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Planning Beef Production: An Application of Dynamic Programming

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The determination of an optimal feeding and marketing strategy for a beef grazing enterprise is formulated as a dynamic programming problem. It is supposed that a decision has to be made from time to time as to whether to market animals at their current weights or to retain them for further fattening; if the former policy is adopted, a further judgment is required as to whether to purchase replacement cattle immediately or at some later time. To obtain a realistic feeding and marketing model, liveweight gains and saleyard prices are assumed to be dependent on the time of the year and the breed of the cattle, saleyard prices also being dependent on liveweight.

1 Introduction

Over the past decade or so, a number of researchers have developed optimizing models relevant to the feeding and marketing of beef cattle, using one of three approaches — production and profit function analysis [5, 13, 14, 21, 36]; simulation [1, 3, 4, 7-12, 15-19, 24, 30, 34, 37-40]; or dynamic programming [25-27, 29, 31].

In this paper, the determination of optimal feeding and marketing strategies for beef cattle grazing pasture is formulated as a dynamic programming problem, and two models are discussed. The material presented in this paper forms part of a larger study which consists of an attack on three related problems — the yield of pasture expressed as a function of rainfall and of usage of superphosphate, the short-term forecasting of cattle prices, and the decision as to whether to market a particular group of cattle at their current weight or to continue feeding them.

2 A Simple Model for Planning Beef Production

Consider the following decision process for planning beef production, adapted from Bellman [6, p. 46]. Suppose that one has a herd of cattle, the total liveweight of the animals at a certain time being z kg, and the

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prerogative, at that time and at the end of each succeeding period (say, a month or a quarter), of sending one part of the herd to market, and retaining the other part for fattening purposes. Assume that the total liveweight of the cattle sent to market at the end of period i is y_i kg, yielding a gross income \$ $f(y_i)$. Suppose that the cattle retained have a liveweight totalling x_i kg at the end of period i and ax_i kg at the end of period $i+1$, so that a represents the ratio of the liveweight of the cattle at the end of one period to their liveweight at the end of the preceding period. If no stock remain after n periods, thereby giving rise to a finite planning horizon, what policy of feeding and marketing should be pursued in order to maximize gross income over the n periods?

Bellman's principle of optimality can be used to solve such a problem. Defining $g_{n-i}(ax_i)$, $i = 0, 1, 2, \ldots, n-1$, as the gross income obtained in periods $i+1, i+2, \ldots, n$, and putting $z = ax_o$, one obtains

$$g_1(ax_{n-1}) = f(y_n)$$

$$g_{n-i}(ax_i) = \max_{0 \leqslant y_{i+1} \leqslant ax_i} [f(y_{i+1}) + g_{n-i-1}(ax_{i+1})] \quad i = 0, 1, 2, \ldots, n = 2$$

3 A More Realistic Model for Planning Beef Production

The above treatment of optimization of gross income from grazing is somewhat idealized. To begin with, a farmer would not normally allow his stock to continually decrease in number as the above model has assumed. Rather, he would seek to replace the animals sold with store cattle, though he may not do this immediately. Such a decision would depend on the price of store cattle at the time, his projection as to what cattle prices shall be in the future, and the quantity of feed he has available.

The simple model for planning beef production assumed that a is constant over all periods and all animals. This assumption is quite unrealistic. The value of a varies both among animals of the same breed and among breeds. For a given animal not suffering from ill-thrift, the value of a depends on the quantity and the quality of pasture available, and this in turn is dependent on uncontrollable weather factors and on controllable factors such as composition of pasture, superphosphate usage, and stocking rate. Suppose, however, that for a particular breed of cattle the ratio of the liveweight at the end of period i to that at the end of the preceding period is designated by a_i, $i = 1, 2, 3, \ldots, n$.

Just as a is not constant, so also the saleyard price per kg varies, and it is not realistic to assume that the same function f can be used to compute the gross income per kg from the sale of cattle at all times. The factors which determine the saleyard prices of cattle have been the subject of recent research [33, 35], but it will be assumed here that it is possible to express gross income from marketing fat cattle in terms of saleyard prices designated by \$$b_i$/kg (carcase weight), $i = 1, 2, \ldots, n$ for a breed whose dressing percentage is d%. In fact, the dressing percentage, which is the ratio of carcase weight to liveweight expressed as a percentage, is influenced not only by the breed of cattle, but also by their age, their weight, and their gut-fill, but since the maximum variation in dressing percentages is of the order of a couple of percentage units, it was assumed in this model that d was constant for a given breed.

Under these assumptions, application of Bellman's principle of optimality gives

$$g_1\ (a_n(x_{n-1} + p_{n-1})) = b_n\ (d/100)\ y_n$$

$$g_{n-i}(a_{i+1}(x_i + p_i)) =$$

$$\max_{\substack{0 \leqslant y_{i+1} \leqslant a_{i+1}(x_i + p_i) \\ 0 \leqslant p_i \leqslant a_1 x_o - x_i}} [b_{i+1}(d/100)y_{i+1} + g_{n-i-1}(a_{i+2}(x_{i+1} + p_{i+1}))],$$

$$i = 0, 1, 2, \ldots, n-2$$

where $p_i$kg, $i = 1, 2, \ldots, n-1$, represent the total liveweight of cattle purchased at the end of each of the first $n - 1$ periods, and $p_o = 0$. The function $g_{n-i}(.)$ has a similar meaning to that given in Section 2.

This model is capable of amendment so that it will maximize net income rather than gross income; all that is entailed is to subtract from the saleyard price received for animals their purchase price, their production costs (including veterinary supplies, freight, and marketing charges), and an appropriate proportion of the annual costs of production (such as the opportunity cost of capital, cost of materials, labour charges, service charges, depreciation, interest paid, and rent paid).

4 Application of the Latter Model to Field Data

4.1 The Data

The data required to illustrate how the latter model can be applied to actual data to maximize profit from any beef cattle grazing enterprise includes the saleyard prices of the various grades of cattle in each period, the costs of production of the cattle, and their liveweight gain in each period.

An Officer of the Pastoral Research Station at Hamilton, Victoria has supplied the following liveweight data in kilograms on a quarterly basis for calves born May-June 1972.

Breed \ Date	Feb 1973	May 1973	Aug 1973	Nov 1973	Feb 1974	May 1974	Aug 1974	Nov 1974	Feb 1975
Hereford X Hereford	234	258	331	433	467	487	497	593	639
Hereford X Friesian	289	306	376	459	495	506	524	614	649

The following quarterly saleyard prices of cattle from February 1973 to February 1975 were obtained from publications of the Australian Meat Board and from the Department of Agriculture:

Date \ Grade	Store Weaners	Yearlings	Light Bullocks	Medium/Heavy Bullocks
	($/head)	(cents/kg dressed weight)		
Feb 1973	50	67.5	75.1	76.6
May 1973	55	74.7	96.8	98.1
Aug 1973	70	88.7	101.8	102.0
Nov 1973	75	95.6	105.8	106.9
Feb 1974	70	94.2	86.7	84.9
May 1974	65	93.1	77.6	64.5
Aug 1974	45	68.8	52.3	48.0
Nov 1974	30	42.6	34.0	32.6
Feb 1975	25	33.0	30.2	28.4

For the purposes of this example, a ratio of carcase weight to liveweight of 60% was assumed, regardless of breed, age, or liveweight.

4.2 Analysis of the Data

At the end of each quarter, one of three decisions was made for each animal on the farm — to retain the animal for further fattening, to sell the animal and immediately replace it by another of store weaner weight, or to sell the animal and not replace it for at least a quarter; in the latter case, the decision at the end of the next quarter became one of purchasing an animal of store weaner weight or of not purchasing such an animal.

Regardless of which of these decisions was made, it was assumed that its effect on the annual cost of production would be insignificant, but a purchasing cost of $10 was set to cover freight and veterinary charges and a selling cost of $10 to cover freight and marketing charges.

Agricultural advisers intending to use the model as a management tool should consider carefully the assumption that the weight of individual animals comprising the herd does not effect the annual cost of production so long as their total weight does not exceed some predetermined amount, which has been designated earlier as z kg.

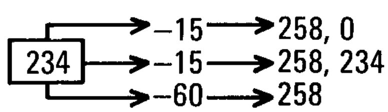
Animals of different liveweights consume different quantities of food in order to achieve a given weight gain, and the amounts of food consumed are not in proportion to their weights; for example, it can be deduced from [22] that the average daily requirements of good quality pasture to achieve a weight gain of 0.5 kg/day would be 4.2 kg DM/day for an animal of liveweight 300 kg and 7.0 kg DM/day for one of liveweight 600 kg. Further, as not all animals are equally efficient in converting food consumed into liveweight gain, there would be a variation in intake between animals of a particular liveweight to achieve a given weight gain.

An annual grazing charge forms part of the annual cost of production, as the latter includes an opportunity cost of the land and the cost of materials such as fertilizer which are used to maintain the pasture; an appropriate proportion of this annual grazing charge is subtracted from the gross income to obtain the net income.

If the annual cost of production does not include the purchase of hay, grain, *etc.* from outside the farm, the implication is that only part of the available acreage of pasture will be grazed during the periods of peak grass growth. In late spring, the output from the remainder would be cut and stored, then fed out at other times of the year; pasture conserved in autumn in closed-off paddocks could be released for grazing during winter.

In practice, it would seem appropriate to determine for a given farm the approximate area of land which will usually supply sufficient grass to enable an average animal of given liveweight to achieve a given weight gain. To illustrate the data required and the calculations involved, the average seasonal pasture growth at Ballarat has been determined from pasture trials, and averaged over the year produces approximately 19 kg/ha/day of dry matter. Allowing for spoilage of some 40% of pasture by cattle, the quantity of pasture available would be of the order of 11 kg/ha/day of dry matter. If cattle are assumed to graze as close to the ground as a mower can cut in pasture trials, then it can be deduced from [22] that cattle of liveweight 600 kg achieving a weight gain of 0.5 kg/day would be stocked at approximately 1.5 animals/ha. The conservative farmer, realizing that there will be seasons when the pasture growth is less than average, might be prepared to accept a lower stocking rate, say 1 animal/ha, and conserve pasture produced in excess of his requirements in better years for use in poorer years.

Figure 1 displays the possible feeding and marketing policies for a Hereford X Hereford animal for two years from February 1973 to February 1975. Each node is represented by a box showing the liveweight of an animal and the profit generated by each of a number of actions, the actions being numbered from top to bottom. So, for instance, the first node represented by



indicates that an animal of liveweight 234 kg at the beginning of the quarter will have a liveweight of 258 kg at the end of the quarter. If the animal is sold, the profit generated is —$15, regardless of whether the animal is not replaced (action 1) or is replaced by a store weaner of liveweight 234 kg (action 2); if the animal is retained for further fattening (action 3), the profit generated by this action is —$60.

At this point, three observations should be made concerning Figure 1. Firstly, although the diagram relates to the possible feeding and marketing policies for just one animal, the policies would clearly be the same for each animal in a herd of cattle of uniform composition. Secondly, an animal purchased as a store weaner after the initial purchase in February 1973 has been assumed to gain weight in the same proportion as an animal purchased in February 1973 would have gained weight. And finally, any animal sold which has a carcase weight less than 160 kg is sold as a store; those having a carcase weight between 160 kg and 200 kg are sold as

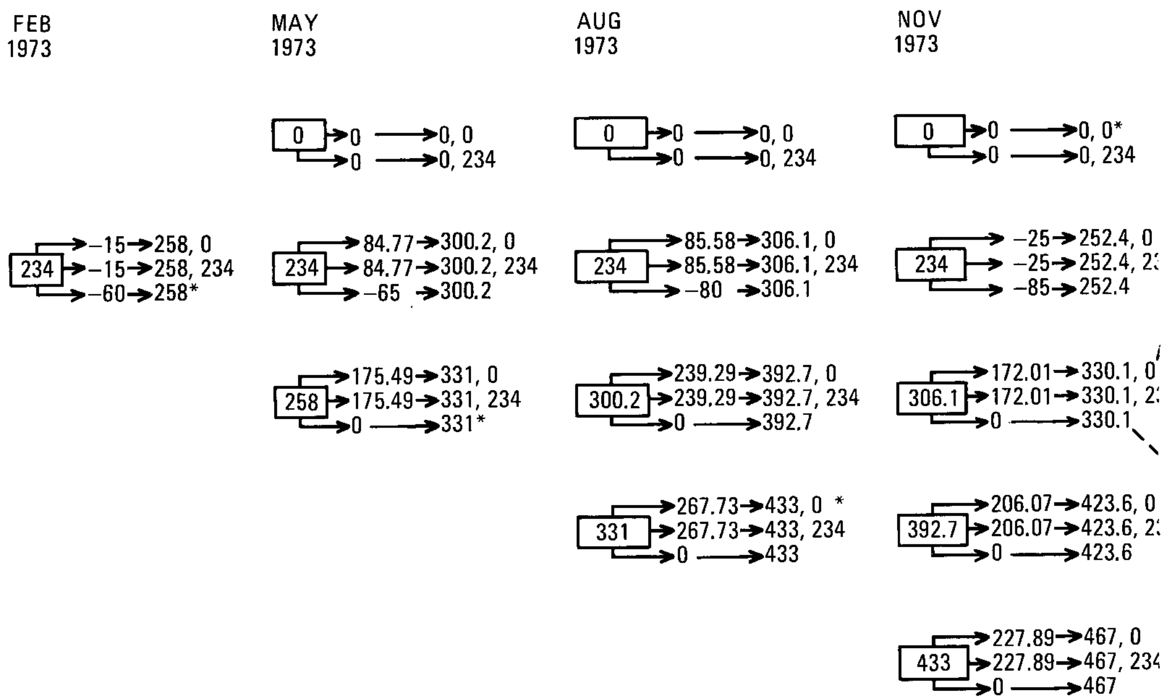


Figure 1 : Possible feeding and marketing policies for a Hereford X Hereford animal, February 1973 to February 1975.

Note : The connection between nodes at two consecutive times is shown by the presence of similar numbers at the right hand end of any stage and at a node of the following stage. Some typical examples of connections (in this case, between November 1973 and February 1974) are shown by dotted lines.

EB
974
MAY
1974
AUG
1974
NOV
1974
FEE
197
0
0 → 0, 0 *
0 → 0, 234
0
0 → 0,0
0 → 0,234*
0
0 → 0,0
(0) → 0,234
0
0 → 0, 0*
−25 → 244, 0
234
−25 → 244, 234
−80 → 244
−40 → 238.8, 0
234
−40 → 238.8, 234
−75 → 238.8
6.36 → 279.2, 0*
234
6.36 → 279.2, 234
−55 → 279.2
−25 → 252.2,
234
78.78 → 263.2, 0
252.4
78.78 → 263.2, 234
0 → 263.2
35 → 249, 0
244.0
35 → 249, 234
0 → 249
62.82 → 284.9, 0
238.8
62.82 → 284.9, 234
0 → 284.9
49.58 → 300.9,
279.2
150.31 → 344.3, 0
330.1
150.31 → 344.3, 234
0 → 344.3
75.85 → 268.6, 0
263.2
75.85 → 268.6, 234
0 → 268.6
65.94 → 297.1, 0
249.0
65.94 → 297.1, 234
0 → 297.1
50.78 → 307.0,
284.9
160.94 → 441.7, 0
423.6
160.94 → 441.7, 234
0 → 441.7
100.27 → 351.4, 0
344.3
100.27 → 351.4, 234
0 → 351.4
71.92 → 320.5, 0
268.6
71.92 → 320.5, 234
0 → 320.5
53.40 → 320.2,
297.1
178.47 → 487, 0
467
178.47 → 487, 234
0 → 487
119.83 → 450.8, 0
441.7
119.83 → 450.8, 234
0 → 450.8
73.25 → 419.2, 0
351.4
73.25 → 419.2, 234
0 → 419.2
52.57 → 345.3,
320.5
133.14 → 497, 0
487
133.14 → 497, 234
0 → 497
95.19 → 537.8, 0
450.8
95.19 → 537.8, 234
0 → 537.8
66.97 → 451.7,
419.2
105.99 → 593, 0
497
105.99 → 593, 234
0 → 593
88.76 → 579.6,
537.8
98.89 → 639, 0
593

yearlings, whilst light bullocks are those whose carcase weight is between 200 kg and 250 kg, and medium/heavy bullocks are those whose carcase weight exceeds 250 kg.

The decision made here to classify animals according to their weight is one which was adopted by the Australian Meat Board in the quotation of market prices during the period to which the data applies. In practice, there is no precise line of demarcation between one grade of cattle and the next, and to prevent abrupt differences in the value of an animal as its classification varied, a buffer zone of 5 kg was applied on both sides of the weights at which a change in classification of an animal occurred. To take an example: in February 1975, light bullocks were quoted at 30.2 cents/kg dressed weight and medium/heavy bullocks at 28.4 cents/kg dressed weight; instead of a sudden drop of 1.8 cents/kg occurring at 250 kg dressed weight, the buffer zone of 245 kg to 255 kg allows a decrease of 0.18 cents/kg dressed weight throughout the 10 kg range of the buffer zone.

The feeding and marketing policy to follow in order to maximize the profit on Hereford X Hereford animals over the two-year period from February 1973 to February 1975 was determined by using the computer software system entitled Dynacode [20].[1] Asterisks in Figure 1 indicate optimal actions.

By varying the costs associated with buying and selling, it was shown that the frequency of marketing recommended by this model was dependent on these costs as well as on saleyard prices and cattle weights.

To show how this model may be applied to the typical farm situation in which the cattle herd is not of uniform composition, a feeding and marketing policy to optimize profit for Hereford X Friesian animals was determined in a manner similar to that presented for Hereford X Hereford cattle. Then the One-at-a-Time method [32, pp. 131-144] was used to determine the feeding and marketing strategy to maximize the total profit for the two types of animals; the use of this method enabled the Dynacode package to be retained to determine feeding and marketing policies to optimize total profit. In theory, the One-at-a-Time method can be extended to maximize profit for a herd comprised of any number of different types of cattle; however, the greater this number becomes, the more tedious is the computation.

5 Model Validation

To assess the closeness with which the model represents reality, one possible course of action is to compare model sales with actual sales over the period to which the data used to test the model applies. The model recommends sales twice during the period, in November 1973 and in November 1974. Qualitatively, these marketing times are reasonable, because they come at the end of spring, when the animals have achieved their highest quarterly weight gain for the year. Quantitatively, these marketing times are in agreement with those adopted by the majority of

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[1] Dynacode is available on the Cyber 72 computer at the Royal Melbourne Institute of Technology, and the execution time required to determine the optimal feeding and marketing strategy was 0.864 seconds.

Victorian farmers; quarterly beef production figures for Victoria obtained from the Australian Bureau of Statistics show that the greatest quarterly production in each of the years 1973 and 1974 occurred in November. Therefore, using data which is now known with certainty, the model makes recommendations as to timing of marketing of cattle which are in agreement with actions taken by farmers when they were using forecast prices.

A second approach to validation is to compare the model against existing models. It has been indicated in Section 1 that there have been three approaches taken to the determination of optimal feeding and marketing policies for beef cattle — production and profit function analysis, simulation, and dynamic programming. What advantages does dynamic programming have over the other two methods? Comparing the dynamic programming technique with methods in which livestock marketing decisions are a by-product of optimal feeding models based on regression equations, Kennedy was led to observe that the latter "cannot be so successfully applied when the total feeding period per animal is relatively long, and the seasonal variations of cattle . . . prices are considerable" [26, p. 147]. In their determination of an optimal feeding and marketing strategy for broiler production, Kennedy *et al* were able to compare simulation and dynamic programming as solution techniques, and they concluded that "a more powerful technique than exhaustive search is required . . . DP is a potential solution technique" [28, p. 19]. Kennedy's conclusion that "DP has been found to be a flexible tool for dealing with the dynamic problems of beef production" [26, p. 158] is one with which we would agree.

To compare our dynamic programming model for the determination of optimal feeding and marketing policies for beef cattle grazing pasture with DP models developed by others, we begin by noting that the models of Meyer and Newett [29] and of Nelson [31] apply to feedlot management; because grazing is not taken into account in their models, a comparison between our model and their models is not instructive. On the other hand, Kennedy's model, described in [25], [26], and [27], in addition to optimizing feeding and marketing policies for cattle continuously yarded, also considers grazing, and comparisons may be made between Kennedy's model and our own. Kennedy assumed that liveweight return per kilogram is a function only of the time of the year and liveweight; we have considered liveweight gains and saleyard prices to be dependent on the time of the year, liveweight gains to also depend on the breed of cattle, and saleyard prices to also depend on liveweight. Kennedy's model did not recommend an optimal grassland management policy as we have done; instead he compared the results from a range of grassland management policies. The differences between Kennedy's model and our own should not be allowed to cloud the fact that each model is self-consistent; they emphasize that prospective users of any model must examine the assumptions on which a model is based to ascertain how closely these assumptions fit the situation to which they propose to apply the model.

6 Conclusion

In this paper, dynamic programming has been applied to the determination of optimal feeding and marketing strategies for pasture-fed beef cattle. The model developed in this paper incorporates variable liveweight gains and saleyard prices, and takes into consideration buying and selling costs. It has been tested with field data, as explained in Section 4, and found to recommend the frequency of marketing which will maximize profit for grazing beef cattle; the results obtained have found acceptance among those with expertise in agriculture.

7 Implications for Further Research

It is proposed to incorporate this work into a larger project which has two other facets namely

(i) an investigation of the yield of pasture as a function of rainfall and superphosphate usage; this investigation will have as its objective the determination of the probabilities of various carrying capacities of a given area of land over a given interval of time.

(ii) the short-term forecasting of cattle prices.

Stochastic dynamic programming will then be used to predict optimal feeding and marketing policies for pasture-fed beef cattle.

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