# Agricultural Economics Report

REPORT NO. 349 DECEMBER 1978

# SPECIFICATION OF MICRO RISK MODELS FOR FARM MANAGEMENT AND POLICY RESEARCH

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## Introduction

This paper illustrates a new methodology for finding preferred action choice(s) under uncertainty for well-defined classes of decision-makers. The methodology does not replace the expected utility maximizing rule, which has been used to identify preferred action choices; rather it extends the rule's accuracy as a positivistic tool and its reliability as a normative guide.

The methodology used in this paper does, however, alter the kinds of information needed about decision-makers. Hence, it is related to a practical question facing the Western Regional Research Committee (W-149). This committee is considering estimating utility functions for a large number of decision-makers differentiated by geographic area, commodity produced, farm size, wealth, and other variables, and they want to know the value of such a project. Because there was no consensus among the committee, a subcommittee was appointed to explore the question in more detail. This paper can be considered part of that exploration.

The remainder of this paper has four parts. In the first, we compare our current interest in utility functions with an earlier interest in production functions and note some similarities. The second part describes how stochastic dominance with respect to a function can be used to identify preferred action choices under uncertainty. The third part illustrates the criterion with a numerical example. And the fourth part concludes with our recommendation for future work on specifying micro risk models.

## Utility Functions in Perspective

The question facing the (W-149) committee of whether or not to estimate von Neumann-Morgenstern utility functions has a familiar ring, but in a different tune and for a different function. It seems reminiscent of our professional interest in production functions during the decade of the sixties (Heady and

Dillon). The interest was both theoretical and empirical. Theoretical interest focused on derivation of expansion paths, isoquants, and profit maximizing solutions for different kinds and shapes of production functions. Empirical work emphasized the statistical estimation of the parameters of production functions for various crop and livestock enterprises.

Output response patterns could be adequately described by a statistically fitted production function only when elaborate control procedures were undertaken. The input being varied had to be carefully measured. Fertilizer, for example, was in some cases applied by hand using small buckets. Plots had to be set up in fields with uniform soil type. Despite researchers' best efforts, however, it was impossible to hold constant all factors not being systematically varied. Weather, insect and pest damage, soil type, and past cropping patterns were factors which often varied in even the most carefully controlled experiments.

At the farm level, unexplained variation in the response of output to changes in input levels was even greater. Inputs were less carefully applied, and the number of factors not held constant was usually large. But no matter, scientists had already fitted a single valued production function to experimental data, and from that single valued function they could make approximate recommendations. On the average input level $X_1$ would result in output level $Y_1$ (Figure 1). Once input and output prices were known with certainty, the profit maximizing levels of factor usage and output could easily be determined.

Our analytic skills have improved since then, and we are no longer willing to assume that production responses are described by single valued functions (Pope and Just). We now recognize explicitly that in response to a given level of usage for an input $X_1$, say fertilizer, farm managers actually face a range of possible output response levels between $\bar{Y}_1$ and $Y_1^*$ with probability $f(Y_1|X_1)$ rather than a single valued level of output (Figure 2).

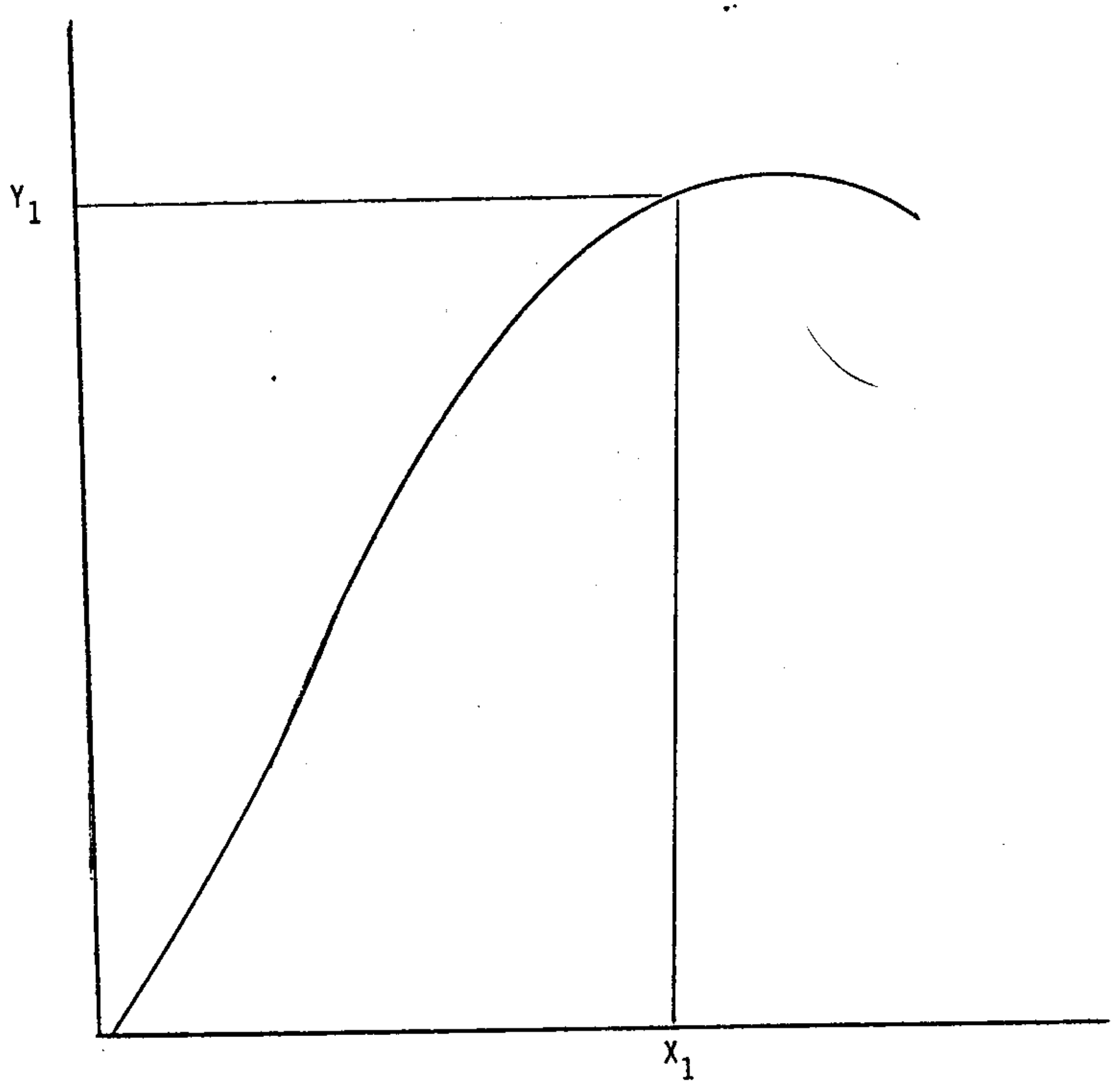


FIGURE 1. A Single Valued Production Function Measuring Maximum Output y for a Specified Input x.

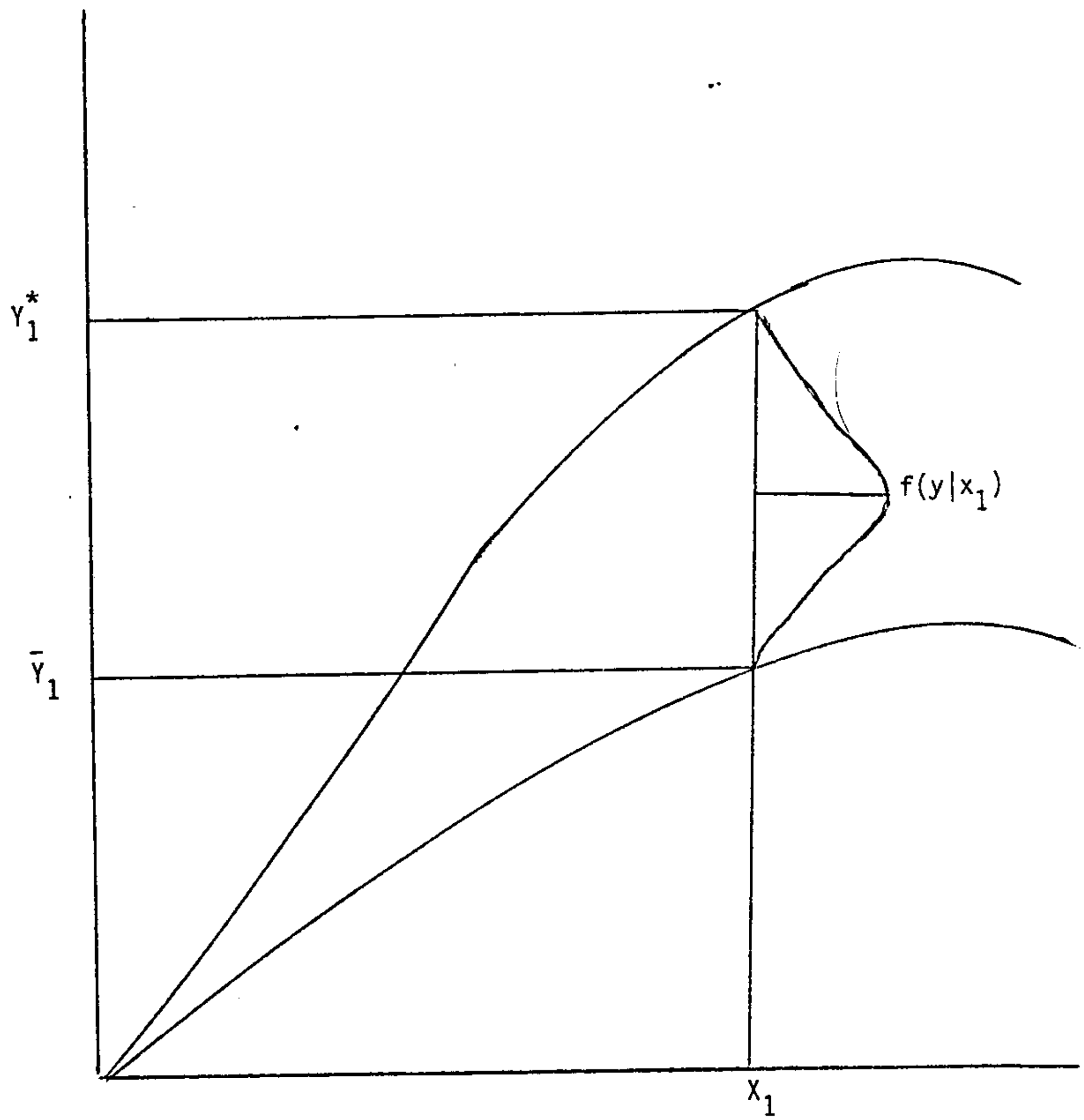


FIGURE 2. A Multivalued Production Function with Output y for a Given Input $X_1$, Described by Probability Density Function f(y).

Previously we did not account for this variance in output; now we do. The explicit consideration of uncertainty has complicated our analysis of economic problems, but it has also made it more realistic (Hiebert, Talpaz and Taylor, Robison and Black). When prices and the production function are known with certainty, the profit maximizing solution is preferred by decision-makers for whom more is better. Once the certainty assumption is dropped, once uncertainty with respect to output and n many user prices is recognized, the expected utility maximizing solution becomes the preferred action choice. If we are to identify that solution, however, we must have a single valued utility function.

Agricultural economists have estimated utility functions for farmers in a number of settings (Officer and Halter). Estimation techniques and interview procedures have been carefully refined, but the estimation of von Neumann-Morgenstern utility functions has proven to be at least as difficult as the estimation of single valued production functions. While production outcomes are usually physical and susceptible to measurement, preferences are not manifest in physical quantities and any utility measures assigned them are completely arbitrary. Moreover, the theory for assigning utility valuations to preference orderings influenced by multiple factors is not well-developed. We usually relate preferences to a single variable, wealth, ignoring the influence of goals other than increase in wealth and of other factors such as age and family status. Finally, because each subject is unique, replication of experiments is not possible.

Due largely to these problems encountered when estimating utility functions, positivistic risk models based on the expected utility hypothesis have not proven to be particularly effective as predictive tools, though they may perform better than models which maximize expected profit (Lin, Dean, and Moore). Our own experience with an expected utility maximizing rule illustrates this point.

In an advanced farm management class students were asked to rank three action choices described by probability density functions. Later they were asked to derive their own von Neumann-Morgenstern utility functions and to order the same three action choices again using the expected utility maximizing rule. Of the 11 students who completed the assignment, only 2 had rankings based on the expected utility rule which matched those of their a priori evaluations.

We need to ask, then, whether we can in fact derive single valued utility functions defined over wealth which rank uncertain outcomes in a way consistent with the underlying preferences of particular decision-makers. If this is our goal, it is similar to the one held earlier by those concerned with the estimation of single valued production functions, and we are proceeding, just as we did before, by attempting to hold more factors constant and to reduce "measurement" errors through the development of more realistic interview procedures. How much can we refine our techniques, however? How close can we come to holding enough factors constant to permit us to say that we have found a "true" single valued utility function which accurately reflects preferences?

While we believe that efforts to improve our ability to represent decision-maker preferences with empirically derived utility functions, we also believe that analysts must recognize that utility functions cannot be known with certainty. At best, we can be reasonably confident only that a given decision-maker's preferences are represented by utility measures falling within some interval such as that between $\bar{U}(W)$ and $U^*(W)$ in Figure 3.

The problem of decision-making under uncertainty now becomes one of ordering action choices with uncertain outcomes for a decision-maker whose utility function may take any shape, but which falls within the confidence interval defined by $\bar{U}(W)$ and $U^*(W)$. Obviously, this is a more difficult task than merely finding the action choice which maximizes the expected utility associated with

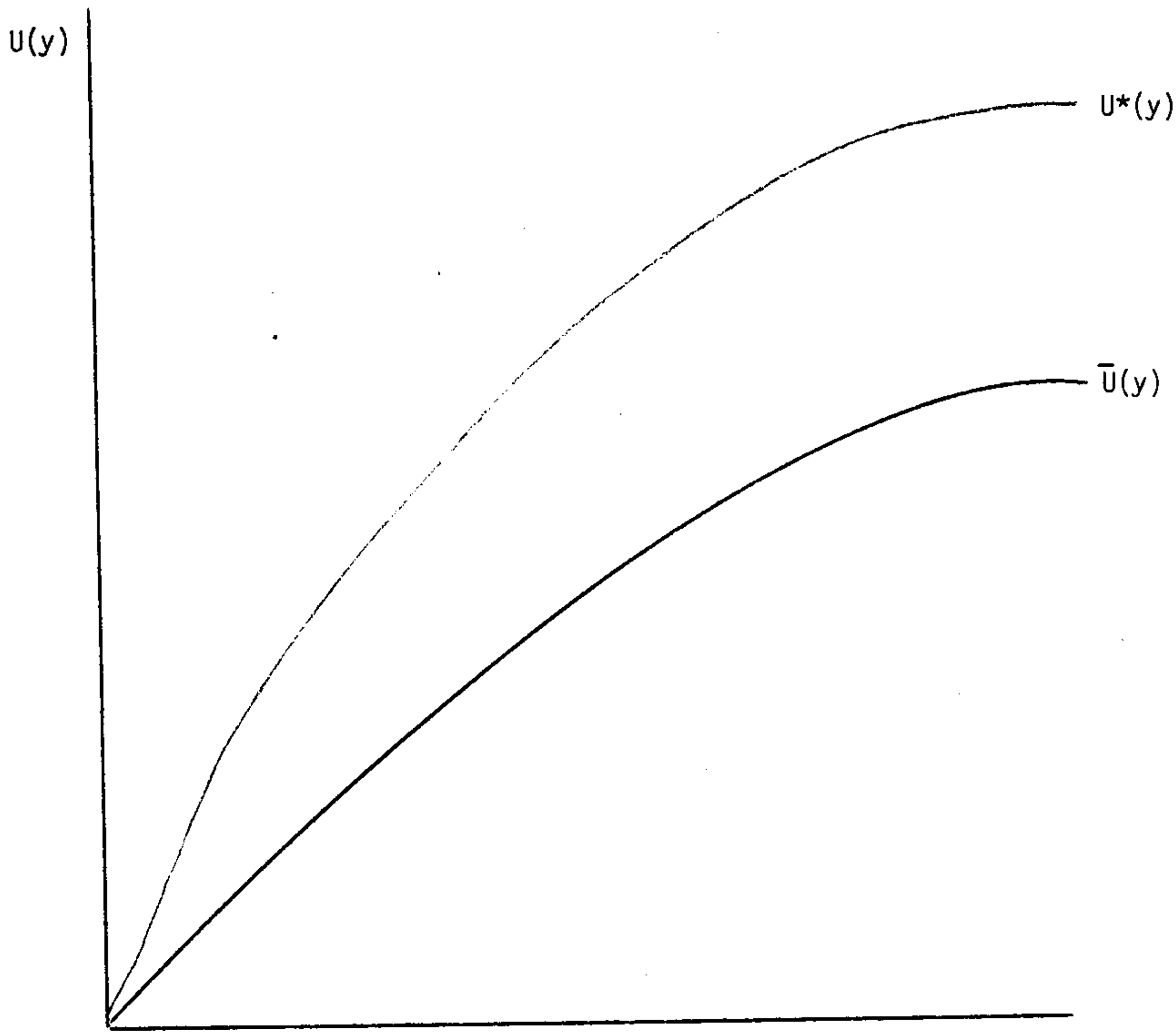


FIGURE 3. Interval Valued Utility Functions Valued over Outcomes y.

a single valued utility function, but by addressing the problem in this way we should be able to improve the predictive and prescriptive power of our decision analysis.

## A New Methodology for Ordering Choices under Uncertainty

In this section, we introduce a technique developed by Jack Meyer, stochastic dominance with respect to a function, which can be used to solve decision problems in which neither the outcomes of a given action choice nor the utility function of the decision-maker is known with certainty. We begin our discussion, however, with a brief review of the concept of absolute risk aversion, which we relate to two well-known efficiency criteria: first and second degree stochastic dominance.

It should be recalled that the scale and origin of von Neumann-Morgenstern utility functions are arbitrary; i.e., utility function U (X) would order action choices in the same way as utility function $U^*(X) = a + b\,U(X)$, where a and b are constants and $b > 0$. Therefore, the interval described by upper and lower confidence bounds for an empirically derived utility function are not unique, a fact which further complicates our decision problem. Arrow and Pratt, in separate articles show, however, that by dividing the second derivative of the utility function by the first derivative, a ratio is obtained which does uniquely represent preferences. This ratio, the absolute risk aversion function or $R_A(X)$, is defined in equation 1:

$$(1)\quad R_A(X) = -U''(X)/U'(X)$$

Since $R_A(X)$ is derived from a utility function, it is defined over all values of wealth for which the utility function is defined and twice differentiable. Hence, nearly all utility functions can be represented by absolute risk aversion functions. For example, the log utility function has an absolute risk aversion function which is equal to:

$$(2)\quad R_A(X) = 1/X \qquad \text{where } X \neq 0$$

For the negative exponential utility function, $1 - e^{-\lambda X}$ the coefficient of absolute risk aversion is:

$$(3)\quad R_A(X) = \lambda$$

Plotted in Figure 4, $R_A(X)$ functions derived from these utility functions are single level in the space of all possible absolute risk aversion values. As such, they identify a class of investors. Most efficiency criteria also identify a class of investors by their risk aversion coefficients, but such classes are generally much less restrictive than those associated with a single utility function.

Consider how some popular efficiency criteria identify classes of decision-makers by values of their absolute risk aversion function. First degree stochastic (FSD) dominance over action choices, based on cumulative density functions of action choices (Hadar and Russell; Hanock and Levy), applies for all decision-makers who prefer more to less; i.e., those for whom marginal utility is positive ($U' > 0$). Hence, the value on $R_A(X)$ is unrestricted. Second degree stochastic dominance (SSD) is more restrictive than FSD requiring not only positive marginal utility ($U' > 0$), but also risk aversion ($U'' < 0$). As a result, the class of agents for which SSD applies is the class with positive values of $R_A(X)$ for all values of X. This, incidentally, excludes decision-makers who possess Friedman-Savage type utility functions whose value for $R_A(X)$ initially is negative.

A natural extension of stochastic dominance would be something in between a single line representing a utility function, and stochastic dominance which

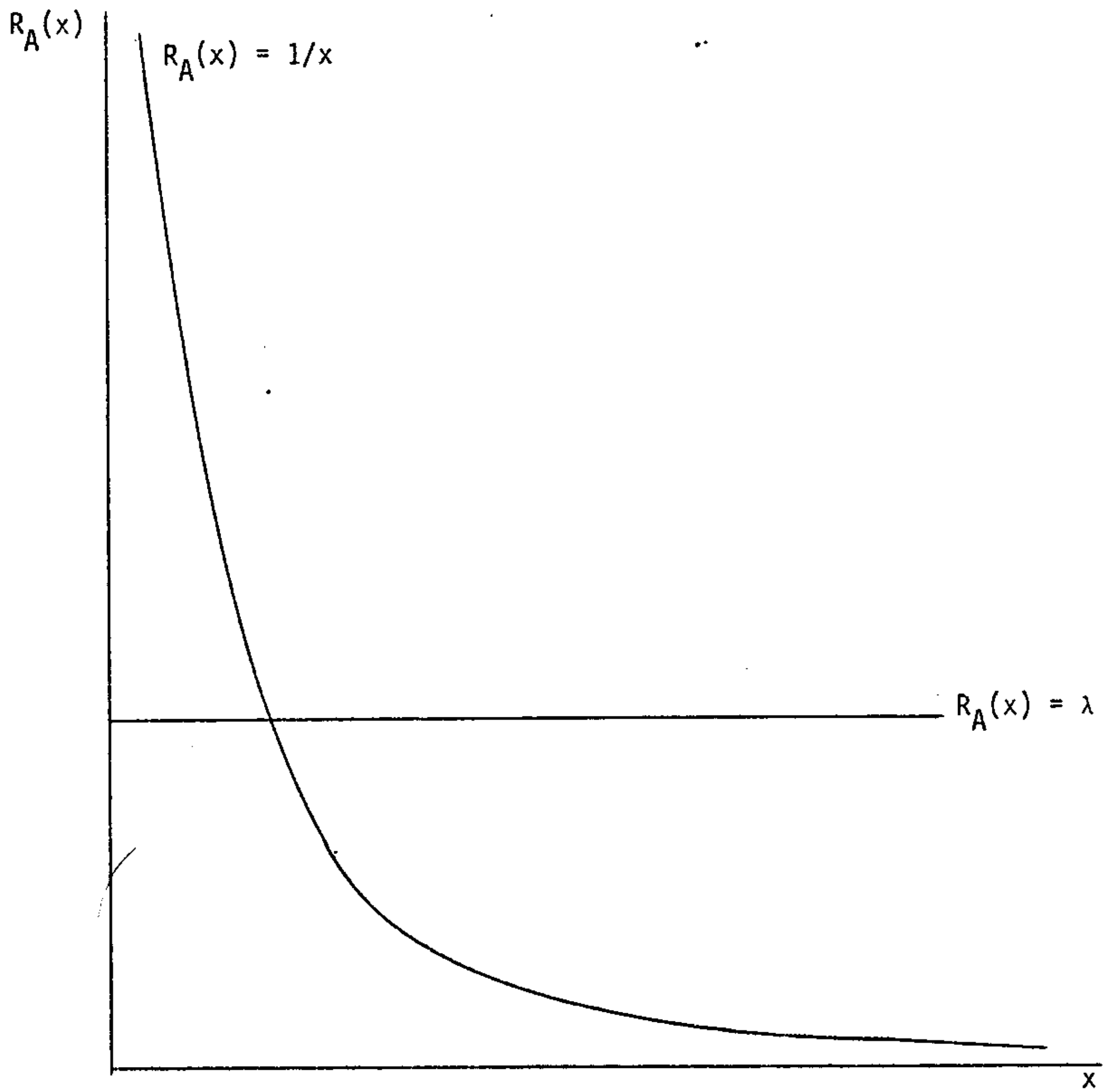


FIGURE 4. Mappings of Utility Functions into $R_A(x)$ Space for Log and Negative Exponential Utility Functions.

holds for the whole space or positive half-space of values of $R_A(X)$. This is the alternative supplied by Meyer. He provided a methodology for ordering action choices for a class of agents bounded by an upper and lower absolute risk aversion function. That is, he has developed an efficiency criterion for the class of agents whose risk aversion measures are bounded by:

(4) $R_A(X)_L < R_A(X) < R_A(X)_u$

Graphically, this might include the class described in Figure 5.

Like first and second degree stochastic dominance criteria, Meyer's criterion involves a comparison of cumulative density functions of outcomes associated with particular action choices. It provides an ordering which is consistent with the postulates of expected utility theory for all distributions. Unfortunately, closed form solutions for a generalized decision problem involving a confidence interval in the utility function are not possible unless the risk aversion function is unbounded from above or below. Otherwise, the procedure requires that the problem be solved using optimal control techniques. These are described by Meyer and will not be reviewed here. What we offer is an example of how stochastic dominance with respect to a function can be used to solve a practical decision problem.

## A Practical Application of Stochastic Dominance with Respect to a Function

In this section, we present results based on an application of Meyer's criterion to a problem involving the selection of an investment portfolio. These results are of particular interest for several reasons. First, they demonstrate that stochastic dominance with respect to a function is, in fact, a powerful tool for ordering action choices under risk. Second, they provide rather dramatic evidence of some of the shortcomings of EV portfolio analysis.

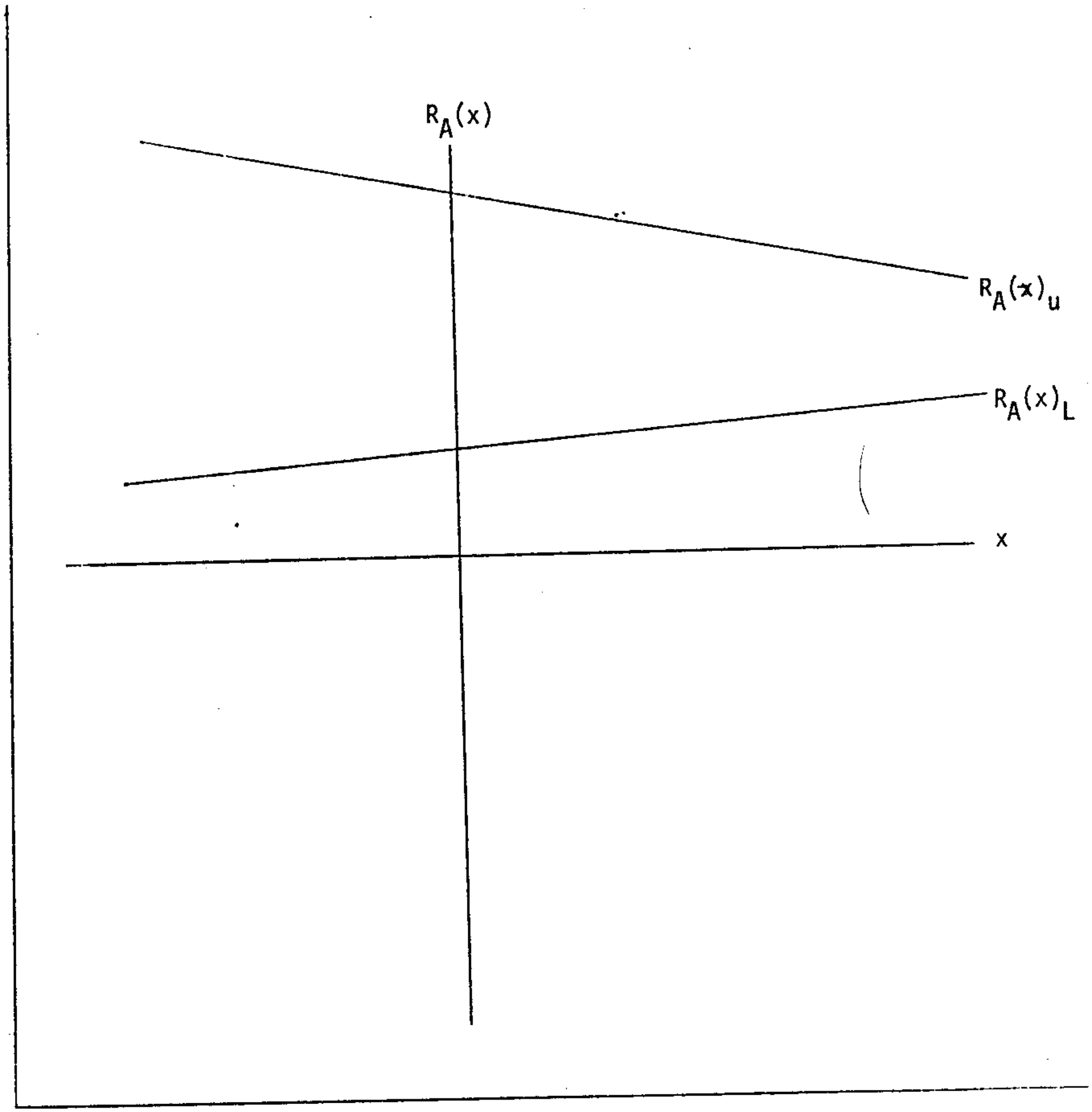


FIGURE 5. A Class of Agents Whose Risk Aversion Coefficients Bounded by $R_A(x)_u$ and $R_A(x)_L$.

Finally, they serve as a starting point for a discussion of a still more powerful approach to the problem of prescribing action choices under risk.

In our example a utility maximizing investor must choose a portfolio comprised of two assets, one of which has a stochastic return and is, therefore, risky. The investor has initial holdings of each asset valued at $5,000 and cash holdings of $250. He can acquire additional holdings of either asset using his cash holdings or borrowed funds. Additional holdings of one asset can also be purchased with funds obtained by liquidating some or all of the initial holdings of the other asset. Both assets are lumpy; they can be bought and sold only in $50 units. Both are also somewhat illiquid, since a fee proportional to their value is charged when they are sold. There is a ceiling on the amount of credit obtainable, and a lower bound in the size of loans. The cost of borrowing increases along a step function as the amount of credit used increases. A final factor which affects the investor's choice is the demand for cash balances, which is also stochastic. The cost of adjusting to a new portfolio plus the demand for cash balances must not exceed the liquid wealth of the investor, which is defined as the sum of current cash balances, borrowed funds, and revenue from asset liquidation.

More formally stated, our problem is:

(5) $\text{Max } E\,[U(y)]$

s.t.

$$X_1 + X_2 \leq DW\text{-}CD + (1-C_3)\,X_3 + (1-C_4)\,X_4 + X_5$$

$$X_5 \leq CL$$

$$X_1X_3 = 0,\ X_2X_4 = 0$$

$$X_1, X_3, X_4 = 0,\ 50,\ 100,\ 150, \ldots 5{,}000$$

$$X_2 = 0,\ 50,\ 100,\ 150, \ldots 4{,}000$$

$$X_5 = 0,\ 100,\ 101,\ 102, \ldots 7{,}500$$

where

$y$ = change in net worth

$X_1$ = amount of asset 1 acquired

$X_2$ = amount of asset 2 acquired

$X_3$ = amount of asset 1 liquidated

$X_4$ = amount of asset 2 liquidated

$X_5$ = amount of funds borrowed

DW = cash available for investment = \$250

CD = desired cash reserve for meeting cash demands = \$668.00[1]

CL = limitation on credit = \$7,500

$C_3$ = fee for liquidating asset 1 = 7%

$C_4$ = fee for liquidating asset 2 = 3%

The first constraint requires that liquid wealth be sufficient to meet the cost of adjustment to the new portfolio. The second limits credit. The two multiplicative equality constraints ensure that the prescribed action choice does not call for both the acquisition and liquidation of the same asset. The remaining constraints define permissible levels for each activity. The change in net worth, y, associated with a particular action choice is represented by the following expression:

$$(6) \quad y = r_1 (X_{10} + X_1 - X_3) + r_2 (X_{20} + X_2 - X_4) + (1-C_3) X_3 + (1-C_4) X_4 - R(X_5) - CEX$$

where

$X_{10}$ = initial holdings of asset 1 = \$5,000

$X_{20}$ = initial holdings of asset 2 = \$5,000

$r_1$ = rate of return on asset 1 $\sim N(.05,.0016)$

$r_2$ = rate of return on asset 2 = .025

CEX = cash expenditures $\Gamma$ (4,100)

$R(X_5)$= interest charge

$$R(X_5) = \begin{cases} .02X_5\,, & X_5 \le \$3{,}000 \\ 60 + .025(X_5 - 3{,}000); & \$3{,}000 < X_5 \le \$5{,}000 \\ 110 + .03(X_5 - 5{,}000); & \$5{,}000 < X_5 \le \$7{,}500 \end{cases}$$

and other variables are defined as above.

Application of the Meyer criterion involves the binary comparison of distribution of outcomes associated with different action choices. For our purposes, we chose to compare distributions associated with five portfolios which are efficient according to the commonly used E-V criterion. The problem specified above, though relatively small in dimension, is rather complex and is not suited for optimization by quadratic programming -- the technique usually employed to identify E-V efficient portfolios. Using a revised version of a Monte Carlo programming algorithm developed by Donaldson and Webster, however, we were able to overcome this difficulty.[2]

Monte Carlo programming is a random search procedure. Feasible portfolios are generated sequentially in a completely random fashion. The outcome of each new portfolio, measured in terms of some single valued objective function, is compared to the outcome associated with the previous best portfolio. If a sufficiently large number of portfolios is examined, one which is optimal or nearly optimal, should be identified.[3]

To generate a set of E-V efficient portfolios, we specified the following quadratic objective function:

$$(7)\quad Z = E(y) - \frac{\lambda}{2}\,Var(y)$$
$$= E(r_1)\,(X_{10} + X_1 - X_3) + r_2\,(X_{20} + X_2 - X_4)$$
$$+ (1-C_3)\,X_3 + (1-C_4)\,X_4 - R(X_5) - E(CEX)$$
$$- \frac{\lambda}{2}\,[(X_{10} + X_1 - X_3)^2\,Var(r_1) + Var(CEX)]$$

Using Monte Carlo programming to maximize this objective function for several values of $\lambda$, portfolios at different points along the E-V frontier were identified. Activity levels for the five E-V efficient plans used in the analysis below are given in Table 1.

Distributions of outcomes associated with these plans were generated in the following manner. Two series of ten random numbers, the first having a normal distribution with mean .05 and variance .0016 and the second having a gamma distribution with mean 400 and variance 40,000, were generated independently using techniques discussed in Manetsch and Park. By substituting a value from the first series for $r_1$ and a value from the second for CEX into equation 6, a sample outcome for a given plan was computed. In this way ten sample outcomes for each plan were generated. The means and standard deviation for the sample distributions are also given in Table 1.

The five plans were then ordered for 11 groups of investors using the Meyer criterion. The first group includes investors who are risk neutral as well as some who are mildly risk loving and mildly risk averse. The 11th group includes investors who are highly risk averse. Before discussing the results themselves, it should be noted that the coefficients of absolute risk aversion which are used to define groups of investors correspond exactly to the parameter $\lambda$ of the negative exponential utility function used to determine E-V efficient portfolios (Freund). If the E-V criterion orders action choices in a manner consistent with the postulates of expected utility, then we should find that the optimal portfolio for a given value of $\lambda$ is not dominated for the class of investors including those with absolute risk aversion coefficients equal to that value. Because we have no reason to believe that the distribution of outcomes will be normal, however, this expectation may not be well founded. It has been demonstrated that the E-V criterion be consistent with the postulates of expected

Table 1. E-V Efficient Portfolios Derived Using Monte Carlo Programming Methods

| Portfolio | Activity Level $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | Expected Change in Net Worth | Standard Deviation of Change in Net Worth | $\lambda$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 5000 | 0 | 0 | 250 | 5175 | 229.46 | 462.14 | 0 |
| 2 | 4350 | 0 | 0 | 0 | 4768 | 209.57 | 437.94 | .001 |
| 3 | 2950 | 1200 | 0 | 0 | 4568 | 148.31 | 386.50 | .002 |
| 4 | 0 | 3200 | 0 | 0 | 3618 | 19.23 | 284.22 | .01 |
| 5 | 0 | 4000 | 4300 | 0 | 419 | -490.35 | 174.76 | .09 |

utility theory only when outcomes are normally distributed (Samuelson) or when the utility function is quadratic (Tobin).

The set of portfolios which is efficient under the Meyer criterion is presented in Table 2 for each of the 11 classes of investors. Portfolio 1 dominates all others for the first class of investors. This is consistent with the results of the E-V analysis, since under that criterion we expect the first portfolio to be preferred to all others by risk neutral investors. Portfolios 1 and 2 are efficient for the second class of investors. This contradicts the results of our E-V analysis, since it indicates that either portfolio 2 or 3 should be preferred by each investor in the class and that both of these portfolios should dominate portfolio 1. Portfolio 2 is efficient for classes 3 and 4, and portfolios 2 and 3 are efficient for class 5. Because we have not considered E-V efficient portfolios associated with risk aversion coefficients within the ranges defining these classes of investors, we cannot compare these results with those of the E-V analysis. Portfolio 3 dominates all others for investor classes 6 through 11. This clearly contradicts the results of the E-V analysis, since we would expect either portfolio 4 or portfolio 5 to be efficient for at least some investors in each of these classes. Closer inspection of the simple distribution of outcomes for portfolios 3, 4, and 5 indicates that the former dominates each of the latter two by first degree stochastic dominance -- i.e., that all investors who prefer more to less would prefer portfolio 3 to either of the other two. Clearly the E-V criterion is not a valid one for ordering the action choices facing these classes of investors in this problem. Its failure is almost certainly attributable to the non-normality of the distribution of outcomes associated with the different portfolios.

On the other hand, the Meyer criterion has given us an ordering of the five portfolios which is consistent with the postulates of expected utility

Table 2 .

Efficient Portfolios for Classes of Decision Makers Identified by Ranges on the Value of Absolute Risk Aversion Coefficients

| | Range of Risk Aversion Coefficients | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) -.001 to .001 | (2) .001 to .003 | (3) .003 to .005 | (4) .005 to .007 | (5) .007 to .009 | (6) .009 to .011 | (7) .011 to .013 | (8) .013 to .015 | (9) .015 to .017 | (10) .017 to .019 | (11) .019 to .500 |
| Non-Dominated Portfolios Efficient Portfolios | 1 | 1<br>2 | 2 | 2 | 2<br>3 | 3 | 3 | 3 | 3 | 3 | 3 |

for all distributions. Furthermore, it has enabled us to make that ordering without specifying a utility function or an exact value of the decision-maker's risk aversion coefficient. All that is needed to order risky action choices is a range -- a confidence interval -- within which we are reasonably certain that the decision-maker's risk aversion coefficient falls. We need not, then, focus our empirical efforts on the identification of a functional form for different investor's utility function and on the estimation of the parameters of these functions. Rather, we need to develop better empirical tools for the estimation of absolute risk aversion coefficients for different classes of individuals.

The Meyer criterion is not without shortcomings, however. If the range of absolute risk aversion coefficients defining a particular class of actors is too large, it may not provide a complete ordering of action choices. We see this problem in our results above for investor classes 2 and 5. A more serious difficulty arises because the Meyer criterion operates by making binary comparisons. For most problems the number of action choices is large and pair-wise comparison of each choice to all others may not be possible. We can reduce the set of action choices through the application of some other criterion -- such as the E-V criterion -- but we have no assurance that such a rule will not eliminate the best action choice.

Little can be done to resolve the first problem mentioned above. It can be corrected only by reducing the range of risk aversion coefficients defining each class of investors. We believe we can suggest at least a partial solution to the second and more important problem, however. We propose to incorporate the Meyer criterion into a Monte Carlo programming model <u>as the evaluative criterion</u> -- i.e., a subroutine which applies the Meyer criterion will replace the objective function as a means for determining whether a new portfolio generated at random is preferred to the previous best portfolio. Since the Meyer criterion can be shown to be fully transitive in its ordering of action choices, this technique can be used to find a set of efficient portfolios which, as the number of portfolios examined increases, should include a portfolio approaching the action choice which maximizes expected utility for each member of a particular class of investors.

## Conclusion

In this paper we have compared our current interest in utility functions to our earlier interest in production functions. Neither type of function can be estimated without error because of the number of excluded variables not held constant. Earlier we ignored the error term associated with production functions, treating production levels as single valued outcomes defined over variable input levels. Later we accounted for the stochastic nature of production in our decision models by introducing single valued utility functions with non-constant marginal utilities of wealth. The Meyer criterion, stochastic dominance with respect to a function, allows us to refine further our analysis of decision-making under uncertainty. It permits us to relax the earlier requirement of a single valued utility function, replacing it with well-defined upper and lower bounds on the absolute risk aversion function, $R_A(X)$. In effect, it permits us to solve decision problems for well-defined classes of investors. Defining investor classes using the $R_A(X)$ measure, as noted above, has already been used to identify classes of decision-makers who act in accordance with the well-known stochastic dominance efficiency criteria. Meyer's criterion allows us to narrow the bounds on those classes and is therefore a more discriminating decision tool.

Our empirical results demonstrate that stochastic dominance with respect to a function can be applied in realistic decision situations. Furthermore, they demonstrate that some very commonly used techniques such as E-V analysis may not be very effective when the assumption of normally distributed outcomes cannot be made.

Applying the Meyer criterion introduces the new problem of how to place bounds in the absolute risk aversion function. One method for doing this might involve the determination of confidence limits in the parameters of empirically estimated utility functions which are also parameters of the absolute risk aversion functions. Alternatively, simple values of the absolute risk aversion function could be obtained using an approximation formula derived by Pratt. If reasonable bounds in $R_A(X)$ can be found, thereby explicitly recognizing the fact that utility functions cannot be known with certainty, we will probably have a more effective analytical tool for both predicting and prescribing preferred action choices.

The above results have some implication for the question being considered by the W-149 committee. Is it worthwhile to estimate utility functions? We are concerned that a single valued utility function may not be an accurate reflection of investor preferences because of important variables which may affect preferences and which are not held constant. Perhaps what is really needed is some way to identify decision-makers by more flexible measures, such as bounds on $R_A(X)$ as Meyer's criterion allows.

Footnotes

[1]Cash demand was assumed to have a gamma distribution with mean 400 and variance 40,000. By setting CD at $668.00, we ensured that the probability of cash demand exceeding cash reserve is only .10.

[2]For other applications of Monte Carlo programming, see Anderson, Dent and Byrne, Dent and Thompson, and Carlson, <u>et. al</u>.

[3]For a problem the size of that considered here, inspection of 1,000 plans was considered to be adequate.

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