Supply and Demand Stabilization and Welfare in Agriculture: Some Dynamic Considerations — A Comment

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In a recent paper published in the Review, Wolanowski and Strong (WS) ask themselves whether stabilisation of the random elements affecting the supply and demand of a commodity will alter the welfare received by producers and consumers. They consider a partial equilibrium cobweb-type model with linear supply and demand functions having additive stochastic terms and assume that producers plan production on the basis of the Nerlovian adaptive expectations model, viz.

\begin{align*}
(1) \quad & x(t) = a + b p^*(t) + \nu(t) \\
(2) \quad & p^*(t) = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i p(t-1-i) \\
(3) \quad & p(t) = \alpha - \beta x(t) + u(t)
\end{align*}

where $x(t)$ is quantity produced and also consumed, $p^*(t)$ is expected price used by producers, $p(t)$ is realised market price, $a$, $b$, $\alpha$, $\beta$ and $\lambda$ are known parameters, and $u(t)$ and $\nu(t)$ are random terms with zero mean and finite variances. Welfare of producers and consumers is measured by producers’ and consumers’ surplus, respectively. In the paper they argue that the expected welfare of both producers and consumers, and by implication also aggregate welfare, is greater the more variable the random terms in both the demand and supply functions (as measured by the variance terms for $u(t)$ and $\nu(t)$).

One of the principal findings of the WS analysis — that stabilisation reduces aggregate welfare — is contrary to the findings of other studies of commodity stabilisation. While other studies with slightly different models reach different answers about the distribution of welfare gains, they all conclude that stabilisation leads to an increase in the aggregate welfare level — for a survey see Turnovsky [2] and for a model closest to that used by WS see Turnovsky [1].

The purpose of this comment is to explore the reasons for the atypical WS conclusion. It is contended that WS take an inappropriate measure of producer surplus which is different from realised producer surplus. Taking realised producer surplus it is then shown that stabilisation will benefit producers and yield a net gain in aggregate economic surplus.

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Consider in Figure 1 two situations; in Figure 1 (a) producer's expected price \( p^* \) exceeds the realised price \( p \) and in Figure 1 (b) the reverse occurs. Now, using the cobweb model the quantity produced \( x \) is determined by \( p^* \) on the supply curve. The predetermined quantity \( x \) then is sold to clear with realised price being given by \( p \) on the demand curve.

In terms of consumer surplus, I am in agreement with the measure used by WS in their formulae (9). In terms of Figure 1 consumer surplus is given by area \( a + b + c \) in Figure 1 (a) and by area \( f \) in Figure 1 (b).

The measure of producer surplus used by WS is given by their formulae (21). In effect they specify producer surplus with respect to the realised market price \( p(t) \) and the quantity which would be supplied at \( p(t) \). These are the areas \( d \) in Figure 1 (a) and \( h + g + i + j \) in Figure 1 (b).

But, and this is the critical point of my objection with the WS analysis, actual quantity supplied by producers is determined not with respect to \( p(t) \) but with respect to \( p^*(t) \). Then, in Figure 1 (a) for quantity produced \( x(t) \) and realised price \( p(t) \) producer surplus is given by area \( d - c - e \). Here the WS analysis over-estimates the area of producer surplus by \( c + e \). In Figure 1 (b), for quantity produced \( x(t) \) and realised price \( p(t) \) producer surplus is given by area \( g + h \). The WS analysis includes also area \( i + j \), but the output required to earn this surplus is not even produced.

In more general terms, realised producer surplus is given by

\[
G_p = \frac{1}{2} (p^* - p_{\text{min}}) x - (p^* - p) x
\]

where \( p^* \) is expected price formed as in (2), \( p_{\text{min}} \) is the price at which the supply curve cuts the vertical axis for zero quantity, \( x \) is actual quantity produced, and \( p \) is realised price.

The problem now is to evaluate the expected value of \( G_p \) in (4) in terms of the variance terms \( \sigma_3^2 \) and \( \sigma_1^2 \). Intuitively one expects to find \( dEG_p/da_2^2 < 0 \) and \( dEG_p/da_3^2 < 0 \). As we show next, aggregate welfare is increased by stabilisation, and since, as argued by WS, consumers lose, producers must gain from stabilisation.

Let us consider aggregate surplus given by the sum of producer surplus and consumer surplus. Aggregate surplus is given by areas \( a + b + d - e \) in Figure 1 (a) and \( f + g + h \) in Figure 1 (b). More generally, aggregate surplus is given by the formulae

\[
G = \frac{1}{2} \left[ (p_{\text{max}} - p_{\text{min}}) \bar{x} + (p^* - p) (\bar{x} - x) \right]
\]

where \( p_{\text{max}} \) is the price at which the demand curve cuts the vertical axis for zero quantity, \( p_{\text{min}} \) is the price at which the supply curve cuts the vertical axis for zero quantity, \( p^* \) is forecast price, \( p \) is realised price, \( x \) is actual quantity produced, and \( \bar{x} \) is the quantity at which the supply and demand curves intersect. The first right hand term is constant and independent of production and consumption decisions. The second term is always non-positive and is a function of the price forecast used by producers.

Now, aggregate surplus is maximised when \( p^* = p \), that is, when the producer forecast price and the realised price coincide. The loss of potential surplus is given by the second right hand term of (5). With some algebraic manipulation this loss term can be expressed as
(6) \( L = \frac{1}{2}(p^* - p)(\bar{x} - x) = -b \beta / 2(b + \beta)(p^* - p)^2 \)

where \( b \) is the supply equation price parameter and \( \beta \) is the demand equation price parameter. The effect of stabilisation of the random supply and demand shift terms on expected aggregate surplus then can be assessed from evaluation of expected value of \((p^* - p)^2\).

From (1), (2) and (3)

(7) \( p(t) = \alpha - \beta a - \beta b \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i p(t - i) - \beta v(t) + u(t) \)

By back substitution in (7) \( p(t) \) can be expressed as a linear function of the random terms \( u(t), u(t - 1), \ldots, u(t - \infty), v(t), v(t - 1), \ldots, v(t - \infty) \). Assuming a non-explosive cobweb model the relative weights on the random terms will decline with more ancient observations. Using (2) and (7) the term \( p^* - p \) in (6) can be expressed as a linear function of the current and past values of the random terms \( u \) and \( v \).

To illustrate, take the special case of naive expectations where \( \lambda = 1 \) so that \( p^*(t) = p(t - l) \). Here we can show that

\[
p^*(t) - p(t - l) = \xi(t) - (1 + \beta b \lambda \sum_{i=0}^{\infty} (-1)^i(\beta b)^i \xi(t - l - i)
\]

with \( \xi(t) = u(t) - \beta v(t) \).

Then, returning to (6) and assuming serial independence of the \( u(t) \) and \( v(t) \) terms, the expected value of \( L \) can be shown to be a positive linear function of the variance terms \( \sigma^2_u \) and \( \sigma^2_v \). That is, aggregate society welfare is increased by stabilising the random terms causing shifts of the demand for and supply of the commodity.

This result is at direct variance with that obtained by WS but it is consistent with that obtained in other studies of commodity stabilisation.

The intuitive reason for the result is as follows. Greater variability of shifts of the supply and demand for a commodity cause greater absolute errors, on average, in the error of price forecasts. Formally, variability of the distribution of \( p^*(t) - p(t) \) is positively related to variability of the distribution of \( u(t) \) and \( v(t) \). Stabilisation, by reducing the error of producers' price forecasts, leads to production decisions which are closer to the \textit{ex post} optimum than they otherwise would be. That is, in an uncertain world stabilisation facilitates decision making by reducing the magnitude of random information signals used by producers.

References


**Figure 1(a)**

- Consumer Surp. = \(a + b + c\)
- Producer Surp. = \(d \cdot c \cdot e\)
- Agg. Surplus = \(a + b + d \cdot e\)

**Figure 1(b)**

- Consumer Surp. = \(f\)
- Producer Surp. = \(h + g + i\)
- Agg. Surplus = \(f + g + h\)