Using alterative whole-farm modelling approaches to assess farm enterprise selection, risk and welfare

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Using an expected mean-variance model the changes in farm enterprise levels and indirect utility were examined under conditions of risk aversion, budget constraints and gross margin variance. An extension of the comparative statics of the expected mean-variance model was adopted by introducing a budget constraint into the constrained optimisation problem. A 10-year expected mean-variance whole-farm model was solved for a farm in the wheat-sheep zone of Australia to provide an empirical example. Results were obtained using no planning horizon (the static model) and then with a five-year rolling planning horizon (the dynamic model). In addition, enterprise levels were constrained to match levels observed on the farm so as to compare incomes between the constrained and unconstrained models.

For a cash constrained, risk averse, farmer it was found that they are likely to have larger expenditures than less risk averse operators in order to obtain the same indirect utility. Enterprise levels differed between the dynamic and static models, and a dynamic model was used to help explain inter-temporal decision-making. Risk aversion reduced the set of possible welfare improving production activities available to a farmer.

Keywords: Whole-farm modelling; enterprise selection; risk.

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1. Introduction

There are many whole-farm methodologies available to help better understand farmer decision-making (Hardaker *et al.*, 2004). Two common issues in whole-farm modelling are how to account for risk and what length of planning horizon to use to best capture reality (Pannell *et al.*, 2000). Expected utility theory is often used to account for risk (Patten *et al.*, 1988), for example, setting up a whole-farm model using a negative exponential utility function with linear constraints (Lien and Hardaker, 2001; Torkamani, 2005; Lien *et al.*, 2009). As this form of utility function is increasing in terms of net income, maximising either net income or expected utility will provide the same level of enterprise mixes.[1] With a negative exponential utility function, risk aversion simply affects the value of the objective function. Assessing enterprise mixes can be important as there are often a number of production decisions that provide similar financial outcomes (Pannell, 2006), but a farmer may decide on what enterprise mix to choose based on personal preferences.

Another method used in whole-farm modelling is the 'expected mean-variance' (EV) model. The EV model can be used to find a set of activities that maximises expected total returns for different levels of variance of the total return (Markowitz, 1952). Using a quadratic objective function (a risk weighted net income equation) subject to a set of linear constraints the EV model can identify optimal enterprise levels. This approach has been used in numerous whole-farm modelling studies (Freund, 1956; Scott and Baker, 1972; Lin *et al.*, 1974; Manos *et al.*, 1986; Batterham *et al.*, 1993; Crisostomo *et al.*, 1993; Lansink,

[1] An enterprise is a component of a farm business, for example, a farm may have a sheep enterprise, a cattle enterprise and a grain cropping enterprise. Farm enterprise and farm activity are equivalent terms in this study.

1999; Nartea and Webster, 2008). An EV model is used in this study to explore how enterprise mixes change given uncertain outcomes.[2] The approach is designed to help better understand farmer decision-making.

There are three objectives in this paper. Firstly, to examine the comparative statics of an EV model with a budget constraint and assess how changes to gross margin variance, budget allocation and risk aversion impacts on enterprise levels and indirect utility.[3] Secondly, to compare enterprise mixes and objective function values between a whole-farm model with a five-year rolling planning horizon (dynamic EV model) and a whole-farm model with no planning horizon (static EV model). Thirdly, to compare observed farmer practice with an optimisation program's predicted results. The 'observed model' was the 'predictive model' constrained so as to closely match the set of case-study farm data. This allowed a 'comparable' objective function value to be calculated. To meet these objectives data from a case-study farm are used.

The case-study farm was from the wheat-sheep zone of Australia and was used to highlight how payoffs and enterprise mixes differ between using a dynamic model and using a static model, and that a dynamic model may more closely represent the farmer's behaviour. In addition, it can potentially be shown that the farmer's risk attitude may be limiting

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[2] Using the EV approach will yield results that include the combination of activities that maximizes expected utility only if the farmer's utility function is quadratic or if enterprise gross margins are normally distributed (Hardaker *et al.*, 1991). The normality assumption may be reasonable, particularly if the number of risky prospects is not too small (Anderson *et al.*, 1977). In addition, previous studies have concluded that the EV approach is robust to violations of the normality assumption (Kroll *et al.*, 1984).

[3] Comparative statics is the comparison of two different economic outcomes, before and after a change in some underlying exogenous parameter (Mas-Colell *et al.*, 1995).

production opportunities and that increased risk aversion requires larger budget expenditures to maintain a given level of indirect utility.

This paper builds on the work of Coyle (1992) and Coyle (1999) who examined the effects of output price variance on the comparative static solutions of an unconstrained linear EV model. Examining the comparative statics of a constrained EV model (model with a budget constraint) allows for the effects of changing budget allocations and changing risk aversion levels on indirect utility and enterprise levels to be measured. This is important as cash is often a limiting factor in determining farm production plans. Chavas (1987) explored the comparative statics of maximising the expected utility of wealth subject to a production frontier. The approach in this paper is also designed to assess constrained choices under risk, however, an EV objective function was maximised subject to budget availability, and then an empirical example considered.

A second contribution of this paper is to compare the differences between using a dynamic whole-farm modelling approach versus a static approach. Static models are frequently used in whole-farm planning (Pannell *et al.*, 2000; Janssen and van Ittersum, 2007), however, often agricultural production and storage choices are made in one year that subsequently affect future years, for example, livestock breeding and retaining a buffer stock of livestock feed. Several models have incorporated tactical farm management responses (dynamic models) (Kingwell *et al.*, 1993; Kingwell, 1994; Pannell and Nordblom, 1998). Although Janssen and van Ittersum (2007) find that 11 of 48 whole-farm models they reviewed were dynamic, the literature on comparing the benefit of using a dynamic model versus a static model is limited. One exception is Kingwell et al. (1993) who found that by incorporating tactical responses incomes increased by 20 per cent in a Western Australian situation. In this paper the suggestion of Kingwell et al. (1993) to incorporate risk aversion into the modelling approach is used. Pannell et al. (2000) find that a static model that incorporates risk has a 9 per cent lower certainty equivalent value than a dynamic model that incorporates risk. Additional empirical evidence is added to the debate on differences between static and dynamic models by examining how enterprise mixes, not only objective function values, change between these two approaches.

Results from mathematical programming models have rarely been used in practical agricultural situations (McCown and Parton, 2006).[4] However, such models have greatly strengthened understanding of the systems involved from an educational and research point of view rather than providing 'solutions' at the farm decision-making point. Farmers may not wish to adopt optimised plans and agricultural practitioners have found neither need nor want to use formal approaches in on-farm planning. Musshoff and Hirschauer (2007) identify an income gap between observed and predicted models, and in this paper the change in this gap with changes to risk aversion is examined.

[4] One exception has been the development of minimum-cost feed mixes in the livestock feed industry, for example, Gradiz et al. (2007).

2. Case-study farm

The case-study farm was located in central western NSW, Australia, and was chosen to reflect the conditions found on a typical farm in Australia's wheat-sheep zone (ABARE, 2009). The farm was family-owned and 3990 ha in area. The farmer specialised in wool production, and had, on average, 3000 merino breeder ewes and up to 200 breeder beef cows. The farmer preferred to specialise in wool production and had not diversified into crossbred sheep, even though such diversification was becoming common in the wheat-sheep zone (Perry, 2005). The farmer grew some winter crops, for example, wheat and barley and always planted 120 ha of forages oats and 200 ha of lucerne each year for livestock feed.

Detailed interviews with the farmer provided 10 years of data from 1993 to 2002 on farm activities and management (Table 1). Sample means and variance-covariances, based on the historical time series of gross margins, were derived and were used as inputs into the empirical models. When converting the livestock gross margins to a per hectare basis, merino sheep were found to be a lower-return, lower-risk enterprise, relative to beef cattle.

The merino gross margin was calculated for a self-replacing merino flock and included the sale of lambs, wool and cull-for-age ewes. The beef gross margin was for a self-replacing Hereford herd selling weaners after 15 months. Variable costs were $9337 per 1000 sheep and were $5518 per 100 head of cattle. The variance of gross margins per 1000 sheep was $17 159 883 and the variance of gross margins per 100 cattle was $86 694 448. The covariance between these two farm enterprises was $12 634 663. These data were used in the comparative statics and whole-farm modelling analyses.

Table 1 Distribution of enterprise gross margins in $ per unit per year for the case-study farm[a]

Year	Merino sheep	Crossbred sheep	Beef cattle	Wheat	Barley	Oats	Lupins
1993	18	33	115	147	149	110	197
1994	23	26	40	-34	-30	-47	-72
1995	29	34	156	126	128	128	131
1996	25	43	173	114	107	140	213
1997	21	36	15	60	64	139	113
1998	25	32	103	55	70	108	141
1999	19	38	159	174	162	149	106
2000	18	28	183	-30	-36	33	208
2001	30	43	202	64	55	167	194
2002	42	53	188	-24	-27	-45	69
Mean	25	37	133	65	64	88	130
SD	7	8	64	76	75	79	86

[a] Crop enterprise gross margins are in $/ha and sheep and cattle are in $/head. All enterprise gross margins were directly observed from the farm, except for crossbred lambs. Crossbred sheep gross margins were calculated using NSW Industry and Investment gross margins and Meat and Livestock Australia data. SD is standard deviation.

3. Comparative statics

The EV model is typically used to derive a frontier that shows the trade-offs between the expected income and the variance of the income. In this section, instead of focusing on deriving the EV frontier, a comparative statics analysis based on the EV model's objective function was used to assess how changing enterprise variances, budget allocations and risk preferences affect optimal enterprise levels and indirect utility.[5] This was done algebraically and empirically (using reasonable ranges of coefficient values).

3.1. Theory

Assume that a farmer has two enterprises (x_1 and x_2) and that the enterprise gross margins are c_1 and c_2. The farmer's objective is to maximise Z (the risk weighted objective function):

$$Z = c_1 x_1 + c_2 x_2 - \frac{\alpha}{2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} q_1 & q_{12} \\ q_{21} & q_2 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \tag{1}$$

The sum of the first two terms in Equation (1) is the total gross margin. The term α is the farmer's absolute risk aversion coefficient and the larger the value of α the more conservative the farmer. The gross margin variances of x_1 and x_2 are q_1 and q_2, and q_{12} is the gross margin covariance between x_1 and x_2, with $q_{12} = q_{21}$. The variance-covariance matrix of gross margins is $\begin{bmatrix} q_1 & q_{12} \\ q_{21} & q_2 \end{bmatrix}$.

[5] All of the comparative statics were conducted using Mathematica (Wolfram, 1988). The complete Mathematica code is available from the authors upon request.

The envelope theorem was used by Coyle (1992) to show how optimal enterprise levels change when output prices vary (although no budget constraint was used).[6] In this analysis, a budget constraint is imposed. To produce a unit of x_1 and x_2 requires a_1 and a_2 units of cash, so that the budget constraint is:

$$a_1x_1 + a_2x_2 \le b. \tag{2}$$

The Lagrangian function for this problem is:

$$L = Z + \lambda(b - a_1x_1 + a_2x_2). \tag{3}$$

The solutions for the three endogenous variables, x_1, x_2 and λ, were obtained by simultaneously solving the three first-order conditions of Equation (3). The optimal levels of x_1, x_2 and λ are:

$$x_1^* = \frac{a_2{}^2c_1 - a_1a_2c_2 - a_2bq_{12}\alpha + a_1bq_2\alpha}{(a_2{}^2q_1 - 2a_1a_2q_{12} + a_1{}^2q_2)\alpha}, \tag{4}$$

$$x_2^* = \frac{a_1{}^2c_2 - a_1a_2c_1 - a_1bq_{12}\alpha + a_2bq_1\alpha}{(a_2{}^2q_1 - 2a_1a_2q_{12} + a_1{}^2q_2)\alpha}, \text{ and} \tag{5}$$

$$\lambda^* = \frac{a_2c_2q_1 - a_2c_1q_{12} - a_1c_2q_{12} + a_1c_1q_2 + b(q_{12}{}^2 - q_1q_2)\alpha}{a_2{}^2q_1 - 2a_1a_2q_{12} + a_1{}^2q_2}. \tag{6}$$

Substituting these three values into Equation (1) gives the indirect objective function (indirect utility), which is known as the value function (V):

[6] The envelope theorem allows for the effect of a change in an exogenous variable on the optimised value of an objective function to be found by taking the partial derivative of the Lagrangian function with respect to that exogenous variable at the optimal solution of the problem at hand.

$$V = \frac{\left(a_2c_1 - a_1c_2\right)^2 + 2b(a_2c_2q_1 - a_2c_1q_{12} - a_1c_2q_{12})\alpha + (q_{12}{}^2 - q_1q_2)\alpha^2b^2}{\left(a_2{}^2q_1 - 2a_1a_2q_{12} + a_1{}^2q_2\right)2\alpha}. \tag{7}$$

The maximum value of the objective function for a given set of parameter values is represented by Equation (7). Using the envelope theorem (Varian, 1992), the effect of a change in q_1 on indirect utility can be found by taking the partial derivative of V with respect to q_1 (Equation 8).

$$\frac{\partial \mathrm{V}}{\partial \mathrm{q}_1} = -\frac{\left(a_2{}^2c_1 + a_1q_2b\alpha - a_2\left(a_1c_2 + q_{12}b\alpha\right)\right)^2}{2\alpha\left(a_2{}^2q_1 - 2a_1a_2q_{12} + a_1{}^2q_2\right)^2} \tag{8}$$

The sign of Equation (8) was evaluated by using a set of *a priori* assumptions about the signs of the parameters. Positive values were assigned to a_1, a_2, q_1, q_2, c_1, c_2, b and α and the expected sign of q_{12} was left unrestricted. For all values of the parameters in Equation (8) a rise in q_1 will lead to a fall in V. This implies that increased gross margin variance of x_1 reduces the farmer's indirect utility regardless of the farmer's level of risk aversion or available cash.

To determine how x_1^* and x_2^* change when q_1 increases the total derivative of the first-order conditions of Equation (3) were taken with respect to q_1. Cramer's rule was used to solve the resulting set of equations (assuming $\frac{dq_{12}}{dq_1} \neq 0$). Thus:

$$
\begin{bmatrix} \alpha q_1 & \alpha q_{12} & a_1 \\ \alpha q_{12} & \alpha q_2 & a_2 \\ a_1 & a_2 & 0 \end{bmatrix}
\begin{bmatrix} \dfrac{dx_1^*}{dq_1} \\ \dfrac{dx_2^*}{dq_1} \\ \dfrac{d\lambda^*}{dq_1} \end{bmatrix}
=
\begin{bmatrix} -\alpha\left(x_1 + x_2\dfrac{dq_{12}}{dq_1}\right) \\ -x_1\alpha\dfrac{dq_{12}}{dq_1} \\ 0 \end{bmatrix}. \tag{9}
$$

If any of the following conditions hold then $\frac{dx_1^*}{dq_1} < 0$ and $\frac{dx_2^*}{dq_1} > 0$:

$$
\frac{a_2 q_1}{a_1} + \frac{a_1 q_2}{a_2} < 2q_{12} \ \textit{and}\ a_1x_1 > a_2x_2 \ \textit{and}\ dt[q_{12}, q_1] > \frac{a_2x_1}{a_1x_1 - a_2x_2}, \tag{10}
$$

$$
\frac{a_2 q_1}{a_1} + \frac{a_1 q_2}{a_2} < 2q_{12} \ \textit{and}\ a_1x_1 < a_2x_2 \ \textit{and}\ dt[q_{12}, q_1] < \frac{a_2x_1}{a_1x_1 - a_2x_2}, \tag{11}
$$

$$
\frac{a_2 q_1}{a_1} + \frac{a_1 q_2}{a_2} > 2q_{12} \ \textit{and}\ a_1x_1 < a_2x_2 \ \textit{and}\ dt[q_{12}, q_1] > \frac{a_2x_1}{a_1x_1 - a_2x_2}, \text{ or} \tag{12}
$$

$$
\frac{a_2 q_1}{a_1} + \frac{a_1 q_2}{a_2} > 2q_{12} \ \textit{and}\ a_1x_1 > a_2x_2 \ \textit{and}\ dt[q_{12}, q_1] < \frac{a_2x_1}{a_1x_1 - a_2x_2}. \tag{13}
$$

If q_{12} is negative, then when a_1x_1 is greater than a_2x_2 any rise in q_1 will see x_1 fall and x_2 rise only when $dt[q_{12}, q_1] < \frac{a_2x_1}{a_1x_1 - a_2x_2}$, and determining the sign of this term is essentially an empirical matter. When q_{12} is positive, the change in sign will depend on numerous interactions and no general conclusion can be assigned.

Lambda represents how much the objective function changes if there is an extra dollar of budget allocation available to spend on producing x_1 and x_2. The change in lambda when q_1 changes (Equation 14) measures how the value of an extra dollar changes when the variance of x_1 increases.

$$\frac{d\lambda^*}{dq_1} = \frac{2(a_2 q_{12} - a_1 q_2) x_1 \alpha}{a_2^2 q_1 - 2a_1 a_2 q_{12} + a_1^2 q_2} \tag{14}$$

Signing comparative statics results will ultimately depend on the numerical values of the parameters in the above equations. It is not uncommon to be unable to algebraically sign a solution for a farm household economics problem (Barnum and Squire, 1980).

3.2. Empirical application

To empirically test how changes in enterprise variances, risk attitudes and budgets alter the value function and the optimal enterprise levels, data from the case-study farm were used as inputs into Equation (3). In this section, x_1 is set to 1000 merino sheep and x_2 is set to 100 beef cattle.

The budget was set at the level of the average observed enterprise levels multiplied by the enterprise costs for merino sheep and beef cattle. Following the procedure in McCarl and Bessler (1989), the value of α was set at 0.0002. In this approach, the upper limit of α was established so that the certainty equivalent for the farmer, ignoring wealth, was non-negative.

The change in x_1^* and x_2^* when q_1 and q_2 vary is presented in Figures 1 and 2. Variance was varied from 50 per cent to 200 per cent of the observed variance, whilst keeping all else constant. As q_1 increases x_1^* declines and x_2^* rises. Changes in variance of the cattle gross margins, q_2, have a stronger effect on x_1^* and x_2^* than do changes in the variance of sheep gross margins, q_1.

With an increased budget allocation, x_1 increases at a faster rate than x_2 (Figure 3). At fixed levels of the parameters, increased budgets result in cattle numbers, x_2, remaining fairly constant. Thus, such a farmer appears to have a limited willingness to increase cattle numbers as the budget increases and prefers to focus on the lower gross margin, lower variance, enterprise, x_1.

As the absolute risk aversion coefficient increases more budget is allocated to the lower variance enterprise, x_1 (Figure 4). There are clearly levels of risk aversion that result in increased budget allocation reducing the value of the value function (Figure 5). The marginal effect of a greater budget allocation on the value function can be negative, and this depends on the farmer's absolute risk aversion coefficient (Figure 5). To obtain a higher value function as α is increased the amount of cash also needs to rise. There is thus a substitution between risk aversion and budget allocation. With a falling α, less cash is required to obtain the maximum value of the value function. When $0 < \alpha < 0.00015$ more budget is preferred. When $\alpha > 0.00015$ an increased expenditure can reduce the value function.

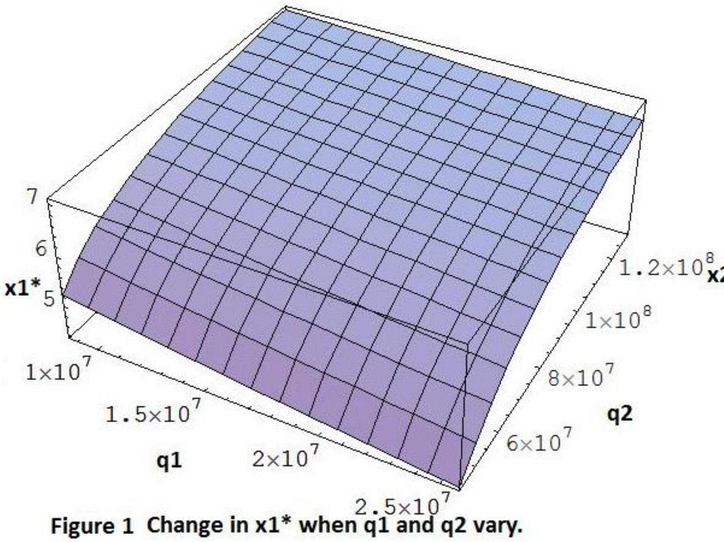

Figure 1 Change in x1* when q1 and q2 vary.

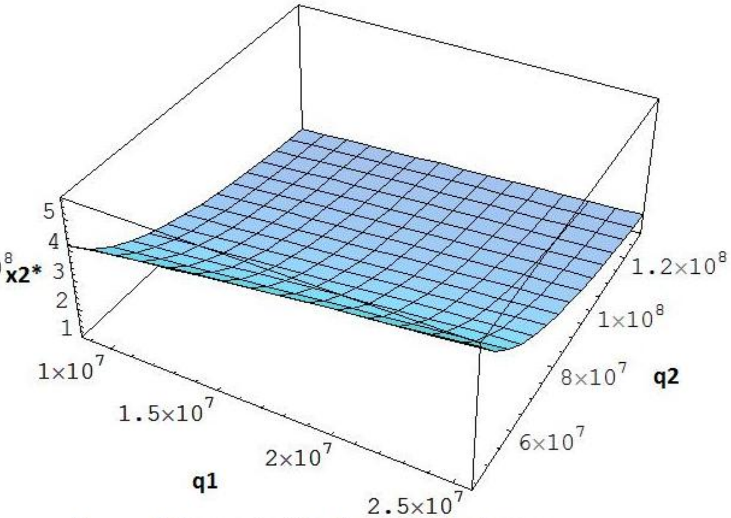

Figure 2 Change in x2* when q1 and q2 vary.

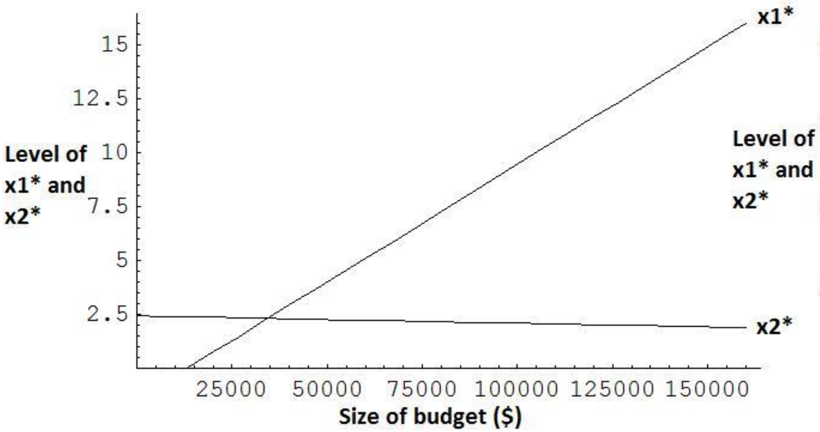

Figure 3 Change in activity levels when budget changes.

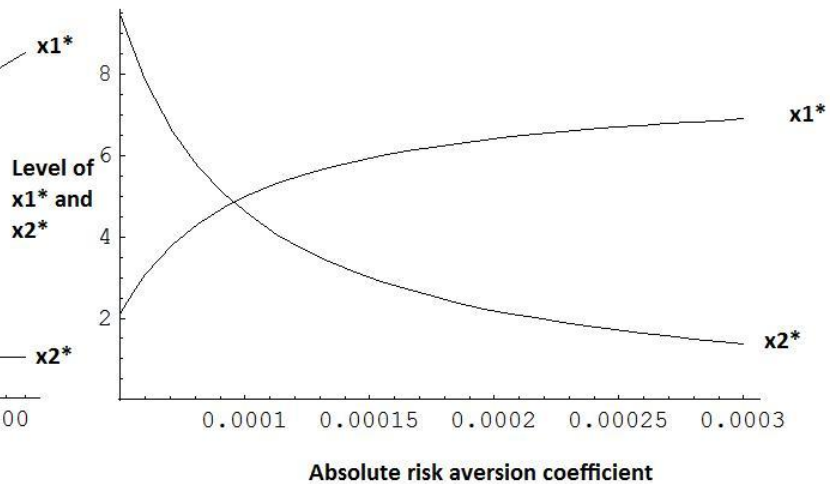

Figure 4 Change in activity levels when absolute risk aversion coefficient changes.

As x_1^* and x_2^* are a function of b and α there will be different combinations of b and α that result in the same gross margin (Figure 6). A more risk averse farmer is likely to need greater expenditure to have the same gross margin than a less risk averse farmer. This is because the risk averse farmer prefers more x_1 and less x_2 (Figure 4). However, x_2 has a lower per unit variable cost.

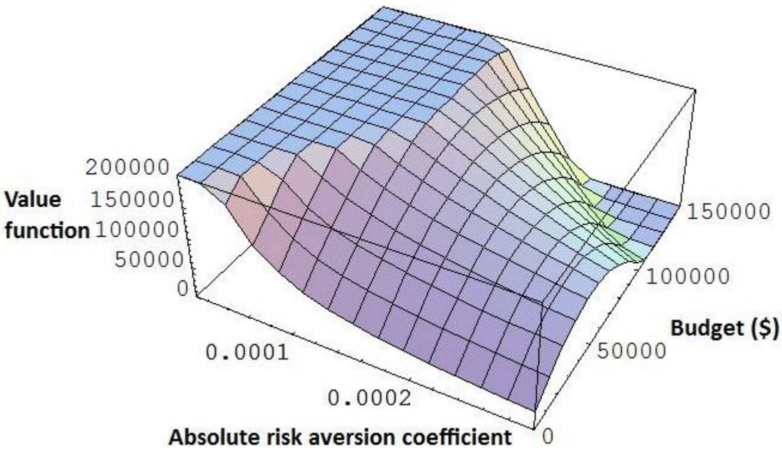

Figure 5 Changes in value function when absolute risk aversion coefficient and budget vary.

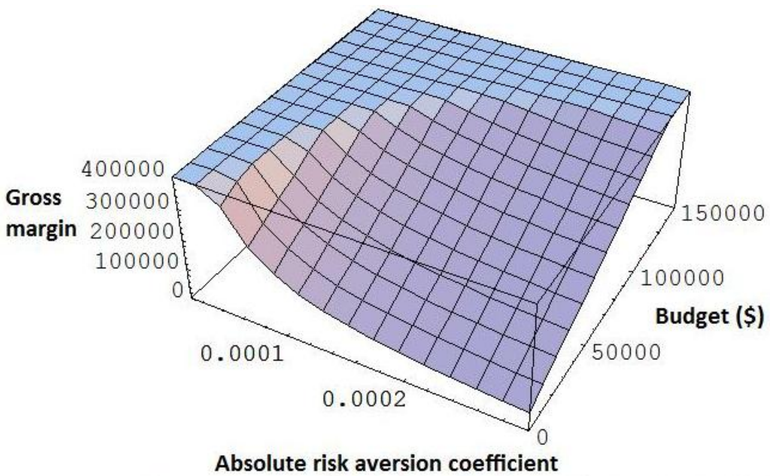

Figure 6 Changes in gross margin when absolute risk aversion coefficient and budget vary.

To measure how variability affects enterprise mixes and the objective function, comparative statics was used to examine changes in enterprise variances. Variance changes as gross margins vary between individual years. An alternative method, as opposed to changing variances in a static framework, was to assess risk in a farming systems model over multiple years that incorporate varying gross margins. In the comparative statics analysis only two enterprises were considered in one time period. In reality, family farms are more complex. Risk can be accounted for in a somewhat different manner in multi-period whole-farm models. In the whole-farm multi-period approach ten years are modelled so that there is less interest in how variances change as gross margins change each year and more emphasis on how the variation will affect optimal decision-making through time.

4. Whole-farm modelling approach

Whole-farm EV models were developed for the case-study farm to illustrate the differences between dynamic and static models and the differences between observed and predicted farm plans and to also assess how different budgets and levels of risk aversion alter indirect utility. The models were written and solved in the general algebraic modelling system (GAMS) (Brooke *et al.*, 1992). The complete GAMS codes and data files are available from the authors upon request. The treatment of time in the models is summarised in Figure 7.

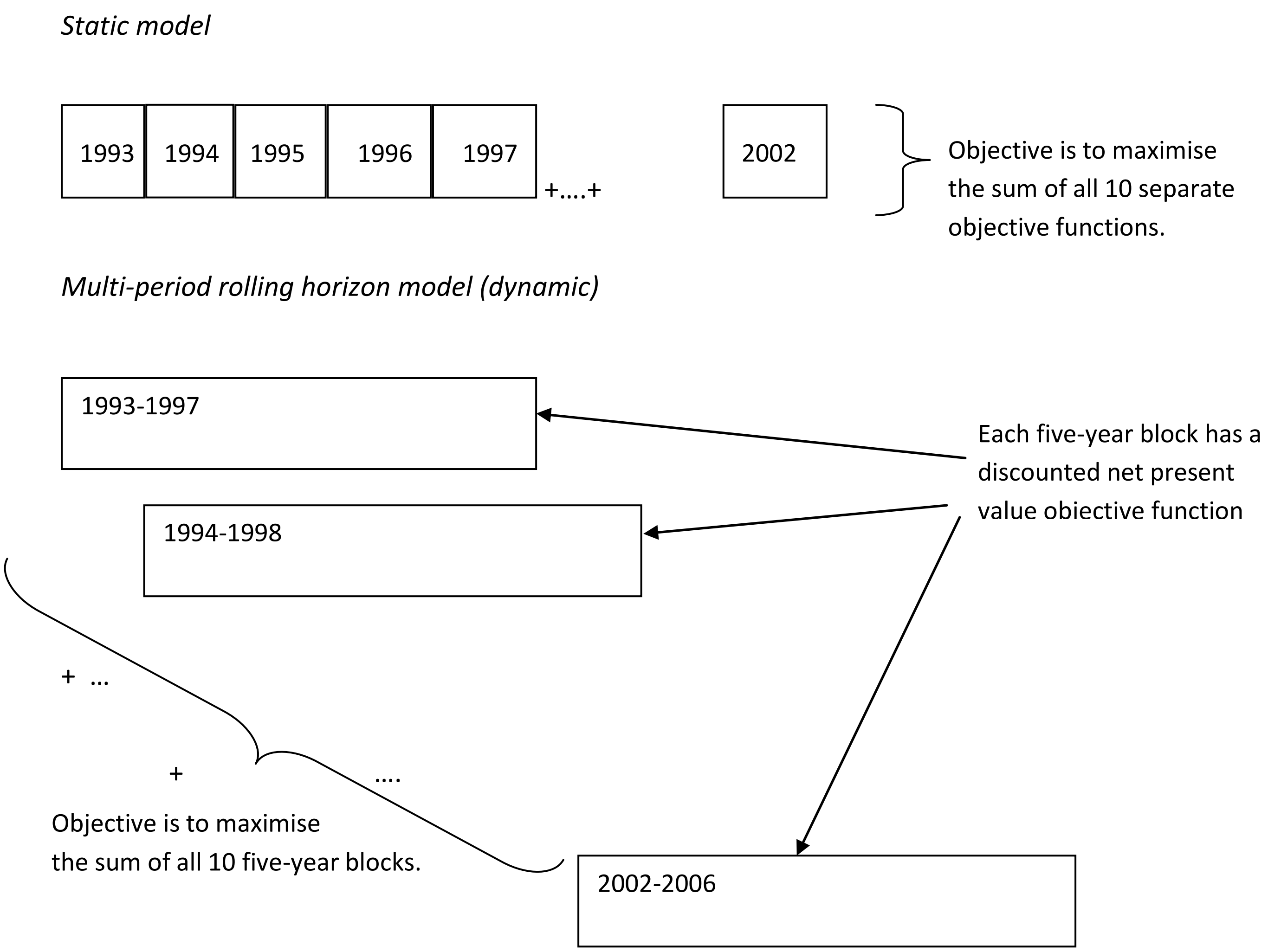

Figure 7 Dynamic and static model objective functions.

4.1. Static model

The objective function of the static model was derived so as to maximise the sum of 10 separate EV models. The 10 models represent 10 years (1993-2002). There is no feedback between the years in this model (that is, no planning horizon). In 1994, the farmer only considers current prices and yields to maximise the current year's objective function. The formulation of the model is as follows:

Maximise $\text{obj}_{static} = Z_y + ... + Z_{y+9}$ (15)

subject to:

$$\sum_{y=1993}^{2002} \sum_{i=1}^{I} x_{iy} A_{iy} \leq b_y. \quad (16)$$

Where:

$$Z_y = \sum_{i=1}^{I} x_{iy} GM_{iy} - 0.5\alpha x_{iy} Q x_{iy}, \quad (17)$$

i = farm enterprise (i = 1,...,I),

y = year (y = 1993, y+1 = 1994, ... ,y+13 = 2006),

α = absolute risk aversion coefficient,

x_{iy} = an $n_{iy} \times 1$ vector of enterprise levels in year y,

GM_{iy} = an $n_{iy} \times 1$ vector of enterprise gross margins in year y,

Q = variance-covariance matrix for the average year's activity gross margins,

A = an $n_{iy} \times m_{iy}$ matrix of technical coefficients in year y,

b = an $m_{iy} \times 1$ vector of resource stocks in year y, and

$x_{iy} \geq 0$.

4.2 Dynamic model

The objective function of the dynamic model was derived so as to maximise the sum of expected utility over 10 five-year time periods (1993–1997, 1998–2003, … , 2002–2006). The expected utility in each five-year period was the discounted sum of the individual year's EV model objective functions. An EV-function for 10 separate years was used and cash, feed and ending livestock numbers were carried over between years. The formulation of the model was as follows:

$$\text{maximize } obj_{dynamic} = \sum_{n=1}^{10} O_n \qquad (18)$$

subject to eqation (14) and

$$\sum_{y=1}^{14} \sum_{i=1}^{I} L_{iy} x_{iy} + x_{iy+1} A_{iy+1} \leq b_{y+1}. \qquad (19)$$

Where:

L_{iy} is a set of y matrices linking years,

$$O_n = \frac{Z_y}{(1+d)^0} + \ldots + \frac{Z_{y+4}}{(1+d)^4} + \frac{V_{y+4}}{(1+d)^4} \qquad (20)$$

$+\ldots+$

$$O_{n+9} = \frac{Z_{y+9}}{(1+d)^0} + \ldots + \frac{Z_{y+13}}{(1+d)^4} + \frac{V_{y+13}}{(1+d)^4},$$

d = discount rate (set at 6%), and

V = discounted value of breeder livestock held at the end of each five-year period.

If only $O_1 + O_5$ was maximised the solutions for enterprise levels in each year would differ from the solutions in $obj_{dynamic}$. If only $O_1 + O_5$ was maximised decisions on enterprise levels in 1996 would be based on what will happen only in 1997, and this implicitly assumes that events and decisions after 1997 have no consequences, that is, the farm stops functioning. Maximising $obj_{dynamic}$ means that decisions on enterprise levels in 1996 are based on what will happen in 1997 to 2000. The case-study farm data are for the years 1993 to 2002. To permit the five-year rolling horizon results to be comparable with the static model from 1993 to 2002, the 2003 to 2006 period had the same price and yield parameters as in the period 1993 to 1996.

Two experiments were conducted on the static and dynamic models:

- Restrict each model to the enterprise mixes that match the observed farmer practice of not producing crossbred lambs, not having more than 200 breeder cows and growing 200 ha of lucerne and 120 ha of forage oats each year.

- Five levels of the absolute risk aversion coefficient and five levels of available cash were used to examine the effects of different levels of risk aversion and available cash on enterprise levels and the objective function.

4.3 Activities and constraints

The main groups of activities in the models were as follows:

- Livestock activities: sheep and cattle. To represent flock and herd dynamics, the sheep flock and cattle herd was divided into various classes consisting of representative animals. The sheep enterprise had a merino flock at the core, ewes could be joined to merino or border leister rams and the offspring were either merino or crossbred lambs. Lambs were either sold at five months, and merino lambs could be retained to replace cull-for-age breeders. Beef cattle weaners were retained for 15 months and then sold, or used to replace cull-for-age breeders.
- Cash crop activities: wheat, barley, lupins and oats. Summer crops were not grown on the farm.
- Forage crop activities: Native pasture, forage oats and lucerne. Wheat, lupins and oats grain were also grown as forage crops. Forage oats were grazed, and grain yields were lower than oats grown purely for grain. Grain oats were not grazed, and were grown to provide grain for livestock feed in low rainfall years.
- Supplementary feeding: blocks were specified for each month (32 per cent crude protein), oats, lupins and wheat grain could also be purchased.
- Labour: each month labour could be hired in and off-farm employment could be obtained.

The main constraints were as follows:

- Land constraint. The farm was 3990 ha. Such an area is close to the district average (ABARE, 2003). Land clearing restrictions and soil types constrained the farm to 680 ha of arable land.
- Rotation limits. Best practice was to follow a grain-legume crop rotation, for example, wheat-lupins-wheat. Therefore, no more than 50 per cent of the arable area could be allocated to any one crop in a single year.
- Livestock feed. Livestock were required to obtain a set amount of energy (megajoules) and crude protein each month. Energy demands for the different classes of livestock were calculated using Pond et al. (1995) and NSW Industry and Investment fact sheets.
- Seasonal labour constraints. There were two workers on the case-study farm and the maximum amount of family labour was set at 500 hours a month. Technical input-output coefficients for seasonal labour requirements per unit of enterprise were assumed fixed and were based on data in Turvey (1988). Family labour could be supplemented with hired labour. The maximum amount of hired labour available was restricted by the available cash. Off-farm employment was set at a maximum of 200 hours a month, as full-employment was not realistic and some unemployment occurred.
- Cash. A limit of $150 000 was assumed to be available to spend on agricultural activities. This matches with the observed farmer spending.
- Livestock equilibrium conditions. Culling of merino ewes was assumed to occur after six years. Therefore, in the static model, and in order to maintain a self-replacing

flock, either 18 per cent of merino lambs were retained or 18 per cent of the flock was bought to maintain a constant flock size. If crossbred lambs were produced more breeders were purchased to maintain a merino breeder base. Cows were culled after 10 years, therefore, to maintain a self-replacing herd, either 10 per cent of calves were retained or 10 per cent of the herd was bought to maintain a constant herd size. A lambing rate of 100 per cent for merino lambs and 110 per cent for crossbred lambs was used. Breeder cows had a 100 per cent calving rate.

Additional conditions in the dynamic model:

- The number of breeder ewes and breeder cows at the start of a year were required to equal the ending stock from the previous year, plus merino lambs or calves retained.
- If there was any cash left over in December of a specific year it was carried forward to the following January.
- Forage crops could be grown for feed supply and could be stockpiled and carried forward to future years. The maximum amount of grain and hay that could be stored at any one time was 300 tonnes. This amount matched the size of the on-farm storage facilities.

5. Results

The average values of the objective function in the static and dynamic models for different values of the absolute risk aversion coefficient are given in Table 2. The enterprise mixes associated with different models are given in Tables 3 and 4. Differences between the gross margins and the variance for the predicted and observed models are in Figure 9. Changes in the dynamic model's objective function when the budget was varied for different levels of risk aversion are presented in Figure 10.

5.1 Dynamic versus static models

The value of the objective function varied between the years using alternative modelling approaches (Figure 8), however, the differences are not statistically significant (P-value = 0.44). In some years, the static model had higher returns compared to the first year in the dynamic model (for example, 1995), and vice-versa (for example, 1997). As the dynamic model accommodated inter-temporal trade-offs, its objective function could be lower in the first year but higher in future years.

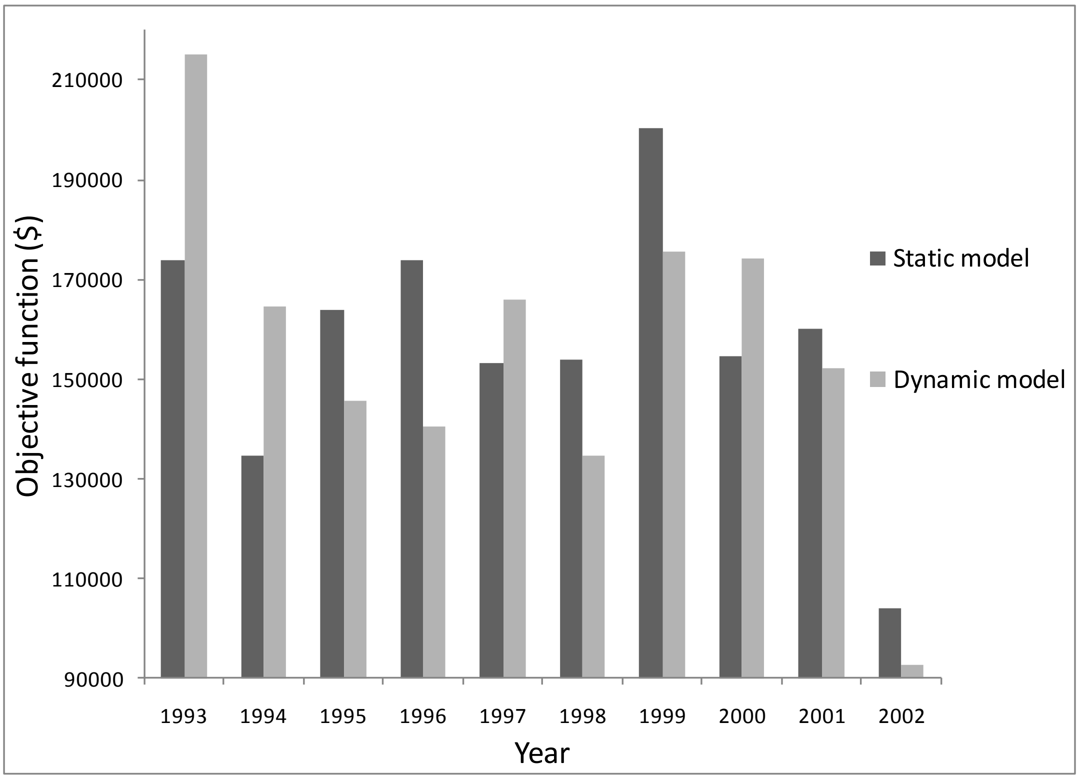

Figure 8 Value of the predicted models' objective functions for the static model and the first year of the dynamic model.

Table 2 Estimated average annual objective function values ($'000) for different models and different levels of risk aversion[a]

Dynamic or static model	Observed or predicted model	Level of risk aversion[b]		
		1.2×10^{-4}	2×10^{-4}	2.8×10^{-4}
Static	Observed	134	118	106
	Predicted	181	157	142
Dynamic	Observed	126	113	105
	Predicted	180	156	139

[a] The above results are for a budget of $150 000 (as observed on the case-study farm) and a comparison of the first year of each of the five-year models with the static model for the equivalent year.

[b] Risk aversion was measured as the value of α (Equation (1)) and the larger the value the greater the risk aversion.

Enterprise levels differed between the static model and the first year of the dynamic model. With no planning horizon (Table 3), the average area of forages grown each year was less than that in the rolling planning horizon model (Table 4). The static model (Table 3) had, on average, more sheep and less cattle each year than in the dynamic model (Table 4). For the dynamic model there appeared to be inter-temporal trade-offs between the time periods that influenced enterprise mixes. For example, having less sheep in the first year and growing more forages increased livestock numbers in future years so that the objective function over the whole time period was maximised.

Table 3 Static model enterprise levels for the average year and budget of $150 000 for different levels of risk aversion and different model types

Enterprise (units)	Model	Level of risk aversion		
		$1.2x10^{-4}$	$2x10^{-4}$	$2.8x10^{-4}$
Merino lambs (1000 head)	Observed	2.9	3.1	3.3
	Predicted	0	0.5	0.9
Crossbred lambs (1000 head)	Observed	0	0	0
	Predicted	2.9	2.4	2.2
Calves (100 head)	Observed	1.4	1.3	1.2
	Predicted	1.6	1.5	1.3
Area of grain crops (ha)	Observed	307	277	264
	Predicted	235	185	166
Area forage crops (ha)	Observed	353	359	359
	Predicted	278	237	210

Table 4 Dynamic model enterprise levels for the average year and budget of \$150 000 for different levels of risk aversion and different model type

Enterprise (units)	Model	Level of risk aversion		
		1.2×10^{-4}	2×10^{-4}	2.8×10^{-4}
Merino lambs (1000 head)	Observed	2.4	2.2	2.1
	Predicted	0.4	0.4	0.5
Crossbred lambs (1000 head)	Observed	0	0	0
	Predicted	1.6	1.6	1.6
Calves (100 head)	Observed	1.9	1.9	1.8
	Predicted	2.7	2.3	2.1
Area grain crops (ha)	Observed	160	169	170
	Predicted	115	92	86
Area forage crops (Ha)	Observed	503	494	488
	Predicted	554	511	485

The first year in each dynamic model had, on average, less sheep, more cattle and a greater area planted to forage crops (P-value < 0.05), relative to the same year in the static model. To examine how these differences in enterprise mixes influenced the farmer decisions in future years, total returns and total enterprise levels for the whole five-year planning horizon for the dynamic model were compared with the five static models for the same period, using 1993-1997 as an example. To allow for comparisons the static model had a discounted objective function.

The discounted sum of all five objective functions was higher in the dynamic model than the static model (Table 5). The value of the objective function was higher in the dynamic model when more than the current year's enterprise levels were considered. In the dynamic model, more forage crop were grown as the fodder could be carried forward to future years. Sheep numbers were similar across the five years (Table 5) and cattle numbers were higher in the dynamic model. In an average year, the farmer decisions in the dynamic model were less reliant on buying-in livestock feed (227 tonnes versus 312 tonnes).

Table 5 Predicted model's objective function and enterprise levels for the average year and budget of $150 000 using a five year dynamic model (1993-1997) compared to five individual static models for the same years

Variable	Model	
	Static	Dynamic
Objective function ($'000)	196	261
Crossbred lambs (1000 head)	3.88	3.98
Calves (100 head)	1.03	1.82
Area grain crops (ha)	156	75
Area forage crops (ha)	446	604
Feed purchased (tonnes)	312	227

5.2 Comparison of predicted and observed models

From comparing the observed model data with the predicted model the predicted model yielded a higher objective function than the observed model (Table 2). It is common for an optimisation model to over-predict income, for example, Musshoff and Hirschauer (2007) found that optimised programs produced a gross margin 15 per cent higher than those realised by farmers. In the approach used in this paper, the gaps between the farmer's observed program and the optimized program fell as the level of risk aversion rose (Figure 10). The average gap between observed returns and optimised returns was 18 per cent, with a range from 42 per cent (not very risk averse) to 4 per cent (highly risk averse).

When the restriction of having the dynamic model mimic observed farmer practices was removed some of the enterprise combinations changed (Table 4). Crossbred sheep came into the farm plan, the area of forage crops fell and cattle also replaced sheep.

The decision-maker could maintain an acceptable level of variance (V_1 or V_2) and improve the total gross margin by moving into the feasible solution spaces in Figure 9. In this situation, the gross margin could rise without an increase in variance. As risk aversion declined (viewed as having a larger minimum acceptable variance), the feasible solution space increased, $A_2 > A_1$.

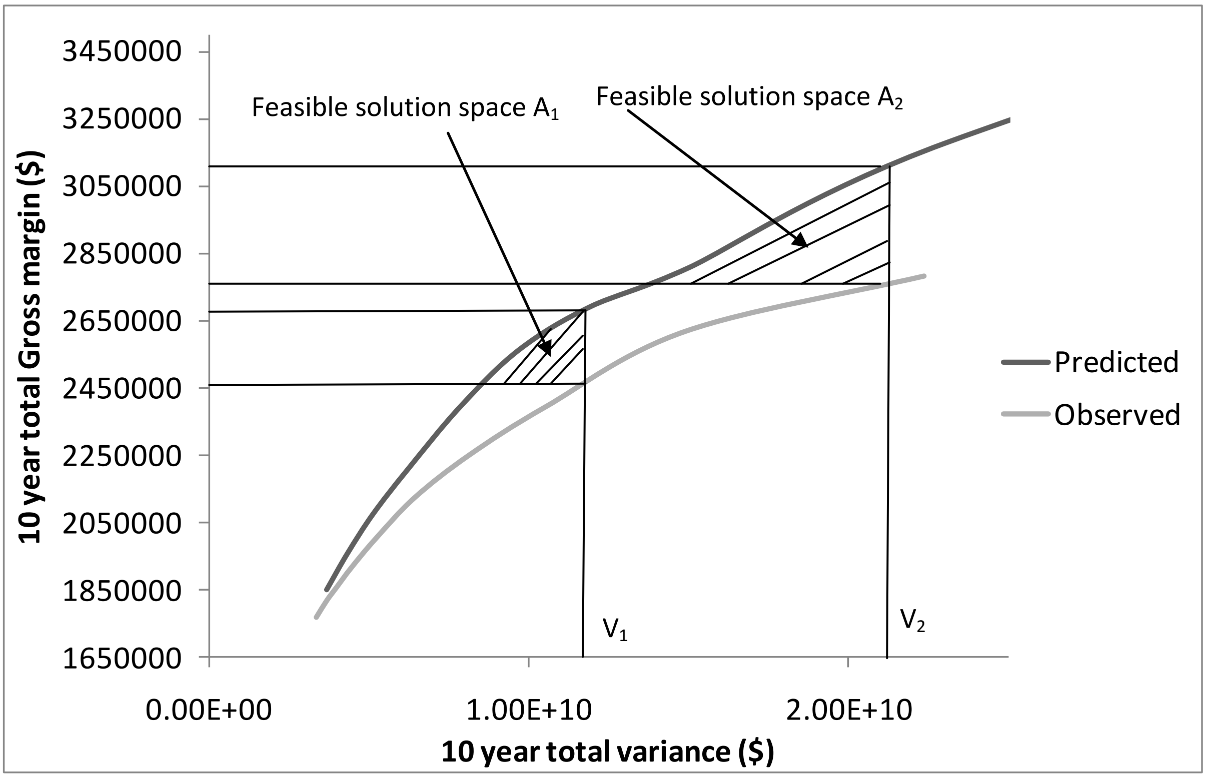

Figure 9 Expected variance-gross margin tradeoffs for two five-year blocks in the predicted dynamic model, using a budget of $150 000.

5.3 Budget size, risk aversion and objective function

Using the whole-farm dynamic model the changes in objective function as the budget allocation changed were compared for different values of α. Diminishing marginal returns to the size of the budget allocation were observed. As farmers become less risk averse, a smaller budget is needed to achieve a given objective function value (Figure 10). To achieve an objective function value of approximately $155 000, a farmer with α = 0.0002 requires approximately $150 000 each year, whilst a farmers with α = 0.00024 requires approximately $180 000 each year. The results between the comparative statics analysis (Figure 5) and the whole-farm modelling (Figure 9) were similar but not directly comparable as the whole-farm approach had many more activities and constraints.

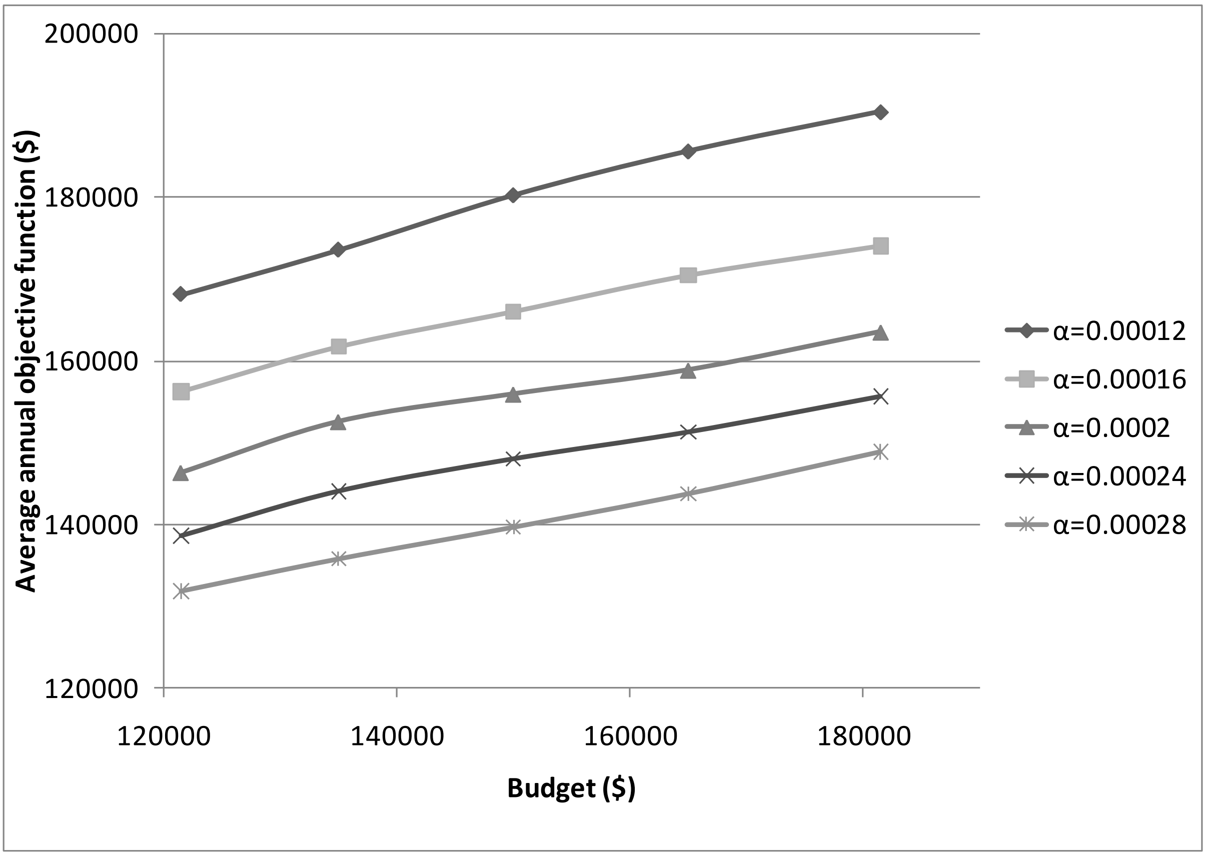

Figure 10 Objective function for the first year in the predicted dynamic model for different budgets and different levels of absolute risk aversion coefficient (α).

6. Discussion

Farmers that have lower levels of wealth are often more risk averse (Pannell *et al.*, 2000). In this study, comparative static and whole-farm analyses showed that if a farmer is more risk averse they will require a larger budget to obtain the same objective function compared to a less risk averse farmer (Figures 5 and 10). This implies that smaller operators require a relatively larger budget allocation to obtain the same objective function value as larger operators. Smaller farmers (viewed as farmers who prefer low levels of income variance) appear to have fewer options in changing enterprise mixes ($A_1 < A_2$ in Figure 9). This implies that average farm sizes may continue to increase as larger operators are more flexible with their production choices. Being less risk averse increases production possibilities and options to improve income without raising the variance of returns. These results are based on data from one farm and a larger sample size would allow testing to see if the results hold for different farm types.

Conventional theory suggests that a larger budget allocation is always preferred to less; however, this is the case if profit maximisation is being pursued. Profit maximisation is often assumed but not tested (Just, 2003). In addition, farmers often have multiple non-profit maximising objectives (Wallace and Moss, 2002). If the farmer's objective is to maximise a function such as Equation (1) then output levels may appear lower compared to those under profit maximisation, as a greater budget allocation will always lead to a non-decreasing gross margin but not necessarily non-decreasing indirect utility. The indirect utility objective function is non-increasing in enterprise variance (Equation 7). If farmers do not purely seek to maximise total gross margins but rather they wish to maximise a risk weighted objective function (Equation 1), the variance of enterprise returns is then important to consider in decision-making. Although illustrative results cannot provide a general proof, it is emphasised that cross-price variances can also have strong effects on enterprise levels. Often, falling own-prices are seen as the reason for farmers moving out of an enterprise; in the example in this paper, the variance of an alternative enterprise appears to play an important role in determining enterprise mixes.

Just (2003) argues that in risk research, profit maximization is often assumed to be the farmer's objective but is not tested. In this study it has been found that the case-study farm did not carry out activities that maximized a risk weighted objective function (predicted model enterprise levels differed from observed model enterprise levels). Farmers may have multiple non-financial objectives that require assessment, for example, maintaining flock genetics or focusing on activities that coincide with management skills so as to reduce stress in day-to-day activities.

The optimised model's total gross margin was closest to the observed model's total gross margin at the highest level of risk aversion (Figure 9). Thus, the case-study farmer could be viewed as having a low willingness to accept income variance. This suggests that the farmer was risk averse and it may also partially explain why less risky activities were chosen. For example, maintaining production in activities that are well understood, that is, not switching into crossbred sheep, and not adopting high cattle numbers as their gross margins were found to be more variable.

As the study focus was on one farm, the scope for providing broad-scale farm management advice appears limited. However, the results provide an indication of how alternative modelling approaches may affect welfare and enterprise choices made on a typical farm in the wheat-sheep zone of Australia. The results imply that when researchers compare dynamic and static models that comparing individual years may not provide a complete picture of enterprise selection decisions, particularly if inter-temporal feedbacks are important. A more useful approach appears to be to compare a block of years so that the full effects of forward planning can be appreciated.

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