (nor can we reject the hypothesis that $\lambda = -1$). This again points out one difficulty of the Box-Cox technique in that a flat likelihood function makes it quite difficult to discriminate between alternative functional forms. Nevertheless, we feel that the estimates generated using the Box-Cox generalized functional form are still superior to those that result from simply specifying a functional form *a priori*, and at little increase in computational burden. However, we admit that one may just as easily interpret our results as a justification of the usual double-log formation.

A major problem with our study is the relative lack of explanatory power of our estimated equations as seen by the low $\bar{R}^2$'s. We feel that this is due to our use of quarterly data as compared to the results of Chang [2] who obtained much higher $\bar{R}^2$'s with annual data. Much of the variation in the data from quarter to quarter becomes obscured when aggregated to form annual data. One should note that for our preferred beef equation, the coefficients of all quarterly dummies are significant. However, further work with various dynamic models is surely warranted.

The income elasticity of demand for beef at the sample means using the results of regression 6 is 0.418, while for the double-log version of 6, the income elasticity is 0.462. This in itself is not a particularly useful comparison, but examination of the income elasticity estimates over the sample period 1962(1)-1975(2) is particularly enlightening. In Table 5 we present some estimated income elasticities for selected observations over the sample period using the estimated coefficient of income from regression 6. Of most importance is that the estimated elasticity seems to be declining over time, a result more in line with what we would expect in a developed economy such as Australia. The sharp increase in the income elasticity for 1975(1) is explained by the fact that the quantity of beef consumed began increasing markedly in 1974 for reasons we believe having little to do with increasing income.

6. From 0.1873 in 1962(1) to 0.2381 in 1975(2). These were calculated in each time period using equation (3) with actual income and estimated meat demand since we are interested in elasticities on the estimated demand function.

Table 3: Demand for Beef Using Box-Cox Technique
(t statistics in parentheses)

Regression Number	Constant	Beef Price (P_B)	Income (Y)	Food Price (P_F)	Adj. Meat Price (P_A)	P_B/P_A	P_B/P_F	1st Q (D_1)	2nd Q (D_2)	3rd Q (D_3)	$\hat{\lambda}$	$\bar{R}^2$	D—W
5	11.23 (.805)	—3.232 (—5.310)	.9667 (.903)	—3.394 (—.646)	.9704 (2.113)			.040 (1.49)	.077 (3.08)	.045 (1.82)	—.4	.5253	2.462
6	2.678 (2.528)	—2.435 (—6.766)	1.164 (3.405)		.7415 (2.055)			.067 (2.78)	.112 (4.74)	.070 (2.86)	—.3	.5352	2.491
7	—2.054 (—.673)		2.743 (.995)			—.0899 (—4.431)		.011 (1.75)	.021 (3.43)	.010 (1.60)	—.9	.3688	1.891
8	3.074 (2.736)	—2.637 (—6.195)	1.755 (3.592)					.058 (3.01)	.095 (5.09)	.063 (3.23)	—.4	.5005	2.321
9	5.579 (3.209)	—8.437 (—5.381)			3.715 (2.205)			.007 (1.66)	.016 (3.74)	.008 (1.80)	—1.0	.4431	2.145
10	23.33 (3.822)			—13.49 (—4.027)	1.303 (2.389)		—.4253 (—6.649)	.017 (1.28)	.048 (3.54)	.024 (1.71)	—.5	.5240	2.369
11	—21.27 (—1.37)		3.991 (1.741)	10.29 (1.326)		—.1591 (—3.276)		.039 (2.18)	.059 (3.45)	.045 (2.10)	—.6	.3810	2.005

Table 4: Detailed Empirical Results for Preferred Demand for Beef Equation
(t statistics in parentheses)

$\hat{\lambda}$	Constant	P_B	Y	P_A	D_1	D_2	D_3	$\bar{R}^2$	D—W	$L(\hat{\lambda})$
1.0	15.34	—.191	.021	.051	1.79	2.86	1.83	.5731	2.523	—21.166
	(6.26)	(—7.33)	(4.90)	(2.09)	(2.84)	(4.60)	(2.83)			
0.5	7.32	—.507	.098	.145	.501	.818	.517	.5617	2.511	—18.770
	(5.87)	(—7.19)	(4.33)	(2.13)	(2.84)	(4.70)	(2.86)			
0.0	3.95	—1.35	.462	.403	.142	.236	.148	.5463	2.499	—17.538
	(4.17)	(—6.96)	(3.75)	(2.10)	(2.82)	(4.74)	(2.87)			
—0.2	3.09	—2.00	.856	.606	.086	.144	.089	.5390	2.493	—17.355
	(3.11)	(—6.83)	(3.52)	(2.07)	(2.79)	(4.74)	(2.86)			
—0.3	2.68	—2.44	1.16	.742	.067	.112	.070	.5352	2.491	—17.327
	(2.53)	(—6.77)	(3.41)	(2.06)	(2.78)	(4.74)	(2.86)			
—0.4	2.25	—2.96	1.58	.907	.052	.088	.055	.5312	2.488	—17.341
	(1.94)	(—6.69)	(3.29)	(2.04)	(2.77)	(4.73)	(2.85)			
—0.6	1.26	—4.39	2.92	1.36	.032	.054	.033	.5277	2.484	—17.488
	(.834)	(—6.55)	(3.07)	(1.99)	(2.75)	(4.71)	(2.84)			
—1.0	—2.39	—9.67	9.92	3.01	.012	.020	.012	.5043	2.475	—18.238
	(—.695)	(—6.23)	(2.63)	(1.87)	(2.68)	(4.65)	(2.80)			
—2.0	—75.46	—70.11	200.68	21.32	.001	.002	.001	.4510	2.459	—22.441
	(—1.32)	(—5.36)	(1.67)	(1.49)	(2.49)	(4.40)	(2.67)			

Table 5: Income and Price Elasticity Estimates ($\hat{\lambda} = -0.3$) for Beef *

Period	*Income Elasticity*	*Price Elasticity*
1962(1)	0.4579	—1.3523
1964(2)	0.4605	—1.3869
1966(3)	0.4266	—1.2599
1968(4)	0.3847	—1.2135
1971(1)	0.4004	—1.2282
1973(2)	0.4005	—1.2576
1975(1)	0.4447	—1.5604

* See footnote 6 for method of estimation.

Table 5 also contains estimated price elasticities in the same selected periods, and it can be seen that they also seem to be declining over time, although in 1975(1), the highest estimate was found. However, it should be noted that in 1974 relative beef prices began a very precipitous decline which could explain the sudden turn-up in price elasticity.

5 Comments

We have attempted to show that by using the generalised functional form developed by Box and Cox, Australian demand for meat and beef equations can be estimated which yield more reasonable results in light of *a priori* expectations. Our meat results were not particularly encouraging, but demand for beef equations resulted in income and price elasticity estimates that, in general, were declining over the sample period as opposed to the constant elasticity estimates generated by a double-log equation. This result, it was argued, is more in keeping with what we would expect in a developed economy such as Australia.

What we are not claiming is that our demand estimates are far superior to any that have been previously generated in other studies, particularly since we could not reject the hypothesis that the functional form is double-log. Certainly, one should not ignore the problem of simultaneity when estimating demand for meat equations[7]. A useful extension of the work here would be to apply Spitzer's [11] recently developed simultaneous equation Box-Cox procedure in the context of Australian demand for meat.

We have also chosen to ignore the possibility of autocorrelated errors in the demand equations. In a recent paper, Savin and White [9] argue that one should test for both functional form and autocorrelation simultaneously, presenting examples in which testing for one or the other alone yields misleading results. Quite possibly, our inability to reject hypotheses about alternative functional forms may be a result of our ignoring the problem of autocorrelation. Therefore, further work on Australian demand for meat using the Box-Cox transform should consider their procedures. An even more general Box-Cox technique would be to allow the transformation parameters (λ's) to take on different values for each of the variables in the equation, although the computational burden increases substantially. Alternatively, one could employ the more general Box-Tukey transform suggested and used by Gaudry and Wills [4] in the estimation of travel demand models. All or some of these alternative approaches may yield

7. For example, see Main, Reynolds and White [7] and Throsby [12].

much better results, especially when combined in a dynamic framework. We leave this work to later research.

Our goal was simply to illustrate the use of the Box-Cox generalised functional form estimation method as applied to Australian demand for meat equations. As mentioned above further work is needed to account for simultaneity and autocorrelation in the Box-Cox framework. Along with these problems, dynamic models need to be developed which take into consideration, among other things, the role of habit in the consumption of beef. It is hoped that this short exploratory paper will stimulate interest in the further use of the techniques outlined here among those actively engaged in empirical studies, not just in the area of meat demand, but in any area where the functional form of the equations to be estimated is usually specified arbitrarily.

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