Damaged Goods: A resource depletion model of addictive consumption

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Abstract
Economic research on the consumption of harmful goods focuses principally on the addictive nature of consumption rather than its impacts on health, despite medical research showing that consumers primarily consider health effects when making decisions about addictive behaviour. In this paper, the standard rational addiction model is recast in terms of a resource depletion problem, where the resource in question is a depletable stock of health, and the time horizon is finite. Analysis of the prototype health depletion model finds two types of consumption path, one that is compatible with the results from rational addiction and one that is not. Several extensions to the prototype model are explored.

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I. Introduction

Not all goods are good. There are many consumption items that provide the user a degree of pleasure but at some cost to the consumer’s health. The examples of alcohol and cigarettes cause substantial harm to their consumers (Baldacchino 2002; Tetley 2002) as well as imposing large costs on the economy as a whole. An Australian Government report estimated that the social cost to the economy of drug abuse in 2004-05 was $56 billion (Collins and Lapsley 2008b). Cigarette consumption contributed $31 billion to this total, and alcohol contributed $15 billion (Collins and Lapsley 2008a).

Considering this problem from an economic perspective, it is interesting to ask why consumers persist in consuming these goods despite their well documented deleterious health effects. It is clear that these goods have intertemporal effects since the consumption of the good harms the consumer’s health over time as well as the good’s addictive nature making it more attractive to the consumer in the future. Psychological and medical evidence suggests that consumers consider this problem from the perspective of damaging their health, rather than from awareness of their level of addiction. Consumers alter their behaviour based on feedback from their health, rather than their level of addiction. This suggests that the modeling approach should start from the basis of a consumer making optimal consumption decisions with reference to their health.

Previous attempts to model the consumption of harmfully addictive goods have focused on the level of addiction of the consumer. Over the past three decades, a set of models has been developed to explain these consumption decisions within the framework of rational self-interested behaviour. The cornerstone of this set of models is the rational addiction model, alluded to in the seminal paper ‘De gustibus non est disputandum’ (Stigler and Becker 1977) and codified in Gary Becker and Kevin Murphy’s paper ‘A theory of rational addiction’ (1988). In Becker and Murphy’s paper, the term rational addiction refers to a consumer choosing to consume the addictive good in the full knowledge of the good’s effects – the consumer consumes the good if it increases his utility.

This paper presents a model of optimal health depletion in which a consumer chooses an optimal path of consumption of a harmfully addictive good in order to maximise his lifetime utility. This optimisation decision is made with reference to a renewable stock of health.
Modelling the consumption of harmfully addictive goods in this way creates a problem with roots in natural resource economics, specifically the economics of renewable resources.

The first section of this paper will describe the connections between the rational addiction and resource depletion models. It will show how both models can yield similar results, but the resource depletion model focuses attention on several aspects of the consumption of harmfully addictive goods that are under-emphasised in the rational addiction model. In order to show how the models are similar, the key components of both will be unpacked and compared. A general model describing the consumption of harmfully addictive goods as a resource depletion problem will be outlined. To highlight the similarities between the models the results from a simplified version of the resource depletion model will be discussed and compared to known results from the rational addiction model before presenting possible further directions for, and extensions to, the resource depletion model.

II. Linking rational addiction to resource depletion

The rational addiction model has all the basic components of a dynamic consumption model: utility maximising behaviour through time; stable preferences; time-consistent discounting of the future; and an intertemporal budget constraint that allows for borrowing and lending (refer to Lambert (1985), p.168, for a simple example). Unlike a standard dynamic consumption model though, the rational addiction model has stable but intertemporally-dependent preferences; that is, the consumption of a unit of the good now will affect the utility derived from consuming an identical unit of the good in the future (Ryder and Heal 1973). The preferences are stable in the sense that they do not change capriciously, rather they adjust through the consumption, or abstinence from consumption, of the harmfully addictive good.

The key feature of the rational addiction model is the existence of intertemporally linked preferences, which allows the modeling of key features of addiction such as tolerance, difficulty of cessation, and withdrawal. The model posits the existence of a stock of ‘addiction capital’ as the mechanism by which consumption decisions affect future utilities. As the harmfully addictive substance is consumed, the consumer’s stock of addiction capital is built up. The size of the stock naturally decays and if no more of the substance is consumed then the stock will asymptotically decline to zero. The stock of addiction capital
affects the consumer’s utility in two ways. First, an increase in the stock of addictive capital decreases the consumer’s utility. Second, an increase in the stock of addiction capital increases the marginal utility of consuming another unit of the addictive good relative to non-addictive goods. The first feature represents the fact that the good is harmful, and the second effect is addiction, which means that past consumption raises current consumption.

The effects of the stock of addiction capital in the rational addiction model occur at the margin. There are no thresholds beyond which the qualitative effects of consumption on utility change substantially. Although in most analyses of rational addiction there are multiple steady states of consumption, these refer to the interaction between the stock of the addictive good and the consumption of that good. There is no level of addiction capital that leads to irreversible injury or death, for example. Thus it is implicitly assumed that there are no intrinsic bounds on the size of the stock of addiction capital, and that a consumer could, in theory at least, grow the stock of addictive capital to any arbitrarily large size. In practice, however, the consumer’s ability to accumulate addiction will be bounded by some form of budget constraint. Similarly, a consumer could theoretically cease consumption and revert back to a ‘clean’ state, as if the consumer had never used any of the harmful good at all. There are no thresholds or hysteresis.

Beneficial addictive goods can also be considered in the rational addiction model\(^1\). This paper will not consider beneficially addictive goods because they demonstrate no tension between utility now and disutility in the future.

**From rational addiction to a resource depletion model**

The implicit assumption of unbounded potential growth of addiction is challenged in this paper. It is clear that the consumption of harmfully addictive goods cannot continue indefinitely without bound because the consumption of these goods has potentially severe health consequences that will eventually prevent further consumption, either through severe illness or death. The consumption of cigarettes, for example, has been shown to lead to lung cancer and chronic obstructive pulmonary disease – a catch-all term for emphysema,

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\(^1\) These goods include activities such as art appreciation, learning an instrument or playing a sport (Stigler and Becker 1977). As a consumer engages more with these activities, the more proficient the consumer becomes, in turn raising their stock of addiction capital. In this case though, an increased stock of addiction capital increases the consumer’s utility directly and also increases the marginal utility of consuming another unit of the good. Beneficially addictive goods are usually healthy pursuits, so the stock of addictive capital need not be bounded.
bronchiolitis, and bronchitis – to name the two main consequences (Tetley 2002). Regular smoking results in a decrease of a regular smoker’s life expectancy by 10 years on average (Doll et al. 2004). The consumption of alcohol is linked to increased risk of accident, cirrhosis of the liver, heart disease and lung cancer (Baldacchino 2002) and a two year reduction in life expectancy (Mäkelä 1998).

Why do people become concerned about their addictions and decide to change their behaviour? Young et al. (2010) cite several studies that find that ‘concerns about health, persistent messages from family and friends, repeated advice from health professionals, and the cost of cigarettes are the reasons most often cited by smokers as catalysts for quitting smoking’. Furthermore, Vangeli and West (2008) find that just fewer than 65 per cent of attempts to quit smoking were triggered by health concerns. The remainder of the responses were dominated by cost concerns and peer pressure. No respondents cited concerns about their level of addiction as a trigger for attempting to quit.

An alternative framework with which to think about the consumption of harmfully addictive goods is found in the resource depletion literature.

The working hypothesis is that the addictive good starts to adversely affect the consumer’s health which provides an impetus for changes in behaviour. Conversely, people do not change their addictive behaviour simply because they are addicted, because the characteristic of addiction is to keep the person addicted. People will be motivated by thinking about the state of their health rather than their level of addiction. Thus changing the perspective of the problem from accumulating an unbounded stock of addiction to depleting a finite stock of health has the attractive property that it is more closely aligned to the experiences of the consumer.

A resource depletion problem entails finding the path of extraction of a finite resource that maximises the net present value of that extraction. This definition has two noteworthy components for the analysis of addiction: optimal consumption must be defined dynamically, and extraction of the resource cannot continue indefinitely. If some of the resource is extracted today, there will be less of the resource available for extraction in the
future. This property holds for both exhaustible and renewable resources\(^2\) (as defined by Sweeney (1993)), but not for expendable resources – resources that replenish so quickly that present extraction does not affect future extraction\(^3\).

An analogue can be drawn between a resource depletion problem and the consumption of a harmfully addictive good. Consider a consumer who has a stock of health – a broadly defined term where a full stock of health represents a consumer who is at their physical peak, and an empty stock of health represents death. This stock is clearly finite. As a consumer consumes harmful goods the stock of health is depleted. If a diminished health stock reduces the consumer’s utility, and an increase in the stock of health increases the marginal utility of consumption of the harmful good (a lower level of health makes consumption more attractive), then the resource depletion analogue begins to look very similar to the rational addiction model, except that the consumer is now drawing down a finite stock rather than accumulating an unbounded stock. This approach will hereafter be referred to as the health depletion model.

III. The health depletion and rational addiction models

This section shows how the health depletion model is a reformulation of the rational addiction model. The rational addiction model is provided as a reference point. The health depletion model is then presented and some of the main departures from the rational addiction model are discussed.

Figure 1.1 shows the structure of the rational addiction model and figure 1.2 shows the structure of the health depletion model. The conceptual difference between the two models is that the definition of the stock variable has changed. Although both stocks occupy the same location within the conceptual model, the signs of the derivatives associated with the stocks changes. The implications of this change are explored in section IV below.

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\(^2\) Because the stock of the resource is finite and current consumption lessens future consumption possibilities, there exists a set of consumption paths that lead to the stock being exhausted within the problem’s time horizon. Thus a natural lower bound exists on the evolution of the resource stock.

\(^3\) Expendable resources can satisfactorily be analysed as a static problem.
Outline of the rational addiction model

In the rational addiction model the consumer is maximizing the net present value of utility subject to the time path of the stock of addiction capital and an intertemporal budget constraint (for a more thorough account see Becker and Murphy (1988) and Clarke and Danilkina (2006)). The consumer’s goal is to

\[
\text{Maximise } U(0) = \int_0^T e^{-\alpha t} u(y(t), c(t), s(t)) dt \quad (1)
\]

subject to

\[
s(t) = c(t) - \delta s(t) - m[D(t)] \quad (2)
\]

and

\[
\int_0^T e^{-\rho t} \left[ y(t) + p_c(t) c(t) + p_d(t) D(t) \right] dt \leq A_0 + \int_0^T e^{-\rho t} w(s(t)) dt \quad (3)
\]

where

\[ U(0) \] is the present value of utility at some initial time \( t=0 \);

\[ u(0) \] is instantaneous utility;
\( T \) is the length of life of the individual;
\( c(t) \) is the consumption of an addictive good at time \( t \);
\( y(t) \) is the consumption of some composite numeraire good at time \( t \);
\( s(t) \) is the stock of addiction capital or level of addiction at time \( t \);
\( \sigma \) is the constant rate of time preference;
\( \delta \) is the constant depreciation rate of the level of addiction;
\( D(t) \) is effort towards reducing the level of addictive dependence;
\( m \) is the effect of \( D(t) \) on the stock of addiction capital;
\( r \) is the constant interest rate;
\( p_c(t) \) is price of addictive consumption in terms of the numeraire good at \( t \);
\( p_d(t) \) is the price of effort reducing the degree of addictive dependence in terms of the numeraire good at time \( t \);
\( A_0 \) is the initial value of the consumer’s assets;
\( w(s(t)) \) equals the wages at time \( t \) as a function of the consumer’s degree of addiction.

The instantaneous utility function, \( u \), is assumed to be a strongly concave function of \( y, c \) and \( s \), which is increasing in both the consumption goods, \( y \) and \( c \), and decreasing in the stock of addiction capital, \( s \). Furthermore, \( u_{cs} \) is positive which implies that a greater stock of addiction capital increases the marginal utility of consumption of the addictive good – this feature creates the addictive effect of \( c(t) \). The earnings function, \( w(s(t)) \), is assumed concave and decreasing in \( s \) – addiction reduces earnings at a diminishing rate.

Given these restrictions, consumers in this model can exhibit a wide range of different behaviours. They can choose not to consume any addictive good at all, or decrease their consumption through time. Alternatively the consumption can approach a steady-state where the level of consumption remains constant and does not increase the stock of addiction capital (Clarke & Danilkina 2006). Restrictions are usually placed on the model so
that the steady-state corresponding to the largest stock of addiction capital is stable, thus preventing run-away growth in the stock of addiction capital.

Papers in the rational addiction literature tend to either assume an infinite planning horizon or do not address the consumer’s behaviour at the terminal time (Becker & Murphy (1988); Clarke (2000); Leonard (1989); Orphanides & Zervos (1995), (1998)). This helps improve the tractability of the analysis, but does so at the expense of realism, thus avoiding the problem of what happens to the consumer at the end of his life.

**Outline of the health depletion model**

The suggested optimisation problem for the health depletion model is similar to the set up for the rational addiction model. The differences are that the stock of addiction capital has been replaced with a finite, depletable stock of health. To reflect the uncertainty surrounding the consumer’s moment of death, a function indicating the probability of living beyond t has also been introduced.

The consumer wishes to

\[
\text{Maximise } U(0) = \int_0^T e^{-\sigma t} u(y(t), c(t), L(t)) \Phi(\psi(c(t), L(t), t)) dt
\]  

subject to

\[
\dot{L} = g(L(t), t) - h(L(t), c(t), t) + k(d(t))
\]
and

\[
\int_0^T e^{-\tau t} [y(t) + p_c(t)c(t) + p_d(t)d(t)] dt \leq A_0 + \int_0^T e^{-\tau t} w(L(t)) dt
\]

With \(c(t), y(t), d(t) \geq 0; 0 \leq L(t) \leq \bar{L}; L(0) = L_0\)

\(L(t)\) is the stock of health at time t;

\(\Phi(.)\) is the probability of living beyond t;

\(\psi(.)\) is a function that modifies the probability of dying at time t;

\(d(t)\) is effort towards increasing the stock of health;

\(g(.)\) is the intrinsic ability of the body to heal itself in the absence of exogenous shocks;
h(.) is the damage done to the body by consuming the addictive good;

k(.) is the effect of d(t) on the stock of addiction capital;

w(L(t)) equals the wages at time t as a function of the consumer’s stock of health;

L̅ is the maximum value the health stock can obtain – the consumer’s peak health.

All other variables retain their meaning from the rational addiction model.

**The uncertain lifespan**

An agent in this model knows that his lifespan will be finite, but he does not know, a priori, when the moment of his death will occur. An approach to this problem is suggested in Kamien and Schwarz (1991, p61-62) and expanded upon by Levy (2002). Let \( F(t) \) be the probability of the agent dying at time \( t \), \( F'(t) \) be the associated probability density function, and \( T \) be an upper bound on possible lifetime (a time by which point the agent is sure that he will be dead). The choice of this time is somewhat arbitrary with the only requirement being that \( F(T)=1 \). Then \( 1 - F(t) = \int_t^T F'(s) ds \) is the probability of living until at least \( t \). If the individual dies at time \( t \), the total lifetime utility will consist of the discounted stream of utility from the consumption path up to \( t \). Hence the individual’s problem is to:

\[
\text{maximise } \int_0^T F'(t) \left[ \int_0^t e^{-\sigma s} u(y(s), c(s), L(s)) \, ds \right] \, dt \tag{7}
\]

which can be rewritten more simply, using integration by parts, as:

\[
\int_0^T e^{-\sigma t} u(y(t), c(t), L(t)) \left[ 1 - F(t) \right] \, dt \tag{8}
\]

The probability that the individual lives until at least \( t \), that is \( 1 - F(t) \), is written as \( \Phi(t) \) in equation (4) for simplicity. The derivation of equation (8) from equation (7) is provided in the mathematical appendix to Levy (2002).

Levy (2002) extends this approach by using a probability function that includes a function of the stock of addictive capital as one of its arguments (Levy uses weight as the addictive capital). In Levy’s model, the probability of death at time \( t \) increases with the quadratic deviation of weight from the optimum weight. The inclusion of \( \psi \) in equation (4) represents the possibility of having the probability of death being not only a function of time, but also the control and state variables, \( \psi(t) = \psi(c(t), L(t), t) \). This approach is appropriate because
the consumption of goods such as alcohol and cigarettes have a proven detrimental impact on expected lifespan (Doll et al. 2004; Mäkelä 1998).

**The effect of the stock of health**

In a rational addiction model an increase in consumption of the harmfully addictive good increases the addiction stock which in turn decreases utility. In the health depletion model the consumption of the harmfully addictive good decreases the stock of health, which reduces the ability of the consumer to derive utility from consumption, both addictive and composite.

As the consumer consumes the addictive good health decreases from its maximum and declines toward zero. This description is in terms of effects on utility with respect to decreases in health, but in order to discuss constraints on the derivatives of utility with respect to health the reverse case needs to be considered, i.e. the effects of increases in health on utility. A marginal increase in health increases the instantaneous utility of consumption\(^4\), \(u_L > 0\). The stock of health is constrained to lie between zero and some maximum (peak health), i.e. \(0 < L < \bar{L}\). Without loss of generality, the maximum stock, \(\bar{L}\), can be normalised to 1. If the consumer drives his stock of health to zero, he will have no capacity to obtain utility, thus it is assumed that \(u(0, c) = 0\).

Furthermore, it is expected that the agent’s life ends when the stock of health falls to zero. If the consumer has no health then death is the natural consequence. This effect needs to be incorporated into the model. A possible approach is to include the stock of organ capacity into the probability of living beyond \(t\), \(\Phi(\psi(.)\), such that \(\psi(.) = \psi(L(t))\). This implies that \(\Phi_L(t) > 0\), and when \(L(t) = 0\); \(\Phi(t) = 0\).

**The equation of motion for the stock of health**

If damaged, the human body is usually able to heal itself, at least to some extent, in the absence of further damage. The equation of motion for organ ability is thus divided into three sections: the intrinsic ability of the body to heal itself; the detrimental effect of consuming the harmfully addictive good; and the effect of external activities to promote healing.

\(^4\) The utility function is assumed to be additively inseparable in consumption and health.
Because \( L \) is constrained between 0 and 1, the intrinsic ability of the organ to heal itself must approach a steady state at \( L = 1 \). The simplest way of modelling this growth is with the logistic growth function. Thus \( g(L(t), t)) = L(1 - L) \), where \( g(.) \) is dimensionless. The use of a logistic growth function is relatively common in bioeconomics (Wilén 1985).

To motivate discussion of the effects of a harmfully addictive good on the health of the consumer the example of alcohol will be examined. Alcohol represents the case where consumption of the harmfully addictive good causes harm in a smooth and continuous manner. The discussion of a discontinuous case, represented by cigarette addiction, is postponed to section VI.

**Modelling alcohol addiction**

The consumption of alcohol damages ‘nearly every organ and system of the body’ (Baldacchino 2002, p.19). According to Maher (1997, p.5) ‘[t]he liver is particularly susceptible to alcohol-related injury because it is the primary site of alcohol metabolism’. The model will therefore be developed based on the effects of alcohol on the health of the liver.

The working assumptions for the effect of consumption of alcohol on the health of the liver are:

- The consumption of alcohol at any moment will reduce the growth of the stock of liver capacity
- If no alcohol is being consumed, no harm is being done to the liver and the liver will regenerate itself
- The stock of liver capacity will decrease if the harm being caused by the consumption of alcohol is greater than the body’s ability to regenerate the liver
- Consuming a large amount of alcohol does proportionally more harm than consuming a small quantity at a given moment
- As the consumer’s health declines, the harm done by the consumption of alcohol is proportionally greater, i.e. people with healthy livers are better able to process alcohol than people with unhealthy livers (Diehl 1993)

Recall that the consumption of alcohol affects the growth of the stock of liver health through the harm function, and a positive value for the harm function represents a decrease
in the value of $L$. The assumptions above suggest, respectively, the following restrictions to the form of the harm equation:

- $h_c > 0$
- $h(L, 0) = 0$
- if $h(L, c) > g(L)$ and $k(d(t)) = 0$ then $\dot{L} < 0$
- $h_{cc} > 0$
- $h_L < 0$

The simplified model that will be compared to the rational addiction model in section IV uses this set of assumptions.

**IV. Simplified Health Depletion Model**

The health depletion model is simplified in order to enable a basic level of analysis. It still retains the characteristics identified in figure 1.2. It still captures the most important characteristics of a resource depletion model as well as paralleling the core components of the rational addiction model as described in figure 1.1. The theory of optimal control is used to generate the results.

**Background on optimal control**

The goal of dynamic optimisation in this context is to find the time path of consumption that maximises the consumer’s life-time utility (Weber 2005). The modern approach to solving dynamic optimisation problems of this nature is to use optimal control theory, which is a refinement of the classical calculus of variations (Dorfman 1969). In an optimal control problem, there is a set of variables that describe the state of the system, $s(t)$, for instance: the level of capital in an economy; the amount of ore in a mine; and for the problems discussed in this paper, the stock of health and the stock of addiction capital. The evolution of these state variables may depend on the value of the state variable, time and other variables that are under the control of the consumer. Once the values of the control variables, $c(t)$, are defined over the horizon of the problem, values of the state variables will also be defined subject to suitable boundary conditions. Thus, the consumer can choose time paths for the control variables that maximise the consumer’s life-time utility.
The key result from optimal control theory is the Pontryagin maximum principle\(^5\) which provides the conditions necessary for an optimal solution to a control problem. The maximum principle uses an equation called the Hamiltonian defined as:

\[
\mathcal{H}(c(t), s(t), \lambda(t), t) = u(c(t), s(t), t) + \lambda(t) f(c(t), s(t), t)
\]

Where \(u(\cdot)\) is the instantaneous utility at time \(t\), \(f(\cdot)\) is the time derivative of the state variable, and \(\lambda(t)\) is the costate variable, which has a similar interpretation to the Lagrange multiplier in static optimisation – economically it is interpreted as the shadow price of the stock at time \(t\). The maximum principle requires that \(c(t)\) maximises the Hamiltonian, \(\frac{\partial \mathcal{H}(t)}{\partial c(t)} = 0\), with the state and costate variables satisfying a pair of differential equations,

\[
\dot{s}(t) = \frac{\partial \mathcal{H}(t)}{\partial \lambda(t)} \quad \text{and} \quad \dot{\lambda}(t) = -\frac{\partial \mathcal{H}(t)}{\partial s(t)}.
\]

If the utility and state equations are non-linear then finding an explicit analytical solution becomes difficult, if not impossible. In this case it is necessary to use qualitative techniques to describe the intuition behind the solution\(^6\). This is the approach used for analyzing the models in this paper.

**Assumptions behind the model**

In order to make the analysis more tractable, several simplifications have been imposed on the health depletion model described above. These simplifications are described below.

1. The composite consumption good has been omitted from them model. The model only has one source of utility, namely, the utility derived from the consumption of the addictive good. Essentially, this assumes that the addictive good and the composites are not substitutable so their consumption decisions can be made independently. Also it assumes that the wealth effects of consuming the addictive good are so small that they can be neglected.

2. The budget constraint has been omitted. This assumption follows from the assumption that there are negligible wealth effects. Thus the consumer can consume as much of

\(^5\) The theory of optimal control was developed by L. S. Pontryagin and his colleagues in the Soviet Union, as well by Robert Bellman in the United States (Dorfman 1969).

\(^6\) For an accessible introduction to optimal control see (Weber 2005). For a thorough, accessible textbook on the subject see (Kamien and Schwartz 1991) or (Leonard and Van Long 1992). For a particularly rigourous treatment see (Seierstad and Sydsæter 1987).
the addictive good as desired without reducing his consumption of other goods. Furthermore the price of the good has been omitted. If the price of the good is constant across the planning horizon, the interpretation of the utility is as net utility. The marginal utility of consumption is reduced by a constant amount. These two assumptions impose substantial limits on the usefulness of the model since price effects cannot be analysed. An extended version of the model that incorporated price and budget effects would need to be analysed before normative predictions regarding policy could be made.

3. The consumer is now assumed to know terminal time with certainty. Furthermore the decisions of the consumer will affect neither the terminal time, nor the probability of dying before the terminal time. This is quite a restrictive assumption for the reasons discussed earlier. The differences in the solutions between a certain and an uncertain terminal time are discussed in the further direction section.

4. The expenditure on effort to heal the body has been removed from the simplified model. Growth in the stock of health can only occur endogenously. The assumption is limiting since the consumer may likely engage in activities such as rehab or detoxification when their health is low and recovery activities, such as exercise or eating healthily, to offset consumption when their health is high.

5. The equation of motion for the stock of health is assumed to be autonomous. This substantially simplifies the analysis, as well as allowing the possibility of phase plane analysis. To prevent the stock of health growing indefinitely if no addictive good is consumed, \( g(L) \) is assumed to produce bounded growth in \( L \). For simplicity logistic growth is assumed, i.e. \( g(L) = rL(K - L) \).

6. The time dependencies of the variables have been removed to visually simplify the analysis. This has only been done when it causes no ambiguity. Following the discussion in the modelling alcohol section the following assumptions are employed in the simplified model: \( u_c > 0; \ u_{cc} < 0; \ u_L > 0; \ h_c > 0; \ h_{cc} > 0; \ h_L < 0; \) and \( g_L \) depends on the value of \( L \). When \( L < \frac{K}{2} \) \( g_L \) will be positive. When \( L > \frac{K}{2} \) \( g_L \) is negative. This result depends on the assumption of logistic growth.
7. Finally, it is assumed that the utility function and the harm function are additively inseparable. Utility cannot be derived from consumption if the consumer has no health. The consumer does not value health directly; rather he values health for its capacity to increase utility from consumption. Similarly for harm, the consumer will not be harmed if he is not consuming any addictive good.

**Simplified model**

The consumer’s problem in the simplified model is to

\[
\text{maximise } \int_0^T e^{-\rho t} u(c, L) \, dt \tag{9}
\]

subject to

\[
\dot{L} = g(L) - h(L, c) \tag{10}
\]

and \(c(t), L(t) > 0; L(0) = L_0\)

The current valued Hamiltonian associated with (9)-(10) is given by:

\[
\mathcal{H}(c, L, \lambda) = u(c, L) + \lambda(g(L) - h(L, c)) \tag{11}
\]

with transversality condition \(e^{-\rho T} \lambda(T)L(T) = 0\)

Note that a current value Hamiltonian is defined with reference to value at time \(t\). In particular \(\lambda\) represents the future value of the stock at time \(t\) valued at time \(t\). The current valued Hamiltonian is related to a present valued Hamiltonian by \(\mathcal{H}_{\text{present}}(c, L, \pi) = e^{-\rho t}\mathcal{H}_{\text{current}}(c, L, \lambda)\), with \(\lambda = e^{\rho t}\pi\) (see Leonard & Van Long (1992), p149).

Taking the first order conditions yields:

\[
\frac{\partial \mathcal{H}}{\partial c} = u_c - \lambda h_c = 0 \tag{12}
\]

\[
\frac{\partial \mathcal{H}}{\partial \lambda} = g(L) - h(L, c) = \dot{L} \tag{13}
\]

\[
-\frac{\partial \mathcal{H}}{\partial L} = -[u_L + \lambda(g_L - h_L)] + \rho \lambda = \dot{\lambda} \tag{14}
\]

To conduct qualitative analysis it is important to find an expression for \(\dot{c}\), the time derivative of consumption. Finding this provides information about the slope of the consumption path, and thus the effect of the parameters on consumption.

Rearranging equation (12) yields
\( \lambda = \frac{u_c}{h_c} \) \hspace{1cm} \text{(15)}

Taking the time derivative of (15) gives another expression for \( \dot{\lambda} \)

\[ \dot{\lambda} = \frac{d}{dc}\left( \frac{u_c}{h_c} \right) \cdot \dot{c} \] \hspace{1cm} \text{(16)}

The expression in equation (16), \( \frac{d}{dc}\left( \frac{u_c}{h_c} \right) \), can be rewritten as \( \frac{h_c u_{cc} - u_c h_{cc}}{h_c^2} \) using the quotient rule. Using the assumptions enumerated earlier, it can be shown that this expression is always negative, as follows

\[ \frac{h_c [+] u_{cc} [-] - u_c [+] h_{cc} [+] }{h_c^2 [+] } < 0 \]

Thus equation (16) shows that if the value of health is increasing then the rate of consumption must be decreasing. This result is compatible with standard economic analysis, where if the full price of consumption (marginal cost and user cost) increases, then consumption will decrease.

Equating (14) and (16) and substituting in (15) gives the expression for the time derivative of consumption

\[ \dot{c} = \frac{1}{\frac{d}{dc}\left( \frac{u_c}{h_c} \right)}\left[ \frac{u_c}{h_c} (\rho - [g_L - h_L]) - u_L \right] \] \hspace{1cm} \text{(17)}

The rate of consumption will be negative only if the discount rate is greater than the intrinsic net growth rate of the stock. Considering the signs of the derivatives in (17), \( \dot{c} \) will only be negative if the expression in the square brackets is positive, \( \frac{u_c}{h_c} (\rho - [g_L - h_L]) - u_L \). Since \( u_L, u_c \), and \( h_c \) are all positive, the inequality can only be satisfied if \( \rho > [g_L - h_L] \). The rate of consumption will not necessarily be positive if the reverse inequality holds.

**The transversality condition**

Considering the transversality conditions it can be shown that the final stock of health must be equal to zero. The transversality condition for the current valued Hamiltonian is given by

\[ e^{-\rho^T \lambda(T)} L(t) = 0 \]
Assume that \( \lambda(T) = 0 \). Using (15), this assumption implies that \( \frac{u_c(T)}{h_c(T)} = 0 \), which can only be true if \( u_c(T) = 0 \) or \( h_c(T) = 0 \). By assumption \( u_c(t) > 0 \), and since the model is referring to a physical system (a good causing harm to the body) \( u_c(T) \neq \infty \), contradicting the assumption that \( \lambda(T) = 0 \). Clearly, \( e^{-\rho T} \neq 0 \) since the time horizon of the problem is assumed finite.

Thus, the transversality condition can only be satisfied if \( L(T) = 0 \), that is, the consumer has exhausted his health by the time of his death. This also makes intuitive sense. The consumer has no value for health after he dies, and having health always provides utility, so the consumer has an incentive to completely extract all the possible utility from his health before he dies.

**Phase plane**

In order to discuss the time path of consumption and health, it will be useful to develop a phase plane in stock-consumption space. To do so, simple functional forms need to be assumed for the instantaneous utility function and the equation of motion for health. Two simple functions that satisfy the assumptions are: \( u(c, L) = L\sqrt{c} \); and \( \dot{L} = L(K - L) - \frac{c^2}{L} \). Here, \( K \) is the maximum allowable quantity of health. Substituting these equations into (9) and (10) and following the same solution method give the equations of motion for the stock and consumption. Setting these differential equations to zero and plotting them in \( (L, c) \) space yields the nullclines of the system. Figure 2 shows these nullclines, the steady states of the system, and two example trajectories – labeled \( L_{\text{low}} \) and \( L_{\text{high}} \).

Figure 2 shows that there are two qualitatively different paths of consumption that the consumer can choose. If the consumer is endowed with a high initial stock of health, the optimal consumption path will resemble \( L_{\text{high}} \). The consumer will have a monotonically decreasing consumption trajectory. Along this path the discount rate will be greater than the growth rate of the stock. Alternatively, if the consumer begins the problem with a small stock of health, he will increase consumption, but at such a rate as to allow the stock to grow. There is a critical moment when the consumer switches from an increasing consumption path to a decreasing consumption path, which he will then follow until the terminal time. This critical point occurs when the \( L_{\text{low}} \) trajectory intersects with the consumption nullcline.
The discount rate is a measure of the impatience of the individual. Increasing the discount rate will skew consumption towards the start of the planning horizon. This can be seen through equations (14) and (16). In (14) an increase in the discount rate will increase the growth of the value of the stock, which decreases the slope of the time path of consumption in equation (16). If the consumer is following a $L_{\text{high}}$ style consumption path, which always has a negative slope, a decrease in $\hat{c}$ for all $t$ moves consumption towards the start of the planning horizon. The initial rate of consumption will be higher and the rate will decline more quickly – see figure 3.

**Figure 2 – Trajectories in stock-consumption space**

Path $L_{\text{low}}$ is a representative optimal trajectory with a low $L_0$
Path $L_{\text{high}}$ is a representative optimal trajectory with a high $L_0$
As mentioned earlier, if the consumer is following a downwards sloping trajectory, the discount rate will be greater than the intrinsic rate of growth in the stock. Thus when the consumer is following a \( L_{\text{high}} \) style trajectory, the discount rate is greater than the growth rate, which implies that the consumer’s utility is reduced by delaying consumption. The consumer’s incentive is to deplete the stock as fast as possible – assuming that the growth rate and the discount rate are the only objects of interest. This is a common result in the optimal harvesting literature (Wilen 1985).

![Figure 3 – Effect of discount rate on a \( L_{\text{high}} \) trajectory](image)

When the consumer is following a \( L_{\text{low}} \) style path, the time path of consumption will be increasing before the critical point. Along this section of the path the discount rate will be less than the composite of the net intrinsic rate of growth plus the ratio of the value of the stock now to the value of the stock in the future, i.e. \( \rho < (g_L - h_L) + \frac{u_L}{x} \). This suggests that the consumer will derive greater utility from letting the stock appreciate in order to consume more later, rather than consuming as fast as possible now.

When consumption approaches some utility maximising optimal steady state with an infinite time horizon, the problem is said to display a turnpike result. In a finite horizon problem, a turnpike result means that the consumption trajectory approaches the optimal steady state before turning away to satisfy the appropriate boundary condition (Wilen 1985). Turnpike results are usually presented in the context of intertemporally independent utility functions. These results, however, can also be found in intertemporally dependent problems as shown by Samuelson (1971). The optimal trajectories in the health depletion
problem display turnpike results, in that they move towards the optimal steady state (marked ‘stable steady state’ in Figure 2), before turning away to satisfy the terminal condition, \( L(0) = 0 \). Interestingly, unlike most turnpike results, the optimal steady state in this model is locally stable, rather than a local saddle. This result is driven by the high level of non-linearity displayed by the consumption nullcline.

Finally consider how the value of the resource changes through time. The value of the resource at time \( t \), represented by \( \lambda \), is a measure of the scarcity of the resource. An increase in scarcity will increase the value of \( \lambda \), ceteris paribus. Taking (14) and dividing by \( \lambda \) yields:

\[
\frac{\lambda}{\lambda} = p - \left[ \frac{u^L}{\lambda} + (g_L - h_L) \right]
\]

The LHS of this expression is the percentage change of the value of the resource through time. It is a generalisation of the simple Hotelling rule, in which the value of the resource increases at the rate of discount (Hartwick and Olewiler 1998). The value of the resource does not grow as quickly in this model. There are two reasons for this. First, since the stock of health is growing, the growth in the stock offsets the increase in scarcity due to consumption. Second, health provides utility directly to the consumer through its impact on the marginal utility of consumption (\( u_{cL} > 0 \)). Thus, the consumer has an extra incentive to conserve the resource. Increasing the instantaneous marginal value of the resource smooths consumption, leading to an effect similar to a reduction in the discount rate (see Figure 3).

V. Discussion of the results

Comparison to rational addiction

The basic definition of addiction in Becker and Murphy’s model (1988) is that ‘a person is potentially addicted to [the addictive good] if an increase in his current consumption of [the addictive good] increases his future consumption of [the addictive good]’. The consumption of goods in the present needs to be a complement to the consumption of goods in the future for the consumer to display addictive behaviour. This characteristic is called adjacent complementarity (Ryder & Heal 1973). This paper does not rigorously analyse the health depletion model for adjacent complementarity. However, some inferences can be drawn using Becker and Murphy’s basic definition of addictive behaviour.
The analysis shows two key results that parallel rational addiction and one that is ambiguous.

Becker and Murphy observe that analysing a non-linear rational addiction model will yield two stable steady states: one with a high level of addiction and a high level of consumption of the addictive good; and another where the consumer is ‘clean’ (no addiction and no consumption of the addictive good). In comparison, the health depletion model has three stable steady states when using an infinite horizon: one with both c and L positive; one with maximum health and no consumption of the addictive good; and one with a positive level of addictive consumption and an exhausted health stock. The state with maximum health corresponds to the ‘clean’ steady state in the Becker and Murphy model, where the consumer has the minimum stock of addictive capital and abstains from consumption. The positive steady state corresponds to the positive steady state in the Becker and Murphy model where the consumer is able to maintain a controlled and sustainable level of addiction. The path that leads to the exhaustion of health is perhaps the best analogue of addictive behaviour. Here the consumer chooses a consumption path that leads to his eventual demise, even though there are sustainable paths he could take.

The effect of marginal utility of health creates an effect that is analogous to addiction. Recall that the utility function is additively inseparable in its arguments. Furthermore, since \( u_L \) and \( u_c \) are positive the cross derivative, \( u_{cL} \), is also likely to be positive. Considering equation (17), increases in \( u_L \) will increase the slope of the consumption path, consequently causing the consumer to consume more of the addictive good in the future.

For a consumer to be addicted to a good, his consumption of the good must be increasing. Thus if the consumer is following the \( L_{\text{high}} \) type path, where consumption is always decreasing, the consumer cannot be displaying addictive behaviour. The \( L_{\text{low}} \) path is a better candidate for addictive behaviour since the consumption increases along the optimal path, although this is merely an observation and does not guarantee that the consumer is displaying addictive behaviour.

**Implications of the results**

The consumption paths from the health depletion model have natural interpretations in terms of consumer behaviour. The positive steady state represents a consumer who consumes the addictive good ‘responsibly’ meaning that their consumption does not cause
long-term harm. The steady state with full health and no addictive consumption reflects the behaviour of a teetotaller who completely abstains from consumption. In this case such a consumer would be receiving no utility, but that is an artefact caused by the exclusion of the non-addictive ‘composite consumption’ good from the model. Finally, the steady state where the consumer exhausts his health shows uncontrolled, self-destructive consumption of the addictive good. This behaviour corresponds to ‘chronic addiction’, defined by the Health Officers Council of British Columbia (2005) as ‘use that has become habitual and compulsive despite negative health and social effects’.

The particular consumption path of the addictive good the consumer will choose will depend on the parameters of the model. For instance, consumers with higher discount rates are more likely to be chronic addicts. A useful extension to this work would be to find empirical economic and scientific data on the factors that cause consumers to follow one of the three consumption patterns described above. These causes could be compared to the effects of the parameters in the model as a test of the plausibility of the model’s descriptive ability.

VI. Further directions
The links between the results from the simplified health depletion model, the rational addiction model and the research on the health impact from addiction are sufficiently strong that further investigation is warranted. Several avenues for further investigation are proposed below.

Effects of uncertainty
The possibility of the consumer facing uncertainty about his time of death was discussed in section III. It would be interesting for future work to analyse the effect of this uncertainty on the optimal decisions of consumers. Clarke (2000) analyses an infinite horizon rational addiction model with a mortality hazard. The key insight in Clarke’s paper is that consumers will reduce addictive consumption when consumption independent-risks are reduced and when consumption-dependent risks are increased. This result is driven by the fact that risk enters the planning problem by altering the consumer’s discount rate. If the risk of death increases the effect is the same as an increase in the discount rate, that is, consumption is skewed towards the present. In the health depletion model, if the consumption of the
addictive good were linked positively to mortality risk with a large marginal effect, it would be likely that the consumer would decrease the rate of consumption. This assumption may be plausible for alcohol since a substantial number of road accidents involve drunk drivers. Cigarettes, however, may not display this effect since mortality risk for smoking tends to be linked to cumulative consumption rather than instantaneous consumption.

The analysis of the health depletion model assumed that the consumer was going to die with certainty at a particular terminal time. Even if the moment of death is faced with uncertainty the terminal time is still fixed. The model could be extended by allowing the consumer to optimally pick the terminal time within some upper bound, thus transforming the model into a truncated horizontal line problem.

**The effect of age on health**

The assumption was made that the consumer’s health would respond identically regardless of the age of the consumer. The implication is that a 25 year old would have the same response to an addictive good as a 70 year old, which is clearly not the case. The solution would be to allow for the maximum stock to decline with age. The decline of the body is a natural consequence of aging. For example, at $t = 0$ the organ would be able to recover to full capacity after a shock, but at $t = 50$ the organ may only be able to recover to 80 per cent capacity.

**Path dependence of consumption**

The consumption of the addictive good could not only reduce the maximum health stock, but it could also retard the ability of the organ to regenerate after a shock. An extra stock could be introduced, $r(t)$, the stock of regenerative ability, such that $g(L(t), t)$ becomes $g(L(t), r(t), t)$. An extra equation of motion would be introduced where consumption of the addictive good reduces this stock, e.g. $\dot{r} = -c(t)$. This stock would enter the function $g(\cdot)$ such that a reduction in the stock, $r(t)$, reduces the maximum potential steady state of organ capacity.

**Modelling cigarette addiction**

As noted in the introduction, cigarette consumption imposed the largest social cost to economy from drug abuse in 2004-05 (Collins and Lapsley 2008b). The case of cigarettes is used to illustrate a case where there is a risk of a threshold effect in the level of health. This represents the risk of being diagnosed with cancer. Here the stock of health declines
gradually until the onset of cancer when the stock of health declines rapidly to a much lower level.

Smoking cigarettes not only damages the lungs directly through chronic obstructive pulmonary disease (Tetley 2002), but also increases the risk of the smoker developing lung cancer (Gilbert 2004). The advent of cancer leads to a rapid decrease in lung function (Hong and Tsao 2008). According to the European Consensus Statement on Lung Cancer ‘there is a lag time of many years between beginning smoking and the clinical manifestation of cancer’ (Biesalski et al. 1998, p.168), thus it is assumed that the development of cancer is linked to the total number of cigarettes smoked, rather than the number smoked at any given time. It is the cumulative, not instantaneous, consumption of cigarettes that causes cancer. It is also assumed that the body loses its ability to regenerate itself once the consumer has cancer; spontaneous remission is an extremely rare event (Horino et al. 2006).

The modelling of lung degradation due to smoking consists of two parts: an instantaneous damage effect identical to the way alcohol affects the liver; and a threshold effect where the accumulation of cigarette consumption leads to a sudden drop in health. In order to model this second effect, a second stock variable is introduced; namely, cumulative cigarette consumption. The total stock at any time \( t \) is given by the following integral:

\[
S(t) = \int_0^t c(\tau) d\tau
\]  

(19)

This stock would appear in the optimal control problem formulation as another equation of motion with appropriate boundary conditions \( S = c(t) \) and \( S(0) = 0 \).

The stock of cumulative cigarette consumption affects lung capacity through the harm function. The harm function is divided into two parts: the effect of instantaneous consumption; and the harm caused by cancer. Thus the harm function is rewritten as

\[
h(L, c, t) = h(L, c, t) + \hat{h}(S)
\]

The shape of the \( h(.) \) function for cigarettes is identical to the \( h(.) \) function described above for alcohol. The \( \hat{h}(S) \) function exists to force the stock of lung capacity towards zero once some threshold stock, \( S^* \), has been reached. Furthermore, once the threshold has been reached, the \( \hat{h}(S) \) function offsets the intrinsic ability for the organ to heal itself, so the stock of lung capacity is static if no cigarettes are being consumed.
Figure 4 shows the indicative shape of the $\hat{h}(S)$ function. Before the critical $S$ value is reached, the harm caused by cancer is zero. Beyond the threshold, the damage caused by cancer rises rapidly and falls just as rapidly. This serves to force the stock of lung capacity quickly towards zero. The harm caused by cancer does not fall then to zero, but to a point such that $\hat{h}'(S) = -g(L, t)$. Since $S$ cannot decrease ($c(t) \geq 0$), this restriction makes it impossible for the consumer to recover from the effects of cancer once the critical threshold has been reached, without some external intervention.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image}
\caption{The indicative shape of the threshold harm function}
\end{figure}

\section*{VII. Conclusion}
Starting from the basis that consumers are motivated by the state of their health when considering consuming harmfully addictive goods, this paper sketched a resource depletion style model that focuses on the consumer’s stock of health and which is shown could account for addictive behaviour.

Analysis of the simplified health depletion model with a finite time horizon showed that a consumer would consume along one of two possible paths: either a Hotelling style depletion path or a turnpike style path were the consumer allows his health to improve before depleting it. In both cases the consumer would fully deplete his health by the end of the planning horizon. A comparison of the infinite horizon version of these paths to the stylised path from Becker and Murphy’s rational addiction model found that there appear to be some promising parallels between the two models. The results from the simplified health
depletion model also demonstrated known responses to addictive consumption such as ‘responsible’ consumption, teetotaling and chronic addiction.

However, the analysis of these parallels is tentative and a more rigorous comparison between the health depletion model, the rational addiction model and research on addiction in the health literature is necessary to determine a more precise understanding of where the models are in agreement or are dissimilar.
References


Collins, D. J., & Lapsley, Helen M. (2008). The avoidable costs of alcohol abuse in Australia and the potential benefits of effective policies to reduce the social costs of alcohol (p. 51).


